Deformation Analyses of Florida Highway Subgrade Sand Subjected to Repeated Load Triaxial Tests

NORMAN D. PUMPHREY, JR., AND RODNEY W. LENTZ

Laboratory repeated load triaxial tests are conducted to estimate the effects of highway traffic on the permanent and resilient deformation of a subgrade sand commonly used as a foundation for a flexible highway pavement structure in Florida. Combinations of confining stress and cyclic principal stress difference (test variables) and of dry unit weight and moisture content (sample variables) are used for each sample and loaded to 10,000 cycles. Confining stress, cyclic principal stress difference, and dry unit weight are correlated with permanent strain and resilient modulus and thus affect deformation properties of these soils. However, moisture content correlates with neither permanent strain nor resilient modulus. Static tests are conducted on various samples similar to those used in the cyclic tests. These results are used to normalize cyclic stress and strain at 10,000 cycles. A regression model is developed for predicting cyclic strain in the sand from static tests by using a technique previously developed by Lentz. The model is compared to one for Michigan highway subgrade sand. Using a general linear test at a level of significance of 0.05, it is found that the data sets from each sand cannot be pooled to form one regression model. Values of resilient modulus after 10,000 loading cycles for the Florida sand ranged from 16,000 psi (110 240 kPa) at 5 psi (34.5 kPa) confining stress to 56,000 psi (385 840 kPa) at 50 psi (344.5 kPa) confining stress.

In many locations around the country and the world, rutting of flexible highway pavement is of major concern when useful pavement life is evaluated. One of the natural consequences of severe pavement rutting is that water will remain in puddles on the roadway, and hydroplaning—the loss of vehicle control due to excess water between tires and pavement—becomes a hazardous possibility. Yoder and Witzczak (1) and Terrel and Rimsrong (2) state that rutting is due to the consolidation (densification) of one or more layers of pavement under repeated wheel loads, especially the heavier loads induced by multiaxle and multiwheel vehicles. Traffic-induced vibrations and the actual wheel load stresses are transmitted through the pavement layers, thus causing the consolidation. In many pavement design procedures, protection of the subgrade from these excess stresses and vibrations caused by traffic is a major design consideration.

Another important aspect of pavement design is the resiliency characteristics of the pavement layers, that is, the amount of deformation under a given load that is elastic. It is particularly important in design concepts using the linear elastic-layered models. The Asphalt Institute (3) has incorporated an elastic-layered theory model in its design by using subgrade resilient modulus as a design input.

Therefore, this investigation was undertaken to examine a Florida quartz sand commonly found as a subgrade material in parts of the Florida panhandle. The primary objectives of this study were (a) to investigate permanent and resilient deformation characteristics of the soil and (b) to analyze a technique to predict permanent deformation under repeated load laboratory tests using only static load triaxial tests. Both cyclic and static load tests were conducted on sand samples under various combinations of confining stress, cyclic principal stress difference, dry unit weight, and moisture content.

LITERATURE SURVEY

Permanent Deformation

Permanent deformation may be defined as the amount of deflection occurring under a given load that is not recoverable when the load is removed. Yoder and Witzczak (1) state that permanent deformation is the result of two different mechanisms—densification and repetitive shear deformations (plastic flow).

Deformation due to plastic flow is the primary criterion upon which many current pavement designs are based. Two general design techniques—empirical approaches and rational methods—currently exist. Empirical approaches involve the correlation of pavement deformations to some predetermined condition of failure. Two common empirical methods are the empirical-strength procedures and the limited-subgrade-strain method. In both techniques the deformation is controlled by varying the thickness of pavement layers and by regulating the material quality based on an index test (California bearing ratio, resilient modulus, etc.). These methods involve the same basic assumptions, except that the strength approach assumes that all deformation occurs in the subgrade layer, whereas in the limited-subgrade-strain method, the deformation may occur in all layers. The major disadvantage with these methods is that neither can predict the amount of permanent strain that will occur under a given number of loadings. Purely rational design techniques have not yet enjoyed wide acceptance in practice, but interest is increasing as these methods begin to more closely model actual field conditions and as computer accessibility rises.

If empirical approaches are dominant, how might the pave-

N. D. Pumphrey, Jr., School of Civil Engineering, Purdue University, West Lafayette, Ind. 47907. R. W. Lentz, Department of Civil Engineering, University of Missouri-Rolla, Rolla, Mo. 65401.
ment design engineer predict this permanent strain (\( \varepsilon_p \))? Cyclic or repeated load triaxial tests of soil and granular base course material have recently been the most widely accepted tests for measuring permanent deformation. Some researchers (1, 4) have found the relationship between permanent strain and number of load cycles (N) to be linear on a log-log plot. Therefore, the deformation law can be written as

\[
\varepsilon_p = aN^b
\]

(1)

where \( a \) is the ordinate intercept at \( N = 1 \) and \( b \) is the slope of the plot. However, Barksdale (5) and Lentz (6) found that a linear relationship exists on the \( \varepsilon_p - \log N \) plot. This law can be expressed as

\[
\varepsilon_p = a + b (\log N)
\]

(2)

Parameter \( a \) is the ordinate intercept at \( N = 1 \) and \( b \) is the slope of the straight-line plot.

Diyaljee and Raymond (4) developed a constitutive equation relating laboratory permanent strain of a highway soil to the ratio of repeated load deviator stress, to static failure deviator stress, and to the number of cycles to which the soil is subjected.

McVay and Taesiri (8) conducted compression cyclic triaxial tests and compression-extension cyclic triaxial tests on a Florida sand and compared results from both tests on permanent and resilient strain characteristics.

Lentz (6) used various normalization procedures of cyclic deviator stress and permanent strain plots to develop a technique for predicting laboratory permanent strain at a particular number of loading cycles using only static triaxial tests. The basic requirement was that the dry unit weight and moisture content be the same for the static tests as that which would be analyzed in a repeated load triaxial tests. After his analysis was complete, Lentz (6) had selected peak static strength as the value for normalizing cyclic principal stress difference and static strain at 95 percent of the peak static strength for normalizing permanent strain. The resulting plot of normalized stress (ordinate) versus normalized strain at the end of 10,000 loading cycles (abscissa) fit a hyperbolic function with a correlation coefficient of 0.98 when least-squares regression was performed. The equation of the curve was

\[
\sigma_d/S_d = \left( \varepsilon_p/\varepsilon_{0.95S_d} \right) \left[ n + m \left( \varepsilon_p/\varepsilon_{0.95S_d} \right) \right]
\]

(3)

\[
\varepsilon_{0.95S_d} = \text{strain at 95 percent of the peak static strength of the soil,}
\]

\[
\sigma_d = \text{cyclic principal stress difference,}
\]

\[
S_d = \text{peak static strength of the soil, and}
\]

\[
n, m = \text{regression constants,}
\]

This curve could then be used by conducting a single static test to find the values of \( S_d \) and \( \varepsilon_{0.95S_d} \), assuming a cyclic deviator stress that could be applied and calculating from the equation or reading from the plot the value of permanent strain to be expected in the sample.

**Resilient Deformation**

Resiliency is the deformatonal component of the soil that will be recovered after a load has been applied and removed from a sample. This resilient component can be used in the material characterization of a soil.

Laboratory testing to determine soil resiliency consists of a series of repeated load tests. The resilient modulus (\( M_r \)) is one method of material characterization in which the aspect of recoverable soil deflection is considered. It can be defined as follows (1):

\[
M_r = (\sigma_1 - \sigma_3)/\varepsilon_r
\]

(6)

where

\[
\sigma_1 = \text{total peak vertical stress,}
\]

\[
\sigma_3 = \text{confining stress, and}
\]

\[
\varepsilon_r = \text{recoverable strain at a given stress repetition.}
\]

**Factors Affecting Cyclic Loading Properties of Cohesionless Soils**

Substantial research has been conducted to determine the effect of various variables on cohesionless soils subjected to repeated loads. These factors may be divided into the categories of test and sample variables.

Test variables can be controlled in the testing procedure and are virtually unaffected by a change in the soil sample being tested. Researchers (5; 6-8; 9, pp. 365–383; 10-18, pp. 341–345; 19) have identified many variables that affect both permanent and resilient deformation of a soil sample, including confining stress, number of loadings, cyclic principal stress difference, load duration and frequency, stress history, loading wave form, and sample end restraint. Most researchers agree that the first three have the greatest effect on a sample.

Sample variables affect strength and deformation characteristics of the soil because of a change in the soil sample itself. These factors include density or dry unit weight, degree of saturation, aggregate gradation or fines content, specimen size, and particle size, shape, and roughness (1, 5, 6, 9, 12, 13, 19-22, pp. 63-74). Density of the soil sample is the most important of these variables.

**SAMPLE MATERIAL, PREPARATION, AND TESTING**

Highway subgrade sand was obtained from a borrow pit in Leon County, Florida. The sand had been used as subgrade fill material for an urban widening project in Tallahassee. The
classification of the material was a uniform, fine sand, or AASHTO classification A-3, with a coefficient of curvature of 0.92 and a coefficient of uniformity of 1.74. Particle sizes ranged from the No. 10 sieve to just below the No. 200 sieve, with approximately 1 or 2 percent of the material passing the No. 200 sieve.

Standard (AASHTO T-99) and modified (AASHTO T-180) compaction tests were conducted to determine maximum dry unit weight and optimum moisture content, and the results are plotted in Figure 1. The material was then compacted into a 4-in. (101-mm) diameter triaxial sample mold and prepared for testing (23).

![Graph showing compaction tests on highway subgrade sand, Leon County, Florida.](image)

**FIGURE 1** Compaction tests on highway subgrade sand, Leon County, Florida.

Several of the test and sample variables mentioned previously were selected for study. Of primary interest were confining stress, cyclic principal stress difference, dry unit weight, and moisture content. Various combinations of these factors were tested in cyclic triaxial tests. Cyclic principal stress difference was set at different percentages of the peak static soil strength ($S_d$) determined from samples tested at similar dry unit weight, moisture content, and confining stress combinations.

An inverted haversine wave form of 0.1-sec duration was used for all repeated load tests. This period is roughly equivalent to the time in which a vehicle traveling 30 mph (48 km/hr) affects a point in the top of the subgrade of a flexible pavement structure. The 0.1 sec was followed by a 0.9-sec rest period to allow proper damping of the load before the following load was applied. Therefore, a frequency of one load per second resulted. All cyclic tests were continued to 10,250 cycles.

### RESULTS AND DISCUSSION

As stated briefly earlier in this paper, the primary objectives of this study were (a) to investigate the permanent deformation characteristics of the highway subgrade sand, (b) to examine resilient properties and particularly to determine the resilient modulus of the sand, and (c) to examine a technique (6) developed to predict permanent deformation of soil samples in cyclic triaxial tests from static triaxial test information.

#### Permanent Deformation

Data from cyclic triaxial loading tests were used to evaluate the permanent deformation characteristics of the highway subgrade sand under various conditions of confining pressure, cyclic principal stress difference, dry unit weight, and moisture content.

A plot of permanent strain ($\varepsilon_p$) versus the logarithm of the number of cycles ($N$) was constructed for each cyclic test sample. The results fit the relationship found in Equation 2. Typical plots may be found in Figure 2.

Of all the test variables that can affect the behavior of granular soil in cyclic loading tests, the confining stress ($\sigma_3$) is the most important (5, 12, 15). Increasing confining stress can...
greatly increase the strength and resilient modulus of a soil and decrease the permanent strain at a given level of cyclic principal stress difference. The confining stress in a pavement layer depends on the position with respect to the applied wheel load, the magnitude of wheel load, and resilient modulus. Thus, this study included testing at confining stresses of 5, 25, and 50 psi (34.5, 172.3, and 344.5 kPa) for the highway subgrade sand.

The permanent strain values for the 10,000th cycle of each sand sample were determined. The results are summarized in Figure 3 for comparison of the confining stress effects. For lower values of cyclic principal stress difference ($\sigma_d$), the permanent strain increased slightly or remained constant for larger confining stress values. However, for the $\sigma_d$ of 0.75 $S_d$, the $e_p$ decreased substantially as confining pressure increased.

Cyclic principal stress difference, as would be expected, had the greatest effect on the permanent strain of the soil. Increasing $\sigma_d$ results in an increase in the permanent strain for constant confining stress. Figure 3 contains a comparison of the effects on permanent strain for stress ratios of 0.40, 0.60, and 0.75. [Stress ratio is defined as the cyclic principal stress difference divided by the peak static test stress ($S_d$) performed on a similar sample.] At each of the confining stresses, the sample strain increased as the stress ratio increased. The difference between the strains at stress ratios of 0.40 and 0.75 was significantly less for the samples subjected to higher confining stress than for those subjected to the lower confining pressure. This observation may be explained by the fact that the higher confining stress causes increased interparticle friction and aggregate interlock, resulting in less movement under load.

Of the sample variables, density or dry unit weight ($\gamma_d$) has probably the greatest effect on the permanent strain during cyclic loading of a soil. Figure 4 shows the relationship between density and permanent strain for the highway subgrade sand. Three dry unit weights were used in testing of the highway subgrade sand—95 percent of AASHTO T-99 (standard compaction), AASHTO T-99, and AASHTO T-180 (modified compaction). The standard and modified samples were compacted and tested at their respective optimum moisture contents. The 95 percent standard compaction samples were densified at the moisture content used as optimum for the standard method. As expected, the high soil density resulted in less permanent soil strain for both the optimum and below-optimum moisture samples. This result is reasonable because higher compactive effort decreases the volume of voids in a soil, resulting in more particle contacts and greater aggregate interlock. Thus, the soil particles are less likely to reorient themselves during loading.

Tests were conducted on two different moisture content levels (3 percent below optimum and at optimum) of the sand. Preliminary plans included testing samples at 3 percent above optimum; however, samples could not be compacted to the required density using the tamping method, so this moisture condition was eliminated from the program.

Figure 4 shows the effect of moisture content on permanent strain for the sand. At the higher densities, increased moisture in the sample appeared to cause an increase in the strain. At the lower density, the strains were virtually identical. The trend at densities outside of the range tested is not known and might be difficult to predict based on the limited results obtained during this testing program. The densities used in these tests, however, included the range that is likely to be encountered in the field.

**Resilient Deformation**

Another area of interest in this study program was the determination of the resilient modulus ($M_R$) of the subgrade sand. As with the permanent deformation discussed in the previous section, several test and sample variables and their effects on the resilient modulus were considered.

A plot of the logarithm of resilient modulus versus the logarithm of the number of cycles was constructed for each
cyclic test sample. Figure 5 contains typical plots of the data, showing a very slight increase in $M_R$ as the number of loadings increases.

A relationship between confining stress and resilient modulus for the highway subgrade sand may be observed in Figure 6. This figure contains the values of $M_R$ calculated from the sample regression equations for load cycle 10,000. At each
cyclic principal stress difference, the effect of confining stress on $M_R$ was the same: as confining stress increases, $M_R$ also increases. Values of resilient modulus for the sand were found to be as low as 16,000 psi (110 240 kPa) for the 5-psi (34.5-kPa) confining stress and as high as 56,000 psi (385 840 kPa) for the 50-psi (344.5-kPa) confining stress. Thus, confining stress was shown to have a substantial effect on the value of resilient modulus. The pavement designer, then, is left with the dilemma of which value to use for design of the pavement structure. Values of $M_R$ depend on confining pressure, but confining pressure depends on wheel load, position with respect to the wheel, and $M_R$.

In this research on Florida sand, conflicting results were obtained on the effect of cyclic principal stress difference on resilient modulus at the various confining stress levels. Figure 6 contains a summary of these results. For the confining stresses of 25 psi (172.5 kPa) and 50 psi (344.5 kPa), the resilient modulus continued to increase as the stress ratio on the tested samples became larger. The rate of this increase was slightly larger for the higher of these two confining stresses. However, $M_R$ at the 5-psi (34.5-kPa) confining stress increased to a peak value and then began to decrease as the stress ratio became larger. The stress ratio at which this peak occurs could not be determined exactly from Figure 6, because only three samples were tested at this confining stress, but it appears that it occurred between stress ratios of 0.50 and 0.65. If so, the decrease in resilient modulus may have been connected with the "threshold" stress that causes a sizable increase in permanent strain on a sample. Further study is required to determine what, if any, relationship exists.

Although the exact nature of the relationship between dry unit weight and resilient modulus for the highway subgrade sand was not well defined in this study, the best-fit line tended to have a positive slope, indicating that $M_R$ increased with increasing dry unit weight. Figure 7 contains the plot of points
cyclic principal stress difference and confining stress on resilient modulus at $N = 10,000$. The FIGURE 6 Effect of cyclic principal stress difference and confining stress on resilient modulus at $N = 10,000$.
obtained from the samples tested at different dry unit weights or moisture contents or both.

The effect of moisture content on resilient modulus has been a particularly elusive characteristic for researchers to examine. No definite trend has developed for all materials when this area of concern is analyzed. Figure 7 contains comparisons of highway subgrade sand samples tested cyclically at different levels of moisture content in the sand. Because of the scatter in the points, no satisfactory relationships were found between moisture content and resilient modulus.

Normalizing Effect of Static Stress and Strain

Lentz (6) developed a technique for predicting cumulative permanent strain in a soil sample under cyclic loads based on simple static triaxial laboratory tests on a similar sample. Because test and sample variables that affect permanent strain are essentially the same for both static and cyclic tests, static test results can be used to normalize the cyclic results. By selecting appropriate points from each test type, one may use samples tested at different densities, confining stresses, and cyclic deviator stresses to define one single curve, which may be used as a predictor of permanent strain after a given number of loading cycles.

For the cyclic tests, Lentz (6) selected the cumulative strain and cyclic deviator stress after the 10,000th cycle of load. This point was chosen primarily because most of the samples were loaded no higher than the 10,000th cycle, except for a few special tests on the effects of stress history. Although 10,000 cycles was considerably lower than the number of loadings that can be expected on an average highway pavement, the value was selected (a) because of the time required to conduct the individual tests (e.g., 24 hr would allow only 86,400 loadings at one loading per second) and (b) because at most cyclic deviator stress ratios, the plot of permanent strain versus log number of cycles remains linear beyond 10,000 cycles.

From the static test results, values of static stress difference and strain were required for the normalization of the cyclic values. Lentz (6) selected the peak deviator stress for use in normalizing the cyclic deviator stress. The cyclic stress was divided by the peak deviator stress to achieve the normalization. Thus, this ratio will normally be less than 1 for all cyclic tests. A ratio of more than 1 indicates that a cyclic load larger than the peak static load is being applied, so sample failure and plastic strain will occur after a small number of loads.

A static strain at 95 percent of the peak static deviator stress was selected for use in normalizing the cyclic permanent strain. This value was chosen because (a) it contained a large amount of plastic (permanent) strain because it was within only 5 percent of the peak stress and (b) it was a well-defined value that was readily reproduced by other testing personnel. Normalization was achieved by dividing the cyclic permanent strain by the selected static strain.

Figure 8 shows the Lentz (6) data plotted on the curve of normalized stress versus normalized strain. The solid line represents a hyperbolic function that was used in regression analysis to fit the data points.

Equation 3 is the equation of the hyperbolic curve. Dividing both sides of Equation 3 by $\varepsilon_p/\varepsilon_{0.95S_d}$ and reciprocating both sides gives

$$
\frac{(\varepsilon_p/\varepsilon_{0.95S_d})/(\sigma_d/S_d)}{n + m(\varepsilon_p/\varepsilon_{0.95S_d})} = \varepsilon_p/\varepsilon_{0.95S_d}
$$

which is a linear relationship with normalized strain as the abscissa, with the ratio of normalized strain to normalized stress on the ordinate, and with $n$ and $m$ the y-intercept and slope of the line, respectively.
One potential problem with the Lentz (6) prediction technique was that it was developed by using only one material—a subgrade sand obtained from a test site in the northern part of Michigan's lower peninsula. A primary thrust of this study was to determine whether this technique was applicable to other materials. The subgrade sand obtained from the borrow pit in Leon County, Florida, was the primary comparison material in that sufficient tests were conducted to obtain a statistical relationship between normalized stress and normalized strain. Several samples were run on two other materials, limerock and bank run shell, for an indication of applicability of the technique to other types of granular material with significantly different consistency and gradation.

Figure 8 also shows the data obtained from the testing of the Florida sand. The dashed line is the regression curve obtained for the hyperbolic function. In this same figure two points each for the limerock and bank run shell and one check point for the Michigan subgrade sand are plotted for visual comparison.

Of the 11 data points available for regression analysis on the Florida subgrade sand, 10 were logical in their graph positions, but 1 was found to be an outlier. Because the static stress-strain curve for this sample was much steeper than other curves for the sand, it was believed that some undetected problem had developed during testing. This point, therefore, was determined to be an outlier and was omitted when the best-fit regression line was determined. The remaining points provided a very suitable fit with an \( r^2 \)-value of 0.977, whereas the \( r^2 \)-value was reduced to 0.788 when this outlier was included.

Statistical methods provide useful tools for comparing two separate sets of data points such as the Michigan sand data from Lentz (6) and the Florida sand data from testing done in this study. The basis for comparing these two data sets is the y-intercepts \( n \) and slopes \( m \) of the lines obtained through the least-squares regression analysis, where \( n \) and \( m \) are the regression constants from Equation 7. For the Michigan sand, \( n = 0.1970 \) and \( m = 0.9591 \). Similarly, \( n = 0.1531 \) and \( m = 1.1941 \) for the Florida sand.

Neter and Wasserman (24) described the general linear test that allows a statistical comparison of two separate sets of data and their regression lines.

The general linear test indicated that the two regression lines are not identical. Ninety-five percent confidence interval tests were conducted for the individual parameters—slope \( m \) and y-intercept \( n \)—to determine whether both parameters differed for the two regression lines. It was found that the y-intercepts were essentially the same but that the slopes were different. Although the regression lines from Equation 7 begin at essentially the same level on the axis of normalized strain to normalized stress ratio, this value will increase more rapidly with increasing normalized strain for the Florida sand than for the Michigan sand. That is, for the higher stress ratios, the Florida sand will show significantly more normalized strain, or strain close to the sample failure values for the static tests. This fact would indicate that, of the two materials, the Michigan sand would show greater resistance to rutting in the field. Many additional sands would require testing, however, if a conclusion is desired on the rut susceptibility of these sands in the broad spectrum of sand types.

As noted previously, two samples each of limerock and of bank run shell were also normalized and plotted in Figure 8 to determine whether the relations developed for the sand were good indicators for other coarse-grained materials. Although insufficient tests were conducted to perform a reliable regression analysis, a visual inspection would suggest that the shell and limerock react completely differently from the sand material. It appears that lower stress ratios in the shell and limerock result in higher normalized strain and thus strains closer to the static test failure strain.

**CONCLUSIONS**

1. Increased confining stress causes a slight or no increase in permanent strain for low stress ratios but a substantial decrease in this strain for large stress ratios. The break point appears to occur somewhere between a ratio of 0.60 and 0.75. Resilient modulus increases as the confining stress becomes larger at all levels of stress ratio. The range of the increase also becomes greater as the stress ratio increases.

2. In all cases permanent strain increases with increasing number of loading cycles. Resilient modulus increases slightly as the number of cycles becomes larger.

3. At each confining stress tested, an increase in stress ratio results in an increase in permanent strain in the sample. This increase is substantial for the 5-psi and 25-psi confining stresses, but is small for the 50-psi sample group. For the 25-psi and 50-psi confining stresses, resilient modulus increases with increasing stress ratio. However, at 5 psi the resilient modulus peaks at a stress ratio between 0.50 and 0.65.

4. For both the optimum and the below-optimum moisture groups, the increase in dry unit weight causes a decrease in the permanent strain.

5. An increase in permanent strain is caused by an increase in moisture content for soil densities near the AASHTO T-99 and T-180 densities. At 96.5 percent of AASHTO T-99 density, the permanent strain is virtually the same for both water contents. Therefore, effects of moisture content on permanent strain cannot be confidently stated because of limited data and conflicting results.

6. Permanent strain on a laboratory sample may be predicted from static load tests by using a hyperbolic relationship found on a plot of normalized stress versus normalized strain. However, when separate hyperbolic relationships were compared for the Michigan sand and the Florida sand, it was statistically determined that the two sets of data produced slightly different relationships (at a 0.05 significance level) and cannot be pooled.

7. Values of resilient modulus after 10,000 loading cycles for the Florida sand ranged from 16,000 psi (110 240 kPa) at 5 psi confining stress to 56,000 psi (385 840 kPa) at 50 psi confining stress. Because confining pressure depends on wheel load and resilient modulus and resilient modulus depends on confining pressure, care should be used when the resilient modulus value to be used in pavement design is determined.

**REFERENCES**


