A Model for Predicting Free-Flow Speeds Based on Probabilistic Limiting Velocity Concepts: Theory and Estimation

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Methods for predicting space-mean speeds over a heterogeneous roadway for a freely moving vehicle are needed for many applications in highway planning. Although various shortcomings of a linear specification for the steady-state speed prediction model have been recognized and alternative models based on a limiting speed specification have been proposed, the latter have not previously been rigorously estimated. Presented are the theoretical formulation and empirical estimation of a new probabilistic model for predicting steady-state speeds on a homogeneous road section, based on a limiting velocity approach. The statistical implementation employing a multinomial logit formulation and estimation results using a Brazilian data set in excess of 100,000 observations are presented. Alternative methods have been developed for applying the steady-state speed model to predict speeds over heterogeneous roadways under different informational limitations and accuracy requirements. The speed model, together with related models for predicting fuel consumption and tire wear, have been incorporated in the World Bank Highway Design and Maintenance Standards Model (HDM-III) to provide a basis for engineering-economic analysis of alternative standards of geometric design and pavement design and maintenance for low-volume roads.

A method for predicting the costs of operating a vehicle on a highway of known characteristics is an important tool for highway sector planning and project evaluation. In general, a method for predicting vehicle operating costs per unit roadway distance consists of *(a)* a central model to predict speed and related variables (e.g., power used); and *(b)* a set of interfacing models that would use the predictions from the central model as inputs, and generate predictions of journey time, fuel consumption, tire wear, and vehicle utilization *(1)*.

Some of the desirable properties for the central component, the speed prediction model, are as follows:

- It should be flexible in its input requirements;
- It should be appropriately sensitive to the policy options being evaluated (e.g., design parameters and road maintenance resource expenditure);
- It should be amenable to extrapolation over a reasonable range of the policy variables; and
- It should be readily transferable to other environments.

The purpose of the paper is to describe the limiting speed approach to predicting steady-state speeds of vehicles on a homogeneous road section and its statistical implementation leading to the formulation of a probabilistic limiting speed model. Estimation results using a large data set of speed observations collected in Brazil are presented and discussed.

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The probabilistic limiting speed approach to modeling vehicle operation on a roadway bears a close similarity to the random utility approach to modeling urban travel demand among discrete alternatives. This similarity has been exploited in resolving some of the issues relating to model estimation, aggregation, and transferability.

The steady-state speed prediction model is the centerpiece of the vehicle operating cost module of the World Bank Highway Design and Maintenance Standards Model—Release 3 (HDM-III) *(1–3)*. The linkages between the speed model and various vehicle cost components are indicated in the concluding section.

STEADY-STATE SPEED PREDICTION MODEL

To cope with the diversity of input information availability and output accuracy requirements that the speed prediction model is called on to cope with, it is convenient to structure the process of speed prediction into two components:

1. A model for predicting the vehicle speed on a road section over which the characteristics of interest do not change appreciably. Such a section is referred to as a homogeneous section. The concept of speed used is that of a steady-state speed, and this component of speed prediction may be called a steady-state speed prediction model.
2. A set of procedures with which to apply the steady-state speed model for predicting the speed profile over a heterogeneous roadway by using the available information on the roadway and with the desired degree of accuracy for the particular application. These procedures are called roadway speed prediction methods.

The steady-state speed of an unimpeded vehicle of known attributes traversing a homogeneous road section of known characteristics, located in a fixed overall socioeconomic and traffic environment, may be defined as the speed the vehicle would eventually attain and maintain if the homogeneous road section is indefinitely long. Thus steady-state speed in a fixed environment is a property associated with a given combination of a homogeneous road section and a vehicle.

A homogeneous road section is assumed to be completely defined if its surface type, slope, curvature, superelevation, and surface irregularity measure are specified. The section is assumed to be sufficiently wide so that the road width has no effect on the speed—as was the case with roads used in speed observation in Brazil. It may be observed that the highway characteristics represent the policy variables under the control of the highway planner. These are occasionally referred to as speed-influencing characteristics or road severity factors collectively denoted by the symbol X. It may be noted that the direction of travel on the homogeneous section is part of the description of the section.

The term vehicle is shorthand for operator-vehicle system and the term operator is used to indicate that the speed decision maker may be an individual driver or a transport firm. As for the characteristics of the vehicle, the vehicle class and loading (in the case of a truck) are supposed to be known. Also, a set of technical characteristics of the vehicle—such as unladen weight, drag coefficient, and so on—are assumed to be known or assignable with reasonable accuracy. The technical characteristics of the vehicle are denoted by the symbol Y.

Finally, there is a set of behavioral-technical characteristics of the vehicle, such as used power, perceived friction ratio, desired speed, and so on. These are the estimated parameters of the steady-state speed prediction model and are collectively denoted by the symbol θ. For a given application these parameters may be estimated afresh, calibrated on the basis of limited observations or, in some cases, judgmentally determined.

A specification of a steady-state speed prediction model is a functional form relating the steady-state speed, V (in m/s), of the given vehicle to variables that capture various speed-influencing characteristics of the homogeneous section. The variables considered are given in Table 1.

TABLE 1 VARIABLES CONSIDERED

Speed-Influencing Characteristic	Variable Name	Units	Symbol	Note
Vertical alignment	Gradient or slope	Fraction	GR	With sign
Horizontal alignment	Curvature	rad/km	C	Without orientation
	Superelevation	Fraction	SP	
Surface irregularity	Roughness	m/km IRI	R	See note below
Surface type	–	–	ST	Paved/unpaved

Note: IRI is the International Roughness Index.

The unit of roughness used is the International Roughness Index (IRI), which summarizes the varied wavelengths and amplitudes of the surface irregularities in a slope index that is equivalent to the total axle-body movement (in meters) made by a typical passenger car over a unit distance (in kilometers). The index quantifies the impact of roughness on a moving vehicle in much the same way as roughness causes vehicle costs, and as such is judged to be the most applicable measure of roughness for economic evaluation purposes *(4,5)*.

The models found in the literature may generally be classified into two approaches: direct and latent (or unobservable) variable approaches. The basic specification using the direct approach would be the linear form

$$V = a_0 + a_1GR + a_2C + a_3R \tag{1}$$

where

a_0, a_1, a_2, and a_3 = parameters to be estimated,
GR = gradient or slope (fraction),
C = curvature (rad/km), and
R = roughness (m/km IRI).

The direct approach is not entirely satisfactory for the following reasons:

- It is possible to predict unreasonably low (at times negative) steady-state speeds while using plausible values for the independent variables, especially for low standard roads where these independent variables assume large values.
- The partial derivatives of predicted steady-state speed with respect to each of the road severity factors is constant and, as just noted, in general, negative. The policy implication of this property of linear specifications is that reduction of one of the road severity factors (such as roughness), through a greater investment in road maintenance, will show an increase in speeds, although another factor (such as gradient) may actually be inhibiting speeds.

Although it is possible partially to mitigate these shortcomings by making the functional form nonlinear and by including interaction terms, the approach would still be ad hoc.

DETERMINISTIC LIMITING SPEED MODEL

The alternative approach for specifying a steady-state speed model is to use a set of limiting speeds or constraining speeds as latent or unobservable variables. Instead of associating the steady-state speed directly with the speed-influencing variables X of the homogeneous road sections, these variables are regarded as interacting with the relevant characteristics of the vehicle Y to generate a set of steady-state speed constraints. The resulting steady-state speed of the vehicle on the homogeneous section is then postulated to be the maximum attainable speed subject to these constraints. In symbols,

$$V^{\circ} = \min(\overline{V}_g, \overline{V}_c, \overline{V}_r, \overline{V}_d) \tag{2}$$

where

V° = deterministic steady-state speed;
$\overline{V}_g$ = gradient-limited speed,
$\overline{V}_c$ = curvature-limited speed,
$\overline{V}_r$ = roughness-limited speed, and
$\overline{V}_d$ = desired speed in the absence of road severity factors,

All of the speeds are expressed in meters per second.

Figure 1 shows the resultant steady-state speed, V°, and the limiting speeds as functions of gradient for a laden heavy truck traversing slightly curvy and relatively smooth homogeneous sections. As an example of interpreting the plot, if the vehicle is traveling uphill on a homogeneous section with 1 percent gradient, the values of limiting speeds are as follows: $\overline{V}_g$ = 18 m/sec, $\overline{V}_c$ = 68 m/sec, $\overline{V}_r$ = 52 m/sec, and $\overline{V}_d$ = 24 m/sec.

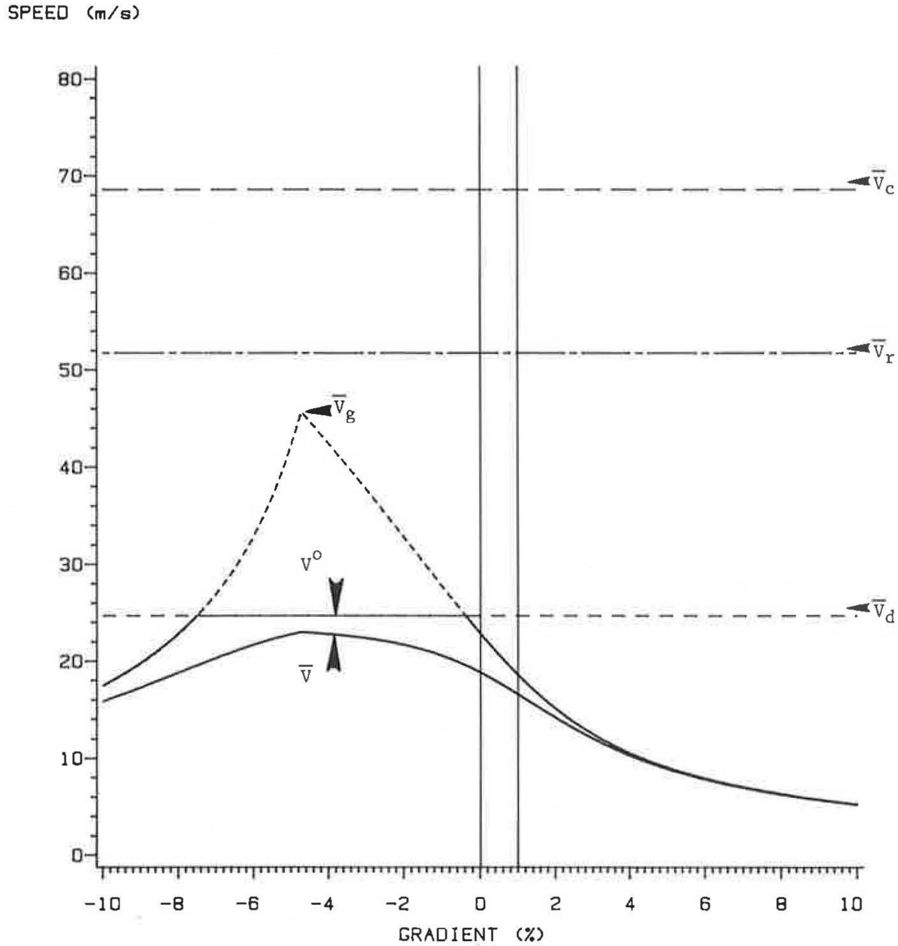

FIGURE 1 Constraining speeds and the steady-state speeds as functions of gradient.

Hence, the steady-state speed V^o is 18 m/s.

The steady-state speed curve has three distinct regimes. For very steep negative gradients (of magnitude greater than 7.5 percent), the steady-state speed is governed by the upward-sloping part of the gradient-limited speed, $\bar{V}_g$, which depends on the braking power used. Over the middle of the gradient range, the steady-state speed is determined by the desired speed, $\bar{V}_d$. Finally, for slightly negative grades and for all positive grades, the downward-sloping part of $\bar{V}_g$, which depends on the used driving power, dominates. In this example, the curvature-limited speed, $\bar{V}_c$, and the roughness-limited speed, $\bar{V}_r$, do not have a discernible influence on the steady-state speed.

Thus, the steady-state speeds predicted by using a limiting velocity formulation show asymptotically consistent behavior. That is, as various severity factors deteriorate from their ideal values, the predicted speeds decrease monotonically but retain plausible values. Further, as will be shown, a considerable amount of scientific, technological, and behavioral information can be incorporated in relating the limiting speeds to road section and vehicle characteristics.

The limiting speed approach to steady-state speed prediction has been used in a number of studies *(6–10)*. The studies differ in the number of limiting speeds used as well as in the way they are related to road and vehicle characteristics. The derivation of the constraining speeds used in this study is described next.

Gradient-Limited Speed, $\bar{V}_g$

The limiting speed governed by the vertical alignment of the homogeneous road section is derived based on two considerations, namely, the driving power used and the braking power used, thus giving rise to two basic limiting speeds, V_{dr} and V_{br}, respectively. $\bar{V}_g$ is taken to be the lower of these two speed constraints. That is,

$$\bar{V}_g = \min(V_{dr}, V_{br}) \tag{3}$$

where V_{dr} is the speed governed by driving power and gradient (m/sec), and V_{br} is the speed governed by braking power and gradient (m/sec).

V_{dr} and V_{br} are derived by first making behavioral assumptions about the use of driving and braking power, respectively, and then relating speeds to powers and gradients through the force-balance relation. Under steady-state conditions, the force balance may be written as

$$1{,}000P/v = mg(GR + CR) + 0.5\rho c_d av^2 \tag{4}$$

where

P = used power (kW);
v = speed (m/sec);

m = vehicle mass (kg), which is $m_0 + m_1$, where m_0 = mass of the empty vehicle (kg) and m_1 = net load (kg);
g = gradient;
GR = section gradient (as a fraction and with sign);
CR = coefficient of rolling resistance (dimensionless);
ρ = mass-density of air (kg/m^3) given by $1.225(1 - 226h10^{-5})^{4.225}$, where h is the elevation of the section over the mean sea level (m);
c_d = drag coefficient of the vehicle (dimensionless); and
a = projected frontal area of the vehicle (m^2).

SPEED GOVERNED BY DRIVING POWER, V_{dr}

V_{dr}, the speed limited by driving power used and gradient, is arrived at based on the assumption that when vertical gradient is the only road severity factor, the vehicle is driven at steady-state speed using a constant level of driving power. Denoting the constant driving power used by PDRIVE (in kW) and substituting PDRIVE for P in the force balance (Equation 4), yields

$$0.5\rho c_d a v^3 + mg(GR + CR)v - 1{,}000\ \text{PDRIVE} = 0 \quad (5)$$

which is a cubic equation in the unknown quantity v. It may be observed that for all values of GR, the number of sign changes in the coefficients of the equation is one and hence, by Descartes's rule of signs, the equation always has exactly one positive root *(11)*. V_{dr} is defined as the unique positive solution to Equation 5.

Cubic equations are generally solved iteratively. However, because the coefficient of v^2 in Equation 5 is zero, the equation has a relatively tractable analytical solution, which is given by Watanatada et al. *(1)*.

Speed Governed by Braking Power, V_{br}

V_{br}, the limiting speed determined by braking power used and gradient, is arrived at on the basis of the postulate that when a vehicle descends a long steep grade its descent speed is controlled by the vehicle braking capability, which results from the use of the vehicle engine retardation power or the regular brakes, or both. Thus, V_{br} is analogous to the concept of braking crawl speed. It is assumed that when negative gradient is the only road severity constraint, the steady-state speed is attained by using a constant level of braking power, a positive quantity by convention denoted by PBRAKE (in kW). Substituting P = –PBRAKE in the force balance (Equation 4),

$$0.5\rho c_d a v^3 + mg(GR + CR)v + 1{,}000\ \text{PBRAKE} = 0 \quad (6)$$

which is again a cubic equation in the unknown speed v. When the value of the effective gradient (i.e., $GR + CR$) is negative, the equation has two distinct positive roots; it is not, in general, possible to identify the physically meaningful solution on a priori grounds. However, because the braking speed constraint is likely to become binding only on steep negative grades where the steady-state speeds will be relatively low, the contribution of air resistance to the force balance may be neglected without serious error. Further, it is expected that the braking speed constraint be inapplicable when positive power is needed to move the vehicle, that is, when the effective gradient is nonnegative. Thus, the limiting speed due to gradient and braking power may be obtained as

$$V_{br} = \begin{cases} \infty & \text{if } GR + CR \geq 0 \\ -1{,}000\ \text{PBRAKE}/[mg(GR + CR)] & \text{if } GR + CR < 0 \end{cases} \quad (7)$$

Figure 2 shows the V_{dr}, V_{br}, and $\bar{V}_g$ curves as functions of gradient for a laden heavy truck on a homogeneous section (with a rolling resistance coefficient of 0.015.).

For the coefficient of rolling resistance, the following relationship expressing it as a function of section roughness, estimated in the Brazil study *(1)*, may be used

$$CR = \begin{cases} 0.0139 + 0.00026R & \text{for buses and trucks} \\ 0.0218 + 0.00061R & \text{for cars and utilities} \end{cases} \quad (8)$$

Curvature-Limited Speed, $\bar{V}_c$

The limiting speed governed by curvature of the homogeneous road section is arrived at from the postulate that when curvature is significant, the speed is limited by the tendency of the wheels to skid. An appropriate measure of the tendency to skid is the ratio of the lateral force on the vehicle to the normal force, which may be termed the perceived friction ratio. Under the assumption that when curvature is the only constraining road severity factor, the steady-state speed of a vehicle is attained by using a constant perceived friction ratio, denoted by FRATIO, an expression for $\bar{V}_c$ may be derived as follows.

The lateral force (LF) and the normal force (NF) on the vehicle are (in newtons)

$$\begin{aligned} LF &= mv^2(C/1{,}000)\cos SP - mg \sin SP \\ &\simeq mv^2(C/1{,}000) - mgSP \end{aligned} \quad (9)$$

and

$$\begin{aligned} NF &= mg \cos SP + mv^2(C/1{,}000) \sin SP \\ &\simeq mg + mv^2(C/1{,}000)SP \end{aligned} \quad (10)$$

where C is the section curvature (rad/km), and SP is the section superelevation (expressed as a fraction).

Thus

$$\begin{aligned} \text{FRATIO} &= \text{LF/NF} \\ &= [(v^2C/g\ 1{,}000) - SP]/[1 + SPv^2C/(g\ 1{,}000)] \\ &\simeq v^2C/(g\ 1{,}000) - SP \end{aligned}$$

Taking $\bar{V}_c$ to be the positive root of the above quadratic in v,

$$\bar{V}_c = [(\text{FRATIO} + SP)\ g1{,}000/C]^{1/2} \quad (11)$$

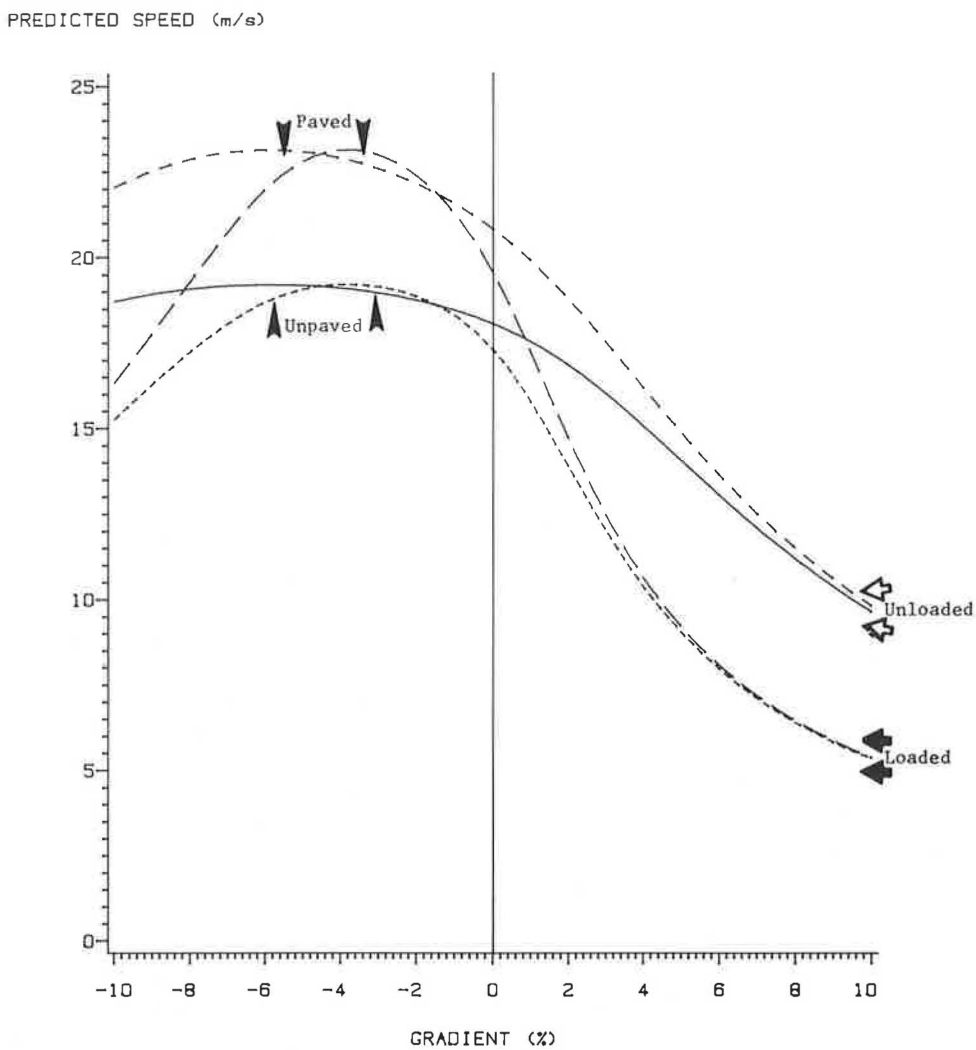

FIGURE 2 Predicted speed of a heavy truck as a function of gradient.

The parameter is called the perceived friction ratio to distinguish it from the actual friction ratio, which is the ratio of the vectorial sum of the lateral and drive forces on the vehicle to the normal force. The curvature-limited speed can also be modeled based on lateral acceleration or sight distance considerations.

From the Brazil data, the FRATIO parameter for a vehicle class has been found to depend on the surface type and, for trucks, the loading condition.

Roughness-Limited Speed, $\overline{V}_r$

This speed constraint is derived based on the notion that when roughness is the only prevailing road severity factor, the vehicle speed is limited by the discomfort sensed or the severity of the ride. An adequate measure of the ride severity for a vehicle with a rigid rear axle is the average rectified velocity (ARV) (in mm/s), defined as the rate of cumulative absolute displacement of the rear-axle relative to the vehicle body. It is approximately proportional to vehicle speed and road roughness, and may be written as

$$\mathrm{ARV} = \mathrm{ARV}(v) = 1.15vR \qquad (12)$$

where R is the section roughness (in m/km IRI), and the constant of proportionality reflects a calibration factor for the Maysmeter-equipped Opala automobile used in the Brazil study and unit conversion factors *(12, 13)*.

When roughness is the only constraining road severity factor, the steady-state speed for a vehicle is assumed to be attained at a constant representative value of average rectified velocity, denoted by ARVREP (in mm/s); under this assumption an expression for $\overline{V}_r$ is achieved by solving

$$\mathrm{ARVREP} = 1.15vR$$

That is,

$$\overline{V}_r = \mathrm{ARVREP}/(1.15R) \qquad (13)$$

It will be seen that, for the Brazil data, the ARVREP parameter does not vary significantly across surface types and load classes, only over the vehicle classes. Thus, given the roughness of the homogeneous section, the roughness-constrained speed, $\bar{V}_r$, may be computed if an estimate of the ARVREP parameter is available.

Desired Speed, $\bar{V}_d$

Finally, $\bar{V}_d$ is the desired speed, that is, the speed at which a vehicle of a given class would be operated in the absence of constraints based on gradient, curvature, and roughness. The desired speed results from the driver's response to psychological, safety, economic, and other considerations (for example, speed limits or even driver's perception of the strictness of enforcement), and, as such, it can be related to a number of factors. (In an extension of the current model using Indian data, it depends on the width class of the homogeneous section.) In the current model, $\bar{V}_d$ has been assumed to be constant for a given surface class, and estimated directly as a model parameter.

PROBABILISTIC LIMITING SPEED MODEL: FORMULATION

Even if the assumption of constant θ were true, the limiting speeds would still vary over different homogeneous sections and over different vehicles of the same class; these variations could be only partially explained by the variation in the observed characteristics of the section *(X)* and vehicle *(Y)*. Some of the important reasons are measurement errors, omission of characteristics of the road section and vehicle, deviations of the characteristics X and Y from the values actually used, the inability of the observer to determine the binding constraint with certainty, and the inability of the modeler to completely specify the decision procedure of the vehicle operator. In sum, the limiting speeds have to be treated as random variables and the parameters have to be estimated on this basis. It is the explicit recognition of the stochastic nature of the constraining speeds that distinguishes the probabilistic steady-state speed prediction model presented here from those of the earlier studies.

This notion is formalized by treating the limiting speeds as random variables (or variates) with means or expected values given by the expressions derived in the deterministic version. For example, denoting the gradient-limited speed variate by $V_g = V_g(X,Y{:}\theta)$, it may be written that

$$V_g(X, Y{:}\ \theta) = \bar{V}_g(X, Y{:}\ \theta)\eta_g(X, Y{:}\ \theta)$$

or, suppressing the arguments for simplicity,

$$V_g = \bar{V}_g\eta_g$$

Treating the other limiting speeds analogously,

$$V_z = \bar{V}_z\eta_z, \quad \text{for } z = g, c, r, \text{ and } d \tag{14}$$

Next, a new random variable V is defined as

$$V = \min\ (V_g, V_c, V_r, V_d) \tag{15}$$

Just as the speed constraint random variables, V may be expressed as

$$V(X,Y{:}\ \theta) = \bar{V}(X, Y{:}\ \theta)\eta(X, Y{:}\ \theta) \text{ or } V = \bar{V}\eta \tag{16}$$

It should be noted that although

$$V = \min\ (V_g, V_c, V_r, V_d)$$

it is generally not the case that

$$\bar{V} = \min\ (\bar{V}_g, \bar{V}_c, \bar{V}_r, \bar{V}_d)$$

In fact,

$$\bar{V} \leq \min\ (\bar{V}_g, \bar{V}_c, \bar{V}_r, \bar{V}_d)$$

with the equality holding if and only if the random variables are perfectly positively correlated or they are degenerate, that is, the variations are all zero. In other words, the mean of the minimum is generally less than the minimum of the means.

Thus the relation between the means of the speed constraints and the mean steady-state speed depends on the assumptions imposed on the joint distribution of the errors of the speed constraint variates. The error structure is specified and the estimation is performed by making use of two well-known distributions (lognormal and normal) and two distributions from a class known as asymptotic extreme value distributions. These are the Weilbull distribution and the Gumbel distribution *(14–16)*.

Just as normal distributions are preserved when the arithmetic operation involved is one of addition, the Weibull and the Gumbel distributions are preserved when the arithmetic operation involved is minimization (or maximization). This property enables one to derive the distribution of the minimum variate as a closed form function.

The disturbances pertaining to a particular speed observation and the associated speed constraints are specified by using three nested components of error. First, there are errors $\varepsilon(X)$, pertaining to the homogeneous section, which include unmeasured characteristics of the section and speed measurement errors. Second, there are errors $\zeta(X,Y)$, pertaining to the particular vehicle observed at that section, which include unmeasured characteristics of the particular vehicle at the section. Finally, given these two errors, there would be errors $\tau_z(X,Y)$, specific to the various speed constraint variates for that speed observation. That is, with η_z as the random part for a given realization of a constraining speed variate v_z for vehicle Y on section X,

$$v_z = \bar{V}_z\eta_z = \bar{V}_z\,[\varepsilon(X)][\zeta(X, Y)][\tau_z(X, Y)] \tag{17}$$

Proceed by imposing fairly standard assumptions of lognormality regarding the first two components of error. The Weibull distribution will be used for the third component to derive the conditional distribution of the observed speed variate. Specifically,

- Errors $\varepsilon(X)$ are independent and have identical lognormal distributions with mean 1.

- Errors $\zeta(X,Y)$ are independent and have identical lognormal distributions with mean 1.
- Errors $\tau_z(X,Y)$ are independent and have identical Weibull distributions with mean 1 and shape parameter β.

Under these three assumptions, by using the properties of the Weibull distribution, the following results are obtained:

- Conditional on $\varepsilon(X)$ and $\zeta(X,Y)$, the attained speed is a variate V, which has a Weibull distribution with a shape parameter β.
- The relationship between the conditional means of the attained speed and the limiting speed variates is

$$\bar{V} = (\bar{V}_g^{-1/\beta} + \bar{V}_c^{-1/\beta} + \bar{V}_r^{-1/\beta} + \bar{V}_d^{-1/\beta})^{-\beta} \quad (18)$$

where $\bar{V}_x = \bar{V}_x(X,Y: \theta)$ and hence, $\bar{V} = \bar{V}(X,Y: \theta, \beta)$.

Thus, the speed observation may be written as a random variable V, with

$$V = \bar{V}\eta = \bar{V}[\varepsilon(X)][\zeta(X, Y)][\tau(X, Y)]$$

where $\tau(X,Y)$ have independent Weibull distributions with mean 1 and shape parameter β, or

$$V = \bar{V}\varepsilon\omega \quad (19)$$

where $\omega = \omega(X,Y) = [\zeta(X,Y)][\tau(X,Y)]$ which is approximately lognormal.

Equation 18, along with the expressions for $\bar{V}_x$ given earlier, constitutes a multinomial logit model that is nonlinear in the parameters θ and β *(17–19)*. Figure 1 shows $\bar{V}$ as a function of gradient. At 1 percent gradient, the predicted speed is approximately 16 m/s.

Equivalently, the model can be expressed in terms of the logarithms of speeds. This version is more convenient for estimation purposes. Assuming that for a given speed observation the logarithms of the constraining speeds have independent Gumbel distributions with identical scale parameters β, the properties of Gumbel distribution can be used to express the model as follows. Defining

$$U = \ln V \quad (20)$$

$$U_z(X, Y: \theta) = \ln U_z(X, Y: \theta), \text{ for } z = g, c, r, \text{ and } d \quad (21)$$

leads to

$$\bar{U} = \ln [\exp(- \bar{U}_g/\beta) + \exp(- \bar{U}_c/\beta) + \exp(- \bar{U}_r/\beta) + (- \bar{U}d/\beta)]^{-\beta} \quad (22)$$

where $\bar{U}_z = \bar{U}_z(X,Y: \theta)$ are the means of the random variables $\bar{U}_z$. Thus by analogy with the speed model, the log speed model may be written as

$$U = \bar{U} + e + w \quad (23)$$

where $e = e(X)$ have independent normal distributions with constant mean and variance, and $\omega = w(X,Y)$ are independent and identically distributed as convolutions of independent normal and Gumbel distributions. The distribution of w is approximately normal.

The estimation problem then is to find estimates θ and β that satisfy a suitable criterion. The two most commonly used criteria for estimating a logit model are maximum likelihood and least squares *(20–22)*. The criterion chosen was least squares.

PROBABILISTIC LIMITING VELOCITY MODEL: ESTIMATION

Data for Estimation

The data for the steady-state speed model were obtained from radar speed observations of vehicles at selected homogeneous road sections over a period of about 1 year. Sections were distinguished by direction of travel. Because the roughness of these sections varied significantly over nonconsecutive observation periods, the unpaved sections were further distinguished by roughness intervals of 4 m/km IRI. This procedure resulted in a total of 216 homogeneous sections with gradient ranging from –9 to 11 percent, curvature from 0 to 50 rad/km, and roughness from 1.5 to 15 m/km IRI.

The observations made for each vehicle sighting were spot speeds on the section, vehicle type, and load condition. The vehicle types observed were categorized into six classes, and the three truck classes were further divided into unloaded and loaded categories. These classes are given in Table 2 with the adopted average vehicle characteristics. The average gross vehicle masses were obtained from a separate axle load study for Brazil. The values of aerodynamic drag coefficient and frontal area were adapted from those for typical makes and models prevalent in Brazil for each vehicle class.

TABLE 2 VEHICLE CLASSES AND CHARACTERISTICS USED IN STEADY-STATE SPEED MODEL ESTIMATION

Vehicle Class	Drag Coefficient, c_d	Projected Frontal Area, a (m^2)	Total Vehicle Mass, m, (kg)
Car	0.50	2.00	1 200
Utility	0.60	3.00	2 000
Bus	0.65	6.30	10 400
Light/medium truck	0.70	4.5	5 400 (unloaded) 11 900 (loaded)
Heavy truck	0.85	5.2	7 900 (unloaded) 19 200 (loaded)
Articulated truck	0.65	5.8	15 900 (unloaded) 37 700 (loaded)

In all, about 100,000 speed observations were included. For each vehicle class, the logarithms of individual speed observations pertaining to a section were averaged, yielding the dependent variable values of the estimation data set.

TABLE 3 ESTIMATION RESULTS OF STEADY-STATE SPEED MODEL FOR SIX VEHICLE CLASSES: ESTIMATES

Vehicle Class	β (a)	PDRIVE, kW (b)	PBRAKE, kW (c)	ARVREP, mm/s (d)	V m/s: Unpaved Surface (e)	V m/s: Paved Surface: Increment over Unpaved Value (f)	V m/s: Paved Surface: Value (e) + (f)	FRATIO: Unpaved Surface (unloaded or loaded vehicle) (g)	FRATIO: Paved Surface, Unloaded Vehicle: Increment over Unpaved Value (h)	FRATIO: Paved Surface, Unloaded Vehicle: Value (g) + (h)	FRATIO: Paved Surface, Loaded Vehicle: Increment over Paved Unloaded (i)	FRATIO: Paved Surface, Loaded Vehicle: Value (g) + (h) + (i)
Car	0.274	26.8	16.0	259.7	22.8	4.5	27.3	0.124	0.144	0.268	–	–
Utility	0.306	32.7	24.0	239.7	21.8	4.6	26.4	0.117	0.104	0.221	–	–
Bus	0.273	83.1	157.3	212.8	19.3	6.7	26.0	0.095	0.138	0.233	–	–
Light/medium truck	0.304	69.7	140.4	194.0	20.0	2.7	22.7	0.099	0.154	0.253	–0.083	0.170
Heavy truck	0.310	79.6	189.2	177.7	20.0	4.7	24.7	0.087	0.205	0.292	–0.107	0.185
Articulated truck	0.244	147.2	368.0[a]	130.9	13.8	9.6	23.4	0.040[a]	0.139	0.179	–0.049	0.130

[a]This parameter for the articulated truck class was exogenously assigned.

Estimation Results

The final results are presented in Table 3. These results consist of six sets of parameter estimates, for cars, utilities, buses, light and medium trucks, heavy trucks, and articulated trucks. The asymptotic *t*-statistics associated with these parameter estimates are given in Table 4 in a similar format. A goodness-of-fit measure analogous to the R^2 value in linear models is also given. This was obtained by regressing the mean observed speeds against predicted speeds.

There were too few observations for articulated trucks to support the determination of all the model parameters and the PBRAKE and FRATIO parameters were assigned the values of 368 kW and 0.40, respectively, when the estimation was carried out.

All the parameter estimates have the expected sign and most have the expected relative magnitudes as individually discussed in the following paragraphs. All but one of the parameter estimates have asymptotic *t*-statistics significant at approximately 5 percent.

Except for the mixed class of light and medium trucks, the magnitudes of the driving power used (PDRIVE) are consistently smaller than the maximum rated power values of the typical vehicles of the respective classes. In fact, there is an approximate relationship between these quantities that can be used to calibrate the PDRIVE parameter for a new vehicle *(1)*.

The magnitudes of the braking power used (PBRAKE) appear to increase with the gross vehicle mass. As would be expected, the greater the mass of a vehicle, the more the braking capability needed to render the vehicle operations safe. An approximate relationship between the braking power used and the gross vehicle mass may also be derived.

The FRATIO estimates, from 0.087 to 0.292, are well below the range of 0.6 to 0.7 found from skid-pad tests of modern high-performance passenger cars. This appears to indicate a large margin of safety within which vehicles are generally

TABLE 4 SOME IMPORTANT STATISTICS ASSOCIATED WITH THE ESTIMATION

Vehicle Class	β (a)	PDRIVE (b)	PBRAKE (c)	ARVREP (d)	V_d: Unpaved Surface (e)	V_d: Increment for Paved Surface (f)	FRATIO: Unpaved Surface (g)	FRATIO: Increment for Paved Unloaded (h)	FRATIO: Further Increment for Paved Loaded (i)	Variances: σ_e^2	Variances: σ_w^2	R^2	No. of Observations	Sum of Squared Residuals
Car	10.9	15.2	7.9	20.3	22.8	4.9	12.4	10.3	–	0.00654	0.0224	0.92	216	1.36
Utility	10.9	19.3	7.9	17.5	17.4	4.0	9.3	7.0	–	0.00808	0.0355	0.89	216	1.68
Bus	6.8	27.3	4.6	12.0	12.0	3.8	5.3	5.3	–	0.02477	0.0276	0.83	216	5.15
Light/medium truck	13.4	44.0	9.5	24.6	18.4	2.8	9.0	7.3	–3.7	0.01574	0.0405	0.87	431	6.64
Heavy truck	9.7	41.6	10.9	19.1	11.6	2.8	3.8	5.9	–3.1	0.02578	0.0369	0.85	381	9.59
Articulated truck	7.0	33.9	–	19.0	11.6	7.0	–	5.0	–1.5	0.03588	0.0365	0.81	232	8.07

Note: The members in the parameter columns are the respective asymptotic *t*-statistics.

operated on public roads. Within a vehicle class, the estimates for unpaved roads are significantly smaller than those for paved roads. Further, for paved road operations, laden trucks have smaller FRATIO estimates than unladen trucks. Finally, across the vehicle classes, the FRATIO estimates tend to vary inversely with the size of the vehicle.

The estimates of the average rectified velocity (ARVREP) show a clear tendency to vary inversely with the vehicle size, the cars having the largest value and the articulated trucks the smallest. This is somewhat surprising because on purely physical reasoning it would be expected that the smaller vehicles be more sensitive to road roughness than the larger ones. The reversal of relative magnitudes is probably explained in part by the higher tire stiffness of larger vehicles and in part by the economic response of the driver to the relatively higher cost impact of roughness on larger vehicles.

The estimates for the desired speed ($\bar{V}_d$) are, as expected, higher for paved roads than for unpaved roads. Moreover, they tend to be larger for smaller vehicles, although they are relatively constant for each surface type for most vehicle classes.

The discriminating power of the models may be seen in Figures 2–4, which show graphs of predicted steady-state speed plotted against the gradient, curvature, and roughness, respectively—for unloaded and loaded heavy trucks—for both paved and unpaved surfaces.

APPLICATIONS AND DIRECTIONS FOR FUTURE RESEARCH

The most important current application of the steady-state speed prediction model has been the development of three methods for predicting speeds on heterogeneous roadways.

1. The micro-transitional roadway speed prediction method, which uses detailed information on the roadway and simulates transitional driver behavior including speed change cycles.
2. The micro-nontransitional method, which requires the characteristics of all the homogeneous sections of the roadway, but does not model transitional driver behavior.
3. The aggregate method, which uses a classification-based aggregation procedure, which is fairly widely used in demand aggregation *(18, 23)*.The method uses summary descriptors of the characteristics of the roadway and generates speed predictions for one-way and round-trip travels on the roadway. The aggregate method, which incorporates the effect of road width

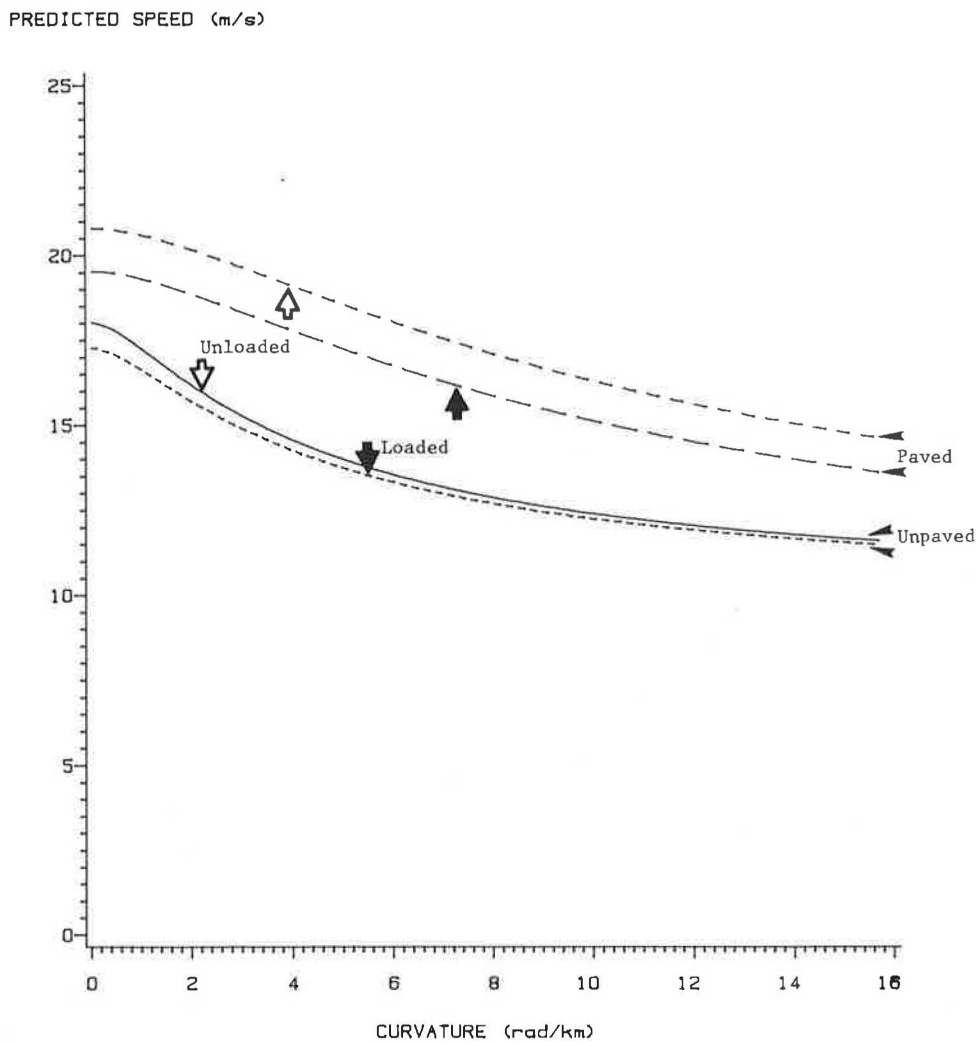

FIGURE 3 Predicted speed of a heavy truck as a function of curvature.

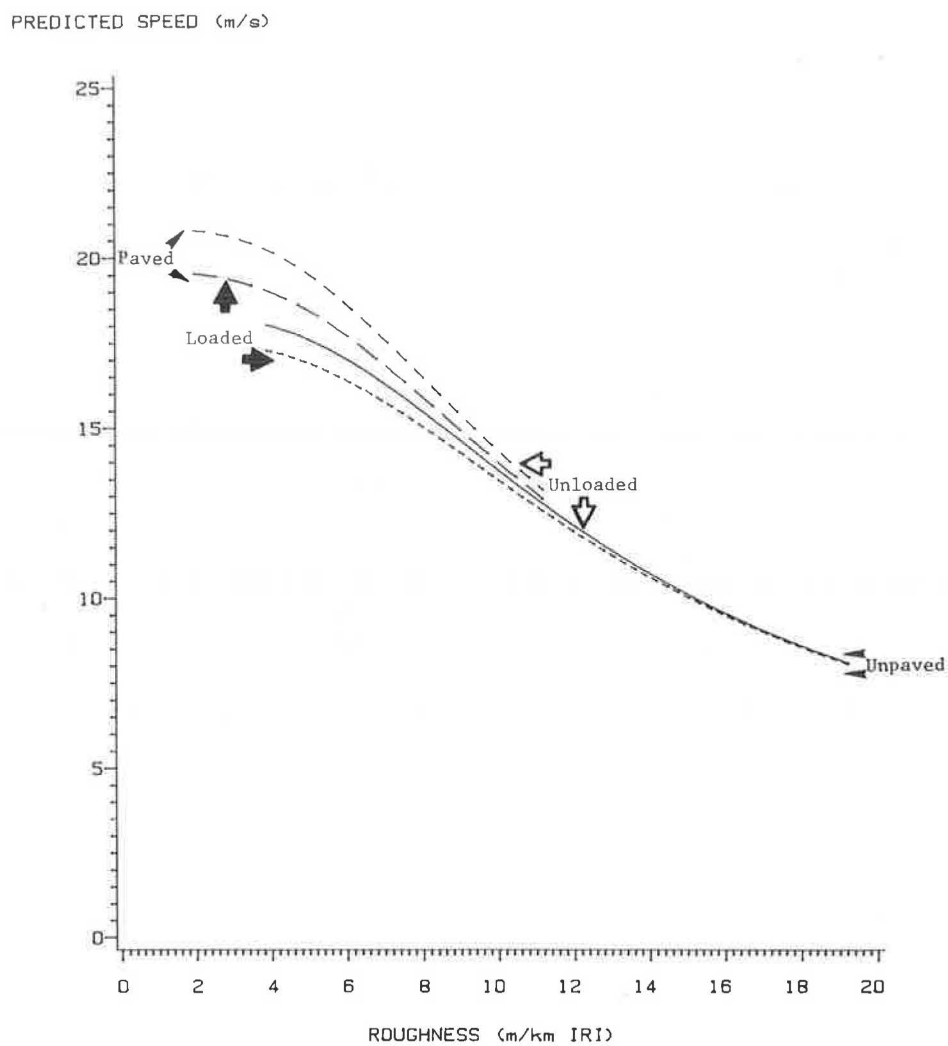

FIGURE 4 Predicted speed of a heavy truck as a function of roughness.

based on data from India, has been selected for use in the HDM-III model *(2)*.

Models have been developed that use predicted speeds and other derivative variables (such as power used, tangential energy, block speed, etc.) as inputs to predict fuel consumption, tire wear, and vehicle utilization. Depending on the method of speed prediction used, these models can provide predictions on a disaggregate or aggregate basis. The aggregate versions of these models have also been implemented in HDM-III.

A methodology to calibrate the parameters of the steady-state speed prediction model for a new environment without full-fledged estimation has been developed based on a nonrandom sampling technique *(2, 24)*.

Potentially the most important area for future application of the speed prediction method is in modeling vehicle interaction and the consequences for vehicle operating costs, both at the disaggregate and aggregate levels.

Possible improvements to the steady-state speed prediction model include the following:

- The relationships between the speed constraints and speed-influencing characteristics of the road section could be further enhanced by taking into account the effect of the following: shoulder width and condition on the desired speed, curvature on the braking capacity, and lateral acceleration and sight distance on perceived friction ratio.
- In regard to the distributional assumptions made in the probabilistic version of the model, the most restrictive ones are independence and equality of the shape parameter for all the speed constraint variates. It would be of interest to test the acceptability of these assumptions by estimating the model parameters under the more general multinomial probit formulation.

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