On Predicting Pavement Surface Distress with Empirical Models of Failure Times

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A statistical procedure that overcomes common difficulties in the development of distress models from empirical data is described and applied in an example. The time at which cracking or raveling appears in bituminous pavements is influenced by both trafficking and weathering and varies across pavements, even under nominally identical conditions. Also, any data set that represents a uniform sample of roads of all ages in a network over a given time period will typically include some unobserved (or "censored") events, that is pavements on which distress began either before or after the observation "window." The method applies failure-time theory, using maximum likelihood methods so that censored data can be included to prevent statistical bias in the predictions. The variability of failure times is represented by a Weibull distribution because that is considered the most appropriate for the concurrent mechanisms of fatigue and weathering and is flexible in shape. An example application of the procedure to a major analysis of Brazilian condition data shows that it permitted the use of a much wider data base than would otherwise be possible and produced important results that quantify strong effects of weathering and variability.

The initiation of surfacing distress such as cracking or raveling marks a significant stage in the deterioration of a pavement. From this point, the rate of deterioration usually accelerates at a rate that varies with traffic, pavement, and climatic conditions. The economic consequences of this are expressed through the secondary impact of surfacing distress on road roughness, which has a significant influence on vehicle operating costs *(1)*.

The timing of periodic maintenance to control the deterioration is thus largely dependent on the time of initiation of distress. Hence the prediction of that time is an important component of pavement management systems and models for the economic evaluation of road maintenance policies and pricing policies. That distress does not occur instantaneously over the entire length of roads under like conditions is also important because the needs for maintenance expenditure are thereby spread over time. Considerable attention was therefore devoted to the modeling of pavement distress from field data in recent World Bank studies *(1,2)*, and in particular to the joint effects of traffic and aging because these influence the allocation of road damage costs between users and society.

The procedure described is a statistical method for estimating empirical prediction models from pavement condition data. The method, based on failure-time theory, incorporates the stochastic variations of pavement behavior and represents the concurrent effects of traffic-related fatigue and time-related aging, which can vary considerably from region to region. The method was developed because the variability evident in real pavement data and the fact that the time of appearance of distress usually cannot be observed on all sections within a finite study period were hindering the analysis of cracking and raveling data from a major United Nations Development Program road costs study in Brazil *(3)*.

One example from the World Bank's analysis of this study *(2)* is given to illustrate the procedure, and guidance is given on its application to other regions.

TRAFFIC AND AGE-RELATED MECHANISMS

Traffic-related cracking occurs through fatigue of bituminous surfacing materials under repeated wheel-load applications. The occurrence of failure, here defined as the first visible crack, is commonly expressed in terms of the cumulative number of load applications and is related to the tensile strains induced in the surfacing under the spectrum of mixed wheel loadings and to the material properties, primarily mixture stiffness and the volume of bituminous binder, by the following general expression:

$$N_f = K \cdot \varepsilon_t^{-n} \quad (1)$$

where

N_f = number of repetitions of load in flexure under strain control to the initiation of fatigue cracking;
ε_t = maximum horizontal tensile strain; and
K, n = estimated constants (values vary with materials and test conditions, but typically n increases from 3 to 7 as the material stiffness increases from soft to stiff or brittle).

The strains induced in the surfacing depend primarily on the tire contact pressure (and on wheel load in the case of thick surfacings), the thickness and stiffness of the surfacing layer, and the stiffness of the underlying pavement. Laboratory testing has indicated considerable stochastic variation in failure lives, which range typically over an order of magnitude or by a factor of three each side of the mean, under controlled conditions.

Due to exposure to air, a bituminous binder hardens over time primarily by oxidation, which consequently reduces the

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fatigue life of the surfacing material. The rate at which this occurs depends on the oxidation resistance of the binder, which varies with the chemical composition and source of crude; on temperature; and on the film thickness, which determines the length of the oxidation path *(4)*. Hardening rates therefore vary with binder source, climate, and material design. Dickinson *(4)* has observed that cracking usually occurs when a binder reaches a critical viscosity at which the binder can no longer sustain the low strain levels associated with daily thermal movements and fracture occurs. By that stage the surfacing is typically 9 years old although, depending on the composition of the binder, the age may range between 6 and 15 years. This cracking by "aging" usually takes the form of irregular or "map" cracking with a spacing greater than 0.5 m and is likely to progress rapidly over the full area of surface. Raveling can also occur at this viscosity through fracture of the binder film on individual particles.

Thus the interaction of oxidation with traffic-related fatigue advances the time at which fracture occurs in all bituminous-surfaced pavements. This interactive effect may be depicted conceptually as is done in Figure 1. The fatigue life of the surfacing (in logarithm of load repetitions), shown by Curve A for the exposed surface and Curve B for the underside, decreases as the surfacing ages due to oxidation. Curve A is initially highest because lower strains are induced at the surface, but oxidation reduces the life at the exposed surface more rapidly than at the underside, as shown. Under high traffic volumes, shown as Curve C, traffic-related fatigue causes cracking first (at 4.5 years in the example), and, depending on the relative disposition of Curves A and B, initiation of cracking may occur either at the surface (as shown) or at the underside. Under very low traffic volumes, or on stiff pavements, Curve C may be lower than Curve D so that initial fracture may be attributable primarily to the age-related mechanism.

The relative influences of traffic and aging are therefore likely to vary considerably across pavements that have different surfacings, materials, and traffic and also across climates, because these factors influence the relative position of the functions shown. Empirical models must represent such interaction and be adaptable, through parameterization or calibration, to other conditions.

FAILURE-TIME THEORY

Concepts

The initiation of distress such as cracking is a discrete but highly variable event. That is, cracking will occur at different times at various locations along a nominally homogeneous road. The first of these times is termed the initiation of cracking. Another pavement of nominally identical properties and traffic will have initiation at a different time (T_i), where i indexes the pavement section.

The time (T_i) or age of the surfacing at "failure" (here defined as the appearance of distress) thus varies in the real world, even under nominally identical conditions. This can be represented by a probability density function [f(t)] as shown by a hypothetical example in Figure 2. In the function drawn, the first crack is unlikely to appear within "A" years of surfacing construction and is nearly certain to appear before the surfacing is "B" years old. On about one-half of all identical pavements, the first crack is likely to have appeared within "C" years. The probability or chance that the pavement will not have cracked by a certain age is represented by the survivor function [$\bar{F}(t)$] in Figure 2*b*. The location of the functions along the time axis and their shapes can be expected to depend on the properties of the pavement and the intensity of traffic and loading stresses to which it is subjected.

In some instances, when modeling fatigue cracking, it may be desirable to use cumulative traffic (for example the log-

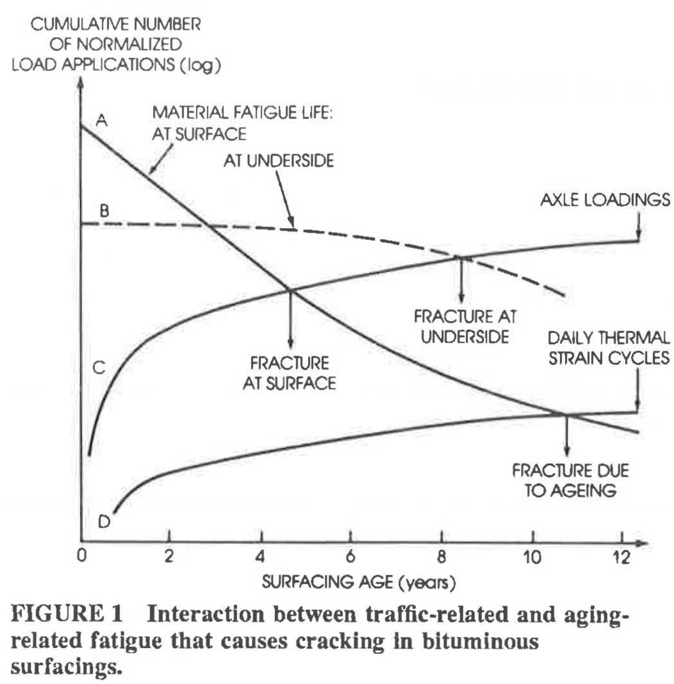


FIGURE 1 Interaction between traffic-related and aging-related fatigue that causes cracking in bituminous surfacings.

arithm of cumulative equivalent standard axles) in place of chronological time, but the modeling principles are similar for both cases. In general, time is the most convenient unit for planning models and is used in the argument that follows.

Because data collection surveys are typically of limited duration, in addition to the considerable variability in failure times, there is the difficulty of unobserved failure events in a typical set of pavement condition data. Among a uniform cross section of pavements with a range of different ages, strengths, and traffic loadings, some of the pavements will have already been cracked on the first survey date, some will begin to crack during the survey period, and on others cracking will begin

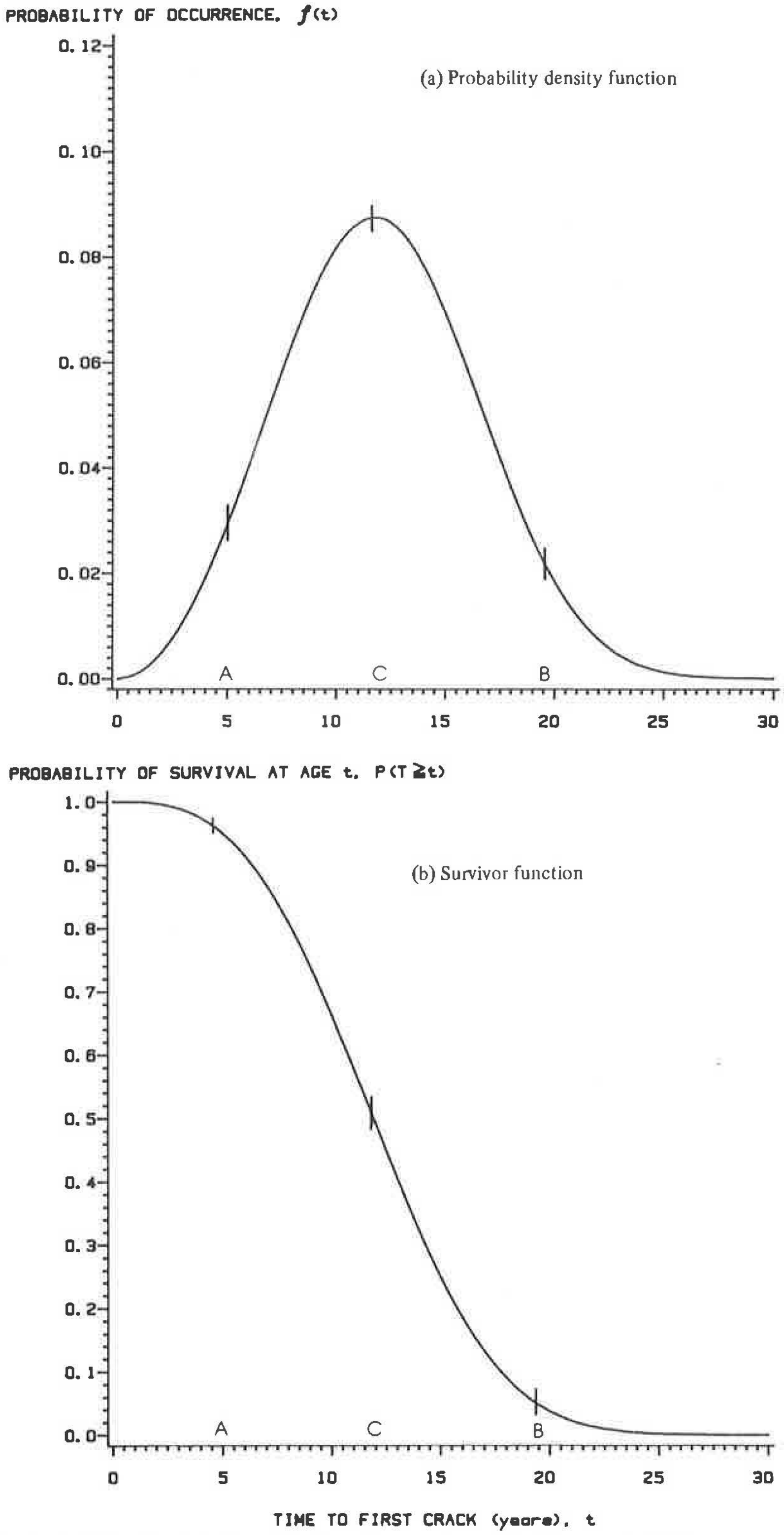


FIGURE 2 Variability of failure times represented by probability density and survivor functions of time to first distress.

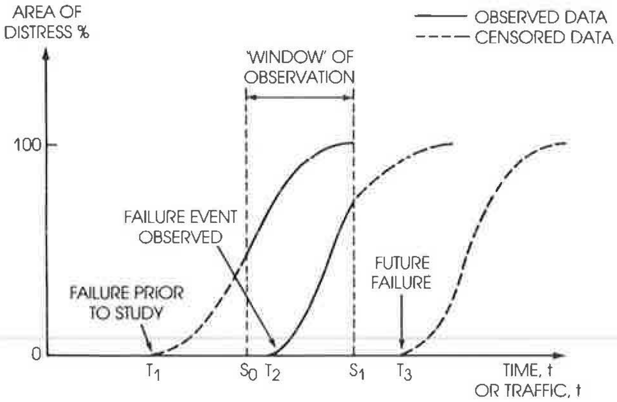


FIGURE 3 Unobserved or censored data on distress initiation and progression: example of three pavement sections with prior, observed, and future failure events, respectively.

only after the end of the survey, as shown in Figure 3. If only the cracking initiation events observed during the survey were included in a statistical analysis, important information about the stochastic and mechanistic properties of the phenomenon coming from the "before" and "after" events might be excluded and thus cause a bias in the model. These latter events, known as "censored data," can be of vital importance particularly in representing long-life pavements in an analysis.

Both features, stochastic variations and censored data, were addressed by developing an estimation procedure based on the principles of failure-time analysis, originally developed to study the reliability of industrial components. The procedure uses the statistical method of maximum likelihood estimation to exploit both censored and uncensored data, as described in the following section, and a flexible distribution that enables the variability of failure times to be determined by the data, as outlined in the succeeding section.

Censored Data: Maximum Likelihood Estimation

T is defined as the time from construction of a section of surfacing to failure, where T is a random variable indexed by i, a section identifier to indicate that the distribution of T depends on section characteristics. The term "failure" is used to describe the first appearance of the mode of pavement distress that is of interest, for example, narrow cracking, wide cracking, or raveling.

T is regarded as a continuous nonnegative random variable and its probability density function is denoted by f(t), its distribution function by F(t), and its survivor function by $\bar{F}(t) = P(T > t) = 1 - F(t)$ (see Figure 2).

Suppose a road section is selected at random, initially observed S_0 years after surfacing, and finally observed $S_1 > S_0$ years after surfacing. One and only one of the following events may be observed:

- $T < S_0$: failure occurred before the road was observed. In this case define $D_1 = 1$, otherwise define $D_1 = 0$.
- $S_0 \leq T \leq S_1$: failure occurred while the road was observed. In this case define $D_2 = 1$, otherwise define $D_2 = 0$. Let z be the observed value of T.
- $S_1 < T$: failure occurred after the road was observed. In this case define $D_3 = 1$, otherwise define $D_3 = 0$.

Let

$$t = S_0 \text{ if } D_1 = 1,$$
$$= z \text{ if } D_2 = 1, \text{ and}$$
$$= S_1 \text{ if } D_3 = 1.$$

By selecting a road at random, values are obtained for D_1, D_2, D_3, and t.

To exploit data on all sections it is necessary to develop a maximum likelihood estimator. Accordingly, consider the joint probability–probability density function of the discrete D_1, D_2, and D_3 and the continuous t.

Standard probability theory gives D_1, D_2, and D_3 as multinomially distributed with $P(D_1 = 1) = F(S_0)$, $P(D_2 = 1) = F(S_1) - F(S_0)$, and $P(D_3 = 1) = \bar{F}(S_1)$. Conditional on $D_1 = 1$ or $D_3 = 1$, t is either S_0 or S_1 with probability 1 in each case. Conditional on $D_2 = 1$, t has the truncated probability density function $f(t)/[F(S_1) - F(S_0)]$. Multiplying marginal and conditional probabilities gives

$$P(D_1 \cap D_2 \cap D_3 \cap t) = F(S_0)^{D_1} \; f(t)^{D_2} \; \bar{F}(S_1)^{D_3}$$
$$= F(t)^{D_1} \; f(t)^{D_2} \; \bar{F}(t)^{D_3} \qquad (2)$$

Now write f(t) as a conditional failure time probability density function, depending on section characteristics x and parameters θ. Then

$$P(D_1 \cap D_2 \cap D_3 \cap t \mid x, \theta) = F(t \mid x, \theta)^{D_1} \; f(t \mid x, \theta)^{D_2} \; \bar{F}(T \mid x, \theta)^{D_3} \qquad (3)$$

Estimation of θ can be achieved by calculating the maximum likelihood estimator. Index t and x by i, which distinguishes sections. Then the probability–probability density function of the observed D_i's, t_i's ***D***, and ***t***, given the x's, ***x***, and θ, is

$$P(\boldsymbol{D} \cap \boldsymbol{t} \mid \boldsymbol{x}, \theta) = \prod_{i=1}^{n} F(t_i \mid x_i, \theta)^{D_{1i}} \, f(t_i \mid x_i, \theta)^{D_{2i}} \, \bar{F}(t_i \mid x_i, \theta)^{D_{3i}} \qquad (4)$$

By taking logs, Equation 5, the log-likelihood function, is obtained. The maximum likelihood estimator (θ) is that value of θ which maximizes Equation 5. θ must be obtained using numerical methods. Under fairly general conditions θ is consistent and efficient. The variance-covariance matrix of θ is estimated by minus the inverse of the Hessian of Equation 5 at θ = θ. See Rao *(5)* or Theil *(6)* for further details of the properties of θ.

$$L(\theta \mid \boldsymbol{D}, \boldsymbol{t}, \boldsymbol{x}) = \sum_{i=1}^{n} \{D_{1i} \log F(t_i \mid x_i, \theta) + D_{2i} \log f(t_i \mid x_i, \theta) + D_{3i} \log \bar{F}(t_i \mid x_i, \theta)\} \qquad (5)$$

The log-likelihood function of Equation 5 is maximized by some variant of the Newton-Raphson procedure. A program was developed that maximizes Equation 5 either by Newton-Raphson as modified by Berndt et al. *(7)* or by steepest ascent.

The Berndt et al. modification of the Newton-Raphson procedure makes use of the identity:

$$E(n^{-1}\delta^2 L/\delta\theta\delta\theta') = -E[n^{-1}(\delta L/\delta\theta)(\delta L/\delta\theta')] \quad (6)$$

so that the matrix of second derivatives

$$E(\delta^2 L/\delta\theta\delta\theta')$$

is estimated by the approximation

$$-n^{-1}\sum_{i=1}^{n}(\delta L_i/\delta\theta)(\delta L_i/\delta\theta')$$

where $(\delta L_i/\delta\theta)$ is the ith term in the summation

$$(\delta L/\delta\theta) = \sum_{i=1}^{n}(\delta L_i/\delta\theta).$$

The program uses analytic expressions for first derivatives of the log-likelihood function. At the termination of the optimization the Hessian associated with the log-likelihood function is calculated by differencing the analytic first derivative vector. Eigenvalues are calculated to check on the definiteness of the Hessian and the Hessian is then inverted to obtain estimates of asymptotic variances and covariances of estimated coefficients. Various predictions are provided for each observation so that the equivalent of residuals can be examined. The program will calculate asymptotic confidence intervals around predictions and provide various graph plots if required.

Asymptotic confidence intervals for expected failure times are provided by exploiting the local large sample linearity of the expression for expected failure time given pavement characteristics and computing the asymptotic variance of the resulting linear approximation; 1.96 times the square root of this variance is then added to and subtracted from the predicted expected failure time to give the required interval.

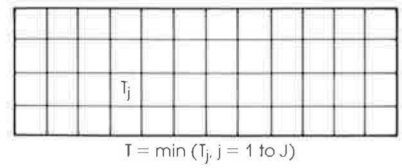
Starting values for the parameters are provided automatically, but there is provision for using a manual start if required. The program is written in SAS's matrix procedure *(8)*.

Variability of Failure Times

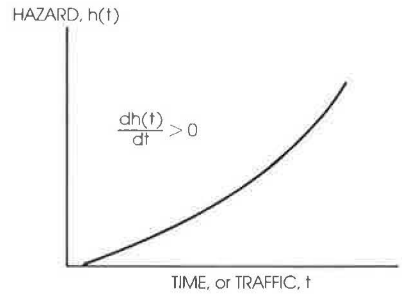
The underlying variation of failure times was assumed in the failure-time model to follow a Weibull distribution [for which general statistical results may be found in Chapter 20, Vol. 1 of Johnson and Kotz *(9)*]. A log normal distribution is also available in the program, but the Weibull distribution was considered the most representative of the joint mechanisms of fatigue and aging for the following reasons.

The time to failure of a section is the first failure to occur among all individual elements of the surfacing, where each fails at a time following some probability law, as shown in Figure 4*a*. The Weibull distribution, a Type-3 extreme value distribution, is suitable for determining the minimum (or limiting distribution) of a series of minima (that is the failure times).

The mechanisms of fatigue under traffic and oxidation, which reduce the available fatigue life, work concurrently, and



(a) Cracking initiation in a nominally homogeneous section represented by elements over which pavement properties are randomly distributed. Failure occurs at the minimum of the failure times of all elements



(b) Fatigue is a hazard which increases monotonically with time or with cumulative number of axle transits (we expect $\beta > 0$)

FIGURE 4 Two hypotheses on the probability of occurrence of crack initiation that indicate a Weibull distribution of failure times.

thus the probability of cracking occurring in the surfacing is expected to increase as the pavement ages. For example, the chances that a pavement will crack in its 15th year, if it has not already cracked by that time, are considered greater than the chances of its cracking in, say, the previous year. This can be described as an increasing "hazard" of cracking. The hazard function [h(t)], a concept used in reliability theory, is proportional to the probability that failure will occur in a short time interval at time t given that it has not occurred previously. It is defined by

$$h(t) = [f(t)/1 - F(t)] = [f(t)/\bar{F}(t)] \quad (7)$$

where

$f(t)$ = probability density function associated with T;
$F(t)$ = probability distribution function, = $P(T \leq t)$; and
$\bar{F}(t)$ = $P(T \geq t)$.

To model fatigue, therefore, the hazard is expected to be an increasing function of time as shown in Figure 4*b*. When the hazard is specified by

$$h(t) = \alpha^{-\beta}\, t^{\beta-1} \qquad \alpha,\ t \geq 0 \text{ and } \beta > 1 \quad (8)$$

the probability distribution function is defined by

$$F(t) = 1 - \exp[-(1/\beta)\, \alpha^{-\beta}\, t^{\beta}] \quad (9)$$

where

α = function of the variables causing failure,

β = curvature of hazard function, and
t = time (or cumulative traffic).

This is the distribution function associated with a Weibull distribution, describing a positively skewed distribution over the nonnegative real axis. The expected, or mean, time to failure [E(T)] is given by

$$E(T) = \alpha\, B(\beta) \tag{10}$$

where B(β) is a constant function of β given by

$$B(\beta) = \beta^{(1-\beta)/\beta}\, \Gamma(1/\beta)$$

and selected values for B(β) are as given in Table 1.

The selection of the type of distribution is important because results are sensitive to it when many of the data have censored values. The Weibull distribution appears to be appropriate for representing the variability of failures for the two conceptual reasons just outlined. It also is flexible because it describes a family of skewed curves for different values of β and α that appear to be realistic for pavement data, as shown in Figure 5. The β parameter determines the shape of the distribution, which becomes narrower as β increases. The α parameter is a scaling function that locates the distribution along the time axis. Because both the β and α parameter values are estimated from the observed data, and neither is fixed, the Weibull model is particularly adaptable to actual circumstances.

The objective of the estimation is not only to determine the average time to failure and its distribution but also to estimate how the expected time to failure depends on pavement and traffic characteristics. These are represented in the α parameter of the model that must be nonnegative. Although other forms could be used, α here was defined by the vector

$$\alpha = \exp\,[X'\gamma]$$
$$= \exp\,(\gamma_0 + \gamma_1 x_1 + \gamma_2 x_2 + \gamma_3 x_3 \ldots) \tag{11}$$

where γ is the vector of coefficients γ_i and X is the vector of parameters (pavement strength, traffic flow, etc.).

Given various pavement, traffic, and environmental parameters (x_i), the model estimates the coefficients (γ_i) and the shape parameter (β), which is assumed constant for all pavements within the data set.

The mean time to failure [E(T) from Equation 10] can be used directly in road deterioration predictions in the same way as the mean time from a deterministic model. In practice, the prediction of the expected time to failure took the following simple form:

$$E(T) = B(\beta)\exp\,[\gamma_0 + \gamma_1 x_1 + \gamma_2 x_2 \ldots] \tag{12}$$

The probability with which failure might occur at some time other than the expected time derives from the value of β, shown for example in the family of probability functions generated by different values of β in Figure 5. At low values of β, say below 2.5, the distributions tend to be widely dispersed and highly skewed with more than half of the observations having values less than the mean but with a number of very late failures. At high values of β, the distributions tend to be more concentrated and centered about the mean.

In general, the failure time (T_p) associated with any probability, $p = P[T \le T_p]$, can be related directly to the mean E(T) by

$$T_p = K(p) \cdot E(T) \tag{13}$$

where $K(p) = [-\beta\, ln(1-p)]^{1/\beta}/B(\beta)$ · K(p) is thus a constant function of β, and values for selected values of probability p are given in Table 1. For example, the median time [M(T)] and lower quartile time [$T_{0.25}$] to failure are given by

$$M(T) = K(0.5)\, E(T) \tag{14a}$$

$$T_{0.25} = K(0.25)\, E(T) \tag{14b}$$

TABLE 1 CONSTANTS DESCRIBING A WEIBULL DISTRIBUTION OF FAILURE TIMES: FUNCTION B AND FACTORS K(p) OF EXPECTED FAILURE TIME E(T) FOR SELECTED PROBABILITIES (p) AND VALUES OF β

BETA	B	SIQF	K05	K10	K25	K50	K75	K90	K95
1.0	1.000	0.549	0.051	0.105	0.288	0.693	1.386	2.303	2.996
1.5	1.183	0.447	0.153	0.247	0.483	0.868	1.377	1.932	2.302
2.0	1.253	0.362	0.256	0.366	0.605	0.939	1.329	1.712	1.953
2.5	1.280	0.300	0.344	0.458	0.685	0.973	1.284	1.573	1.748
3.0	1.288	0.255	0.416	0.529	0.739	0.991	1.249	1.479	1.614
3.5	1.287	0.221	0.476	0.584	0.779	1.001	1.220	1.410	1.521
4.0	1.282	0.195	0.525	0.629	0.808	1.007	1.197	1.359	1.451
4.5	1.275	0.174	0.566	0.665	0.831	1.010	1.178	1.319	1.398
5.0	1.267	0.157	0.601	0.694	0.849	1.012	1.163	1.287	1.356
6.0	1.251	0.131	0.657	0.741	0.876	1.014	1.138	1.239	1.294
7.0	1.235	0.113	0.699	0.775	0.895	1.014	1.120	1.204	1.250
8.0	1.221	0.099	0.733	0.801	0.909	1.014	1.106	1.179	1.218
9.0	1.209	0.088	0.759	0.822	0.919	1.014	1.095	1.159	1.193
10.0	1.198	0.079	0.781	0.839	0.928	1.013	1.086	1.143	1.173
15.0	1.157	0.053	0.850	0.891	0.953	1.011	1.058	1.095	1.114
20.0	1.131	0.039	0.885	0.918	0.965	1.009	1.044	1.071	1.085
25.0	1.113	0.032	0.908	0.934	0.972	1.007	1.035	1.057	1.068
30.0	1.100	0.026	0.923	0.945	0.977	1.006	1.030	1.047	1.056

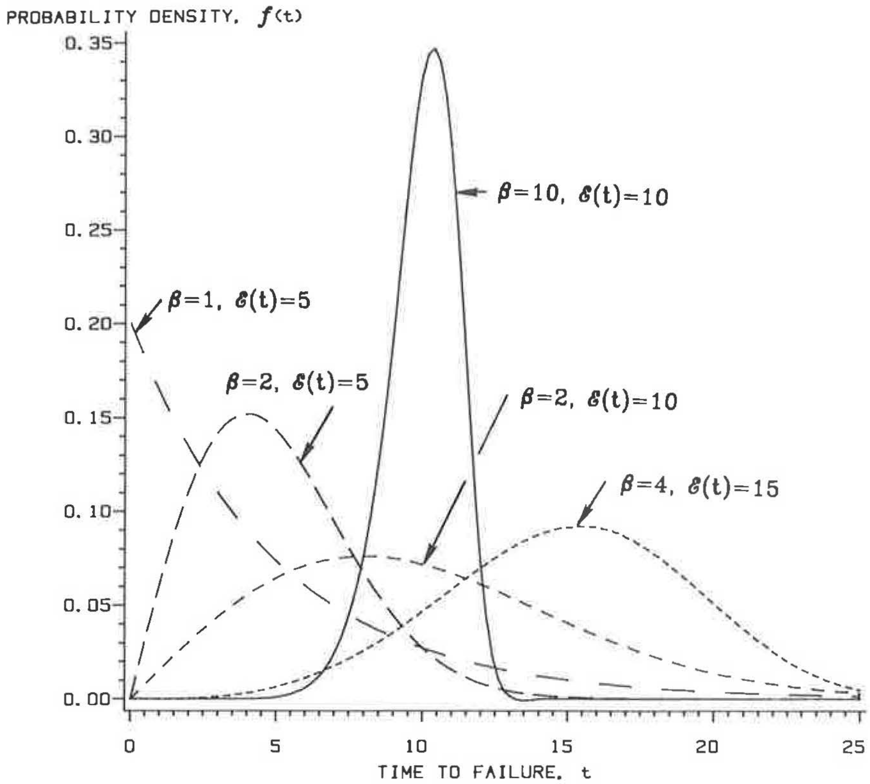


FIGURE 5 Family of probability density functions represented by Weibull distribution of time to failure.

For practical purposes, the width of dispersion is most simply described by the spread of time between two specific probabilities, usually the quartiles. For convenience the semi-interquartile factor (SIQF) is defined to represent the age range between 25 percent and 75 percent failure probabilities, that is the range within which about 50 percent of the pavements may be expected to fail [i.e., $(1 \pm \text{SIQF})\ E(T)$] where

$$\text{SIQF} = (1/2)\ [K(0.75) - K\ (0.25)] \tag{15}$$

Goodness of Fit

The goodness of fit of probabilistic models requires special interpretation because two components are present: the probability of failure occurring by the estimated time and the error of estimate.

First, note that whereas the traditional measures of goodness of fit indicate what proportion of the variation in the dependent variable can be accounted for by variation in the independent variables, in the failure-time models there is an upper bound on this proportion. In principle, the exact time to failure is not predictable because the failure time is a random variable with distributed, positive values; thus R^2 and similar measures can never approach unity. The model explains the variation of failure times that can be attributed to pavement and traffic characteristics, but further than that the influence of chance ensures that 100 identical pavements will crack at 100 different times. No model explains the latter variation, but the Weibull model predicts that it exists and estimates the shape of its distribution through the β parameter. This is typified by the SIQF defined in Equation 15.

Second, the value of log-likelihood (LL) that is maximized in the estimation is not a dimensionless proportion like R^2; its value varies with the magnitude of the dependent variable and with the number of observations, so that it is meaningless to compare LL-values across models or groups of models (except where identical sets of dependent variables are involved). When the dependent variable is of a fixed dimension, it is useful to define an average log-likelihood (LL divided by the number of observations) as a normalized measure of goodness of fit across models having different numbers of observations within the same data set.

Third, because the model used here is not a linear regression model there is no obvious equivalent of the conventionally reported "standard error." To assess the predictive power of the model, confidence intervals (with asymptotic validity) around predicted expected failure times were calculated as described earlier. Because the size of the interval depends in part on the error of estimate of the parameter coefficient, it also varies with the values of the pavement parameters. Thus the intervals do not form a locus of the expected time to failure but instead form loci of the explanatory parameters. As a practical measure, it is possible to compute an "average confidence interval," which is the arithmetic average of the estimated 95th percentile confidence intervals of all observations expressed in units of the dependent variable, for example, ± 1.2 years. Although this is not a precise statistic, it provides a meaningful estimate of the intervals to be expected given the observed ranges of explanatory variables.

Finally, assessing the adequacy of models for censored data remains an open research question, which has recently been addressed by Chesher et al. *(10)*, for example. Future applications of the procedure should consider these developments. In this instance the three practical measures noted previously were used:

- Goodness of fit: average log-likelihood value (for internal best fit for given data set and dependent variable);
- Predictive power: average 95th percentile confidence intervals (average for given data set); and
- Stochastic variation: SIQF; for example, a value of SIQF = 0.36 for $\beta = 2.0$ indicates a fairly wide dispersion, and a value of 0.09 for $\beta = 10$ indicates a very narrow dispersion (meaning that like roads will fail at very similar times).

PRACTICAL APPLICATION

Initiation of Cracking in Surface Treatments

To illustrate the application of the procedure, the empirical estimation of cracking initiation in double surface treatment (DST) (or chip seal) surfacings from data collected in the Brazil Road Costs Study *(3)* is used. The method was applied extensively in analysis of these data as described in full for this and other surface types in Paterson *(2)*; these models have been incorporated in the World Bank Highway Design and Maintenance Model *(1)* for economic evaluation of highway projects and policies.

Observations of initiation of cracking were taken from time-series pavement condition survey data over a 5-year period on 36 independent test sections with original DST surfacings selected within a factorial experimental matrix of traffic volume, surfacing age, base type, and vertical geometry. Initiation of cracking was defined by the occurrence of crocodile cracking of 1 to 3 mm width covering not more than 5 and not less than 0.5 percent of the surfacing area of a subsection. Of the total 102 subsection observations, there were 72 independent traffic-section combinations. The chip seal DST generally had a 14- to 19-mm stone size in the first seal and a 7- to 10-mm size in the second seal. Other characteristics are given in Table 2.

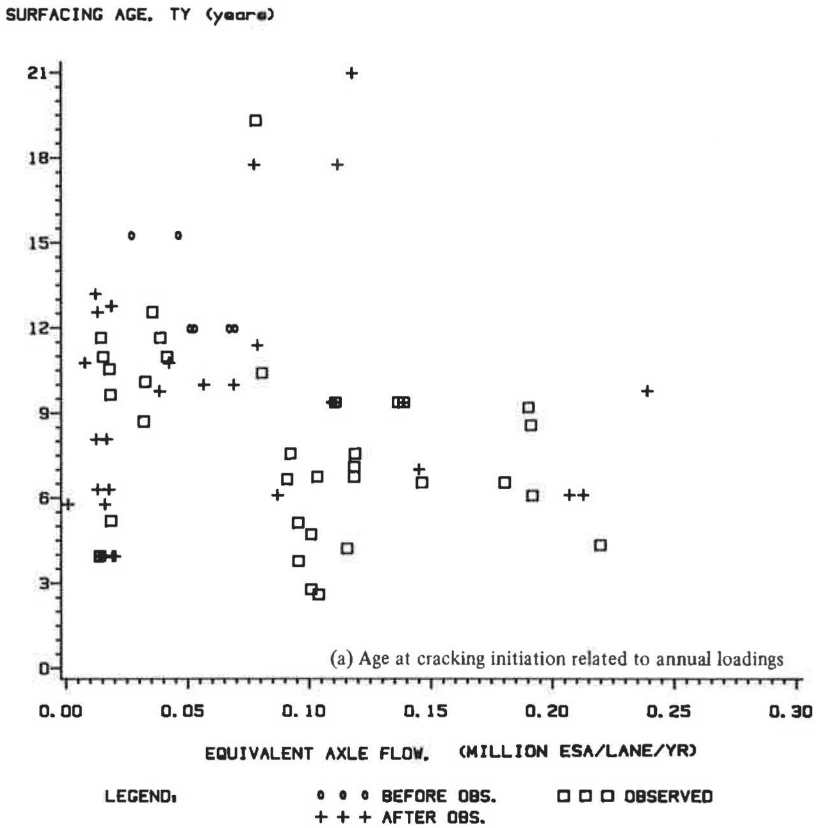
As might be expected for a representative sample of a road network in any given time "window," the actual event of cracking initiation was observed on only 35 of the 102 subsections; 10 had cracked earlier, and the remainder were still uncracked at the date of the last survey. Conventional least squares estimation techniques would therefore have had to exclude two-thirds of the data and operate on only the observed events, whereas all observations could be used in the failure-time estimation procedure.

The surfacing lives observed in the study are shown in Figure 6; symbols are used to indicate whether the initiation occurred before, during, or after the study. The symbol "0" indicates that initiation occurred earlier than, that is below, the point indicated; the symbol "□" indicates an observed initiation; and the symbol "+" indicates that the initiation occurred later than, that is above, the point indicated. A general tendency for the surfacing life to decrease as the annual volume of traffic loading increases can be seen in Figure 6*a*, although there are a number of exceptions and in particular one group of observations has lives longer than 17 years. When the life is expressed in classical fatigue terms of the cumulative number of 80-kN equivalent single axle loads (ESALs) to cracking initiation, as shown in Figure 6*b*, it can be seen that shorter lives are usually associated with higher pavement deflections, but again there are several exceptions and considerable scatter.

Model estimations were made on the data using the failure-time procedure. When time to failure was used as the dependent variable, traffic effects were represented by the rate of trafficking (that is the annual volume per lane) normalized to the average rate over the first 8 years of service in order to avoid the collinearity that occurs between age and traffic volume due to annual traffic growth (old pavements usually carry a higher volume now than when they were new). When cumulative ESALs to failure was the dependent variable, the rate of aging was represented by the inverse of traffic flow (e.g., years per million axles). Benkelman beam deflection, modified structural number, base type and California bearing

TABLE 2 DATA CHARACTERISTICS OF CRACKING INITIATION IN SURFACE TREATMENT SURFACINGS IN BRAZIL ROAD COSTS STUDY

Parameter	Units	Range of Values
Surfacing age	Year	2.7-21
Cumulative loading	Million ESALs/lane	0.005-5.16
Traffic volume (two way)	Vehicles/day	100-2,300
Traffic loading	Million ESALs/lane/year	0.001-0.24
Deflection (Benkelman 80 kN)	mm	0.26-2.02
Modified structural no.		2.93-5.15
Base course CBR	%	32-143
Resilient modulus of surfacing at 30°C (4/36)	GPa	1.6-2.7
No. of sections		36
No. of subsections		102
No. with prior cracking		10
No. observed to crack		35
No. with future cracking		57



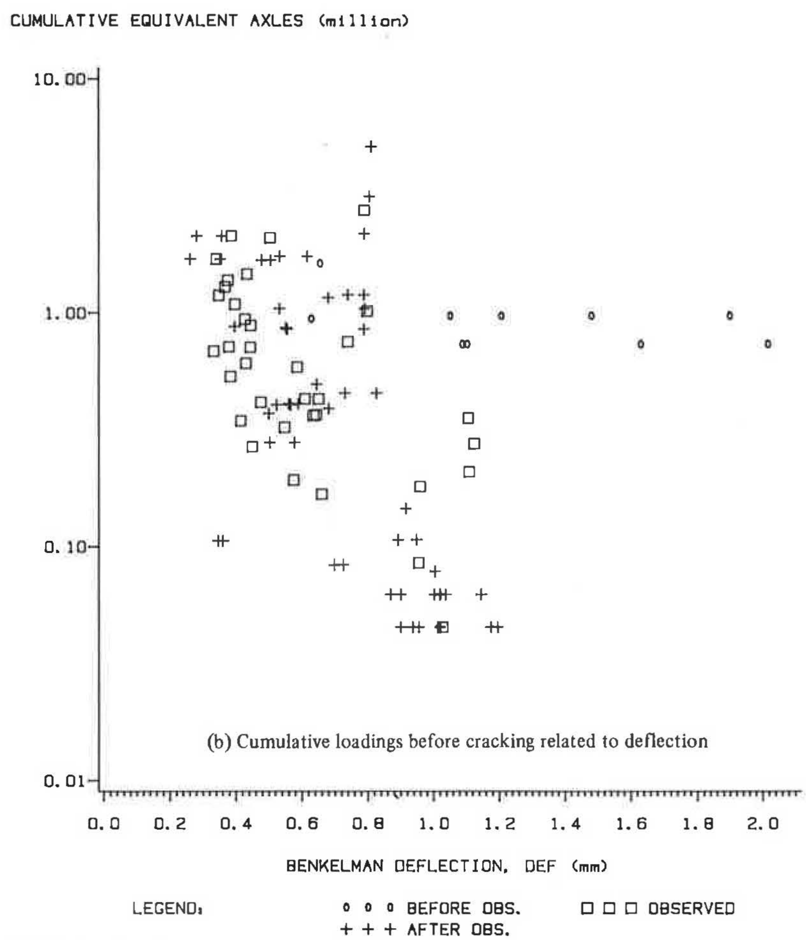


FIGURE 6 Observed lives of double surface treatment surfacings in Brazil as functions of traffic and pavement deflection.

ratio (CBR), construction quality, monthly rainfall, traffic volume (with and without load effects given by ESALs), and surfacing thickness were used as explanatory variables.

The selection of the final models was based on the best statistical fit, as defined by the maximum log-likelihood value and minimum interquartile probability range, and on whether the predictions were reasonable over the range of estimation and beyond. The two with widest application are as follows:

1. $E(T) = 13.2 \exp [-24.3 (1 + CQ) YE2/SNC^2]$ (16)

   SIQF = 0.295
   Average 95 percent confidence interval = 1.73 years
   Average maximum likelihood = –1.27
   $\beta = 2.54$

2. $E(T) = 13.6 \exp [-3.19 (1 + CQ) YE2\ DEF]$ (17)

   SIQF = 0.305
   Average 95 percent confidence interval = 1.73 years
   Average maximum likelihood = –1.27
   $\beta = 2.47$

where

E(T) = expected age of surfacing at initiation of narrow crocodile cracking (1 to 3 mm in width), in years;
YE2 = annual volume of equivalent 80-kN single axles computed with a relative damage power of 2, in million ESALs per lane per year;
SNC = modified structural number of pavement strength including subgrade contribution [see Paterson *(2)*];
DEF = pavement surface deflection by Benkelman beam under 80-kN axle load, in mm; and
CQ = construction quality factor (= 0 for good quality, = 1 when original construction was faulty).

The goodness of fit of the prediction for Model 1, shown in Figure 7, is reasonable but not excellent, reflecting both the large amount of stochastic variation that is observed under real conditions and the difficulties in explaining all of the diverse contributory factors that cause cracking in thin surfacings by practicable measures. The surfacing lives predicted by Equation 16 for a range of pavement strengths and traffic loadings are shown in Figure 8. This shows that the life decreases as the traffic-loading volume increases but at a rate that decreases as the pavement strength (or stiffness) increases. The focal point at an age of 13.2 years represents the effect of aging and oxidation under very low-volume traffic. This point is likely to vary from region to region with the various factors that influence the oxidation rate of the binder, as mentioned earlier.

The variability of cracking failure times due to chance, that is, most of the variance that could not be explained by physical parameters, amounted to a distribution in which half of the

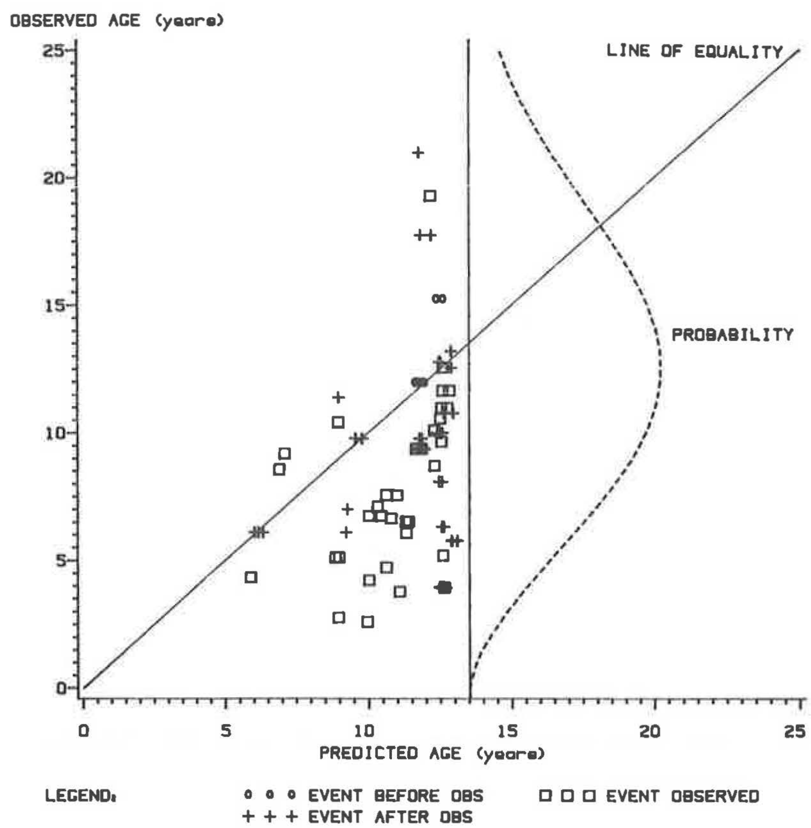


FIGURE 7 Goodness of fit of prediction model for initiation of all cracking for Brazilian data: double surface treatment surfacings on granular base pavements.

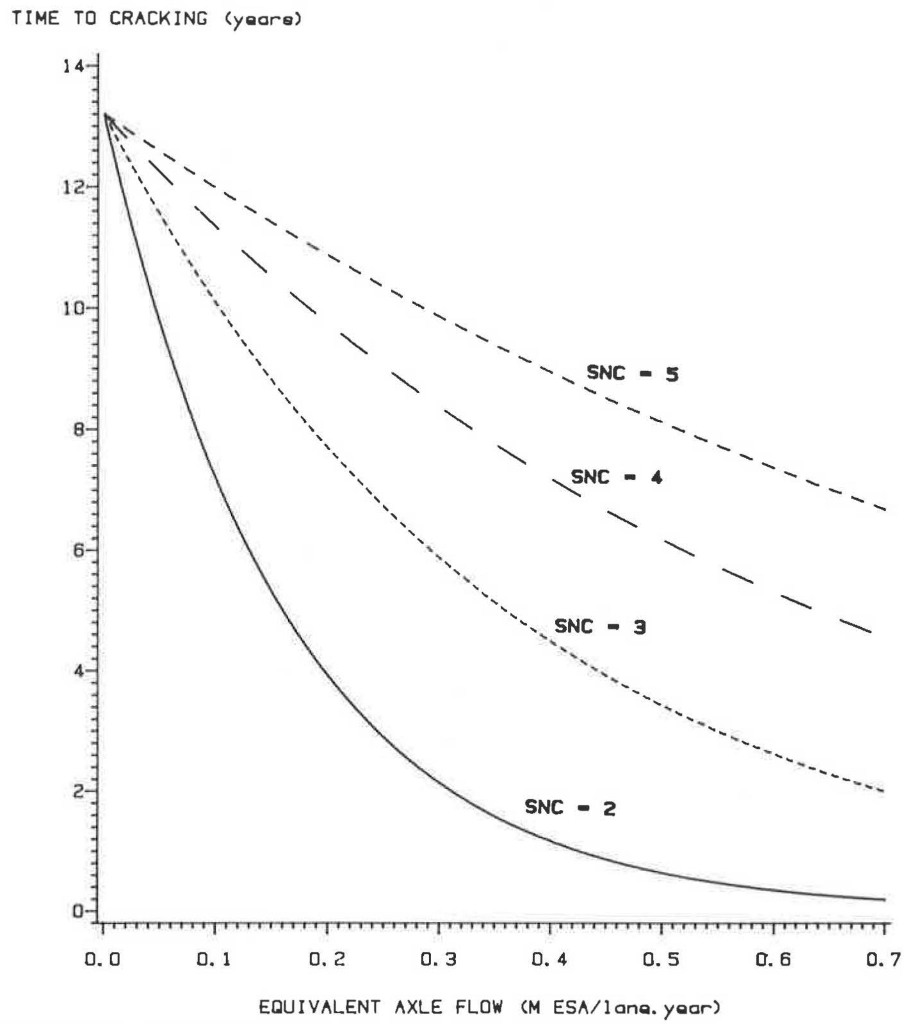


FIGURE 8 Predictions of crack initiation for pavements with original double surface treatment surfacings and granular base in nonfreezing climates.

pavements would fail within ±30 percent of the mean or expected time of failure and the remainder would crack either sooner or later than that range of time. This underlying variability found in the data is shown by the bell curve in Figure 7. This curve is large and graphically illustrates why the empirical analysis of performance data is so difficult.

Calibration or Adaptation of a Model

In a region for which a large, well-structured data set is not available to permit an original estimation of cracking models, it is desirable to calibrate a model such as the present one to satisfy the specific material and climatic conditions of that particular region. In the case presented here, calibration is most effectively achieved by assuming, as a first approximation, that the coefficients of the traffic and pavement parameters are correctly estimated because the function represents an underlying physical mechanism and that it is the aging and oxidation function that most needs to be adapted to local conditions.

On this assumption, calibration can be achieved by assessing the average life of a sample of low-volume pavements in the region and computing an adjustment factor equal to the ratio of the observed low-volume road life ($\bar{T}_L$) to 13.2 years:

$$E(T)' = K_c \, E(T) \qquad (18)$$

where

$E(T)'$ = calibrated estimate of failure time for local conditions,
K_c = local adjustment factor = $\bar{T}_L/13.2$, and
$\bar{T}_L$ = average observed age at failure of local surfacings under low traffic volumes.

CONCLUSION

The development of this procedure, based on the principles of failure-time theory, has provided the statistical means for analyzing surface distress data from pavement performance studies or pavement condition data bases with greatly increased scope. Unobserved cracking events (i.e., censored data) that occur outside the period of data collection are taken into account by the procedure and thus a statistical bias in the results, which might occur were they ignored, is prevented. The underlying variability of failure times due to chance rather than physical properties is quantified by the procedure and the example given shows that the variability can be quite large for cracking failure times. The Weibull distribution chosen for this purpose is highly adapatable to real data and appears to be particularly appropriate to the increasing chance of cracking that occurs as a result of the concurrent effects of fatigue and oxidation.

Although the exposition and example refer to oxidation and aging as the primary time-related mechanism, which was appropriate for the tropical climate of the example application, the model principles apply equally to other time-related environmental mechanisms such as cold-temperature cracking, shrinkage cracking, and so forth.

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