Climatic-Materials-Structural Pavement Analysis Program

BARRY J. DEMPSEY, W. ANDREW HERLACHE, AND ARTI J. PATEL

The Climatic-Materials-Structural (CMS) program has been set up to introduce climatic effects into the analysis of multi-layered flexible pavement systems. The program may be used with selected pavement structural and performance models to analyze a pavement system. It can also be employed as a tool to analyze existing pavement systems in order to obtain estimates of future maintenance requirements. The CMS program is compatible with several pavement structural models for determining radial stresses, strains, and displacements. These structural models include the ILLI–PAVE model, ILLI–PAVE algorithms, and elastic layer analysis. The accuracy of the CMS output depends mainly on the quality of the inputs. It is important that boundary conditions, climatic conditions, and material properties properly represent the system to be analyzed. With representative inputs the CMS program will give realistic values for temperature and moisture profiles and material strength properties. Although future research is required to validate the overall model, the validity of the individual parts of the CMS program has been shown. Analysis of existing pavement systems and comparison of CMS outputs with actual field conditions are recommended. It is expected that further validation studies will confirm the belief that the CMS program provides an economical and realistic means of analyzing multilayered flexible pavement systems by accounting for climatic effects on pavement materials.

The detrimental effects of temperature and moisture are major problems and continue to be a significant cause of pavement deterioration leading to high maintenance costs. In many cases the cost of annual repairs and maintenance is greater than the cost of preventive measures that might have been incorporated into the original pavement design and construction.

In the past completely acceptable techniques, procedures, and criteria were not available for adequately assessing the effects of temperature and moisture changes on pavement systems. Most laboratory durability-testing procedures use arbitrary exposure conditions that are not representative of actual conditions in the field.

Further refinement of analysis procedures must take into account the changing environment in which the pavement system is located. Climatic characteristics (maximum and minimum air temperature, sunshine, wind velocity, precipitation, etc.) are known to vary with geographic location in many states. For example, the average winter temperature for northern Illinois is approximately 25°F, whereas that for southern Illinois is approximately 35°F.

The objective of this study was to develop and implement a computer-based model that would account for the influence of climate on the behavior of flexible pavement systems.

B. J. Dempsey, Department of Civil Engineering, University of Illinois, Urbana, Ill. 61801. W. A. Herlache, Harding Lawson Associates, 20 Hawthorne St., San Francisco, Calif. 94105. A. J. Patel, Illinois Department of Transportation, 2300 S. Dirksen Parkway, Springfield, Ill. 62764.

The model will take climatic parameters and material properties as inputs and from these calculate temperature profiles, moisture profiles, and structural values of the pavement system as they vary with time. The outputs will be organized so that they may be easily input into structural or performance models to fully analyze the performance of a pavement in its proposed environment.

THE PROGRAM

General

The Climatic-Materials-Structural (CMS) pavement analysis program consists of several submodels that are combined to analyze the behavior of multilayered flexible pavement systems. As shown in Figure 1, the climatic model (heat-transfer and moisture models) incorporated into the CMS program takes climatic and material data as inputs and calculates temperature and moisture profiles as they vary with time in a pavement system. This information is used in the material model to calculate the asphalt concrete, base course, subbase, and subgrade stiffness characteristics. This output can then be combined with load data and input into selected structural analysis models to generate data for analyzing flexible pavement behavior.

Heat-Transfer Model

The heat-transfer model used in the CMS program was developed from a similar model previously described by Dempsey *(1)*. The heat-transfer model uses a finite-difference solution to the one-dimensional, Fourier heat-transfer equation for transient heat flow to compute pavement temperatures as a function of time. Energy balance procedures developed by Scott *(2–3)* and Berg *(5)* are used to relate pavement surface temperatures to climatic parameters.

Finite-Difference Pavement System

Figure 2 shows a typical finite-difference pavement system used in the heat-transfer model for computing pavement temperatures. The pavement system consists of a column of nodes that have a cross-sectional area of 1 ft^2.

Nodes 2 through 37 are termed normal nodes. The nodal depth (ΔX) and the number of nodes are chosen so as to ensure mathematical stability and so that the interface between pavement layers will be located at a nodal center. Nodes 2 and 6 are also mixed nodes because the thermal properties of these nodes correspond in part to the thermal properties of the adjacent pavement layers.

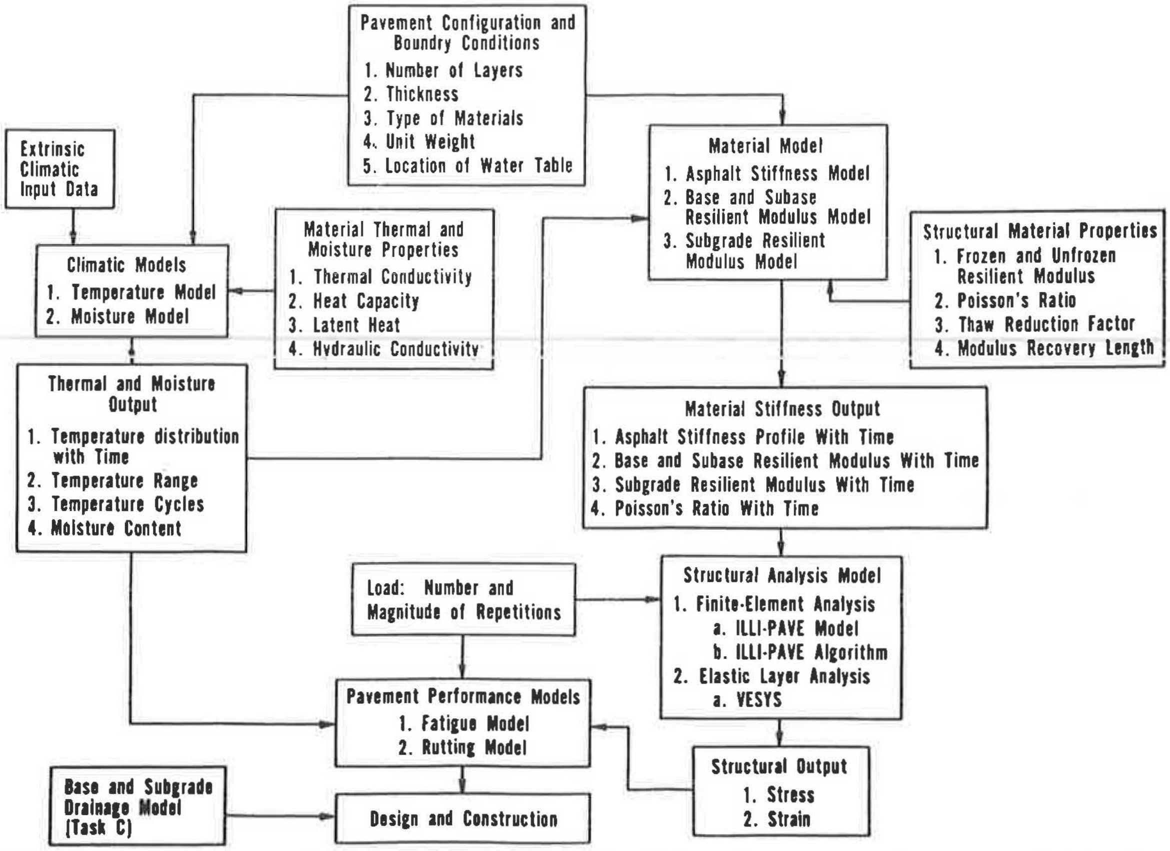


FIGURE 1 CMS program incorporated with structural analysis and pavement performance models to aid in design.

Node 1 consists of one-half of a normal node so that the nodal center will lie on the pavement surface. Node 1 at the pavement surface is the node at which the meteorological parameters are introduced and an energy balance is achieved.

Nodes 38, 39, and 40 are termination nodes and their purpose is to reduce computational time.

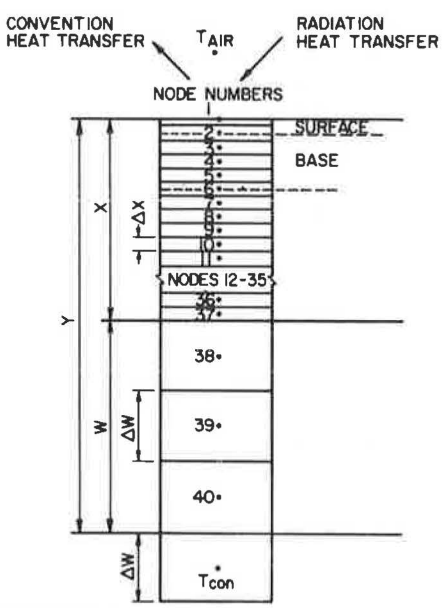


FIGURE 2 Finite-difference pavement system.

The total depth (Y) of the finite-difference pavement system is a variable input parameter in the heat-transfer model. It can be determined from a study of deep soil temperatures at a specified geographic location. For example, studies of soil temperatures in northern Illinois have indicated that the ground temperature remains essentially constant (51°F) at a depth of 144 in.

Finite-Difference Equations

Convection and radiation play a dominant role in transferring heat between the pavement surface and the air, whereas conduction plays a separate role in transferring heat within the pavement system. The general form of the one-dimensional, Fourier equation for conductive heat transfer in the heat-transfer model is expressed as follows (see Appendix for definition of terms):

$$(\partial^2 T/\partial X^2) = (1/\alpha)\,(\partial T/\partial \theta) \tag{1}$$

Dempsey *(1)* has shown that the first and second derivatives in Equation 1 can be replaced by the appropriate finite-difference terms and written as

$$[(T_{n-1} + T_{n+1} - 2T_n)/\Delta X^2] = (1/\alpha)\,[(T'_n - T_n)/\Delta\theta] \tag{2}$$

The thermal diffusivity (α) is equal to $K/C\gamma_d$. By arranging

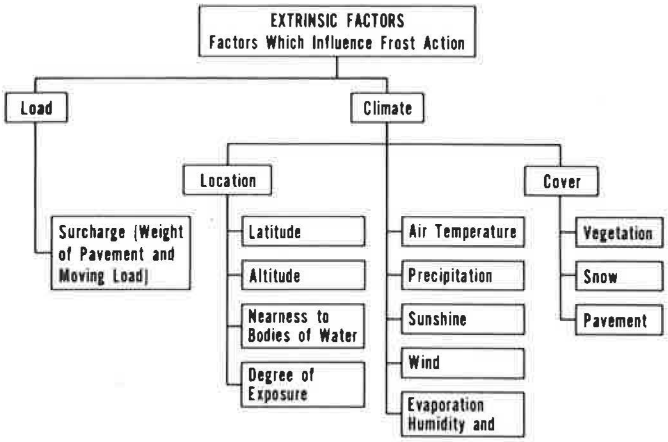


FIGURE 3 Extrinsic factors that influence temperature and frost action.

terms and substituting for α, Equation 2 can be written for the heat balance on an arbitrary interior node as

$$(K/\Delta X)\,(T_{n-1} - T_n) + (K/\Delta X)\,(T_{n+1} - T_n) = (\gamma_d C \Delta X/\Delta\theta)\,(T'_n - T_n) \qquad (3)$$

The terms $(K/\Delta X)\,(T_{n-1} - T_n)$ and $(K/\Delta X)\,(T_{n+1} - T_n)$ are the equations for the thermal conductivity of a nodal volume and the term $(\gamma_d C \Delta X/\Delta\theta)\,(T'_n - T_n)$ is the heat storage in a nodal volume during an incremental time period $(\Delta\theta)$.

A more detailed description of the finite-difference equations used in the heat-transfer model can be found elsewhere *(1, 6)*.

Climatic Parameters

Numerous extrinsic climatic parameters shown in Figure 3 are considered in the heat-transfer model. The most important parameters are those related to the surface node. These are the climatic parameters concerned with the net radiation heat transfer (Q_{rad}) and the convective heat transfer (Q_c) into or out of the pavement system as shown in Figure 4.

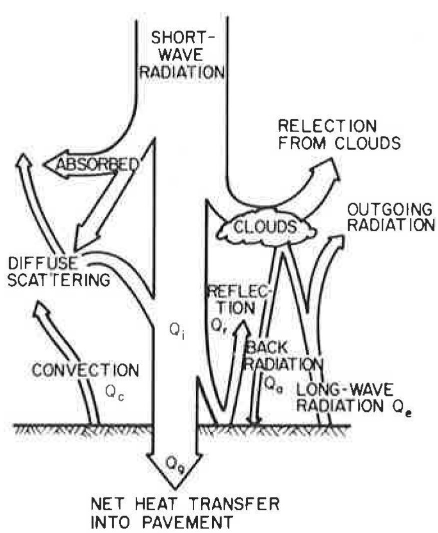


FIGURE 4 Heat transfer between pavement surface and air on a sunny day.

The finite-difference equation for the surface node is ideally suited for use with the meteorological energy balance that can be expressed as follows:

$$Q_i - Q_r + Q_a - Q_e \pm Q_c \pm Q_h \pm Q_g = 0 \qquad (4)$$

The importance of solar radiation in pavement temperature studies has been shown by Straub et al. *(7)* and Aldrich *(8)*. From Equation 4, the net amount of radiation (Q_{rad}) influencing heat transfer at the surface node is expressed as

$$Q_{rad} = Q_i - Q_r + Q_a - Q_e \qquad (5)$$

The amount of incident short-wave radiation used in the energy balance at the surface node is determined by use of a regression equation developed by Baker and Haines *(9)* and expressed as

$$Q_i = R^* \,[A + B\,(S/100)] \qquad (6)$$

The extraterrestrial radiation (R*) can be theoretically calculated for a given location from solar declination, latitude, zenith angle, and solar constant.

In Figure 5, it is observed that the intensity of solar radiation varies parabolically from the time of sunrise to the time of sunset. Based on this observation, the amount of short-wave radiation received at the pavement surface during a finite time increment $(\Delta\theta)$ is calculated by assuming that the extraterrestrial radiation varies in a parabolic manner from the time of sunrise to the time of sunset.

Part of the incident short-wave radiation (Q_i) is lost as reflected short-wave radiation (Q_r). The amount of short-wave radiation reflected is a function of the incident short-wave radiation (Q_i) and the absorptivity (a) of the pavement surface:

$$Q_r = (1 - a)Q_i \qquad (7)$$

From Equations 6 and 7 the net amount of short-wave radiation that enters the energy balance at the pavement surface (Q_s) is derived as follows:

$$Q_s = Q_i - Q_r \qquad (8)$$

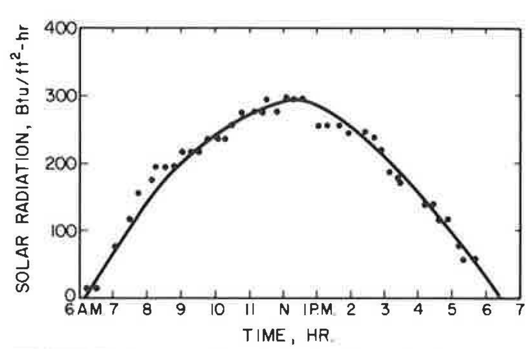


FIGURE 5 Variation in intensity of solar radiation.

By substituting for Q_r in Equation 8, the following equations are obtained:

$$Q_s = a\, Q_i \tag{9}$$

$$Q_s = a\, R^* \,[A + B\,(S/100)] \tag{10}$$

Essentially, Equation 10 considers the influence of cloud cover, reflection from clouds, diffuse scattering, absorption by the atmosphere, and reflection by the pavement surface of the extraterrestrial radiation. Discussion of the range of values for the terms in Equation 10 can be found elsewhere *(1)*.

Using the suggestion of Scott *(4)* that the net long-wave radiation entering the energy balance at the pavement surface be corrected for cloud cover in a manner similar to that used for short-wave radiation, an approach recommended by Geiger *(10)* was used:

$$Q_e = Q_x\,[1 - N\,(\overline{W}/100)] \tag{11}$$

$$Q_a = Q_z\,[1 - N\,(\overline{W}/100)] \tag{12}$$

In Equation 11, Q_x is the long-wave radiation emitted from a unit area of pavement surface with no correction for cloud cover, and, in Equation 12, Q_z is the long-wave back radiation from the atmosphere without a cloud cover correction.

In Equations 11 and 12, N is a cloud-base factor the value of which ranges between approximately 0.90 and 0.80 for cloud heights between approximately 1,000 and 6,000 ft, respectively *(10)*. The percentage of cloud cover $(\overline{W})$ is equal to 0 percent for cloudless days and 100 percent for completely overcast days.

The rate of heat transfer by convection (Q_c) between the pavement surface and the air is computed by the following method for a unit surface area:

$$Q_c = H\,(T_{air} - T_1) \tag{13}$$

The convection coefficient (H) is difficult to estimate because of the many variables involved. Previous studies by Dempsey *(1)* have shown that formulas for estimating the convection coefficient for large flat surfaces can be used for pavement systems.

In the development of the heat-transfer model, the effects of transpiration, condensation, evaporation, and sublimation were neglected because of the uncertainty in predicting their values at this time. Large error was not expected to be created in the energy balance at the pavement surface by assuming Q_h to be zero. Transpiration can be neglected in pavement studies because this is related to vegetation growth. The heat flux resulting from condensation is lost when the condensate evaporates. Heat transfer by evaporation should be minimal if rainwater quickly drains off the pavement surface. Because snow removal from most pavements takes place shortly after the snow has fallen, heat flux caused by sublimation can also be disregarded.

The climatic input for the radiation heat-transfer equations and convective heat-transfer equations can be obtained from weather station records.

Thermal Properties of Pavement Materials

Figure 6 shows the intrinsic factors that influence temperatures and frost problems in pavement systems. The heat-transfer model in the CMS program considers a majority of the factors

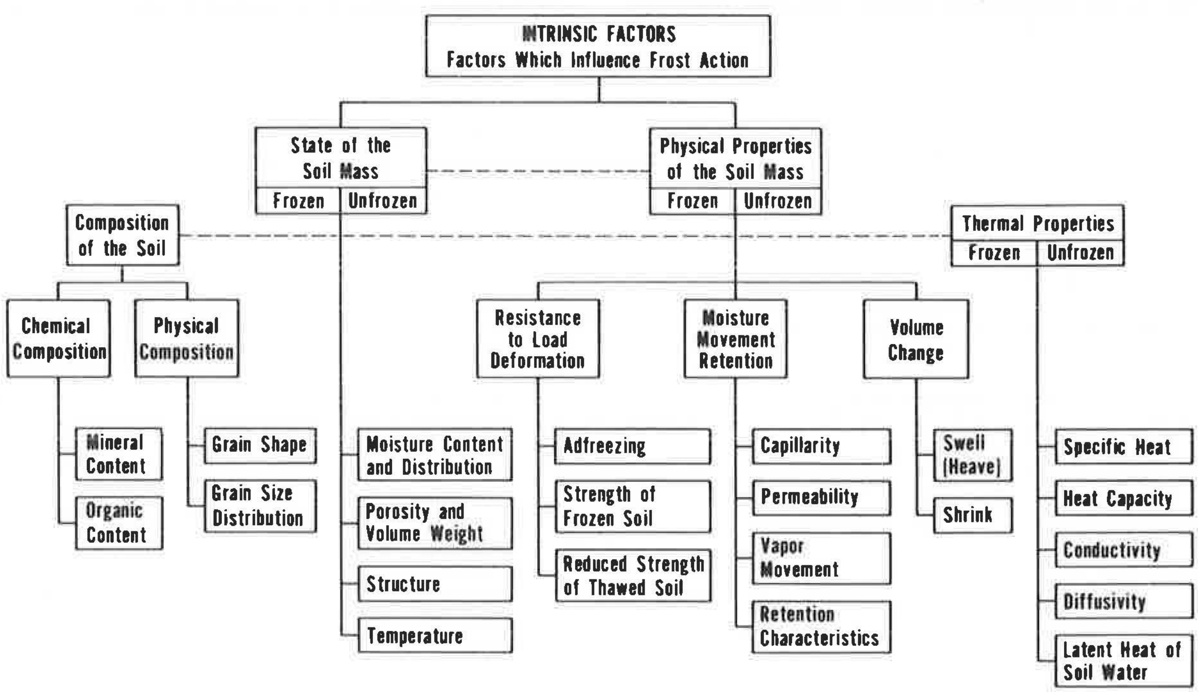


FIGURE 6 Intrinsic factors that influence temperature and frost action.

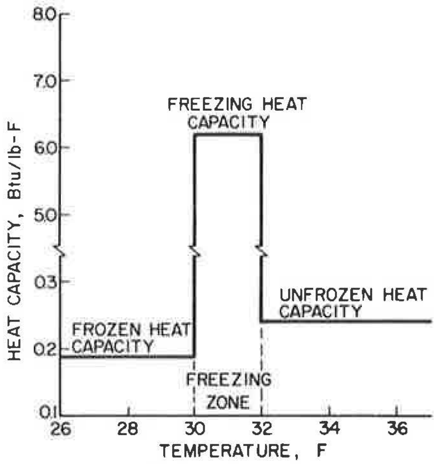


FIGURE 7 Effect of temperature on heat capacity of a granular base material.

The heat capacity of a pavement material during freezing is determined from the latent heat of fusion of the moisture in the material. When the moisture in the pavement freezes, the portion that is about to change phase remains at a constant temperature, the freezing temperature, until the latent heat of fusion is released. The time lag caused by this process retards the rate of frost penetration. The latent heat effect is incorporated into the finite-difference equations by using an approach described by Schenck *(12)*, which makes use of a freezing zone. The freezing zone is a small, hypothetical temperature range over which freezing takes place. Because only moisture effects are considered in this range, the freezing heat capacity in the freezing zone is a function of the moisture content, dry density, and the small freezing temperature range.

A comparison of the freezing heat capacity and unfrozen and frozen heat capacities for a granular base material with about 9 percent moisture is shown in Figure 7. It should be noted that even for a small moisture content the freezing heat capacity of the moisture is far greater than the unfrozen and frozen heat capacities of the material itself.

listed. The most important intrinsic factors considered in the heat-transfer model are the thermal properties of the pavement materials, which include thermal conductivity, heat capacity, and latent heat of fusion. The heat-transfer model recognizes three different sets of thermal properties depending on whether the pavement material is in an unfrozen, freezing, or frozen condition.

The procedures for determining the thermal properties of the pavement materials have been described in detail elsewhere *(1, 6)*. The thermal properties of surface materials are often determined from general tables of physical properties or from scientific research. The methods developed by Kersten *(11)* were found to be well suited for determining the thermal properties of the base, subbase, and subgrade soils.

Validation of the Heat-Transfer Model

The validity of the heat-transfer model was established by using temperature data from the AASHO Road Test at Ottawa, Illinois.

For the purpose of evaluating the heat-transfer model, a winter period at the AASHO Road Test from October 1, 1959, through March 31, 1960, was analyzed. Because pavement temperatures near the surface vary within a given day as well as from day to day, comparisons of theoretical temperatures and measured temperatures were made at 6:00 a.m. and 3:00 p.m.

Figure 8 shows a graphic comparison of measured temperature and theoretical temperature at the middepth of a 6-in.

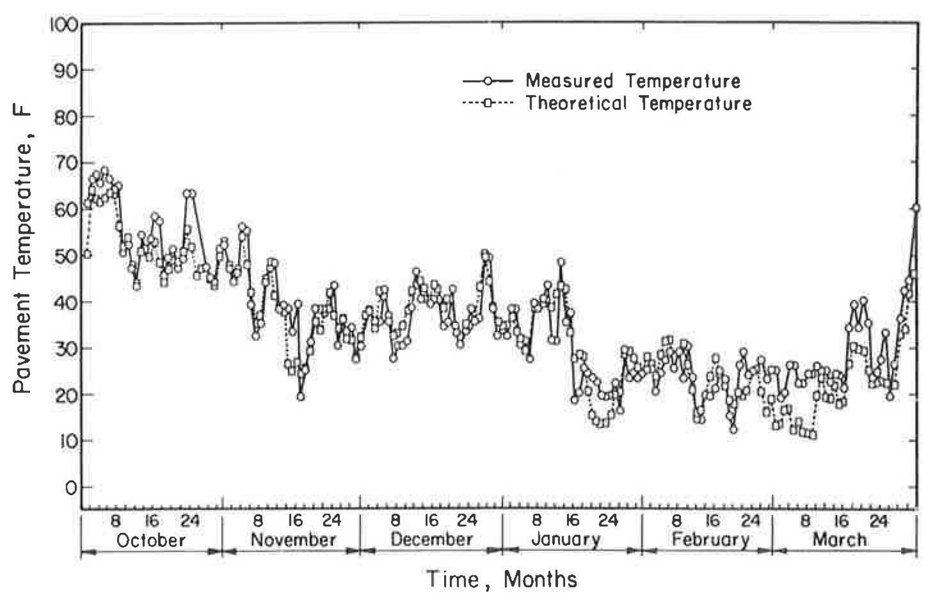


FIGURE 8 Comparison of measured and theoretical temperatures at the 3-in. depth of a 6-in. asphalt-concrete pavement at 0600 hr (AASHO Road Test 1959–1960).

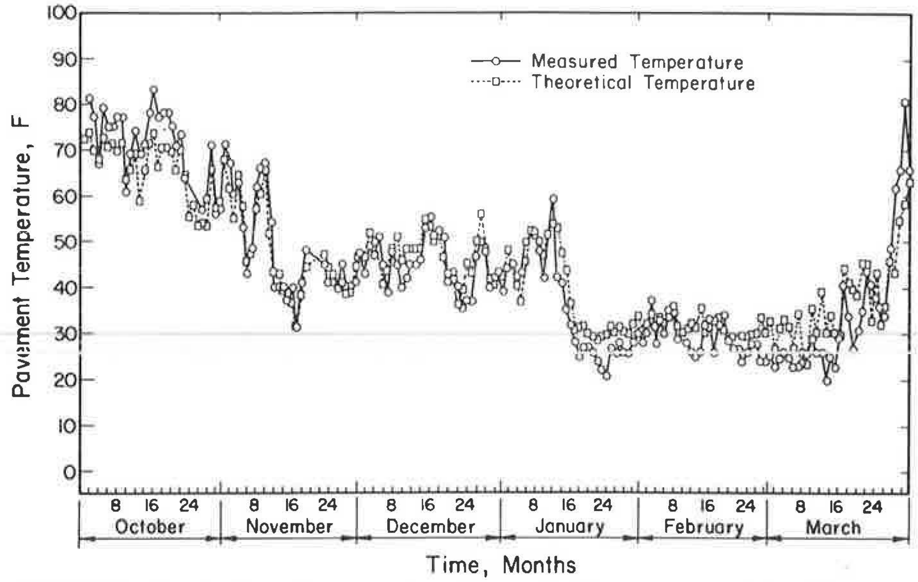


FIGURE 9 Comparison of measured and theoretical temperatures at the 3-in. depth of a 6-in. asphalt-concrete pavement at 1500 hr (AASHO Road Test 1959–1960).

asphalt-concrete pavement surface at 6:00 a.m. The average difference between the measured temperature and the theoretical temperature is 0.97°F.

A typical graphic comparison of measured temperature and theoretical temperature at 3:00 p.m. is shown in Figure 9. The average difference between the measured temperature and the theoretical temperature is 0.73°F at the 3-in. depth.

The validity of the heat-transfer model was further checked by comparing the number of freeze-thaw cycles predicted by the model with the number of freeze-thaw cycles in the actual pavement at the AASHO Road Test. Freezing in the pavement was considered to occur whenever the pavement temperature reached 30°F or less and remained at that temperature for more than 2 hr. Similarly, thawing was considered to occur whenever the pavement temperature exceeded 30°F and remained above that temperature for more than 2 hr.

At a depth of 3 in. in the pavement, 41 freeze-thaw cycles were determined from the theoretical temperatures compared with 38 freeze-thaw cycles determined by analyzing the measured hourly temperatures. At the 6-in. depth, 15 theoretical freeze-thaw cycles were observed compared with 17 freeze-thaw cycles determined from the measured temperatures for the test pavement at the AASHO Road Test.

The excellent comparisons between the theoretical temperatures and the measured temperatures and the good agreement between the number of freeze-thaw cycles at various depths indicated that the heat-transfer model was valid for predicting temperatures for use in frost action and temperature distribution studies of multilayered pavement systems.

Moisture Model

Moisture along with temperature is an important factor that influences the durability and strength parameters of highway soils and materials. Subgrades are generally constructed in the surface soil and they are usually subjected to large moisture content variations. Therefore the prediction of moisture movement and moisture equilibria are of prime importance in the CMS program.

The moisture model in the CMS program was based on procedures developed by the Road Research Laboratory *(13–15)* and a moisture model developed by Dempsey and Elzeftawy *(16)*. The moisture model is essentially an equilibrium model that is based on the following principles *(17)*:

1. The trend in pore water pressure, under certain conditions at a given level of the subgrade, is toward an equilibrium value that depends solely on the height above the groundwater level;
2. A relation exists between the pore water pressure in the soil at a given level and the suction of the soil; and
3. A relation exists between the suction and the water content of the soil.

The conditions for equilibrium to be reached depend on the following assumptions:

1. The temperature of the subgrade is constant, uniform, and above freezing;
2. The subgrade cannot receive moisture by infiltration through the highway pavement or by migration from adjacent soil masses with a higher pore water pressure, nor can it give up moisture by evaporation or migration to adjacent soil masses that have a lower pore water pressure.

In the moisture model the pressure of pore water at any given level must tend toward an equilibrium that cancels out the algebraic sum of the various water potentials. The British Road Research Laboratory has expressed the equilibrium condition as *(17)*:

$$u = -Z \tag{14}$$

In Equation 14, u is the relative pore water pressure, which is negative above the water table, and Z is the height above the groundwater table. Graphically, when the same scale is used for pressures (u) and heights (Z), the function $u = f(Z)$ is a straight line with slope of 45 degrees regardless of the nature and dry density of the various soil layers that make up the mass in question. In principle, estimating the equilibrium pressure profile of a pavement subgrade is dependent on estimating the position of the groundwater level after the pavement is built.

A major input to the moisture model is the soil-water characteristics curve that relates water content of the soil to the water potential or pore pressure and, by Equation 14, to the height above the water table. Croney and Coleman *(18)* have discussed various methods for determining the moisture characteristics curve for soil.

Janssen and Dempsey *(19)* tested a significant number of Illinois soils and determined that an approximate soil-moisture characteristics curve could be estimated from basic soil data. Their studies indicated some similarities for a number of soil-moisture characteristics curves. From a value of zero at saturated water content, the curves first rise almost vertically and show very little moisture change with suction increase (Figure 10). The curves then show a substantial decrease in water content associated with an increase in soil suction of about 100 cm.

The midpoint of the soil-moisture characteristics curve for fine-grained soils can be approximated by determining the water content at 1000 cm of suction (W1000) and the break suction point (LSUC). The following regression equations developed by Janssen and Dempsey *(19)* for AASHTO soil groups A4 through A7 are used in the moisture model.

For A4 and A5 soils:

$$W1000 = 0.496LL + 0.297PL - 0.128SATWAT - 0.579 \quad (15)$$

$$LSUC = 0.568LL + 0.047PL - 0.009LL^2 - 0.082SATWAT - 5.811 \quad (16)$$

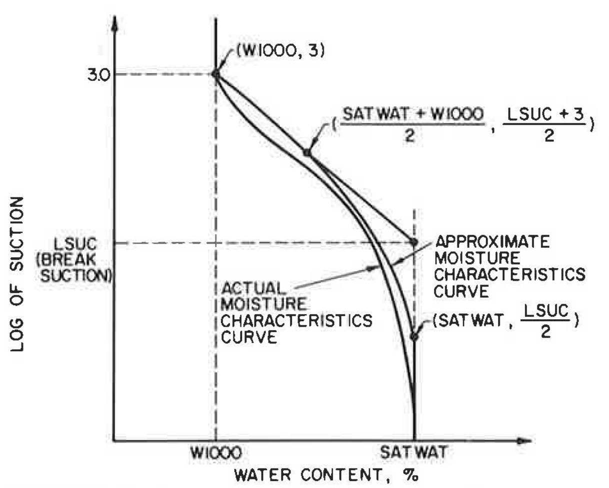


FIGURE 10 Approximation of the soil-moisture characteristics curve used in the CMS program for A4, A5, A6, and A7 soils.

For A6 soils:

$$W1000 = -0.027LL + 0.89PL + 0.0043SATWAT + 0.312 \quad (17)$$

$$LSUC = -0.273PI - 0.008PL - 0.347SATWAT + 12.873 \quad (18)$$

For A7 soils:

$$W1000 = -0.479PI - 0.215PL + 0.0077LL^2 + 0.291SATWAT + 14.46 \quad (19)$$

$$LSUC = -0.432LL - 0.022PI + 0.005LL^2 - 0.106SATWAT + 13.787 \quad (20)$$

From the midpoint of the break suction (SATWAT, LSUC/2) to a point midway between (W1000, Log of suction = 3) [note that Log of suction = 3 is equivalent to a suction of 1000 cm] and (SATWAT, LSUC), the soil-moisture characteristics curve can be modeled as a parabola (Figure 10), and the top portion of the curve can be approximated with a sloping straight line through (W1000, Log of suction = 3).

The midpoint of the soil-moisture characteristics curve for granular soils can be approximated with a series of sloping straight lines. In the program the granular material has been modeled as seen in Figure 11.

For A2 soils no data were available; therefore the water content for an A2 soil is held constant at a value input by the user.

The moisture model used by the CMS program provides a rational approach to moisture prediction in a pavement system. All that is required to predict moisture contents are standard soil properties that can be obtained from simple soil mechanics tests and knowledge of the water table position. This model serves well at the present time, but future research in developing a moisture movement model based on temperature gradients would be useful if simplicity is still maintained.

Material Model

The strength or stiffness of a pavement system is strongly dependent on the climatic conditions to which it is exposed. It has been observed that the stiffness of an asphalt layer varies with temperature, and the resilient modulus of a nonasphalt layer is dependent on its water content and condition (whether frozen, unfrozen, or thaw-recovering). The CMS program accounts for these changes and predicts the asphalt stiffness and the base, subbase, and subgrade resilient moduli on the basis of climatic conditions.

Asphalt Stiffness

The stiffness of the asphalt mixture is determined by using a model developed originally by the Shell Oil Company. The model calculates the asphalt mixture stiffness from the temperature of the asphalt and the percentage by volume of aggregate in the mix. The user must input points that define the

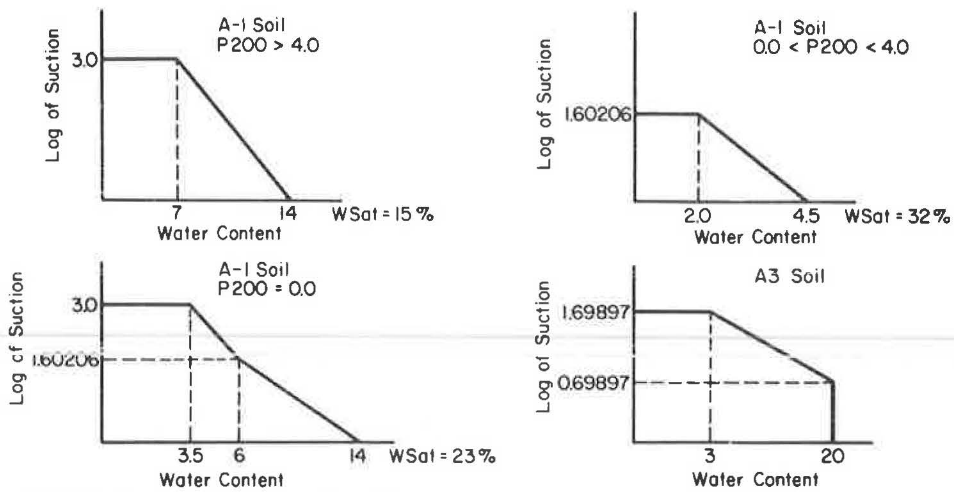


FIGURE 11 Approximate soil-moisture curves used in the CMS program for A1 and A3 soils.

temperature-stiffness relationship for the bitumen. Then, using the temperature values predicted by the heat-transfer model, the program calculates the asphalt mixture stiffness via the following equation developed by Yoder and Witczak *(20)*:

$$S_m = S_b \, [1 + (2.5/n) \, (C_v/1 - C_v)] \quad (21)$$

where

$$n = 0.83 \mathrm{Log}[(4 \times 10^5)/S_b] \text{ and} \quad (22)$$

$$C_v = VOL_{aggregate}/(VOL_{aggregate} + VOL_{asphalt}) \quad (23)$$

The term C_v is modified if the mix has an air void ratio greater than 3 percent as recommended by Van Draat and Sommer *(21)*:

$$C'_v = [C_v/(1 - H_{air})] \quad (24)$$

Here H_{air} is the actual air void content of the mix minus 3 percent.

By this procedure the asphalt stiffness is calculated for each node and then an average is determined for the entire layer as a function of time and temperature.

Base Course and Subbase Resilient Modulus Model

The resilient moduli of coarse-grained soils do not vary throughout the year to the extent that those of fine-grained soils do. For prediction of the resilient modulus, the CMS program categorizes coarse-grained soils into one of two states, frozen or unfrozen. Therefore for base and subbase materials, including stabilized materials, the user must input values for the frozen and unfrozen resilient moduli. The CMS program will then select the appropriate value depending on whether a frozen or unfrozen material condition exists.

Subgrade Resilient Modulus

The resilient modulus of the subgrade varies greatly during the year in most temperate regions. As the subgrade freezes there is a marked increase in the resilient modulus, which indicates a stiffening of the subgrade. Then as the subgrade thaws the resilient modulus drops substantially below its initial unfrozen value. This weakening of the subgrade on thawing is most apparent in fine-grained soils. Therefore the CMS program considers fine-grained soils to be in one of three conditions: frozen, unfrozen, or thaw-recovering. The resilient modulus for the frozen or unfrozen subgrade may be input by the user or calculated within the program. For coarse-grained subgrade (AASHTO classification A1 through A3 soils), the user must input the values for the frozen and unfrozen resilient moduli just as he must for the base and subbase materials.

If the user does not input a value for the unfrozen fine-grained resilient modulus, the program will calculate a value for each node based on its water content. Thompson and Robnett *(22)* derived many regression equations based on data from many Illinois subgrade soils. These equations relate the volumetric water content to the resilient modulus at a repeated deviator stress of 6 psi. The two relationships that are used in the CMS program are

For $\gamma_d \leq 100$ pcf:

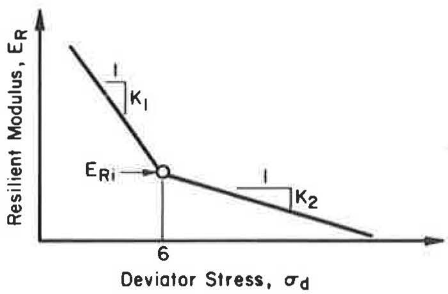
$$E_{Ri} = 27.06 - 0.526\theta \quad (25)$$

For $\gamma_d > 100$ pcf:

$$E_{Ri} = 18.18 - 0.404\theta \quad (26)$$

In these equations the volumetric water content (θ) is input in percentage form, and the resilient modulus at a repeated deviator stress of 6 psi (E_{Ri}) is calculated in ksi. Also, the program does not allow the resilient modulus to drop below 1.0 ksi in the case of exceptionally high water contents.

Thompson and Robnett *(22)* found that the value of the resilient modulus at a deviator stress of 6 psi is the critical point in defining the resilient modulus versus the repeated deviator stress relationship (Figure 12). When this point has been determined, the rest of the curve may be approximated using E_{Ri} versus σ_d curve slopes of K_1 equal to –1.1 ksi/psi and K_2 equal to –0.178 ksi/psi as suggested by Thompson and Robnett *(22)*.



Subgrade Stress – Dependent Model

FIGURE 12 Resilient modulus versus deviator stress.

As the subgrade freezes, the resilient modulus increases to a value approximately two orders of magnitude greater than the unfrozen value. Because the resilient modulus is so high, the pavement deflections will be relatively small. Therefore great accuracy is not required in the estimation of the frozen resilient modulus (small deflections are less critical than larger deflections). If the user does not input a value for the frozen modulus, it will be assigned a value of 100 times the unfrozen value [for more information on the frozen resilient modulus refer to Vinson *(23)* and Vinson et al. *(24)*].

The final condition in which the subgrade may occur is termed thaw-recovery. At the onset of spring thawing the pavement deflections have been observed to be larger than before freezing. Johnson et al. *(25)* have found that for silt subgrades in test pavements in Hanover, New Hampshire, the resilient modulus during thawing can be as little as 1 or 2 percent of the unfrozen value (silty subgrades are most susceptible to this kind of behavior when thawing). This weakening may be attributed to a secondary structure and areas of high moisture content development during ice lens formation. As the pavement is loaded the strength increases, presumably as a result of moisture redistribution and the deterioration of the secondary structure. With repeated loading the resilient modulus nears its unfrozen or prefreezing value *(26)*. When many such cycles are being considered, it is acceptable to assume that the soil regains its full strength.

The CMS program has modeled this behavior with a linear interpolation between the high and low values of the resilient modulus. The equation is

$$E_{Ri}\ (t) = \{[E_{Ri}\ (100 - \text{REDUCT})/100]/\text{RECPER}\}\ t + E_{Ri}\text{REDUCT} \qquad (27)$$

In this relationship, $E_{Ri}(t)$ is the resilient modulus at time t within the recovery period (RECPER). E_{Ri} is the value of the unfrozen resilient modulus discussed previously.

The reduction in the resilient modulus (REDUCT) is taken as a percentage of the unfrozen value. The user may input a value for the reduction factor, or a default value of 10 percent will be used.

It appears that the time required to reach 80 percent recovery varies from roughly 35 to 65 days *(27)*. This length of time is a function of the soil type and the traffic rate or number of loadings. The length of the recovery period (RECPER) may be input by the user, or a default value of 60 days will be used. The value of the resilient modulus will be calculated in this way until the end of the recovery period is reached or until the subgrade refreezes.

At the present time the moisture content of the subgrade is determined as a function of the standard soil properties and the distance to the water table. Because the unfrozen resilient modulus is based on the moisture content, it will remain constant during a period of constant water table depth. In the event that a more complex moisture movement and equilibria model, which allows for dynamic moisture content changes during a time period, is desired, the CMS program can easily be adapted to account for the changes in material properties associated with changes in moisture content.

The preceding approach to modeling the stiffness properties of a pavement system takes into account the climatic effects that result in seasonal variations at a given location. The values obtained simulate the actual conditions more realistically than does an approach that ignores the climatic effects. Therefore it is believed that this model will aid in obtaining more reliable predictions of pavement performance.

Structural Analysis Models

The problem of evaluating pavement performance is complex. The pavement structure is composed of various materials the properties of which vary diurnally, seasonally, and with repetitions of loading. All of these factors must be fully understood and accounted for when predicting pavement performance. Many mechanistic models have been developed to explain, interpret, and predict pavement load-response patterns. For any flexible pavement system, the load-response pattern is dependent on service conditions such as *(a)* stiffness of the asphalt concrete (a function of temperature) and *(b)* moisture content and physical state (frozen, unfrozen, or thaw-recovering) of the underlying layers.

Current methods of analysis consider environment by defining the most critical situation with respect to a particular distress criterion. By evaluating the properties of the materials for the critical situation, data are obtained that provide for analysis only under the most extreme conditions. This approach results in unrealistic analysis, especially if the critical situation has not been properly defined *(28)*. It is evident that there is a need to include the effect of climate on the whole structural section of a pavement throughout its entire spectrum of seasonal variation to permit accurate calculation of stress and deflection trends. The CMS program can use many of the existing pavement structural analysis models to determine radial stresses, strains, and displacements in multilayer flexible pavements as a function of load, climatic exposure conditions, and material properties.

Elastic Layer Analysis

Application of elastic-layered system theory is the most common method used for calculating stresses and strains in multilayered flexible pavement systems *(29)*. Basic assumptions of this approach include *(a)* each layer is composed of materials that are isotropic, homogeneous, and weightless; *(b)* the system acts as a composite system such that there is continuity of stresses or displacements, or both, across the interface; and *(c)* the constituent materials are linearly elastic and can be characterized by a resilient modulus and Poisson's ratio. The use of layered elastic theory for a valid determination of stresses and deflections in a pavement system requires that resilient modulus and Poisson's ratio be properly defined.

The temperature-dependent stiffness values of the asphalt-concrete surface and appropriate resilient modulus and Poisson's ratio values for each of the underlying layers can be established from the CMS program for a specific hour, day, and year. The combined model permits prediction of stresses and deflections in a pavement system as they are influenced by the time-temperature regime.

ILLI–PAVE Model

The finite-element method has been extensively used in analysis of pavement systems in the last 15 years *(30)*. The ILLI–PAVE model is an axisymmetrical solid of revolution based on the finite-element method. The model incorporates nonlinear stress-dependent material models and failure criteria for granular and fine-grained soils. The principal stresses in the granular and subgrade layers are modified at the end of each iteration so they do not exceed the strength of the materials as defined by the Mohr-Coulomb theory of failure *(31)*.

Numerous research efforts have shown evidence of nonlinearity in flexible pavements *(32, 33)*. Therefore this pavement model offers a realistic approach to analyzing nonlinear stress-dependent materials in the pavement-subgrade system. It is possible to generate deflection basins as influenced by the time-temperature and moisture regime for each specific hour, day, and year using ILLI–PAVE and the modular values calculated in the CMS program. The main disadvantage of the finite-element approach, however, is that it will require long and expensive computer runs.

ILLI-PAVE Algorithms

Previous studies *(30, 34)* have demonstrated the validity of the algorithm approach derived from the ILLI–PAVE model. Using

TABLE 1 ILLI–PAVE DEFLECTION BASIN ALGORITHMS

For Conventional Pavements: $\text{Dependent Variable} = E_{Ri} \times p + E_{ac} \times q + T_{ac} \times r + T_{gr} \times s + o$

GROUP	Dependent Variable	o	p	q	r	s
T_{ac} = 1.5 to 3"	LogΔ	2.096	-0.0232	-0.000149	-0.0967	-0.0137
T_{gr} = 4 to 12"	Area	11.18	-0.315	0.00237	1.742	0.187
T_{ac} = 3 to 8"	LogΔ	1.878	-0.0203	-0.000194	-0.0372	-0.00694
T_{gr} = 4 to 24"	Area	13.25	-0.353	0.00383	1.040	0.0958
T_{ac} = 3 to 8"	LogΔ	1.900	-0.0197	-0.0002	-0.0451	-0.00707
T_{gr} = 6 to 18"	Area	13.21	-0.359	-0.00409	0.946	0.127

For Stabilized Pavements: $\text{Dependent Variable} = E_{Ri} \times p + E_{ac} \times t + E_{stab} \times q + T_{ac} \times r + T_{stab} \times s + o$

Dependent Variable	o	p	q	r	s	t
LogΔ	1.690	-0.0238	-0.000125	-0.0207	-0.0367	-0.000276
Area	20.046	-0.398	0.00192	0.0266	0.454	0.00519

Variables:

E_{ac} (ksi)	Modulus of Elasticity of AC Layer
E_{stab} (ksi)	Modulus of Elasticity of Stabilized Layer
E_{Ri} (ksi)	Subgrade Modulus
T_{ac} (in.)	Thickness of AC Layer
T_{gr} (in.)	Thickness of the Granular Layer
T_{stab} (in.)	Thickness of the Stabilized Layer
Δ (mils)	Equivalent 9k Moving Wheel Load Deflection
Area (in.^{2})	Equivalent 9k Moving Wheel Load Deflection Basin Area

multiple regression techniques, Hoffman and Thompson *(30)* developed deflection basin predictive equations as a function of *(a)* the resilient modulus of the asphalt concrete (E_{ac}), *(b)* the resilient modulus of the subgrade soil (E_{Ri}), *(c)* the thickness of the asphalt layer (T_{ac}), and *(d)* the thickness of the granular base (T_{gr}) *(30)*. Table 1 gives the predictive equations developed for conventional and stabilized pavements.

There is general agreement among pavement engineers and researchers that surface deflection can be used to interpret asphalt-concrete fatigue behavior. Thompson *(35)* has shown significant correlations between ILLI–PAVE–calculated surface deflection and asphalt-concrete radial strain for conventional flexible pavements and full-depth asphalt pavements. Through statistical analyses of ILLI–PAVE data the following equations were developed:

Conventional flexible pavements:

$$\text{Log}\varepsilon_{ac} = -1.1296 + 1.1297\text{Log}\Delta \quad (\varepsilon_{ac} \times 10^{-4},\ \Delta = \text{mils}) \qquad (28)$$

Full-depth flexible pavements:

$$\text{Log}\varepsilon_{ac} = 1.53\text{Log}\Delta + 0.319 \quad (\varepsilon_{ac} \times 10^{-6},\ \Delta = \text{mils}) \qquad (29)$$

Based on a linear relationship of a limiting tensile strain criterion for fatigue cracking, the number of equivalent 18-kip single axle loads can be determined using the following equation *(35)*:

$$N_{18} = (6.6 \times 10^{-6})\ (1/\varepsilon_{ac})^{3.16} \qquad (30)$$

Through the use of ILLI–PAVE analysis Thompson *(35)* has also developed fatigue life prediction procedures for flexible pavement systems with intact high-strength stabilized base courses.

APPLICATION OF CMS PROGRAM

Figure 13 shows a partial output from the CMS program using the ILLI–PAVE algorithm analysis for 27 days of climatic data. The flexible pavement system consisted of 8 in. of asphalt concrete over 6 in. of A2 subbase and an A6 subgrade. The strengths of the asphalt-concrete and subgrade layers were obtained through use of the heat-transfer, moisture, and material models in the CMS program. The pavement deflection and deflection basin areas were determined from the algorithms in Table 1.

Based on 27 days of climatic input data, an average deflection of 18.635 mils was determined for the analysis period. From Equation 28 a radial strain of 0.20×10^{3} (0.2021E–03) in./in. was determined at the bottom of the asphalt-concrete layer. From Equation 30 the number of equivalent 18-kip single axle loads to failure at the predicted deflection is about 3.1 million. If the number of equivalent 18-kip single axle loads that actually occurred on the pavement during the study period is known, the percentage of fatigue consumption can be predicted from Minor's formula.

CONCLUSION

The CMS program was developed to introduce climatic effects into the analyses of multilayered flexible pavement systems. Figure 1 shows how this program may be used with selected pavement structural and performance models to analyze a pavement system. It can also be employed as a tool to analyze existing pavement systems in order to obtain estimates of future maintenance requirements.

The accuracy of the CMS output depends mainly on the quality of the inputs. It is important that boundary conditions, climatic conditions, and material properties properly represent the system to be analyzed. With representative inputs it is believed that the CMS program will give realistic values for the temperature profile and the material stiffness properties. The validity of the individual parts that comprise the CMS model has been shown, but the validity of the model in total has not been demonstrated. For this, analysis of existing pavement systems and comparison of the CMS outputs with actual field conditions are required. It is expected that these validation studies will confirm the belief that the CMS model provides an economical and realistic means of analyzing multilayered flex-

PAVEMENT SYSTEM

LAYER	TYPE	THICK.
1	IMPERM	4.00
2	IMPERM	4.00
3	A-2	6.00
4	A-6	130.00

DATE	AVG AC TEMP (C)	AVG AC E (KSI)	AVG SUBGRADE E (KSI)	DEFLECTION (MILS)	AREA (IN)
1	18.16	.1250E+04	.5636E+01	13.671	24.628
2	18.82	.1224E+04	.5636E+01	13.835	24.522
3	20.94	.1151E+04	.5636E+01	14.309	24.222
4	21.86	.1119E+04	.5636E+01	14.519	24.093
5	25.82	.9962E+03	.5636E+01	15.364	23.591
6	26.87	.9655E+03	.5636E+01	15.583	23.465
7	32.74	.8128E+03	.5636E+01	16.718	22.841
8	34.98	.7596E+03	.5636E+01	17.133	22.623
9	42.91	.6051E+03	.5632E+01	18.400	21.993
10	45.12	.5699E+03	.5632E+01	18.700	21.849
11	48.87	.5168E+03	.5623E+01	19.172	21.635
12	50.04	.4988E+03	.5623E+01	19.331	21.561
13	51.34	.4843E+03	.5595E+01	19.485	21.512
14	52.37	.4692E+03	.5595E+01	19.620	21.450
15	52.87	.4651E+03	.5518E+01	19.727	21.461
16	53.90	.4495E+03	.5518E+01	19.869	21.397
17	57.31	.4073E+03	.5441E+01	20.330	21.253
18	57.65	.4030E+03	.5441E+01	20.370	21.235
19	58.74	.3922E+03	.5364E+01	20.544	21.219
20	58.89	.3901E+03	.5364E+01	20.564	21.210
21	59.42	.3854E+03	.5286E+01	20.680	21.218
22	59.54	.3836E+03	.5286E+01	20.698	21.211
23	59.98	.3800E+03	.5209E+01	20.805	21.224
24	60.05	.3786E+03	.5209E+01	20.818	21.218
25	60.46	.3754E+03	.5132E+01	20.922	21.233
26	60.54	.3740E+03	.5132E+01	20.936	21.227
27	60.85	.3717E+03	.5055E+01	21.031	21.246

AVERAGE DEFLECTION OVER ANALYSIS PERIOD (MILS) 18.635

ASPHALT CONCRETE RADIAL STRAIN (IN/IN) .2021E-03

ALLOWABLE NUMBER OF 18K EQAL 3120475

FIGURE 13 Partial output from the combined CMS program and ILLI–PAVE algorithm analysis.

ible pavement systems by accounting for climatic effects on pavement materials.

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APPENDIX

- A = radiation equation constant;
- a = absorptivity of radiation by a surface;
- B = radiation equation constant;
- C_v = volume percentage of aggregate in the asphalt mixture, decimal form;
- C'_v = the modified aggregate content, decimal form;
- E_{ac} = the resilient modulus of the asphalt concrete layer, ksi or psi;
- E_{Ri} = the resilient modulus of the nonasphalt materials at a repeated deviator stress of 6 psi, ksi, or psi;
- $E_{Ri}(t)$ = the resilient modulus at time t within the recovery period, ksi or psi;
- H = convection coefficient, Btu/hr-ft^2-°F;
- H_{air} = the actual air void content of the asphalt mix minus 0.03, decimal form;
- K_1 = slope of E_R versus σ_d for $\sigma_d < 6$ psi, ksi/psi;
- K_2 = slope of E_R versus σ_d for $\sigma_d > 6$ psi, ksi/psi;
- LL = the liquid limit of fine-grained soils, %;
- LSUC = the break suction point on the water content versus suction plot;
- N = cloud-based factor;
- N_{18} = the allowable number of 18-kip single axle loads for a given surface deflection;
- n = term used in the calculation of the asphalt mixture stiffness;
- PI = the plasticity index for fine-grained soils, %;
- PL = the plastic limit for fine-grained soils, %;
- Q_a = heat flux resulting from long-wave radiation emitted by the atmosphere, Btu/ft^2-hr;
- Q_c = heat flux resulting from convective heat transfer, Btu/ft^2-hr;
- Q_e = heat flux resulting from long-wave radiation emitted by the pavement surface, Btu/ft^2-hr;
- Q_g = heat flux conducted into pavement, Btu/ft^2-hr;
- Q_h = heat flux resulting from transpiration, condensation, evaporation, and sublimation, Btu/ft^2-hr;
- Q_i = heat flux resulting from incident short-wave radiation, Btu/ft^2-hr;
- Q_r = heat flux resulting from reflected short-wave radiation, Btu/ft^2-hr;
- Q_{rad} = net radiation flux influencing heat transfer at a surface, Btu/ft^2-hr;
- Q_s = net short-wave radiation entering into the energy balance at the pavement surface, Btu/ft^2-hr;
- Q_x = long-wave radiation emitted from a surface without cloud cover correction, Btu/ft^2-hr;
- Q_z = long-wave back radiation not corrected for cloud cover, Btu/ft^2-hr;
- RECPER = the length of the recovery period for fine-grained soils, days;
- REDUCT = the reduction factor for thawing fine-grained soils, %;
- R^* = extraterrestrial radiation, Btu/ft^2-day;
- S = percentage of possible daily sunshine;
- SATWAT = the saturated water content for fine-grained soils, %
- S_b = the bitumen stiffness, kg/cm^2;
- S_m = the asphalt mixture stiffness, kg/cm^2;
- T_1 = temperature of surface node, °F;
- T_{ac} = the thickness of the asphalt layer, in.;
- T_{air} = air temperature, °F;
- T_{con} = temperature of constant temperature node, °F;
- T_{gr} = the thickness of the granular base, in,;
- T_n = nodal temperature, °F;
- T'_n = nodal temperature after a time step, °F;
- T_{stab} = the thickness of the stabilized base, in.;
- t = the time, days;
- u = the relative pore water pressure (negative above the water table), psi;
- VOL = volume;
- W = total depth of termination nodes, in.;
- $\overline{W}$ = percentage of cloud cover at night;
- ΔW = depth of a termination node, in.;
- W1000 = the water content at a suction of 1000 cm, %;
- w = water content based on dry weight, %;
- X = total depth of normal nodes, in.;
- ΔX = depth of a normal node, in.;
- Y = total depth of finite-difference pavement system, in.;
- Z = the height above the groundwater table, in.;
- α = thermal diffusivity, K/C, ft^2/hr;
- γ = total unit weight, pcf;
- γ_d = dry unit weight, pcf;
- Δ = the equivalent 9-kip moving wheel load deflection, mils;
- ε = emissivity of radiation by a surface;
- ε_{ac} = the asphalt-concrete radial tensile strain;
- θ = the volumetric water content, %;
- $\Delta\theta$ = time step, hr; and
- σ_d = the deviator stress, psi.