expected grout take in each abutment did not have a significant adverse effect on the project budget, but it did validate and ameliorate the condition of voids in the bedrock formation.

SUMMARY

With construction on the bridge phase of the project recently completed, it can be reported that all major design elements of the foundation performed as anticipated, under both construction loading and railroad live loading. Equally important to the city and Conrail was the fact that rail traffic was unimpeded during construction and vehicular traffic on Brighton Avenue was maintained, even though occasionally reduced to one lane during foundation construction. The only stoppage and rerouting of traffic was limited to a portion of one day when superstructure main girder steel was set.

Publication of this paper sponsored by committee on Foundations of Bridges and Other Structures.

Geotechnical Error Analysis

GREGORY B. BAECHER

A simple method is presented for rationalizing the treatment of uncertainties in geotechnical engineering calculations. This method uses a reliability index to express the degree of confidence in a calculation. The reliability index combines the best estimate with a standard deviation reflecting four principal sources of uncertainty, spatial variability, measurement noise, model bias, and limited data. An example involving shallow footing design is used for illustration.

All engineers design in the face of uncertainties—uncertainty about material properties, conditions encountered in service, models used to predict performance, and many others. Traditionally, this uncertainty has been accounted for by conservative design, with the ratio of facility capacity to the demands placed on it—the factor of safety—chosen from common practice. As a general rule, the approach has been serviceable. Significant geotechnical failures occur at a rate of about 1,000 per year. The consequences of these failures, while important financially, are rarely catastrophic.

On the other hand, the strategy of fixed factors of safety has drawbacks. First, because uncertainty is not addressed directly, there is a tendency to be conservative about each of the estimates needed for design (soil properties, loads, etc.). The result is that the overall design factor of safety is unknown. Second, because the estimates of soil properties, loads, and so forth, are conservative and subjective, predictions of facility performance are often not repeatable. The result is poor quality assurance. Third, levels of uncertainty vary from situation to situation, because amounts and quality of data vary, facility uses vary, and so on. The result is that a fixed factor of safety leads to different likelihoods of adverse performance.

Department of Civil Engineering, Massachusetts Institute of Technology, Cambridge, Mass. 02139.

PURPOSE OF ERROR ANALYSIS

The purpose of error analysis is to improve current practice by expressly considering uncertainties. The term error analysis as used here is not what many now call probabilistic design. Geotechnical engineering involves many uncertainties only some of which are explicit. Therefore, probabilities resulting from analysis are not predictions of rates of failure to be experienced in the field. The majority of failures are attributable to unanticipated loads, gross errors, inadequate maintenance, and other factors not accounted for in design *(1)*.

Error analysis in the present context means a logical accounting for the uncertainties inherent in engineering calculations, and decisions that explicitly balance conservatism against those uncertainties. Specifically, error analysis addresses

1. Selection of design parameters from scattered, limited, and possibly biased data; and
2. Economic rationalization of design.

The method is nothing more than a form of accounting in which uncertainties are tabulated and their influence on engineering calculations combined according to well-defined rules.

UNCERTAINTY IN GEOTECHNICAL DESIGN

Uncertainties in geotechnical predictions are of many types. Some can be quantified, some cannot. In an approximate way they may be divided into five groups:

1. Site conditions,
2. Loads,
3. Model inaccuracies,
4. Construction and quality control, and
5. Omissions and gross errors.

The most important of these for engineering analysis are the first three, which are quantifiable and appear in calculations.

Uncertainties in Site Conditions and Models

If attention is restricted to only those uncertainties that affect calculations, namely site conditions and geotechnical models, a further and more specific subdivision of sources of error is possible. This leads to four sources that are the focus of error analysis:

1. Soil variability,
2. Measurement noise,
3. Measurement and model bias, and
4. Statistical error due to limited measurements.

These are the sources of uncertainty that affect calculated predictions. The first two, soil variability and measurement noise, appear as data scatter. The latter two, measurement and model bias and statistical error, cause systematic errors in predictions.

Data Scatter Equals Spatial Variability Plus Measurement Noise

The scatter among geotechnical measurements is often large. This scatter reflects two things: spatial variability of the soil and random measurement error (noise). A major purpose of statistical analysis is to separate real variability from noise, thereby lessening the magnitudes of data scatter and reducing uncertainty.

Systematic Error Equals Measurement Bias Plus Statistical Error

Bias is a systematic error. If strength is underestimated by a 10 percent bias error at one location, it is underestimated by the same 10 percent everywhere. The distinction between spatial variability and bias is important. For example, a 10 percent probability of failure due to soil variability implies that one-tenth of a long embankment will fail. The same probability due to bias implies a one in ten chance that the entire embankment will fail.

In geotechnical parameter estimation, bias is caused by *(a)* measurement techniques and *(b)* statistical estimation error. Measurement bias is common in geotechnical engineering; it is caused by soil disturbance or a difference between how a property is measured and how a structure imposes load. Statistical bias is also common; it is caused by limited data.

Separating the Sources of Uncertainty

Together, data scatter and systematic error constitute the uncertainty of geotechnical calculations. However, the effects of these components differ, as do the way each propagates through an engineering model. The most important concept of uncertainty analysis has nothing to do with mathematics, rather it has to do with separating source of uncertainty.

The methodology presented here is based on separating errors. It treats calculations and modeling. What results is a reliability index summarizing the confidence that can be placed in calculations.

DESCRIBING UNCERTAINTY

Assessments of soil properties for most purposes are adequately expressed by two numbers: a best estimate, and a measure of uncertainty. Here, the average value and standard deviation are used to express the two attributes. When more than one soil property is estimated, another attribute becomes important. This is the association between the uncertainties in different parameter estimates. Here, the correlation coefficient is used to express this association.

Average Equals Best Estimate

The average or mean of a set of measurements $x = (x_1,...,x_n)$ is denoted m_x, and defined as

$$m_x = 1/n \, \Sigma \, x_i = \text{mean} \tag{1}$$

In effect, the mean is the center of gravity of the measurements along the x-axis. It is used as the best single-valued estimate of x, being neither conservative nor unconservative.

Standard Deviation Equals Uncertainty

The standard deviation of the measurements x is their variation with respect to the mean, expressed as the square root of the sum-of-squared variations

$$s_x = [1/n - 1 \, \Sigma \, (x_i - m_x)^2]^{1/2} = \text{standard deviation} \tag{2}$$

In effect, the standard deviation is the root of the moment of inertia of the data about the mean. The proportional uncertainty or standard deviation normalized by the mean is called the coefficient of variation and denoted Ω_x,

$$\Omega_x = s_x/m_x = \text{coefficient of variation} \tag{3}$$

Just as in mechanics where it is convenient to deal with the moment of inertia rather than its square root, so, too, in analyz-

ing uncertainty is it convenient to deal with the square of the standard deviation rather than s_x itself. The square of the standard deviation is called the variance

$$V_x = s_x^2 = \text{variance} \tag{4}$$

Given the similarity of Equations 1 to 4 to mechanical moments, the mean and variance are often called the first and second moments of the uncertainty in an estimate of x.

Correlation Coefficient Equals Association Between Uncertainties

When dealing with two or more soil properties, the uncertainties in estimates may be associated with one another. That is, the uncertainty in one property estimate may not be independent of the uncertainty in the other estimate. Consider the problem of estimating the cohesion and friction parameters of a Mohr-Coulomb strength envelope based on a small number of tests. If the slope of the envelope to the Mohr circles is mistakenly estimated too steeply, then for the line to fit the data the intercept would have to be too small. The reverse is true if the slope is estimated too flat. Thus, uncertainties about the slope and intercept are not independent, they are related to one another.

The correlation coefficient for a set of paired data x,y = $[(x_1,y_1),\ldots,(x_n,y_n)]$ is denoted $r_{x,y}$, and defined as

$$r_{x,y} = 1/n - 2\,\Sigma\,(x_i - m_x/s_x)\,(y_i - m_y/s_y) = \text{correlation coefficient} \tag{5}$$

In effect, the correlation coefficient is equivalent to a normalized product of inertia in solid mechanics. It expresses the degree to which two parameters vary together. The correlation coefficient is nondimensional because deviations of x and y from their respective means are measured in units of the respective standard deviation. For these reasons $r_{x,y}$ is a convenient measure for expressing the degree of association or dependence between the uncertainties in two properties.

The value of $r_{x,y}$ may vary from +1 to −1; $r_{x,y} = +1$ implies a strict linear relation with a positive slope; $r_{x,y} = -1$ implies a strict linear relation with a negative slope; $r_{x,y} = 0$ implies no association at all.

The corresponding dimensional form of Equation 5, that is, using absolute deviations of x and y rather than normalized deviations, is called the covariance of x,y and denoted

$$C_{x,y} = 1/n - 2\,\Sigma\,(x_i - m_x)\,(y_i - m_y) = \text{covariance} \tag{6}$$

From Equations 5 and 6,

$$r_{x,y} = C_{x,y}/s_x s_y \tag{7}$$

Autocorrelation

Thus far the fact that soil properties are spatially variable has been ignored. They have not only magnitude but also location. The spatial quality of soils data has important implications, for it both strongly affects engineering predictions and increases the amount of information that can be squeezed from a testing program. Fortunately, the salient aspects of spatial variability from an error analysis view are easily analyzed using the statistical concept called autocovariance.

In an approximate way, spatial variability of data can be summarized by two measures: the variance of the data about their mean, and the waviness or frequency content of the variability in space. The longer the period of this waviness, the further data may be spatially extrapolated. Autocorrelation is used to measure waviness.

Autocorrelation measures the statistical association between data of the same type measured at separate locations. For example, the properties of two adjacent soil elements tend to be similar. If one is above average, the other tends to be above average, also; they are associated. Conversely, the properties of widely separated elements are not necessarily similar. If one is above average, the other may or may not be; they are not associated. This association of properties in space can be measured by the correlation coefficient of Equation 5. It is called autocorrelation because the data are all of the same type.

For data x_i, where i = the location of the measurement, the autocorrelation of data separated by interval, δ, is

$$R_x(\delta) = (1/V_x)\,(1/n_{\delta-1})\,\Sigma\,(x_i - m_x)\,(x_{i+\delta} - m_x) \tag{8}$$

the sum taken over all pairs of data having separation distance, δ, their number being n_δ. Autocovariance is related to autocorrelation as covariance is to correlation. The autocovariance of data at points separated by distance, δ is,

$$C_x(\delta) = (1/n_{\delta-1})\,\Sigma\,(x_i - m_x)\,(x_{i+\delta} - m_x) \tag{9}$$

Autocorrelation expressed as a function of separation distance, δ, is said to be the autocorrelation function, and autocovariance expressed as a function of distance, δ, is said to be the autocovariance function.

ESTIMATING UNCERTAINTY

Considered in this section are specific procedures for quantifying the uncertainties identified earlier.

Data Scatter: Soil Variability and Measurement Noise

Scatter in soil data reflects two things: real variability and noise. Yet, the amount of scatter is measured by a single number, namely the standard deviation of the data. It is not possible to separate soil variability from noise simply by inspection. Hence another approach to estimating the fraction of data scatter contributed by either of these sources must be used. The most convenient is through the autocovariance function. The autocovariance function reflects the spatial structure of variability in soil property measurements, and this structure differs depending on how the data scatter is divided between soil variability and noise. Each component has a characteristic signature that can be observed in the autocovariance function.

As a good approximation, measurements taken in the laboratory or field can be modeled as

$$z = x + e \tag{10}$$

where z is the measurement, x is the real soil property, and e is random measurement error. After some algebra, the autocovariance function of the set of measurements turns out to be related to the autocovariance functions of x and e by

$$C_z(\delta) = C_x(\delta) + C_e(\delta) \tag{11}$$

The autocovariance of x equals V_x at $\delta = 0$, and approaches 0 as δ increases. The autocovariance of e, on the other hand, equals V_e at $\delta = 0$, but equals 0 for any $\delta \neq 0$; that is, it is a spike. This is a result of the assumption that the error is independent from one test to another. Thus, for $\delta \neq 0$, the covariance of the e's is zero. Therefore, by extrapolating the observed autocovariance function back to the origin, an estimate of V_x and V_e is obtained directly. For typical in situ measurements on soil, measurement error variances have been found to contribute anywhere from 0 to as much as 70 percent of data scatter Baecher et al. *(2)*.

Systematic Error

Systematic error in the statistical estimation of soil parameters is directly calculated from statistical theory. The most significant of these errors is that in the mean of the soil property in the soil mass. As an approximation, although a robust one, the variance of the statistical error in this mean is

$$V_m = V_x/n \tag{12}$$

where n is the number of measurements. Note, although random measurement error can be eliminated from the data scatter variance to yield a reduced uncertainty, it does contribute to statistical error. Its effect on statistical error can only be lessened by making more measurements. The statistical error in other parameters usually has only second-order effect on predictions.

The last of the major sources of uncertainty, measurement bias, is the most difficult to estimate. Usually, the only way to estimate this component is by comparison of predicted with observed performance or by field-scale experiments. This has been done by Bjerrum *(3)* for field-vane strengths of normally consolidated clay, and has been attempted by other researchers for other applications. Such an approach aggregates a large number of uncertainties or biases together, including those due to inaccuracies of theory and method of analysis. Thus, measurement bias and model bias are usually inseparable.

In Bjerrum's work, the joint effect of bias in field-vane data and bias in 2D modified Bishop stability analysis leads to a correction factor, μ, which is the ratio of back-calculated, undrained strength to measured FV strength. The variation of back-calculated μ's about their mean is summarized in a variance, V_μ, which expresses the uncertainty of the bias term.

Estimating Autocovariance

In this section only a simple and often used approach to estimating autocovariances, the moment estimate, is considered. For readers with greater interest, a more detailed discussion of statistical aspects of estimating autocovariance, including maximum likelihood techniques, is presented by DeGroot *(4)*.

Consider the simple case of measurements at equally spaced intervals along a line, as for example in a boring. Presume that the measurements $x = \{x_1,...,x_n\}$ are uncorrupted by measurement error. The observed autocovariance of the measurements at separation, δ, is

$$c_x(\delta) = (1/n_{\delta-1}) \Sigma (x_i - m_x)(x_{i+\delta} - m_x) \tag{13}$$

where n_δ = the number of pairs of data at separation distance, δ.

This is called the sample autocovariance and is used as an estimator of the real autocovariance, $C_x(\delta)$, for separation distance, δ. Statistically, $c_x(\delta)$ is a moment of the sample data that is used to estimate the corresponding moment of the spatial model. Thus, $c_x(\delta)$ is said to be a moment estimator of $C_x(\delta)$, just as the sample variance is said to be a moment estimator of the real soil variance, V_x.

In the general case, measurements are seldom uniformly spaced and, at least in the horizontal plane, seldom lie on a line. For such situations the moment estimator of the autocorrelation function can still be used, but with some alteration. The most common way to accommodate nonuniformly placed measurements is by dividing separation distances into bands, and then taking averages of Equation 13 within those bands. This introduces some bias to the estimate but for most engineering purposes it is sufficiently accurate.

Combining Uncertainties in a Design Profile

The total uncertainty in engineering properties at a point in the soil profile reflects the combination of data scatter and systematic error. Algebraically, this total uncertainty, measured as a variance, is expressed as

$$V = V_1 + V_2 + V_3 + V_4 \tag{14}$$

where the four components of variance summarize, respectively, the four contributions of uncertainty:

V1 = Variance of the spatial variability, (15)

V2 = Variance of the measurement noise, (16)

V3 = Variance of the statistical error, and (17)

V4 = Variance of the measurement and model bias. (18)

For modeling purposes it is often convenient to draw a design profile of soil properties versus depth. About this profile are drawn two sets of standard deviation envelopes. One set describes point-to-point variability around the mean. This is the contribution of spatial variability. The other set describes uncertainty in the mean itself. This is the contribution of systematic error.

UNCERTAINTY IN CALCULATED PREDICTIONS

The preceding discussion used means, standard deviations, and correlations to describe best estimates and uncertainties about soil properties. For engineering analysis, these mean standard deviations and correlations must be accounted for in calculations. This leads to performance predictions that are described by means, standard deviations, and correlations.

The mathematics needed for relating a second-moment description of soil properties, loads, and other input parameters to a corresponding second-moment description of performance predictions are relatively uncomplicated. Schematically, the procedure is shown as follows:

Soil Parameters
↓
Loads → [M O D E L] → Prediction

A model is chosen for calculating performance. For example, this might be Terzaghi's formula for predicting the bearing capacity of a footing. Next, means, standard deviation, and correlations are evaluated for all required input parameters. These means, standard deviations, and correlations are then translated through the model to determine the resulting means, standard deviations, and correlations on performance predictions.

Best Estimate (Mean) Prediction

Operationally, best estimates of soil properties are translated through a model using a first-order approximation. This is simply a linear approximation to the model in the vicinity of the best estimates of the soil parameters. Mathematically, the calculation of some performance prediction y based on a soil parameter x can be expressed as a function

$$y = g(x) \tag{19}$$

By taking a Taylor's series expansion of g(x) at the point m_x and then truncating all but the first two (i.e., linear) terms, the tangent plane at m_x is obtained. For most geotechnical purposes this linearization is sufficiently accurate. Applying rudimentary probability theory leads to the convenient result

$$m_y \doteq g(m_x) \tag{20}$$

where $\doteq$ indicates first-order approximation. In words, the mean or best estimate of the prediction y is the function of the mean or best estimate of the parameter x. This is the normal deterministic solution, using best-estimate soil properties as input.

Uncertainty (Standard Deviation) in Predictions

By similar reasoning, standard deviations on input soil properties x may also be translated through a model y = g(x) to find a corresponding standard deviation on the prediction y. The first-order approximation leads to the relation

$$s_y \doteq (dy/dx)\, s_x \tag{21}$$

where the derivative dy/dx is an influence factor. In words, the standard deviation of the prediction y is the product of the standard deviation of the parameter x and an influence factor equal to the derivative of y with respect to x. The relation is exact when g(x) is linear.

When the prediction y depends on a set of parameters, $x = \{x_1,...,x_n\}$, the equivalent forms of Equations 20 and 21 are

$$m_y \doteq g(m_{x1},...,m_{xn}) \tag{22}$$

$$s_y^2 \doteq \Sigma\, \Sigma\, (dy/dx_i)\, (dy/dx_j)\, C_{xi,xj} \tag{23}$$

Note, when x_i, x_j are mutually independent, $C_{xi,xj} = 0$ for $i \neq j$ and $C_{xi,xj} = s_x^2 = V_x$ for i = j, thus

$$s_y^2 \doteq \Sigma\, \Sigma\, (dy/dx)^2\, V_x \tag{24}$$

Two special cases deserve note because they are so common. When y is a linear combination of a set of independent parameters, $y = \Sigma a_i\, x_i$,

$$V_y = \Sigma\, a_i^2\, V_{xi} \tag{25}$$

when y is a power function of a set of independent parameters, $y = \Pi a_i x_i bi$,

$$\Omega_y^2 \doteq \Sigma\, b_i^2\, \Omega_{xi}^2 \tag{26}$$

Equation 26 pertains to small coefficients of variation, for example, less than 0.2 to 0.3.

Reliability Index, β

In traditional geotechnical analysis, the adequacy of a design is expressed by a factor of safety, defined as the ratio of capacity to demand

$$F = \text{capacity/demand} \tag{27}$$

The factor of safety makes no allowance for uncertainty. When performance is predicted by both a best estimate and a measure of uncertainty, a new and more complete safety index can be used. One such index that combines both best estimate and uncertainty is the reliability index, β

$$\beta = m_y - y_f/s_y \tag{28}$$

where y_f is the limiting state or failure value for the predicted performance, y. In essence, β measures the number of standard deviations separating the best estimate of performance from some unacceptable value. If the predicted variable, y, were, for example, a factor of safety against bearing capacity failure of a footing, then m_y = mean of F, s_y = standard deviation of F, and $y_f = 1.0$.

A lower value of β implies lower reliability. A β = 0 means

that the best estimate of performance just equals the failure criterion, that is, m_y would equal y_f. $\beta > 0$ means that $m_y > y_f$ because the standard deviation is always positive. Typical values of β for common geotechnical design range from about 2 to 3. The reliability index is a useful measure of safety because it balances the safety implied by a best estimate of facility performance against the uncertainty in that prediction. Thus, β can distinguish between the case of high estimated factor of safety with correspondingly high uncertainty and the case of a low estimated factor of safety with correspondingly low uncertainty. Thus, β allows a more comprehensive balancing of design conservatism against uncertainty than does FS alone, and can lead to significant economies on large projects *(5)*. The use of β rather than FS also allows design conservatism to be quantitatively related to the extent of site characterization and testing, thereby allowing a balance to be struck between information gathering and conservatism.

SETTLEMENT OF SHALLOW FOOTINGS ON SAND

The importance of uncertainties and errors is well illustrated by a field case involving shallow footings *(6)*. The case especially shows the usefulness of separating random measurement error from spatial variability when making predictions. The site overlies approximately 10 meters of uniform windblown sand on which a large number of footings were constructed. The site was characterized by SPT blow count measurements. Predictions were made of settlement, and subsequent settlements were measured.

Spatial Variation and Noise in Settlement Predictions

Inspection of the standard penetration test (SPT) data and subsequent settlements reveals an interesting discrepancy. Because footing settlements on sand tend to be proportional to the inverse of average blow count beneath the footing, from Equation 26 it would be expected that the coefficient of variation of the settlements be approximately that of the vertically averaged blow counts. Mathematically, settlement is predicted by a formula of the form

$$\rho \propto (\Delta q/N)\, g(B) \tag{29}$$

where

ρ = settlement,
Δq = net applied stress at the base of the footing,
N = average corrected blow count, and
$g(B)$ = a function of footing width.

Therefore, from Equation 26,

$$\Omega_\rho \doteq \Omega_N \tag{30}$$

but it is not. The coefficient of variation of the vertically averaged blow counts is about 0.50; the coefficient of variation of the settlements is only 0.37. Why the difference?

The best explanation for this apparent inconsistency is found in estimates of measurement noise in the blow count data. Figure 1 shows the horizontal autocorrelation function for the blow count data. By extrapolating this function to the origin, the noise (or high frequency) content of the data is estimated to be about 50 percent of the data scatter variance. This means that

$$\begin{aligned}(\Omega_{soil})^2 &= (\Omega_{data})^2\,(0.5)\\ &= (0.35)^2\end{aligned} \tag{31}$$

which is close to the observed variability of the settlements.

Calculating Footing Settlement

Footing settlement can be predicted by any of a number of equations. Peck and Bazaara's equation is a modification of the Terzaghi and Peck upper envelope

$$\rho = [(2\Delta q/m_N)\,(2B/1 + B)^2\,(1 - 1/4\ D/B)] \tag{32}$$

where

ρ = settlement (inches),
Δq = allowable applied stress (TSF),
m_N = (vertically) averaged corrected blow count, and
B = footing width (ft).

Water table elevation is ignored. The term involving D/B, where D = embedment depth, is a depth correction factor. In the present case D/B = 0.5. For square footings of design width B = 10 ft, the best estimate of ρ at the allowable stress of 3 TSF (6 ksf) is shown in Figure 2.

Spatial Component of Settlement Uncertainty

The variance of ρ due to uncertainty in m_N is calculated by noting that ρ is inversely proportional to m_N. Therefore, from Equation 26

$$\Omega_\rho \doteq \Omega_{mN} \tag{33}$$

m_N is the average blow count within a depth B of the footing and thus its variance and coefficient of variation are less than those of the point-by-point blow counts, N. For this site, blow counts are taken every 5 ft, thus m_N is the average of two measurements. As such, from Equation 12, $V_{mN} = V_N/2$, and Cov $m_N = \sqrt{1/2}\ \Omega_N = (0.71)\ (0.44) = 0.32$. Therefore, Ω_ρ is approximately 0.32. Alternately, Equation 21 could have been used to find the same result with more effort.

The coefficient of variation of ρ calculated above is that responding to spatial variation in the SPT data. This magnitude of variation should be observed among the various footings around the site. In comparison, the observed values of total

settlements for 268 footing at this site have a mean of about 0.35 ft, and a standard deviation of 0.12. Thus, $\Omega_\rho = 0.34$.

Systematic Component of Settlement Uncertainty

In addition to spatial variability, the limited number of borings causes statistical error in the prediction of average settlement. With 50 borings and hence 50 SPT measurements at any elevation, the statistical error in the estimated mean blow count at any elevation in the upper levels is $Vm_N \doteq V_N/50$. This reflects uncertainty on the average settlement of all the footings at the site.

The settlement model itself introduces bias that differs from site to site. For Equation 32 comparison data of predicted versus observed settlements yield a mean bias *(2)* of $m_b = 1.46$ and a standard deviation of $s_b = 1.32$, where b = observed settlement and predicted settlement. Correcting the earlier estimate for this model bias,

$$m'_\rho \doteq b\, m_\rho \tag{34}$$

where m_ρ is the corrected mean settlement. The variance of the corrected settlement is found using Equation 24 as

$$V_\rho \doteq V_b\, m_\rho^2 + m_b^2\, V_\rho \tag{35}$$

The poor correlation of the settlement model to actual footing performance introduces a large model error if data are unavail-

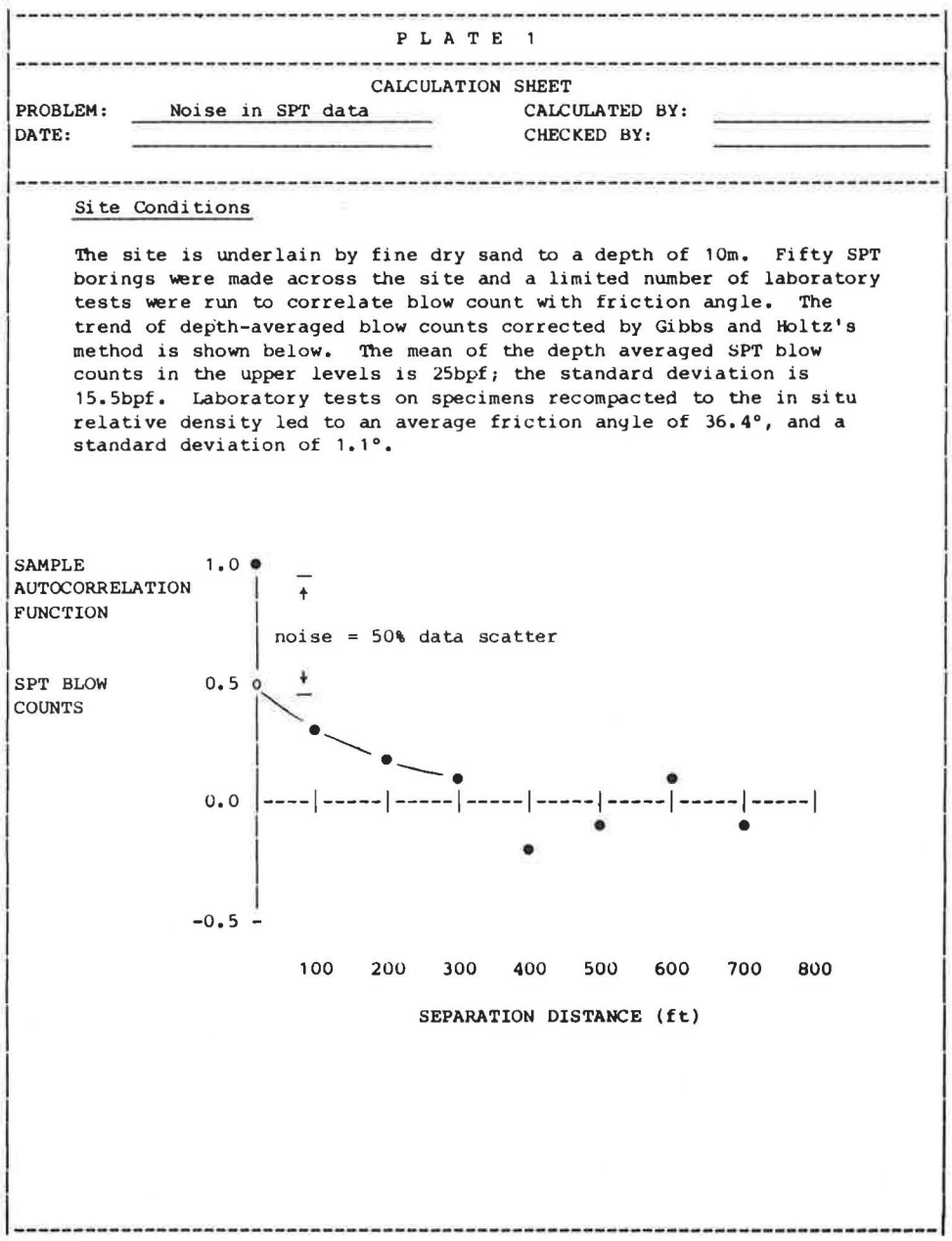


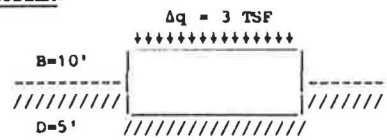
FIGURE 1 Calculation of noise in SPT data.

CALCULATION SHEET

PROBLEM: Footing settlement CALCULATED BY: _____

DATE: _____ CHECKED BY: _____

PROBLEM



allowable ρ = 1"

SOIL PROPERTIES

m_N = vertically averaged SPT blow count, corr'd.

= 25 bpf

(a) BEST ESTIMTE (MEAN) OF SETTLEMENT

$$m_\rho \doteq \left(\frac{2\Delta q}{m_N}\right)\left(\frac{2B}{1+B}\right)^2 \left[1 - \frac{1}{4} D/B\right] = \left(\frac{2 \cdot 3}{25}\right)\left(\frac{2 \cdot 10}{1+10}\right)^2 \left[1 - \frac{1}{4}\, 5/10\right]$$

$$= \underline{\underline{0.70"}}$$

$$m_\rho' \doteq b\, m_\rho$$

$$= (1.46)(0.70") = \underline{\underline{1.02"}}$$

(b) UNCERTAINTY (VARIANCE) OF SETTLEMENT

Spatial Variability

from Eqn. 26, $\Omega_\rho \doteq \Omega_{mN}$

$$\Omega_\rho \doteq \Omega_N = \frac{\sqrt{V[N]}}{m_N} = \frac{\sqrt{V[N]/2}}{m_N} = \frac{\sqrt{11^2/2}}{25} = 0.32$$

$$V_\rho = (\Omega_\rho\, m_\rho)^2 = [(0.32)(0.70")]^2 = \underline{\underline{(0.22")^2}}$$

$$V_{\rho'} = (\Omega_\rho\, m_{\rho'})^2 = [(0.32)(1.02")]^2 = \underline{\underline{(0.32")^2}}$$

Systematic Error

Statistical Estimation Error:

$$\Omega_{mN}^2 = \Omega_N^2/n = (0.32)^2 / 50 = (0.05)^2$$

$$\Omega_\rho = \Omega_{mN} = (0.05)^2$$

$$V_\rho = (\Omega_\rho\, m_\rho)^2 = [(0.05)(0.7)]^2 = (0.04)^2$$

$$V_{\rho'} = (\Omega_\rho\, m_{\rho'})^2 = [(0.05)(1.02)]^2 = (0.05)^2$$

Model Bias:

$$V_{\rho'} = V_b\, m_\rho^2 + m_b^2\, V_\rho \,|$$

$$= (1.32)^2\,(0.70")^2 + (1.46)^2(0.31 \times 0.70")^2$$

$$= 0.85 + 0.10 = (0.98)^2$$

(c) RELIABILITY INDEX

Spatial Variability Alone

$$\beta = \frac{m_\rho - \rho_o}{s_\rho} = \frac{|0.70" - 1"|}{0.22"}$$

$$= \underline{\underline{1.364}} \text{ spatial variability alone}$$

Spatial Variability + Systematic Error

Total uncertainty = spatial variability + systematic error

$$V_\rho = (0.22)^2 + (0.04)^2 = (0.23)^2$$

$$V_{\rho'} = (0.32)^2 + (0.05)^2 + (0.98)^2 = (1.0)^2$$

$$\beta_\rho = \frac{\rho o - m_\rho}{s_\rho} = \frac{1.0" - 0.7"}{0.23"} = \underline{\underline{1.30}} \text{ without model uncertainty}$$

$$\beta_{\rho'} = \frac{m_{\rho'} - \rho_o}{s_{\rho'}} = \frac{1.0" - 1.02"}{0.23"} = \underline{\underline{-0.09}} \text{ with model uncertainty}$$

FIGURE 2 Calculation of footing settlement.

able for calibrating the model to a particular site. This model error is difficult to divide into scatter and systematic parts because data of the type used in Figure 1 are mixtures from many sites and model tests. However, the calculations in Figure 2 attest to the importance of model uncertainty in settlement predictions.

In service, the footings were exposed only to 40 to 70 percent of the allowable load used for predicting settlements. Also, footing dimension and embedments varied. Therefore, the mean predicted settlement and the mean observed are not comparable. However, because Equation 30 is multiplicative, Ω_p should be unaffected by these differences.

CONCLUSION

The purpose of error analysis is to *(a)* identify the sources of uncertainty in engineering calculations, *(b)* estimate the magnitude of error contributed by each source, and *(c)* assess the confidence that should be attached to a calculated prediction. The methodology for performing error analyses is uncomplicated, and its routine use fosters improved quality control and reliability.

ACKNOWLEDGMENTS

The work presented in this paper was funded by the Federal Highway Administration, U.S. Department of Transportation. The author is grateful to Albert F. DiMillio and Jerry A. DiMaggio of the FHWA, and to Douglas Gifford of Haley and Aldrich, Inc. for their helpful comments and suggestions.

REFERENCES

1. L. C. P. Yam et al. *Study of Building Failures—A New Approach.* Document X-2.7. Construction Industry Research and Information Association, London, England, 1980.
2. G. B. Baecher, W. A. Marr, J. S. Lin, and J. Consla. *Critical Parameters for Mine Tailing Embankments.* Bureau of Mines, U.S. Department of the Interior, 1983.
3. L. Bjerrum. Embankments on Soft Ground. *Proc., ASCE Specialty Conference on Earth and Earth Supported Structures,* Vol. 2, Boulder, Colo., 1972, pp. 111–159.
4. D. DeGroot. Maximum Likelihood Estimation of Spatial Correlated Soil Properties. M.S. thesis. Massachusetts Institute of Technology, Cambridge, 1985.
5. Societe d'energie de la Baie James. *Report of the Committee of Specialists on Sensitive Clays of the NBR Complex.* Montreal, Canada, Aug. 1983.
6. C. Hilldale. *A Probabilistic Approach to Estimating Differential Settlement.* M.S. thesis. Massachusetts Institute of Technology, Cambridge, 1971.

Publication of this paper sponsored by Committee on Foundations of Bridges and Other Structures.