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Initial Response of Foundations on Mixed Stratigraphies

CHARLES E. WILLIAMS

A procedure for easily computing the initial settlement of shallow foundations on mixed stratigraphies has been developed. Applicable soil conditions are primarily stiff to hard clays with horizontal layers of dense to very dense sand. The Revised Gibson Model makes use of a simple equation for elastic settlement of axisymmetric footings. An equivalent modulus that accounts for the variations in soil modulus with depth beneath the footing is one of the primary input parameters to the equation. The effect of a sand layer within the foundation soils on initial settlements is included in the procedure by means of an additional factor obtained from parametric charts. Twelve case histories, including elevated and ground storage tanks and multistory buildings, are used to evaluate the effectiveness of the new initial settlement computational method.

The initial settlement component of the response of foundations to applied load is an important design consideration when the supporting soil media are comprised of stiff to hard clays. The presence of competent sand layers within the foundation stratum can effectively "stiffen" the foundation response and should be considered in design.

The Equivalent Gibson Model *(1)* has been shown to be a useful procedure for properly characterizing cohesive foundation media in the Houston, Texas, area and computing expected initial settlements for a large range of foundation sizes. The Equivalent Gibson Model has been expanded to consider the presence of competent sand layers within the supporting soils. The simplicity of the original procedure is maintained by adding only one additional design step involving the use of parametric plots.

The new procedure was evaluated by application to 12 new projects ranging from elevated and ground storage tanks to multistory buildings. Measured initial settlements are compared with those predicted by the Revised Gibson procedure.

PREVIOUS WORK

Initial settlement represents the immediate foundation response to induced shear stresses at constant volume. The remaining two components of settlement due to consolidation and secondary compression involve time-related volume change. For stratigraphies containing moderately to heavily overconsolidated clays, the initial settlement component can account for 30 to 70 percent of the total settlement response *(2)*. Consequently, the expected magnitude of initial settlement for foundations on soil strata with a large percentage of stiff to hard clay layers is a major design consideration. Development of the initial settlement component within the total response of a building foundation to applied load is shown in Figure 1.

Proper design of foundations typically results in contact pressures for footings or mats that do not produce yield zones in the foundation soils. The foundation response is to the left of the "first yield" location shown in Figure 2, which makes it possible to use elastic or linear methods of analysis to predict initial settlements.

Williams and Focht (*1*) recognized that Pleistocene clays in the Houston area typically exhibit an increase in undrained modulus with depth, and that the soil model proposed by

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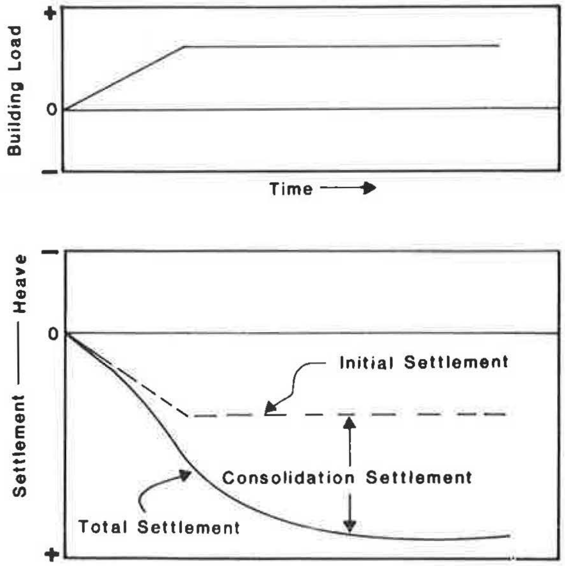

FIGURE 1 Time-settlement response.

Brown and Gibson *(3)* has possible application to such soils. They used the Gibson Soil Model shown in Figure 3 to produce an equivalent constant undrained modulus, $\bar{E}$, for a given foundation width. The equivalent modulus, $\bar{E}$, is defined as the average modulus for a given footing of width, B, which produces the same computed initial settlement as obtained using the Brown and Gibson procedure involving the modulus increasing with depth. The equivalent constant modulus could be input into the classical elastic settlement equation given below to develop an estimate of initial settlement for a given foundation width and applied contact pressure for axisymmetric foundations *(4)*:

$$\rho = PBI/\bar{E} \tag{1}$$

where

ρ = initial settlement,
P = contact pressure,
B = foundation width or diameter, and
I = geometric influence factor.

The Williams and Focht study *(1)* produced the curve shown in Figure 4 in which the equivalent modulus $\bar{E}$ is normalized with respect to the typical undrained shear strength for the foundation soils and related to the width of the foundation. Typical undrained shear strength is defined as the average undrained shear strength over a depth interval of twice the foundation width, with depth measured relative to the foundation bearing level. The band in Figure 4 is converted to a modulus profile with depth and compared in Figure 5 to modulus profiles obtained on similar soils in the Houston area using pressure meter, cyclic triaxial, and reduced cross-hole test data *(1)*.

Several case histories were applied to the model and are plotted in Figure 4. A review of that figure shows a consistent trend toward equivalent modulus values that are 50 to 100 percent higher than the Gibson curves would indicate, for cases in which significant sand layers were present within a depth range of twice the foundation width below the bearing level. Significant sand layers would be defined as relatively continuous cohesionless soil units located within the 2B depth interval beneath the foundation exhibiting a thickness of at least 10 percent of the foundation width. The most logical explanation for the improved settlement response was that the sand layers were mobilizing much higher modulus magnitudes than could be realized in the cohesive strata. The results clearly showed

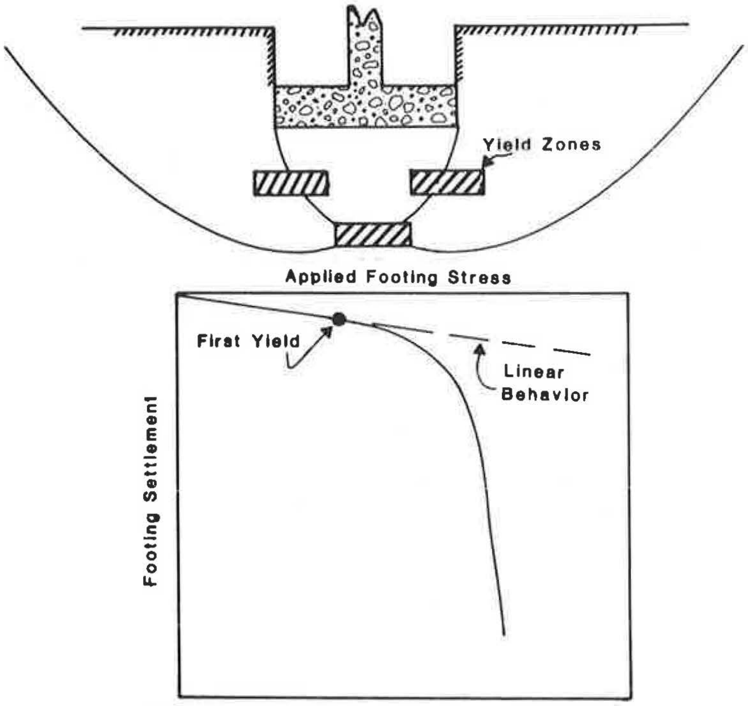

FIGURE 2 Typical load-settlement curve.

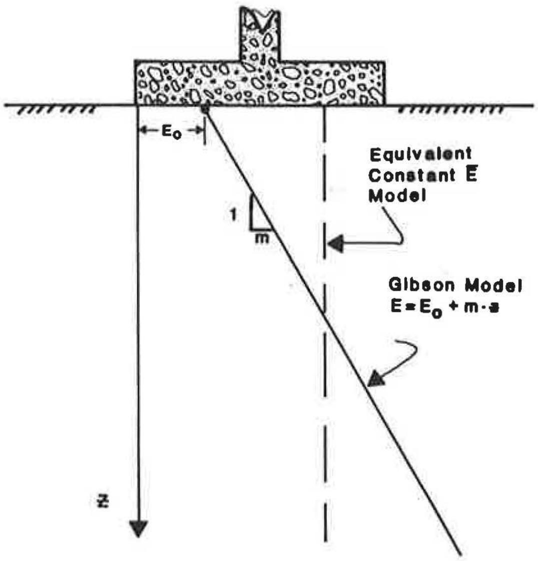

FIGURE 3 Gibson soil model.

that the presence of sand layers could improve foundation response and should be considered in design.

SAND DATA

The silty fine sands typically encountered within Pleistocene sediments in the Houston area are alluvial or deltaic in origin. The buried distributary sands are SM or SP according to the Unified Soils Classification System and may contain fines fractions (silt and clay) of 5 to 40 percent. Generally, less than 10 percent of the sand gradation is coarser than the No. 60 sieve size.

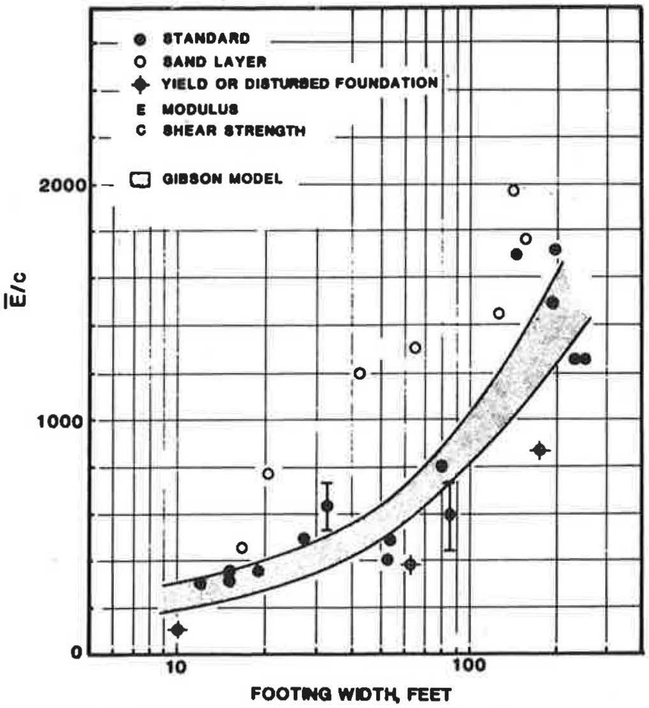

FIGURE 4 Normalized modulus versus foundation width.

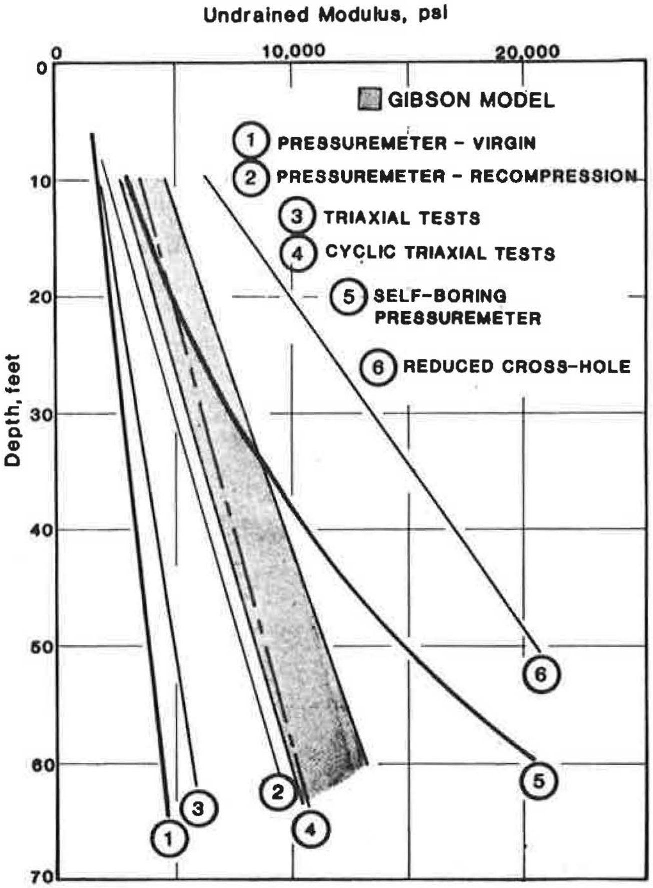

FIGURE 5 Comparison of Gibson model with test data.

Profiles of standard penetration test (SPT) resistance data on sand strata assembled from various subsurface studies in the Houston area are shown in Figure 6. The "dense" category is most commonly encountered and corresponds to a relative density range of 60 to 90 percent based on an empirical correlation between relative density, SPT resistance data, and effective

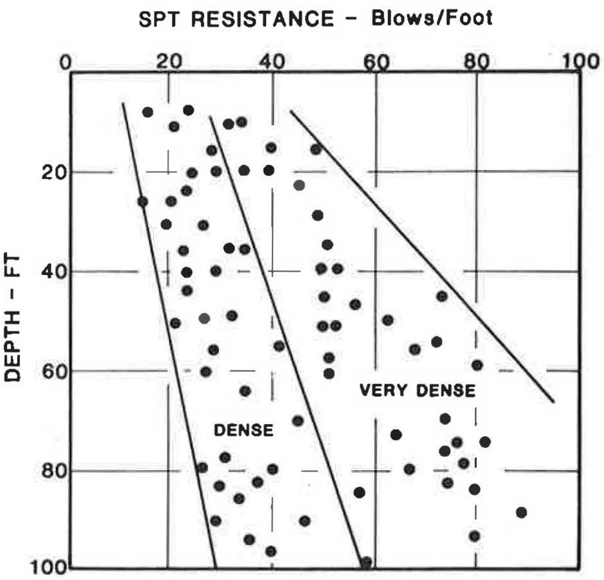

FIGURE 6 Typical sand data.

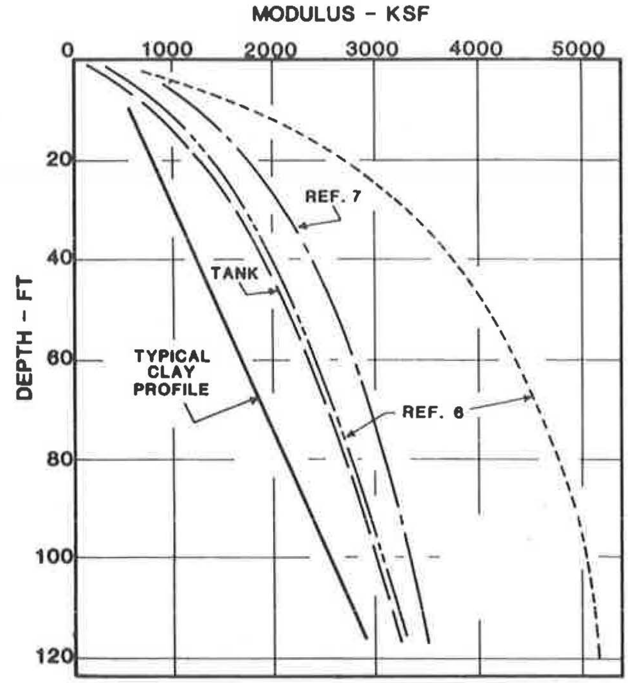

FIGURE 7 Clay and sand modulus profiles.

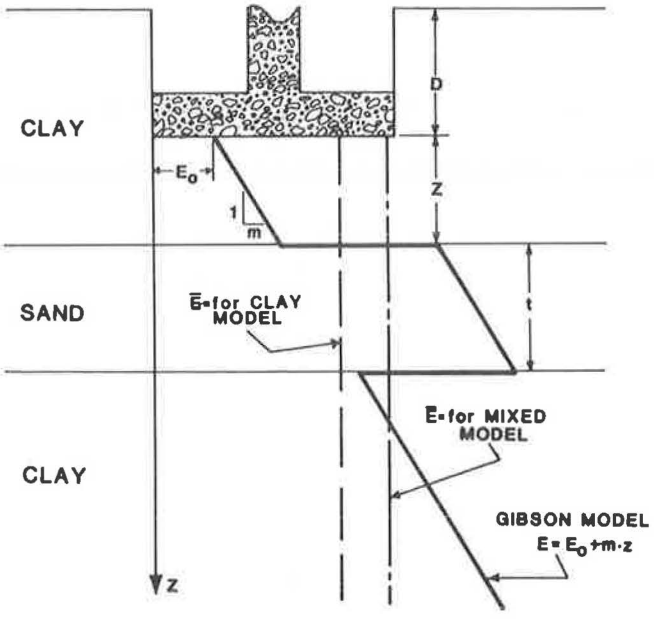

FIGURE 8 Mixed stratigraphy model.

overburden pressure *(5)*. The "very dense" category corresponds to a relative density range above 90 percent and is not encountered as frequently.

The silty sand strata are typically encountered as isolated horizontal soil layers within a primarily cohesive stratigraphy. Commonly observed sand stratum thicknesses range from 5 to 25 ft but can approach 30 to 40 ft near the center of a buried distributary channel. There are areas where two and three buried channels are geologically "stacked" upon each other and produce a total sand stratum thickness in excess of 100 ft.

Profiles of sand modulus with depth developed from a number of sources are shown in Figure 7. The curve labeled "tank" corresponds to a modulus profile backfigured from settlement measurements on a ground storage tank supported on more than 80 ft of sand from the "dense" category. The corresponding factor of safety for this foundation system was in excess of 5. The shape of the modulus profile is in accordance with the classical distribution found to be acceptable for many sands *(6)*. Also shown in Figure 7 is a curve labeled "Reference 7" corresponding to extensive cyclic triaxial and field cross-hole testing of a dense sand *(7)*. The cross-hole modulus values were reduced to 30 percent of calculated magnitudes to account for strain levels typically mobilized by loaded foundations *(8)*. The remaining two profiles are labeled "Reference 6" and correspond to dense and very dense typical sands selected by Hartman for extensive study of sand modulus *(6)*.

A typical clay modulus profile obtained from the Williams and Focht study *(1)* is also shown in Figure 7. Modulus magnitudes at a given depth for the clay profile are much smaller than the dense sands and less than one-half the very dense modulus values.

SAND FOUNDATION MODELS

The Revised Gibson Model is shown schematically in Figure 8. The primarily clay soil profile exhibits an increasing modulus with depth. The sand substratum is characterized in terms of depth below the foundation bearing level and sand layer thickness, and yields a modulus magnitude greater than that for a clay stratum at comparable depth. The equivalent constant modulus, $\bar{E}$, obtained from the mixed stratigraphy model is correspondingly higher than that computed for the homogeneous clay condition for a foundation of given size.

The various methods for computation of settlements of a foundation on sand were reviewed for possible utilization in the Revised Gibson Model procedure. The methods considered are given in Table 1 and are discussed as follows:

- Empirical procedures. The empirical procedures based on SPT or cone data generally model the soil with one representative variable and are not suited to consider layered models.
- Simple elastic models. Simple elastic models are based on a single modulus value for the foundation and cannot handle layered systems.
- Stress-based elastic models. The layered models of Webb and Oweis are useful procedures that have a theoretical base and substantial flexibility in application. However, layer distributions based on stress are greatly affected by differences in layer stiffness.
- Strain-based elastic model. The Schmertmann strain factor procedure is theoretically based and can handle layered systems. The strain influence factor approach is well documented, simple to use, and is relatively insensitive to the effects of embedment on layered soil stratifications.

TABLE 1 METHODS FOR COMPUTING SETTLEMENTS ON SAND FOUNDATIONS

Method	Description	Remarks
Terzaghi and Peck *(9)*	Empirical procedure	Very conservative
Meyerhof *(10, 11)*	Based on SPT data	Moderately conservative
Peck and Bazaraa *(12)*		
Peck et al. *(13)*		
Debeer *(14)*	Semiempirical procedure Based on cone data	Very conservative
D'Appolonia et al. *(15, 16)*	Elastic method with constant modulus	Considers modulus
Webb *(14, 17)*	Elastic layer method, stress-based	Considers modulus with depth
Schultze and Sherif *(14)*	Quasi-elastic method with empirical correlation based on SPT data	Indirectly considers modulus
Oweis *(18)*	Elm elastic model Considers layers, empirically uses SPT data	Complex procedure
Schmertmann *(5, 10, 20)*	Uses strain influence factors and considers layers	Theoretical base, model flexibility, variable modulus

The Schmertmann strain factor approach for computing settlements of foundations on sand is shown in Figure 9. The strain factor is a parameter that characterizes the distribution of vertical strain with depth beneath a footing. The unique strain factor distribution for an axisymmetric footing and the variation of modulus with depth beneath the footing can be input into Schmertmann's equation to compute expected footing settlements. The equation in Figure 9 uses a summation procedure over a depth of twice the footing width and average values of modulus and strain factor for each layer being considered. The equation also uses a factor for embedment and a separate factor for creep; however, these two parameters do not play a role in the utilization of the equation in this paper and have been, correspondingly, deleted.

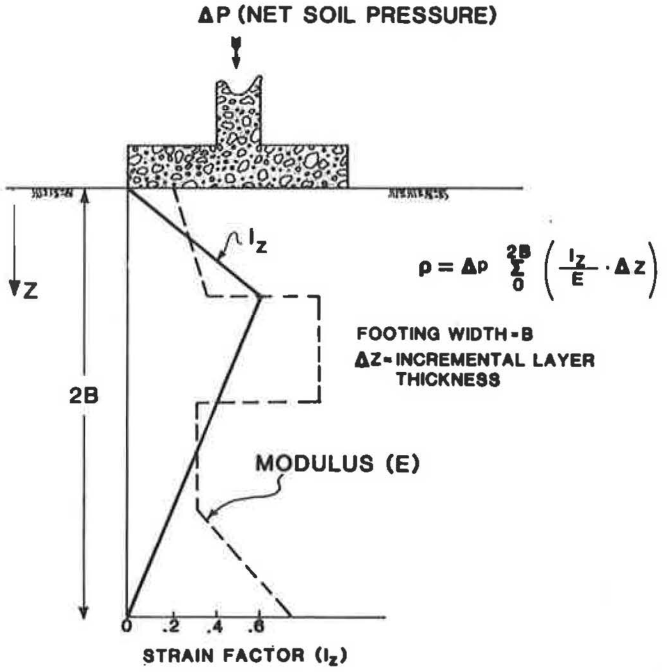

FIGURE 9 Strain factor approach.

Hartman *(6)* evaluated the Schmertmann strain factor approach in detail and found it to be applicable for soils exhibiting modulus magnitudes that increase with depth. Correspondingly, the model should be appropriate for Houston-area stiff to hard clays as well as sand strata. Hartman's findings concerning the insensitivity of the model to footing embedment and relative stiffness effects in layered soils, along with its general applicability to the Revised Gibson Model are summarized as follows:

- Triangular strain factor distribution is appropriate for soils with a nonlinear stress-dependent modulus.
- Mixed stratigraphies with "stiff" layers do not significantly affect strain factor distribution.
- Foundation embedment does not significantly affect strain factor distribution.
- There are "unique" strain factor distributions for rigid and flexible foundation units with axisymmetric geometry.

COMPUTATIONAL PROCEDURE

The Schmertmann procedure is powerful and, using the model shown in Figure 8, could compute settlements for mixed stratigraphies directly, provided detailed modulus data for the given design case were available. However, the intent of this paper is to revise the Equivalent Gibson Model and develop a conceptually simple procedure for computing initial settlements for foundations on mixed soils with a minimal amount of input data. Development of the new procedure involves the following steps:

1. Parametric characterization of a given design condition by foundation width (B), embedment of foundation (D), depth to top of sand layer (Z), thickness of sand layer (t), and competency of the sand layer.

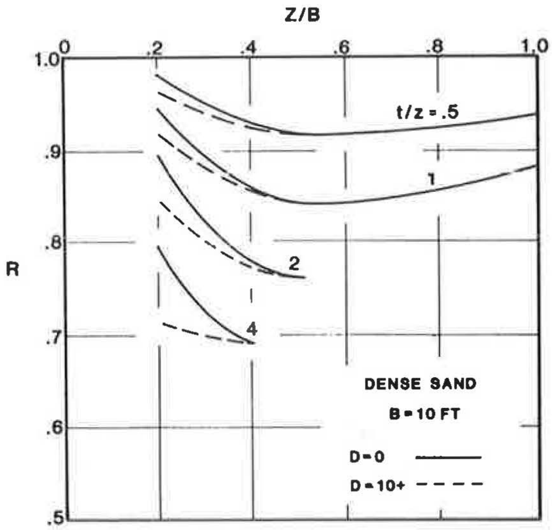

FIGURE 10 **Reduction factor (dense sand, B = 10 ft).**

2. Computation of initial settlements for homogeneous clay profile with given foundation width (B) and embedment (D) using the Schmertmann procedure.
3. Computation of initial settlements for the various mixed soil conditions grouped on the basis of the ratios (Z/B) and (t/Z).
4. Development of a ratio (R) expressed as the initial settlement computed from Step 3 for a mixed soil condition divided by the homogeneous clay initial settlement from Step 2.

Figures 10 through 13 show developed relationships between the ratio (R) and the lumped parameters for depth to sand (Z/B) and sand-layer thickness (t/Z). The curves are grouped into four charts based on competency of the sand layer and size of the foundation. The Poisson's ratio used throughout the development of the computational procedure was 0.40. Parametric studies have revealed this magnitude to yield reasonable results for competent sands *(6)*.

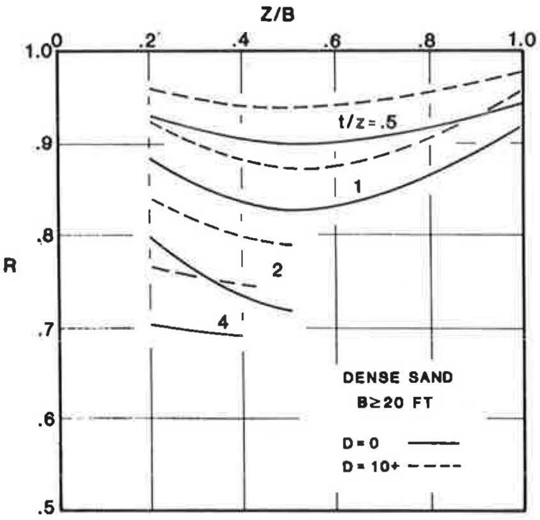

FIGURE 11 **Reduction factor (dense sand, B ≥ 20 ft).**

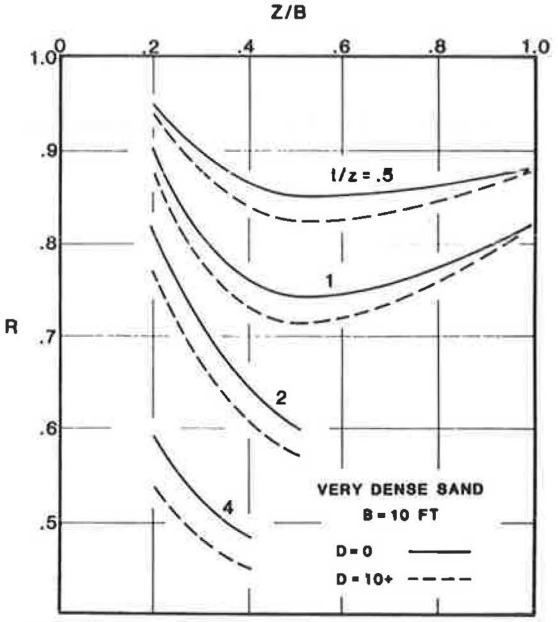

FIGURE 12 **Reduction factor (very dense sand, B = 10 ft).**

The curves in Figures 10 through 13 are generally parallel to the strain factor distribution shown in Figure 9. The maximum effect of the sand layer, interpreted as the lowest ratio (R), is found near a depth to sand (Z) of about 0.5B. The effects of shallower or deeper sands are correspondingly less. Sand layer thickness and competency of the sand also have a direct effect on the ratio (R), with (t/Z) values near 4 in very dense sands producing R values below 0.5.

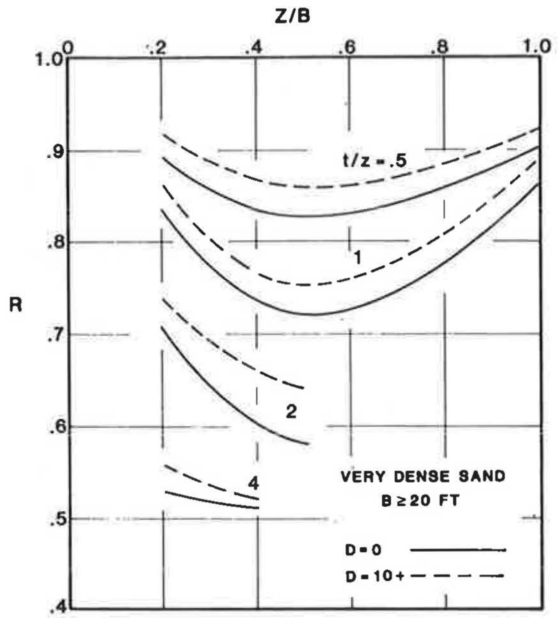

FIGURE 13 **Reduction factor (very dense sand, B ≥ 20 ft).**

TABLE 2 TEST CASES

Case	B (ft)	Z (ft)	T (ft)	Remarks
Ground tank	52	22	30	Very dense sand
Ground tanks	24–59	13	32	Three tanks
Elevated tank	53	12	24	
Elevated tank	53.5	12	9	
Elevated tank	53.5	12	37	Very dense sand
7-story garage	12	7	27+	Very dense sand, deep footings
9-story building	8–15	3	5.5–11.5	Very dense sand, 12 footings
17-story building	34–81	22	7	Two footings
19-story building	16–95	16	20	Very dense sand, 12 footings, deep footings
25-story building	134	15	20	Deep mat
25-story building	141	7	27	Deep mat
28-story building	175	30	20	Shallow mat

The effects of foundation size and embedment are more subtle and are primarily due to the parabolic shape of the sand modulus profile relative to the linear profile adopted for the clay strata. Shallow sands exhibit relatively small modulus magnitudes, which results in surface foundations with no embedment mobilizing larger R values. The effect is most pronounced for small footings and thick, shallow sands. For larger footings, embedment serves to increase R values because the sand and clay modulus profiles converge at depth.

PROCEDURE UTILIZATION

The procedure for utilization of the Revised Gibson Model is as follows:

1. Determine the average foundation width, representative undrained shear strength (c) for the cohesive strata to a depth of twice the footing width, and net increase in soil pressure at the foundation level due to the applied foundation loading.

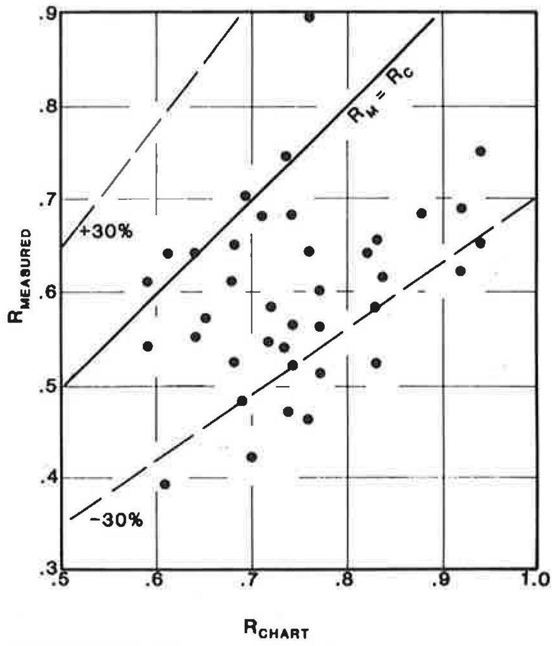

FIGURE 14 Method evaluation.

2. Enter Figure 4 and obtain a representative value of $\bar{E}/c$, which in turn can be converted to an equivalent modulus ($\bar{E}$) by multiplying by the average undrained shear strength (c).
3. Determine an appropriate geometric influence factor (I) and compute a settlement (ρ) based on Equation 1 for initial settlements on half spaces.
4. Characterize the sand substratum as dense or very dense and compute the parameters (Z/B) and (t/Z).
5. Enter the appropriate chart in Figures 10 through 13 and select an R value.
6. Multiply the previously computed settlement by the R value to obtain a modified settlement for the mixed soil condition.
7. The procedure is structured to address only one sand layer. If two distinct sand layers are present within a depth range of 2B beneath the footing, both cases should be addressed separately and the individual R factors should be multiplied together to obtain a final R factor for the entire system.

SETTLEMENT DATA

Twelve projects ranging from elevated and ground storage tanks to multistory buildings were monitored for initial settlements to provide a means for evaluating the new procedure. In some cases the projects were complete and the appropriate data were on file, and in others the new procedure was used to predict settlements during the design phase. Table 2 gives a tabulation of the 12 case histories and the parameters required for input into the Revised Gibson Model.

Values of R were computed for each of the 38 foundations monitored for settlements within the 12 case histories. These R values denoted as R CHART are plotted versus R MEASURED, the R values backfigured from the measured settlements. The comparisons are shown in Figure 14. The distribution of the data points is very encouraging in that most of the results are within 30 percent of R CHART = R MEASURED. A majority of the data points located below the R CHART = R MEASURED line indicate that the Revised

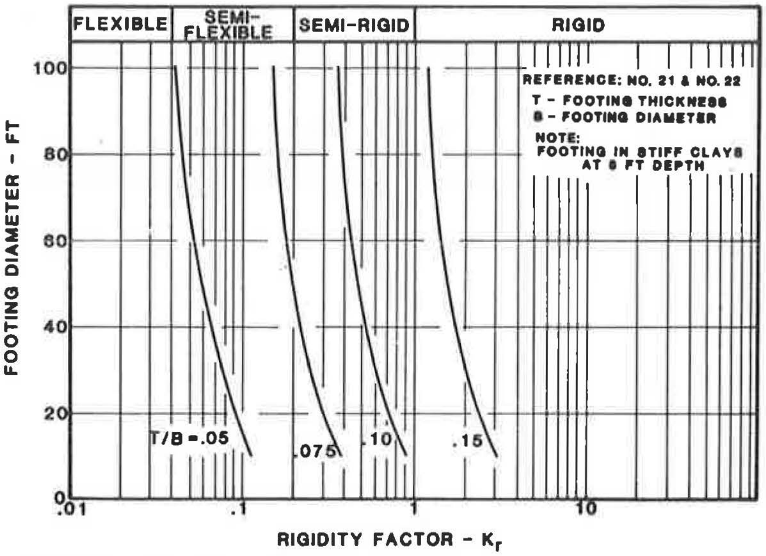

FIGURE 15 Rigidity factor – K_r.

Gibson Model for mixed stratigraphies generally underpredicts the beneficial effects of the sand substratum.

RESULTS OF THE STUDY

The Equivalent Gibson Model procedure has proven to be a simple but systematic approach to computation of initial settlements for foundations on stiff to hard clay stratigraphies that exhibit an increasing undrained modulus with depth. The Revised Gibson Model for layered stratigraphies provides an appropriate extension to the original method. The procedure has a strong theoretical base and is sufficiently detailed to address the major considerations within a foundation engineering design situation, but continues to provide the simplicity of the original Equivalent Gibson Model.

The results in Figure 14 show that the Revised Gibson Model is an effective but conservative procedure for the design cases considered to date. Of the 38 data points, 79 percent are within the ± 30 percent band; and 87 percent of the data base lies below the R CHART = R MEASURED line.

With predicted R values ranging from 0.59 to 0.94 and backfigured R values of 0.39 to 0.90, it is apparent that settlement reductions due to the presence of sand strata are real and can have an effect on foundation planning and design. In one particular case involving value engineering redesign during construction, it was possible to closely map the variable sand substratum thickness with additional subsurface work and custom tailor the individual footing bearing pressures on the basis of the Revised Gibson Model. Had it not been possible to quantify the beneficial effects of the sand stratum on reduced settlements, the redesign would not have been possible.

The new design procedure is most applicable to individual foundation units of less than 50 ft wide and with embedments of 20 ft or less. Larger mat foundations in excess of 100 ft wide were addressed in the case history study; however, understanding of the relative trends of the sand and clay modulus profiles below 100 ft depth is not strong. The semirigid response and complex loading patterns of most large mat foundations justify more detailed analytical procedures involving soil-structure interaction considerations.

It is possible to use data and procedures in this paper as a planning tool for large mat foundations loaded in a complex manner. Modulus profiles can be constructed from information contained in Figure 7, modified as required to reflect specific conditions for the given design case. Figure 15, developed from procedures offered by Terzaghi *(21)* and Brown *(22)*, can be referenced to determine the relative rigidity of the mat foundation, and the flexible and rigid strain factor envelopes in Figure 16 given relative weights based on the rigidity factor computed.

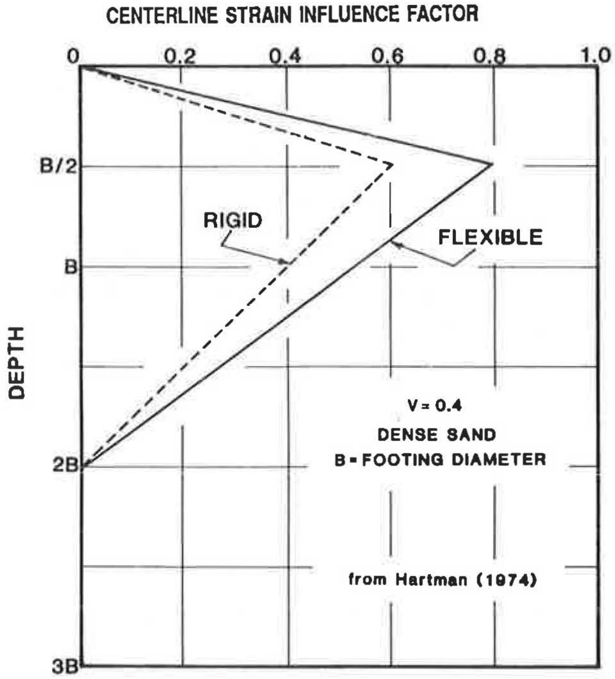

FIGURE 16 Strain factors for axisymmetric loading.

The Schmertmann procedure can then be employed to develop preliminary estimates of expected initial settlements for the mat. This information will be useful for planning purposes and for selection of subgrade modulus values required for the more detailed soil-structure interaction analysis of the mat system.

CONCLUSIONS

The Revised Gibson Model provides a strong extension to the original Equivalent Gibson Model procedure for axisymmetric loading conditions. The procedure has been shown to produce representative but conservative settlement estimates for a wide range of foundation sizes and layered stratigraphies in the Houston area.

The revised method is applicable to mixed stratigraphies comprised of stiff to very stiff clays with dense to very dense sand substrata. Different subsurface conditions will require modifications based on judgment.

The Revised Gibson Model is best suited to individual rigid foundation units of less than 50 ft wide and with embedments of 20 ft or less. Larger mat foundations with semirigid behavior and complex loading patterns should be analyzed by appropriate soil-structure interaction models. However, the procedures given in this paper can be useful in the initial planning of such mat foundation systems. As is the case for most new methods, additional calibration with more case histories would be highly beneficial to verification and expansion of the Revised Gibson Model.

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