Enhanced FREFLO: Modeling of Congested Environments

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Preliminary work with the FREFLO macroscopic freeway traffic simulation model has revealed some limitations in the model's ability to realistically simulate some congested flow conditions. The formulation and implementation of an approach that modifies FREFLO so as to address these limitations is described. This approach yields realistic simulation results for moderate, as well as severe, congested flow conditions. The basic formulation of FREFLO and the modifications under this approach are presented. The problems observed under congested flow conditions are described and simulation results from test networks are shown.

FREFLO, developed by Payne *(1)*, is a macroscopic simulation model of freeway traffic. The model is useful for the analysis of freeway operations because it allows users to simulate a wide range of freeway conditions at modest cost. Thus, it is possible to quantify the effects of projected changes in travel demand, as well as those from proposed operational improvements.

Use of FREFLO in several applications has shown that the model is limited in its ability to realistically simulate congested flow conditions. Various approaches have been suggested by Payne and others to resolve this limitation. The approaches require either some special user inputs for the congested freeway sections *(2)* or more substantial computing time *(3)*.

With support from FHWA, a new approach to this problem was formulated and implemented. The approach described in this paper permits a realistic simulation under moderately, as well as severely, congested flow conditions. At the same time, no foreknowledge of congested locations or special user inputs are required. The approach also preserves the computational efficiency of FREFLO.

BACKGROUND

FREFLO simulates the flow of traffic on freeway networks using an aggregate variable formulation based on suitably modified analogies of fluid flow. The model, successor to MACK II, was developed to evaluate the consequences of different designs, expressed in terms of traffic operational measures of effectiveness.

Initial work with this model revealed that FREFLO was limited in its ability to realistically simulate congested flow conditions *(1)*. This problem was traced to the discontinuity in the flow–density relationship at the onset of congested conditions and to the transformation of the model's formulation from continuous to discrete domain for implementation on a computer.

Payne *(2)* addressed this problem with some modifications to the equilibrium speed–density relationship, calibration of dynamic interaction parameters, and by provision of a discontinuous, speed–density relationship. Preliminary tests with these modifications produced satisfactory results under moderately congested conditions. Problems were still evident under severely congested conditions.

Babcock et al. *(3)* suggested a more fundamental approach to alleviate this problem—spatial discretization of the congested flow sections. Using a test network with bottleneck conditions, it was shown that the simulation accuracy depended on the length of the spatial steps and that very small spatial steps were required to realistically simulate congested conditions. Because such a discrete model would require excessive computing time, two adaptive schemes were presented: heuristic and natural.

The heuristic scheme places subdivisions within a subsection only where and when they are needed. Such determinations are made on the basis of geometry and flow level and are repeated at regular, user-set intervals so that the freeway is always properly, but not excessively, subdivided.

The arbitrary, often conservative, spatial discretization of the heuristic adaption can be avoided by a more natural adaption implemented through reformulation of the model into a Lagrangian (moving) reference frame. The Lagrangian reference frame differs from the Eulerian (stationary) reference frame of FREFLO in that all changes are measured relative to a particle in motion with the local flow. Due to this moving frame, the convection term in the FREFLO dynamic speed relationship is not needed. Thus, discretization becomes a natural part of the state equations.

These adaptive schemes were able to reduce the computing time to a fraction of the time required by the nonadaptive scheme. Of the two schemes, the natural adaption scheme was more accurate and efficient.

The FRECON model *(3)* was developed by implementing the heuristic adaption scheme in FREFLO. It was calibrated and validated using five peak period data sets from Santa Monica Freeway in Los Angeles. With the exception of a complex collector–distributor geometry, the simulation results were generally in agreement with the field data.

Although the spatial discretization approach is appealing, it has some limitations. Despite the adaptive scheme, the computing time requirements for proper discretization in large, congested networks can be costly. Furthermore, FRECON's ability to simulate severely congested flow conditions has not been demonstrated.

FREFLO itself is presently incorporated within the Traf *(4)* simulation system. The system allows FREFLO to interface with other simulation models that can represent neighboring arterials, urban streets, and rural road environments. Within the Traf system, an equilibrium traffic assignment model exists that may be used to provide volume and routing information to FREFLO.

The Traf system was developed and is supported under

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contract to FHWA. It is currently in a limited distribution and has been used by both the Utah and Kansas Departments of Transportation to simulate traffic operations in complex freeway and street networks. During some simulation runs, unrealistically high densities were observed on links in the congested sections of the freeway network.

With support from FHWA, it was decided to modify the current formulation of FREFLO to address the observed deficiencies. These improvements were designed to be transparent to the user in both his input specifications and computer time requirements.

The basic FREFLO formulation and its recent improvements are presented in this paper. The problems observed under congested flow conditions are described. Test results are shown and compared with FRECON and the current version of FREFLO.

FREFLO

FREFLO is based on a continuum representation of traffic, described in terms of the following equations:

$$\frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial x} = f(x,t) \quad (1)$$

$$\frac{\partial u}{\partial t} = u\frac{\partial u}{\partial x} - \frac{1}{T}\left[u - u_e(\rho) + \nu\frac{\partial \rho}{\partial x}\right] \quad (2)$$

[convection] [relaxation to equilibrium] [anticipation]

where

$\rho(x, t)$ = density of vehicles,
$u(x, t)$ = space-mean speed,
$q(x, t)$ = flow rate,
$f(x, t)$ = net flow entering freeway section,
$u_e(\rho)$ = equilibrium speed–density relationship,
T = relaxation coefficient,
ν = anticipation coefficient,
x = position along the freeway, and
t = time.

Equation 1 of the continuum formulation represents the conservation of vehicles. The variable $f(x, t)$ thus corresponds to on-ramp and off-ramp traffic. The dynamic speed–density relationship is contained in Equation 2. The three groups of terms in Equation 2 represent three components of the time rate of change of speed: (a) convection, the effect of vehicles from upstream arriving at this point; (b) relaxation to equilibrium, the effect of drivers adjusting their speeds to the equilibrium speed–density relationship; and (c) anticipation, the effect of drivers reacting to changing travel conditions ahead, that is, to (decrease, increase) speed in anticipation of (higher, lower) density downstream.

The discrete, computer implementable equations from the continuum formulation were derived by spatial averaging and Euler integration *(5)*. Corresponding to the freeway section in Figure 1, the transformed equations are

Conservation:

$$\rho_j^{n+1} = \rho_j^n + \frac{\Delta t}{l_j \Delta x_j}\left[q_{j-1}^n + f_j^{on,n} - q_j^n - f_j^{off,n}\right] \quad (3)$$

Dynamic speed–density:

$$u_j^{n+1} = u_j^n + \Delta t\left[-u_j^n \frac{(u_j^n - u_{j-1}^n)}{\Delta x_j} - \frac{1}{T}\left(u_j^n - u_e(\rho_j^n) + \nu\frac{(\rho_{j+1}^n - \rho_j^n)}{\Delta x_j}\right)\right] \quad (4)$$

where

n = current point in time (superscript);
j = current section (subscript);
l_j = number of lanes along section, j;
Δx_j = length of section, j; and
Δt = time step.

Equation 5 relates volume, density and speed under uniform conditions within a section.

Flow:

$$q_j^{n+1} = \rho_j^n u_j^n \quad (5)$$

This relationship is adapted from several alternatives primarily because of stability considerations *(5)*. The terms "section" and "link" are used interchangeably.

Although Equations 3–5 form the basis of the simulation model, some modifications are required to make provisions for the links at the boundaries of freeway segments, that is, links with no adjoining upstream or downstream links. The boundary conditions are set as follows:

$u_{j-1}^n = u_j^n$ if no adjoining upstream link

$\rho_{j+1}^n = \rho_j^n$ if no adjoining downstream link

Some other modifications in computing upstream speed and downstream density—in the dynamic speed–density equation—are required to represent the general freeway subnetwork link of Traf (see Figure 2) wherein a link can have two feeder and two receiver links *(6)*. This feature allows for the simulation of merging and diverging freeways.

For a general freeway link, the upstream speed and downstream density are computed as the weighted average of the flow according to the following relations:

$$\bar{u}_j^n = \frac{u_{f1}^{n-1} q_{f1}^{out,n-1} P_{f1,j} + u_{f2}^{n-1} q_{f2}^{out,n-1} P_{f2,j}}{q_j^{in,n-1}};$$

$$q_j^{in,n-1} = 0 \quad (6)$$

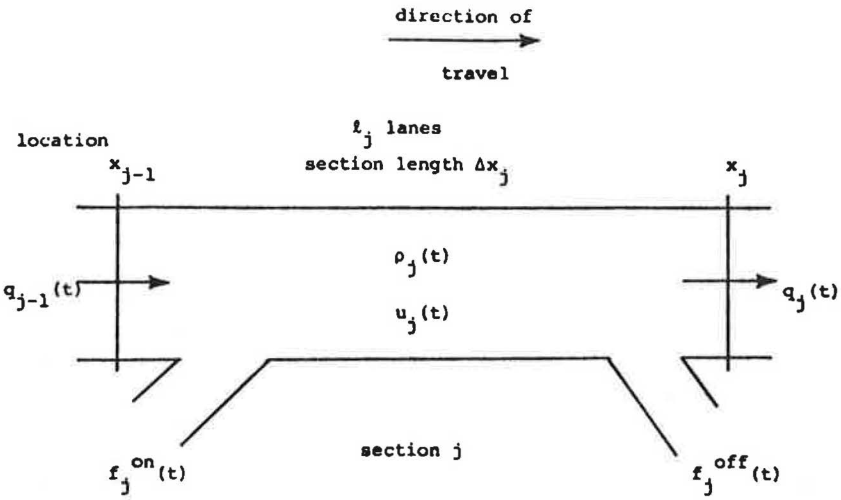

FIGURE 1 Freeway section depicting aggregate variables for a discrete formulation.

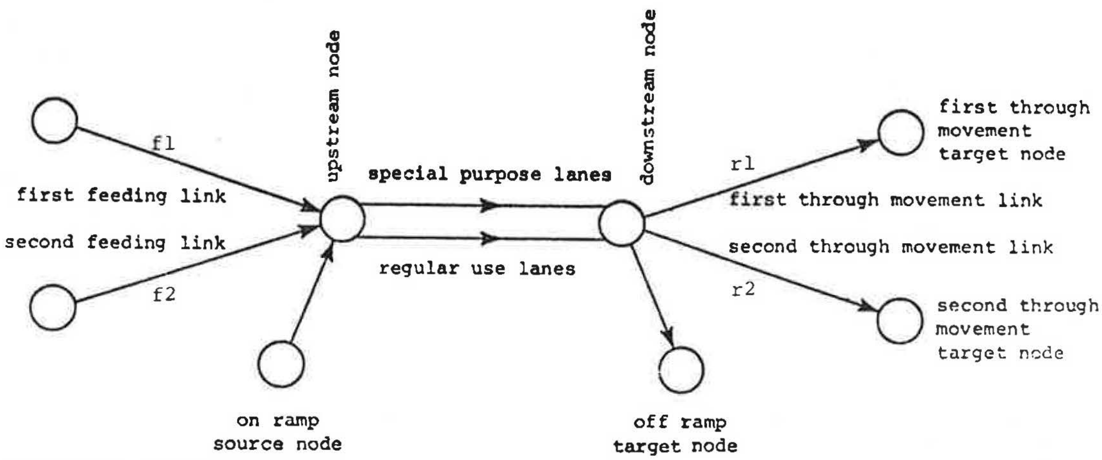

FIGURE 2 General freeway subnetwork link.

$$\bar{u}_j^n = 0; \qquad q_j^{in,\,n-1} = 0 \tag{7}$$

$$\bar{\rho}_j^n = \frac{\rho_{r1}^{n-1}\, q_j^{out,\,n-1}\, P_{j,r1} + \rho_{r2}^{n-1}\, q_j^{out,\,n-1}\, P_{j,r2}}{q_j^{out,\,n-1}}; \qquad q_j^{out,\,n-1} = 0 \tag{8}$$

$$\bar{\rho}_j^n = 0; \qquad q_j^{out,\,n-1} = 0 \tag{9}$$

where

$\bar{u}_j^n$ = space-mean speed of vehicles upstream of link j at the beginning of time step n,
u_j^n = space-mean speed of vehicles on link j at the end of time step n,
$q_j^{in,\,n}$ = entry flow rate on link j over time step n,
$q_j^{out,\,n}$ = exit flow rate on link j over time step n,
$\bar{\rho}_j^n$ = density of vehicles downstream of link j at the beginning of the time step n,
ρ_j^n = density of vehicles on link j at the end of time step n,
$P_{i,j}$ = fraction of flow rate of link i that enters link j,
f_1 = first feeding link,
f_2 = second feeding link,
r_1 = first receiving link, and
r_2 = second receiving link.

PROBLEMS

The departments of transportation in Utah and Kansas have been using Traf to simulate traffic operations on some large networks, which included arterials, freeways, and feeder streets. Due to capacity problems and demand pattern in the freeway corridor, some sections of the freeway network experienced severe congestion. During simulation runs, FREFLO did not reflect the effects of these severely congested conditions as might be expected. Two problems were observed:

1. The densities on the congested freeway links were unrealistically high as more vehicles were allowed to enter a congested link than could be physically accommodated. Consequently, the congestion did not propagate upstream as expected.
2. In cases where one leg of a diverging freeway section experienced severe congestion, the density and entering volumes on the other noncongested leg were much lower than expected.

The simulation results in Table 1 from a test network shown in Figure 3 illustrate these problems. For this test network, the entry volume was set to 2,500 vehicles per hour (vph). The capacity of Sections 1–7 and 8–10 was set to 3,000 vph and 1,000 vph, respectively. The turn specifications from Section 5 to Sections 6 and 8 were such that the expected demand on Section 8 was 1,875 vph. With this demand, capacity flows were expected in Sections 8 through 10 and queuing on Section 5 and upstream. A time slice, Δt, of 4 sec was used.

Because the inflow rate exceeds the outflow capacity, the density on Section 8 increases with time (Table 1). The inflow rate starts to decrease as the feeder link (Section 5) responds to the downstream congestion by reducing the outflow. Due to this reduction in outflow, the density on Section 5 also increases with time.

After only 180 sec of simulation, the density on Section 8 reaches 252 vehicles per lane-mile, which is higher than attainable. This occurs because the anticipation term in the dynamic speed relationship (Equation 4) used by FREFLO underestimates the extent that speeds would be reduced in response to severe downstream congestion. After 260 sec, the outflow from Section 5 has dropped to zero and its density increases rapidly. Although discharge flow is terminated from Section 5, the flow entering downstream Sections 6 and 8 is not immediately reduced. Because volumes entering a section are computed at the beginning of a time slice, feeding flow may lag by one time slice if a link is processed before its feeder(s).

For the next time slice, Equation 9 is applied to compute the downstream density on Sections 6 and 8 as zero because the outflow rate from Section 5 is zero. As a consequence of this spurious result, Section 5 discharges vehicles to both Sections 6 and 8 to the maximum extent possible.

Also, the density on Sections 6 and 8 decreases because flow was terminated from Section 5 during the previous time slice. The indicated decrease in density in Section 6 is unrealistic because under these conditions only the flow entering the congested receiver (Section 8) should be reduced and not the flow entering Section 6. This result occurs because the current formulation of FREFLO does not consider the downstream density of each receiver separately when outflow is computed in a diverging scenario. Instead a single, weighted downstream density is computed based on the flow for each movement. This formulation allows maximum outflow from the feeding section, thereby increasing the density still further on the congested receiver. This problem is exhibited in Table 1 at time 280.

In the subsequent time slices, flow continues to enter Section 8 despite unrealistically high densities within the section. As a

TABLE 1 TEST NETWORK SIMULATION RESULTS—CURRENT FREFLO

Time (sec)	q_5^{in}	q_5^{out}	ρ_5	u_5	q_6^{in}	q_6^{out}	ρ_6	u_6	q_8^{in}	q_8^{out}	ρ_8	u_8
60	2498	2419	23.2	45.2	605	598	6.0	50.3	1814	1048	119.0	8.0
120	2491	2347	30.6	36.9	590	583	6.3	46.2	1771	1030	187.3	5.2
180	2464	2117	40.1	24.1	547	540	6.5	40.9	1638	1019	251.5	3.9
190	2484	2441	42.8	21.5	529	443	6.5	39.8	1588	1012	260.6	3.8
200	2441	1930	45.7	18.4	504	468	5.9	38.7	1512	1015	267.2	3.7
210	2462	1764	49.4	14.9	482	475	6.2	37.6	1447	1012	274.5	3.6
220	2470	1548	54.5	10.7	441	446	6.2	36.2	1323	1008	280.8	3.5
230	2462	1195	61.3	5.8	387	414	6.2	34.7	1161	1004	285.5	3.5
240	2462	648	70.6	0.2	299	374	6.0	33.0	896	990	287.8	3.5
250	2448	14	83.9	0.1	162	302	5.4	31.1	486	983	286.4	3.5
260	2476	0	102.3	0.0	4	216	4.2	30.9	11	972	277.8	3.7
270	2412	1303	120.5	11.6	0	140	0.0	40.8	0	965	0.0	3.9
280	2362	2632	128.6	7.1	326	148	1.6	35.1	977	979	249.4	4.0
290	2333	1570	126.6	3.4	658	256	2.9	33.5	1974	1015	249.4	3.9
300	2354	1224	132.2	5.5	392	385	5.9	32.4	1177	1015	263.4	3.8
360	2290	1526	170.3	4.3	382	382	5.8	32.6	1141	1004	271.6	3.7
540	1519	1498	172.2	4.3	374	374	5.8	32.5	1123	1001	303.5	3.3
720	1523	1494	186.6	4.0	371	371	5.7	32.4	1112	997	335.5	3.0
900	1483	1483	187.7	4.0	371	371	5.8	32.3	1112	997	368.1	2.7

q_j^{in} = entry flow rate on link j (vehicles/hour) q_j^{out} = exiting flow rate on link j (vehicles/hour)

ρ_j = computed density on link j at the beginning of time slice (vehicles/lane-mile)

u_j = computed speed on link j at the end of time slice (miles/hour)

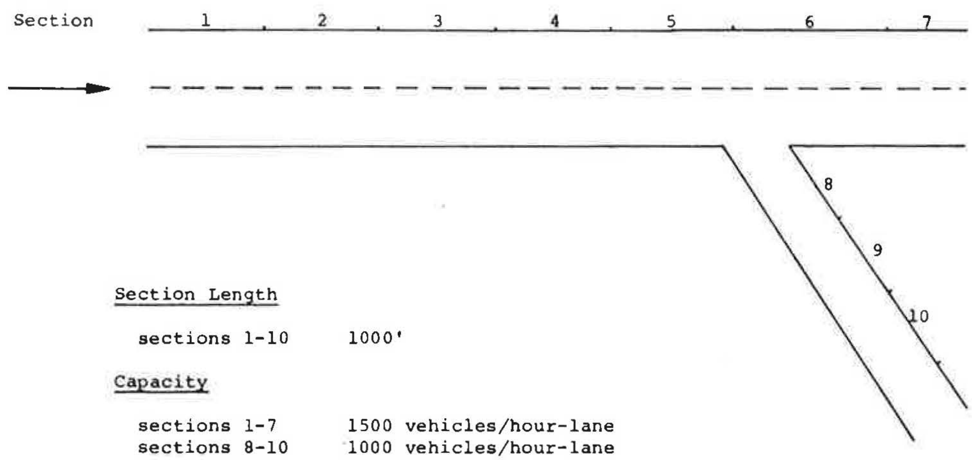

FIGURE 3 Nonlinear test network.

result, congestion propagates upstream much more slowly than expected. The entering volume and density on Section 6, on the other hand, are smaller than expected.

MODIFICATIONS TO FREFLO

The problem of excessively high densities on congested links was addressed through an approach that not only provides a sensible description of traffic operations during congested flow conditions but also eliminates the necessity of specific user inputs for congested freeway sections. The approach also preserves the computational efficiency of FREFLO. Under this approach, flow restrictions were implemented on the congested links and speeds were computed considering congestion on the links rather than using the dynamic speed equation of FREFLO. The densities, of course, were then computed from the conservation equation. Because the dynamic–speed–density equation is not used under severely congested conditions, recalibration and reformulation of FREFLO are not required.

The problems associated with the computation of flow rate for the case of imbalanced downstream density were addressed by weighting the downstream density effect of two discharging movements on the basis of overall turning percentage rather than on the instantaneous flow rates. This weighting scheme ensures that the computed densities reflect the actual conditions downstream of the section. The flow percentage-based weighting scheme is equivalent to the flow rate-based weighting scheme (Equation 8) as long as the outflow from the feeder section is not terminated.

The modifications are applied, in general, whenever the density of a section exceeds a prespecified congestion value, regardless of geometric configuration. In the absence of congestion, simulation is performed using the current FREFLO formulation. The modifications are described in detail elsewhere *(7)*.

SIMULATION RESULTS

The simulation program was modified to include the special congestion scenarios described above. Under these modifications, the special treatment for congested links was applied whenever link density exceeded 120 vehicles per lane-mile. To evaluate the effects of these modifications, traffic flow was simulated for both straight and diverging freeway sections. For both configurations, congestion was created with capacity constraints. The results are described in the following sections.

DIVERGING FREEWAY SECTION

After FREFLO was modified to incorporate the new congestion mechanism, simulation was again performed on the test network shown in Figure 3. The new results are presented in Table 2. Note that all figures represent values associated within a single time slice, Δt, of 4-sec duration; these are not time-averaged values.

In comparison with the original FREFLO results (Table 1), it is seen that the modified FREFLO produced much lower and more realistic densities in the congested Sections 5 and 8. The maximum value of density computed for any one time slice under the new logic was 152 vehicles per lane-mile compared with a maximum value of 368.1 vehicles per lane-mile under current FREFLO.

The congestion on Section 8 also propagates upstream to Section 5 much faster as a result of the modifications. As shown in Table 2, the outflow from Section 5 to Section 8 was reduced to zero at 70 sec of simulation after the density on Section 8 reached a congested level of 122 vehicles per lane-mile. In the original run (Table 1) outflow from Section 5 was not cut off in any time step until 270 sec of simulation; at that time the density on Section 8 had reached 278 vehicles per lane-mile.

With the new logic the density continued to build on Section 5 until it too became congested. Entry flow to Section 5 was ultimately reduced to zero at 220 sec of simulation. In the original run, the entry flow to Section 5 was never restricted to zero.

After Section 8 reached a congested density at 60 sec of simulation, a realistic stop-and-go pattern of entering flow was established under the modified FREFLO. This intermittent entry flow pattern was not observed in the original run.

In addition to these improvements for the congested links, the modified FREFLO also allowed a higher level of flow into uncongested Section 6. With the modifications, the density and entering volumes on Section 6 were less affected by the congestion on Section 8 than they had been under current

TABLE 2 VOLUME LEAVING SECTIONS (vehicles/hour)—CURRENT FREFLO

Time (sec)	q_5^{in}	q_5^{out}	ρ_5	u_5	q_6^{in}	q_6^{out}	ρ_6	u_6	q_8^{in}	q_8^{out}	ρ_8	u_8
30	2491	2419	24.6	48.3	605	590	5.7	51.9	1814	1062	88.9	10.6
40	2498	2419	25.1	47.2	605	598	5.8	51.3	1814	1055	99.9	9.5
50	2491	2412	23.7	46.1	605	598	5.9	50.7	1814	1051	111.0	8.6
60	2498	598	26.3	30.1	605	598	6.0	50.2	1811	1048	122.2	7.8
70	2491	605	40.2	22.4	598	562	6.0	42.8	0	983	133.4	7.9
80	2477	2639	54.1	27.7	605	526	6.2	39.3	0	936	119.0	8.9
90	2441	2898	52.9	29.4	659	558	6.8	41.1	1980	1004	105.2	8.9
100	2441	2884	49.5	31.4	724	644	7.6	42.0	2174	1062	119.5	7.8
110	2470	727	46.2	24.6	720	691	8.1	42.9	2164	1058	135.8	7.0
120	2984	727	59.0	20.2	727	698	8.3	40.2	0	994	152.0	6.9
130	2484	727	71.9	17.1	727	677	8.6	38.4	0	943	137.4	7.7
140	2441	727	84.7	14.9	727	684	8.9	37.1	0	936	123.5	8.6
150	2419	2905	97.3	15.7	724	684	9.2	36.3	0	929	109.7	9.7
160	2398	2927	93.7	16.4	727	698	9.6	36.5	2178	1012	96.0	9.5
200	2470	727	111.4	11.8	734	713	9.9	35.6	0	940	131.5	8.0
210	2426	2934	124.1	12.3	727	713	10.1	35.1	0	932	117.6	9.0
220	0	2826	120.4	13.7	734	720	10.2	35.3	2200	1015	103.9	8.8
300	2876	745	133.5	10.0	752	731	10.4	34.9	0	997	145.5	7.3
600	2506	634	99.4	11.3	637	671	9.8	35.4	0	990	140.0	7.6
900	2232	634	96.5	11.8	634	680	9.8	35.6	0	990	147.0	7.0

q_j^{in} = entry flow rate on link j (vehicles/hour) q_j^{out} = exiting flow rate on link j (vehicles/hour)

ρ_j = computed density on link j at the beginning of time slice (vehicles/lane-mile)

u_j = computed speed on link j at the end of time slice (miles/hour)

FREFLO. The traffic densities in Sections 1–10 under modified FREFLO are given in Table 3. The results show the spread of congestion upstream of Section 8 as expected. Such timely propagation of congestion to upstream sections was not observed under the current version of FREFLO.

Thus, the modified FREFLO allows for stop-and-go operations of traffic only onto the congested branch of a diverging freeway section. Neither the current FREFLO nor FRECON has such a response mechanism; that is, the flow is terminated to both receivers. Therefore, both FRECON and current FREFLO are limited in their applicability for diverging freeway sections operating under congested conditions.

LINEAR FREEWAY SECTION

This test section (Figure 4) was identical to the bottleneck geometry used by Babcock et al. *(4)* in their demonstration of the effect of spatial discretization. The traffic flow in this network simulated for a period of 10 min using both current FREFLO and its modified version. The lane capacity was set to 2,000 vph and entry volume was set to 4,500 vph. With this demand level and capacity specification, capacity flows were expected in Sections 6 through 10 and queueing on sections upstream of Section 6.

Tables 4 through 9 give the simulated output volumes and

TABLE 3 NONLINEAR TEST NETWORK SIMULATION RESULTS—MODIFIED FREFLO

Time (min.)	Section 1	2	3	4	5	6	7	8	9	10
1	24.0	23.9	23.7	23.8	40.2	6.0	5.6	133.4	48.9	30.5
2	24.0	23.9	23.8	25.1	71.9	8.6	6.9	137.4	59.0	38.2
3	24.0	23.9	23.9	28.0	98.6	9.9	7.8	146.1	63.5	44.5
4	24.0	23.9	25.4	67.3	102.9	9.9	8.0	122.2	68.1	49.8
5	24.0	23.9	25.9	69.6	149.1	10.6	18.0	130.8	71.5	54.3
6	24.0	24.7	69.2	94.1	116.8	9.8	8.1	106.2	73.7	58.0
7	24.0	26.0	50.5	117.7	149.5	10.4	8.1	113.5	76.2	61.8
8	24.0	45.4	138.3	102.2	112.3	9.3	7.9	117.0	78.7	64.9
9	32.2	94.2	79.6	87.0	126.1	10.1	7.9	123.1	81.3	67.8
10	40.7	109.3	118.7	93.5	93.5	9.5	8.0	125.5	82.4	70.4

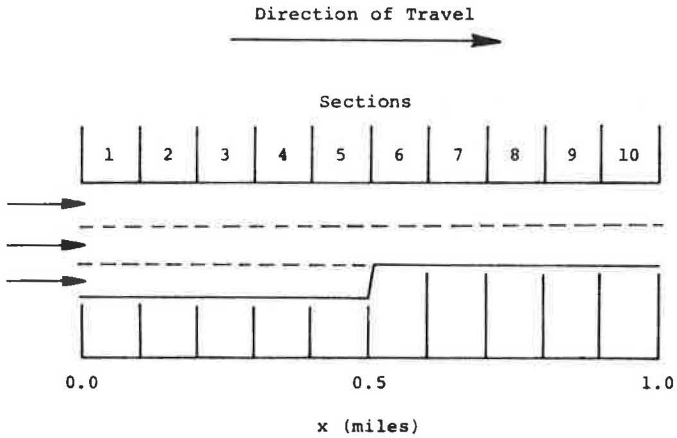

FIGURE 4 Linear test network.

TABLE 4 SECTION DENSITY (vehicles/lane-mile) ON NONLINEAR NETWORK—MODIFIED FREFLO

Time (min.)	Section 1	2	3	4	5	6	7	8	9	10
1	4500	4500	4500	4486	4428	4075	3956	3866	3802	3740
2	4500	4500	4500	4486	4342	4057	3971	3906	3845	3791
3	4500	4500	4500	4471	4144	4024	3978	3928	3877	3827
4	4500	4486	4500	4442	3942	3978	3971	3935	3902	3856
5	4500	4500	4500	4457	4086	4010	3974	3946	3892	3870
6	4500	4500	4500	4428	3956	4014	3982	3964	3924	3877
7	4500	4486	4471	3845	5324	3985	3924	3917	3902	3877
8	4500	4486	4428	4000	3074	3827	3989	3946	3920	3895
9	4500	4500	4442	3917	4295	4158	3982	3949	3920	3888
10	4486	4500	4457	3175	5366	3992	3949	3946	3913	3881

NOTE: These are instantaneous volumes of the indicated times over a time-step of four seconds.

TABLE 5 VOLUME LEAVING SECTIONS (vehicles/hour)—FRECON WITH $\Delta x = 0.01$ mi

Time (min.)	Section 1	2	3	4	5	6	7	8	9	10
1	4468	4465	4463	4461	4368	3971	4000	4000	3999	4000
2	4487	4485	4485	4483	3942	3986	3953	4023	3991	4001
3	4495	4495	4494	4493	3974	3977	3968	3969	4016	3990
4	4498	4498	4498	4490	3973	3975	3975	3967	4020	3966
5	4499	4499	4499	4218	3974	3975	3975	3974	3979	3985
6	4500	4500	4500	3960	3975	3975	3975	3975	3975	4028
7	4500	4500	4500	3975	3975	3975	3975	3975	3975	4001
8	4500	4500	4459	3975	3975	3975	3975	3975	3974	3977
9	4500	4500	4041	3974	3975	3975	3975	3975	3975	3975
10	4500	4500	3977	3975	3975	3975	3975	3975	3975	3975

TABLE 6 VOLUME LEAVING SECTIONS (vehicles/hour)—MODIFIED FREFLO

Time (min.)	Section 1	2	3	4	5	6	7	8	9	10
1	4500	4500	4500	4500	4428	4068	3953	3877	3802	3773
2	4500	4500	4500	4540	4	4136	4010	3931	3877	3823
3	4500	4500	4500	4612	5677	4284	4003	3935	3899	3834
4	4500	4500	4201	4741	4975	3989	3848	3928	3899	3852
5	4500	4500	4597	5634	4	4194	3992	3910	3902	3881
6	4500	4500	4597	7	0	3269	3982	3924	3899	3877
7	4612	5022	5778	5576	4694	4133	3974	3895	3881	3888
8	4244	5522	4746	5335	4946	4108	3956	3920	3899	3888
9	3902	0	5314	4334	4579	3982	3942	3895	3877	3888
10	5065	4	4	5033	3791	4162	3827	3895	3913	3877

NOTE: These are instantaneous volumes of the indicated times over a time-step of four seconds.

TABLE 7 VOLUME (vehicles/hour) LEAVING SECTION 5—MODIFIED FREFLO

Slice	Time (min.) 1	2	3	4	5	6	7	8	9	10
1	4442	4428	2858	5692	5292	4	2927	4622	4907	4716
2	4442	4428	5324	7	5393	3323	5036	7	4792	4640
3	4442	4414	5724	7	5425	4820	5159	0	4676	4532
4	4442	0	5580	2729	5310	5249	5000	3236	7	4453
5	4442	0	5623	5710	4	5080	4849	5044	4	4450
6	4442	2959	5648	5710	4	4612	3802	5198	3265	4460
7	4442	5224	5548	5663	3312	4385	4018	5058	5101	4460
8	4428	5710	7	5681	5137	4536	4288	4918	5198	7
9	4442	5512	7	5706	5152	4655	4568	3334	5072	0
10	4442	5580	2786	7	5022	4810	4784	3438	4889	3053
11	4428	5548	5638	7	5166	4907	4831	3762	3377	5130
12	4428	5263	5695	2686	5209	4950	4824	4180	3550	5184
13	4428	5008	5638	4871	5198	4	4792	4547	3838	5044
14	4428	7	5652	5159	5098	4	4752	4867	4201	4856
15	4428	4	5677	4975	4	0	4694	4946	4579	3791
Average Outflow	4436	3605	4493	3640	4048	3422	4554	3810	3830	3918
Overall Average Outflow					3974					

densities obtained using current FREFLO, modified FREFLO, and FRECON. The volumes are the instantaneous flow rates leaving the downstream boundary of the section and densities are the average section density at the end of the given simulation time.

The results in Tables 4 through 6 show that all three simulations indicate that traffic volume is at (or near) capacity on Sections 6 through 10, as expected. For sections upstream of Section 6, however, different patterns of output volume were generated by the three programs. Note that the simulation results with current FREFLO shown here differ somewhat from the results presented by Babcock et al. *(4)* in their comparison of FRECON and FREFLO because some modifications *(3)* have been incorporated into FREFLO since that time.

With the current FREFLO, the volume allowed to enter a congested section decreases gradually to the capacity of the bottleneck and follows an oscillating pattern thereafter (Sections 4 and 5 in Table 4). That is, the output volume fluctuates

TABLE 8 SECTION DENSITY (vehicles/lane-mile)—CURRENT FREFLO

Time (min.)	Section 1	2	3	4	5	6	7	8	9	10
1	27.2	27.2	27.3	27.7	33.7	105.7	72.0	57.6	48.9	43.8
2	27.2	27.2	27.3	27.9	38.0	130.4	80.2	63.8	54.4	49.0
3	27.2	27.2	27.4	28.6	48.8	146.0	85.3	68.9	59.3	54.0
4	27.2	27.3	27.5	30.7	76.9	144.0	87.4	72.1	63.6	58.8
5	27.2	27.3	27.6	32.8	96.6	147.9	89.0	63.9	65.9	62.0
6	27.2	27.3	27.8	35.5	118.6	152.2	91.6	76.6	68.9	64.7
7	27.2	27.4	28.5	50.1	148.9	126.0	88.0	76.9	70.8	67.8
8	27.3	27.5	31.4	83.9	151.6	121.0	91.1	79.3	73.2	70.3
9	27.3	27.7	33.5	105.5	142.7	125.6	90.6	80.9	76.0	73.6
10	27.4	27.9	36.4	124.4	151.1	119.2	91.2	82.9	78.2	75.9

TABLE 9 SECTION DENSITY (vehicles/lane-mile)—MODIFIED FREFLO

Time (min.)	Section 1	2	3	4	5	6	7	8	9	10
1	27.2	27.2	27.3	27.7	34.8	115.0	74.4	58.7	49.4	43.9
2	27.2	27.3	27.5	29.0	82.7	103.6	79.7	63.5	54.5	49.2
3	27.2	27.3	27.7	32.6	78.8	117.5	78.3	65.6	58.5	53.5
4	27.2	27.3	29.6	70.3	87.3	99.7	74.9	68.7	61.4	57.0
5	27.3	27.6	30.7	120.4	115.9	128.2	82.5	69.1	62.7	59.5
6	27.3	27.7	31.5	109.6	146.3	87.4	85.3	71.1	64.2	61.0
7	29.5	36.9	74.4	79.8	91.2	119.1	80.8	68.9	63.5	61.5
8	32.8	59.3	88.7	81.8	87.9	109.2	80.2	79.3	64.6	61.8
9	33.1	117.2	106.0	71.9	85.7	104.1	80.5	71.0	65.5	63.3
10	44.4	81.0	153.9	115.4	63.9	107.8	79.6	71.2	67.5	64.5

around the capacity of the bottleneck. The oscillations in output volume within each time step are substantial.

The simulation results obtained from FRECON using a spatial step size of 0.01 mi indicate that the output volume from a section upstream of a congested section decreases gradually to the capacity of the bottleneck and remains at that level thereafter with virtually no oscillations (Sections 4 and 5 in Table 5). This implies a complete absence of turbulence at a bottleneck location.

With modified FREFLO, an interesting pattern is revealed as shown in Table 6. The instantaneous volumes entering the congested Sections 2 through 5 upstream of the bottleneck vary widely from zero to above capacity. These oscillations represent the realistic stop-and-go conditions in these sections.

Volumes leaving Section 5 during each 4-sec time slice over the 10-min period are given in Table 7. During the first 72 sec, there are no perturbations in flow because the congestion in Section 6 has not yet propagated upstream to Section 5. Subsequently, the outflow exhibits short-term oscillatory behavior. Observe that the volume leaving Section 5, as tabulated for each minute in Table 6, is actually the output volume during the 15th time slice in each minute, as given in Table 7. The 1-min averaged volumes and the overall 10-min averaged volume are also given in Table 7. The 1-min averages oscillate around the bottleneck capacity, while the long-term, 10-min average of 3,974 vph approximates the bottleneck capacity of 4,000 vph. Thus, the modified FREFLO not only provides a realistic pattern of turbulent flow upstream of the bottleneck but is also able to maintain the desired level of throughput.

The section densities given in Tables 8 and 9 and Figure 5 show the spread of congestion upstream of Section 6 and the presence of an acceleration region downstream in all three simulations.

The simulated section densities under the two versions of FREFLO are similar and are typical for congested conditions (Tables 8 and 9). Although densities on the congested Section 6 for the current FREFLO (Table 8) are somewhat higher than the density normally observed on congested freeways (100–120 vehicles per lane-mile), they are not as high as those computed in the nonlinear test network (Table 1). Note, however, that the modified version (Table 9) propagates disturbances upstream more rapidly than the current version, for reasons given earlier.

The densities on congested sections under FRECON are in the range of 120 to 1,000 vehicles per lane-mile *(3)*. Because

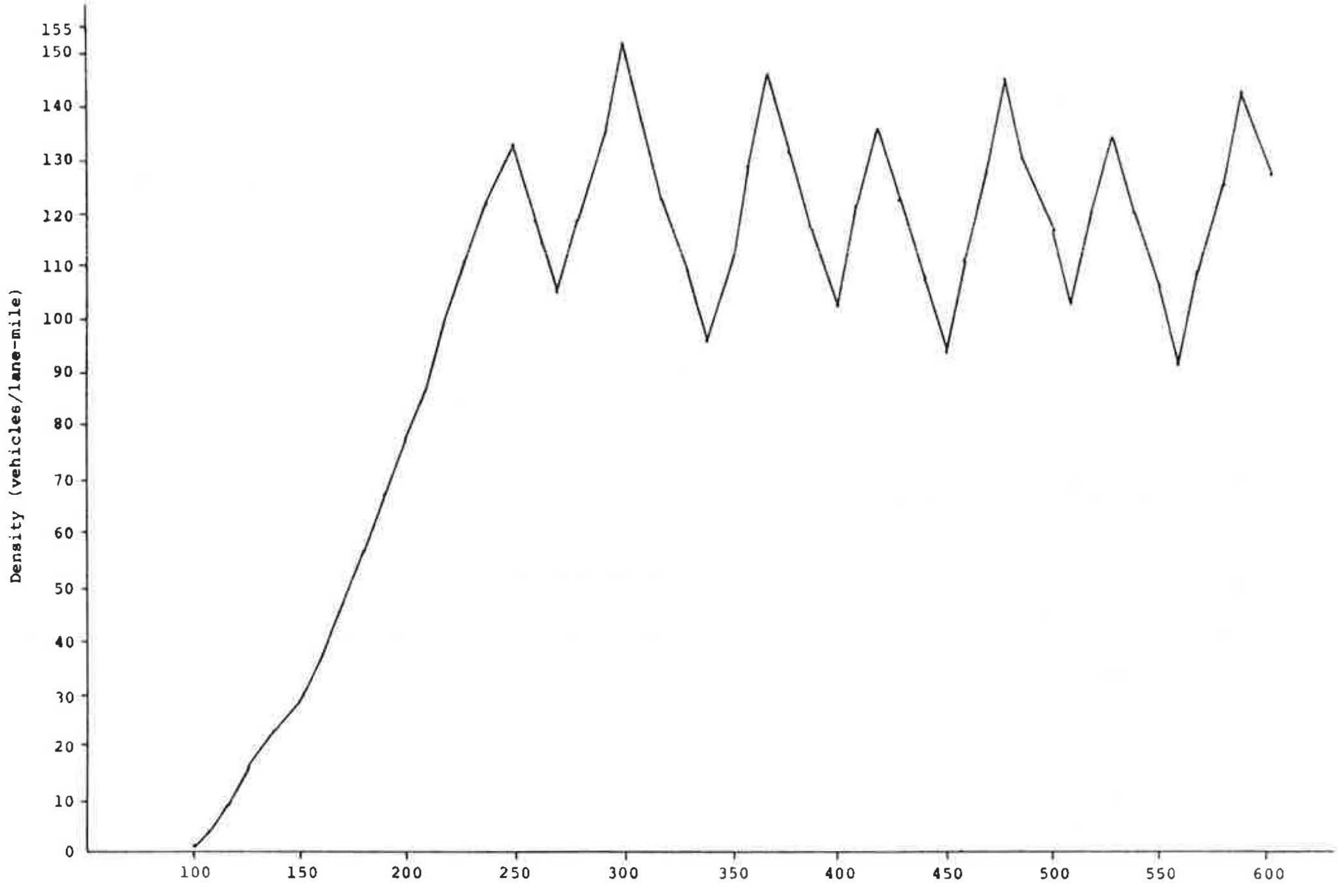

FIGURE 5 Density pattern on a congested section—modified FREFLO.

specific values of densities are not given, no evaluation of the densities produced by this model can be made.

The detailed history of density on a congested section under modified FREFLO is shown in Figure 5. This density gradually increases to the congested density level and oscillates thereafter, reflecting turbulent flow conditions. The amplitude and frequency of these oscillations vary depending on the section length, time slice duration, and outflow rate from feeder link and on whether this link is simulated before its feeder link(s).

CONCLUSIONS

Modifications were introduced to extend the ability of FREFLO to realistically simulate congested conditions for freeway sections of virtually any geometry. Tests of this modified FREFLO model reveal that it is able to:

- Produce values of densities that are much closer to those actually experienced on congested freeway sections,
- Propagate the effects of congestion to upstream sections in a timely manner,
- Produce a realistic stop-and-go pattern of flow entering congested sections, and
- Describe traffic within a diverging freeway section where one branch is congested and the other is not.

With modified FREFLO, the simulation results for the networks of the departments of transportation in Utah and Kansas, and some other test networks, have also been quite satisfactory. The results appear to be quantitatively, as well as qualitatively, accurate. The model is fully responsive to severe and to moderate incidents that block lanes for specified periods of time. The computational performance of FREFLO is not degraded and remains as user-friendly as it was previously.

These modifications have now been introduced as a permanent feature of FREFLO. FREFLO is available under a limited distribution from FHWA.

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Some Measurements of Robertson's Platoon Dispersion Factor

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At the core of TRANSYT, the platoon dispersion model of Robertson is probably the most widely used traffic model in the world. In spite of its wide use, only a relatively small number of studies have tried to calibrate the model for a range of traffic conditions. The default values of the TRANSYT handbooks provide the only information on the value of the dispersion factor available to most users. In the first section of the paper, the sensitivity of the TRANSYT results to the dispersion value is shown. Next, available information on the model is summarized and the results are classified. In the last section of the paper, the results of measurements taken in Karlsruhe and in Pforzheim, West Germany, are reported. The experimental design of the sites was selected to determine the influence of the gradient and the number of lanes on the platoon dispersion.

Robertson's platoon dispersion model *(1)* forms the core of the widely used TRANSYT program in all its different versions *(2, 3)*. Since the initial calibration of the model *(1)* there has been no major effort to calibrate it for a wide range of traffic situations. The studies undertaken to date to calibrate the model will be reported in the third section. In general, the low level of interest was based on the belief that the result of the optimization is only marginally affected by errors in the traffic model. The following section will present some evidence that this belief may be unwarranted in specific situations and could lead to grave errors. The aim of the small series of measurements reported in the last section is to gain a better basis for determination of value of the calibration factor for TRANSYT applications.

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The model will be used with the following notation:

$$q'(i + t) = F * q(i) + (1 - F) * q'(i + t - 1)$$

with

$$t = \beta * T \qquad F = \frac{1}{(1 + \alpha t)}$$

where

T = average travel time between the observation points,
t = average arrival time of the first vehicle of the platoon in intervals of n seconds,
F = smoothing factor,
α, β = calibration constants, and
α_8 = α calibrated with β fixed at 0.8.

SOME EVIDENCE OF THE EFFECTS OF VARIATIONS IN α_8

A real network of 11 nodes was used to study the effect of the variation of the dispersion factor. The network and the calibrated traffic data were available for the evening peak conditions. The calibration was part of a larger study involving TRANSYT as a tool to develop new signal plans for the city of