A Delay Model for Multiway Stop-Sign Intersections

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A limited number of empirical studies have examined the capacity and delay characteristics of multiway stop signs, and some simulation studies have been reported, but no analytical models of delay at multiway stop signs are available in the published literature. The objective of the research reported in this paper, therefore, is the development of such an analytical model of delays experienced at multiway stop-sign intersections. The paper draws on previously reported empirical observations to provide values of critical input parameters, and uses these within the framework of an M/G/1 queueing model to predict delays. Delays at a multiway stop sign are shown to be the result of a set of complex interactions between the flows on all approaches to the intersection. It is shown that there are primary, secondary, and tertiary influences on the delays experienced on the approach; namely, the flow on that approach, the flows on conflicting approaches, and the flows on opposite approaches. In comparison with previously quoted results for multiway stop-sign intersections, the model shows good agreement in terms of capacities and levels of service for various demand splits. What the model adds, however, is the ability to predict levels of performance over a much wider range of operating conditions.

Multiway stop signs are a common form of traffic control at intersections in North America. Whereas British or European practice might be to install a roundabout or miniroundabout at an intersection, it is often the case in North America that stop signs will be installed on all approaches to the intersection with enforcement of the give-way-to-the-right priority after a vehicle has stopped. This usually degenerates to a first-to-the-stopline, first-served priority, with give-way-to-the-right priority being reserved for two conflicting vehicles arriving at the stopline simultaneously.

The conditions under which a multiway stop-sign control is implemented generally fall into two categories. First, at very low-volume intersections, multiway stop signs are often installed for the sake of perceived safety. Although there has been some controversy over this issue, it has been found in several studies *(1)* that multiway stop signs successfully reduce accidents at low-volume intersections.

The second situation under which multiway stop signs may be installed is in urban environments at low- to medium-volume intersections where traffic is heavy enough to warrant some form of control but not enough to warrant traffic signals. In such circumstances the traffic is generally split relatively evenly across all the approaches, thus preventing the specification of major and minor traffic streams that would dictate the use of a conventional major or minor stop- (or yield-) sign control. Because of its position in the traffic control hierarchy, multiway stop-sign control should be evaluated for its effect on traffic delay before implementation. If traffic flows are not sufficiently balanced, then perhaps a major or minor stop-sign control would provide a better solution in terms of overall traffic delay. On the other hand, if the general level of traffic activity is too high, traffic signals may prove to be more efficient (after accounting for the much higher capital costs of the traffic signals).

Despite the need for the evaluation of delays experienced at multiway stop signs, there is surprisingly little in published literature that would enable the calculation of such delays. This is especially noticeable in comparison to the information available on delays at major or minor stop signs and at signalized intersections. A limited number of empirical studies have examined the capacity and delay characteristics of multiway stop signs, and some simulation studies have been reported, but no analytical models of delay are described in the available literature. The objective of the research reported in this paper, therefore, is the development of such an analytical model of delays experienced at multiway stop-sign intersections.

A description of this model requires some explanation of the circumstances that led to its development. The city of Ithaca, New York, has a population of between 30,000 and 50,000 (depending on the time of the academic year). As is common with cities of this size, there is relatively little need, or resources, for a great number of signalized intersections, although there is often a need for some form of intersection traffic control. Depending on the existing patterns of traffic distribution, this control is either a major or minor stop sign or a multiway stop sign. Over a period of years, control systems have been installed to merge with the existing traffic environment, and, conversely, drivers have adapted their route choices to conform to the existing traffic control environment. There are possibilities, however, for major changes in the patterns of traffic distribution within the city caused by the construction of a new bridge which may funnel traffic onto streets currently carrying relatively little traffic. The network effects caused by drivers finding their way from these streets to their destinations may result in major changes in traffic volumes at intersections currently controlled by multiway stop signs. It is therefore necessary to establish the sensitivity of delay at these intersections to changes in traffic flow. Because of the nature of the network effects, the problem is to be addressed by the use of a local area equilibrium assignment network model, which accounts for the delays on links and at intersections. Signalized and major and minor intersections could readily be modeled using existing techniques, but, as mentioned earlier, no comparable analytical technique exists for multiway stop-sign intersections; hence the specific need for the research described in this paper. The model that has subsequently been developed, however, can be used either in the context of a network model or on an individual basis for a specific intersection.

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PREVIOUS RESEARCH ON MULTIWAY STOP SIGNS

As mentioned in the preceding section, there has been little reported research on the delay characteristics of multiway stop-sign intersections. Although a number of studies have examined the safety implications of multiway stop signs, only a handful of studies has examined delays at multiway stop-sign intersections. Byrd and Stafford *(2)* examined the delays experienced at "unwarranted" four-way stop-sign intersections and concluded that care needs to be taken when installing such controls to ensure that the magnitude and distribution of delays are acceptable. However, the study was completely empirical and it would be difficult to generalize the results to other intersections.

Lee and Savur *(3)* mention the results obtained from the application of the TEXAS simulation model to a four-way stop-sign intersection. They provide one graph relating average delay to the total entering traffic at the intersection. However, there are very few details on how the simulation was conducted, and, as will be shown later, it appears that their results are in error when compared with the results obtained from other studies (including the current study).

A more general but somewhat dated study, which has often been quoted, is that by Herbert *(4)*. The results reported by Herbert form the basis of the section on multiway stop signs in the new *Highway Capacity Manual* (HCM) *(5)*. Herbert examined the characteristics of traffic at a limited number of multiway stop-sign intersections, and, from these observations, obtained estimates of intersection capacity at multiway stop-sign intersections under a range of traffic distribution assumptions. The 1985 HCM recommends the capacities given in Table 1, based on Herbert's observations. However, although Herbert's conclusions on intersection capacity are useful, he makes no attempt to specify how the intersection performs in terms of delays experienced in the traffic volume range between zero flow and capacity.

TABLE 1 CAPACITY OF A TWO-BY-TWO LANE FOUR-WAY STOP INTERSECTION

Demand Split	Capacity (vph)
50/50	1,900
55/45	1,800
60/40	1,700
65/35	1,600
70/30	1,500

SOURCE: 1985 HCM, Table 10-5 *(5)*.

The most useful feature of Herbert's study, in the context of the present study, is that he provides estimates of the minimum following headways under various conditions. When all the traffic is on one approach, Herbert quotes a capacity of 1,800 vehicles per hour (vph) (from both directions). This implies that vehicles departing from one stopline leave a gap of 4 sec between the previous vehicle and themselves. When traffic is balanced on both approaches, such that service through the intersection is on a strictly rotating basis, Herbert quotes a capacity of 1,900 vph (from all directions). This implies that when vehicles depart on a rotating basis they do so at a slightly smaller headway of approximately 3.8 sec (between vehicles on alternative approaches). Thus, although the flow on any one approach decreases, the total flow through the intersection increases. When there are two approach lanes from each direction, Herbert reports that the balanced flow capacity is 3,600 vph. This implies a departure headway of 4 sec between alternative approaches, indicating that the extra lane on each approach has increased the clearance time from each direction by 0.2 sec. These minimum departure headways are essential inputs to the model developed in this paper.

In addition to using the capacities derived by Herbert, the 1985 HCM refers to a study by Barton Aschman Associates *(6)* and reports some flow conditions that give rise to a Level of Service C at multiway stop-sign intersections. These results imply approximately equal delays under these flow conditions, but the level of delay implied is not quoted, nor is the method of deriving the level of service explained. The model developed in the present study should provide a rational and theoretically consistent method of deriving such estimates of delay.

DEVELOPMENT OF THE MODEL

In considering the development of the model, two different approaches had to be considered. The first was the development of an analytical model based on the concepts of queueing theory. The second alternative was the development of a discrete event digital simulation model. Development of the simulation model offered several advantages, including a greater degree of realism, more flexibility to alter the model for changing conditions, and less need to make some dubious assumptions in order to keep the model mathematically tractable. The disadvantage of the simulation approach, as always, is that it takes longer to compute an estimate of delay for any given traffic flow condition. Given the background to the study, as described earlier, where the model was to be used as a component of a network model it was imperative that computation times be kept to a minimum. In these circumstances, therefore, the development of an analytical queueing model was clearly desirable, if at all possible. If not possible, the results from a simulation model would have to be summarized by means of empirical mathematical equations of best fit to the simulation results.

The basic queueing model used was an M/G/1 model (negative exponential arrival rates, general distribution of service rates, and a single server). The assumption of random arrivals is probably reasonable under the traffic conditions in which multiway stop-sign intersections would most likely be expected. As noted later in this paper, when traffic flows at the intersection are high (and hence random arrivals may be a questionable assumption), the delay results predicted by the model should not be interpreted literally. The single server is the intersection itself, which processes vehicles from each approach. The queueing discipline is, however, somewhat unusual in that vehicles are processed in order of their arrival at the stopline. This is a form of priority queueing, where priority is assigned to the vehicle that has been waiting the longest at the stopline (which is not necessarily the one that has been waiting the longest in the system).

A number of similar priority queueing systems have been reported in the queueing theory literature, but each has distinct differences from the multiway stop-sign situation. The alternating priority system *(7)* switches priority between queues but only when the queue, which is currently being served, has been exhausted. For an intersection, this would correspond to the queue on one approach being emptied before the alternative approach was served (this would be an approximation to a well-functioning vehicle-actuated traffic signal). A closer approximation to the multiway stop-sign situation is that described by Greenberger *(8)* in the application of a round-robin priority model to computer time-sharing systems. In this model, each queue (computer terminal) is serviced for a finite period of time (called a quantum) before priority is switched to another queue (if indeed there are other queues waiting for service at that time). The original queue is then placed at the end of the line of queues, and service will be resumed on this queue when all other nonempty queues have received a quantum of service. If a service is actually completed within a quantum, then priority switches immediately to the next queue. To apply this model to a multiway stop sign, the quantum would simply need to be made larger than the time required to service one vehicle. In this case, the server would remain on one queue until the vehicle is cleared, whereupon it would switch to another nonempty queue. The problem of applying this model to the multiway stop-sign situation is that this model assumes that all queues (computer terminals) are identical with equal average arrival rates and service rates. Thus the model could only be applied to multiway stop-sign intersections with equal flows on each approach. Both of the aforementioned models belong to a general class of priority queueing models known as machine-shop models *(9)*, wherein a repairperson must attend to machines as they break down. Machines can be assigned different priorities depending on such factors as the type of machine, or the time since breaking down. In addition to the limitations noted earlier, there is one further problem. All these models assume that the single server, by definition, can serve only one machine at a time. However, in the multiway stop-sign situation it is possible for the intersection (the server) to serve two vehicles simultaneously (if they arrive consecutively on nonconflicting approaches, such as northbound and southbound). For the foregoing reasons, the adaptation of an existing general model is not simple and the development of a specific model is warranted.

Use of the M/G/1 queueing model involves the Pollaczek-Khintchine formula whereby

$$L = [2\rho - \rho^2 + \lambda^2 V(s)]/[2(1 - \rho)] \tag{1}$$

where

- L = average number in the system (average number on the approach, including the vehicle at the stopline);
- λ = average arrival rate;
- s = average service time;
- $V(s)$ = variance of service time; and
- ρ = utilization ratio, which equals arrival rate x service time.

Using Little's equation, the average time in the system is then given by

$$W_s = L/\lambda \tag{2}$$

The major problem, then, is finding the average and the variance of the service times on each approach to the intersection. To illustrate the basic concept behind the calculation of service times, consider the simple four-way stop-sign intersection shown in Figure 1, where flow exists only on the northbound and westbound approaches, and where all vehicles are proceeding straight through the intersection. Consider a vehicle

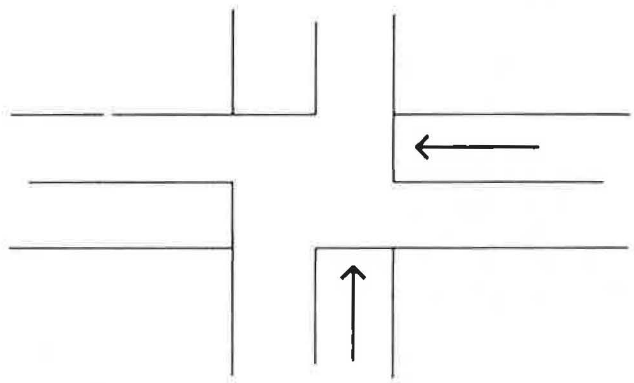

FIGURE 1 Four-way stop signs with flows on two approaches.

that arrives on the northbound approach, then calculate its service time (where service time is defined as the time between this vehicle's departure time and the time at which a vehicle immediately in front could have departed). If there is no vehicle waiting on the westbound approach when this vehicle arrives at the stopline then it can follow the previous northbound vehicle through the intersection at the minimum allowable headway, t_m. From Herbert's *(4)* study, it is assumed that t_m = 4 sec. However, if there is a vehicle waiting on the westbound approach when the northbound vehicle arrives at the stopline, then it must wait for the westbound vehicle to clear the intersection before it can proceed. In turn, the westbound vehicle must have waited for the previous northbound vehicle to clear the intersection. The clearance times for each approach are given, from Herbert's study, as t_c = 3.6 + 0.1 (number of crossflow approach lanes from both directions). Thus, for a simple four-way stop intersection with one lane on each approach, t_c = 3.8 (there being two crossflow lanes, one from each direction). The total clearance time T_c is simply the sum of the clearance times on each approach, and is the service time for a northbound vehicle which arrives when a westbound vehicle is already waiting at the stopline.

The average service time for a northbound vehicle, therefore, is given by

$$s_n = t_m \text{ (probability of no westbound vehicle at stopline)} + T_c \text{ (probability of westbound vehicle at stopline)} \tag{3}$$

The probability of a westbound vehicle being at the stopline when a northbound vehicle arrives depends on the utilization ratio on the westbound approach. The utilization ratio is simply the probability that the system is nonempty at any point in time. Thus,

$$s_n = t_m(1 - \rho_W) + T_c(\rho_W) \quad (4)$$

Thus, the average service time on the northbound approach is a function of the average service time on the westbound approach. However, by symmetry, the average service time on the westbound approach is also a function of the average service time on the northbound approach, whereby

$$s_W = t_m(1 - \rho_n) + T_c(\rho_n) \quad (5)$$

By substituting Equation 5 into Equation 4, and noting that the utilization ratio is calculated as the ratio of the arrival rate over the service rate, or the product of the arrival rate and the average service time, the following expression for the northbound approach service time can be obtained:

$$s_n = (\lambda_W \cdot t_m \cdot T_c + t_m - \lambda_W \cdot t_m^2) \div [1 - \lambda_W \cdot \lambda_n (T_c^2 - 2 \cdot t_m \cdot T_c + t_m^2)] \quad (6)$$

By substituting Equation 4 into Equation 5, a similar expression for the westbound approach service time is obtained. In the simple case just described, where there are only two flows competing for priority at the intersection, it is possible to solve directly for s_n and s_W and then to proceed with the analysis to obtain the delays on each approach. The situation becomes more complex, however, when there are multiple flows at the intersection. For example, consider the more general situation of flows on all approaches to a four-way stop-sign intersection as shown in Figure 2.

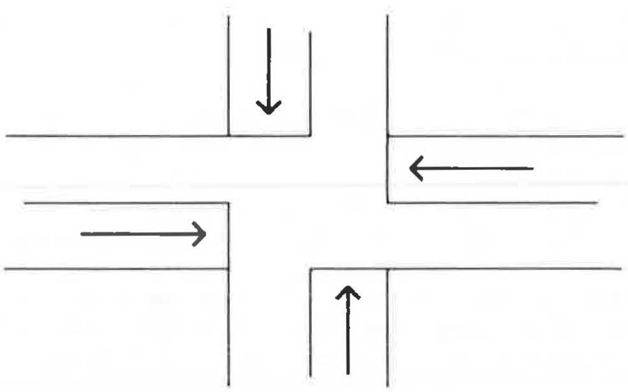

FIGURE 2 Four-way stop signs with flows on all approaches.

Under the conditions of flows on all approaches, the same general equations just outlined still apply. However, when applying Equation 3 to determine the average service time on the northbound approach, the utilization ratio used must apply to the east–west approach as a whole because northbound traffic must give way to vehicles waiting on either the westbound or eastbound approaches. Therefore

$$\rho_{ew} = 1 - (1 - \rho_e) \cdot (1 - \rho_w) \quad (7)$$

By similar reasoning

$$\rho_{ns} = 1 - (1 - \rho_n) \cdot (1 - \rho_s) \quad (8)$$

The average service times on the northbound approach and the southbound approach will therefore be functions of the flows and service rates on both the eastbound and westbound approaches (the service time on the northbound approach will, in fact, be the same as the service time on the southbound approach because both northbound and southbound vehicles must give way to exactly the same eastbound and westbound traffic). The service times on the eastbound and westbound approaches will, in turn, be functions of the flows and service times on the northbound and southbound approaches. These interactions give rise to a series of equations that, even with assumptions of equal flows on all approaches, are mathematically intractable in a closed-form solution.

The problem is further complicated if the possibility of multiple lanes on each approach is allowed. Under these conditions the utilization ratio on the approaches will be given by the following equations if it is assumed that drivers on an approach split equally between the available lanes.

$$\rho_{ew} = 1 - (1 - \rho_e/L)^L \cdot (1 - \rho_w/L)^L \quad (9)$$

$$\rho_{ns} = 1 - (1 - \rho_n/L)^L \cdot (1 - \rho_s/L)^L \quad (10)$$

where L equals the number of lanes on the appropriate approach.

Because these conditions do not yield a closed-form solution to the determination of average service times, it is necessary to adopt an iterative approach to obtaining stable values of the service times on the north–south and east–west approaches. This is done by assuming initial values of the service times on each approach calculating the utilization ratio on each approach (by noting that $\rho_i = \lambda_i \cdot s_i$), and substituting the approach utilization ratios into Equations 9 and 10 to obtain the effective blocking utilization ratios for each approach (that is, the utilization ratio of the east–west approach as perceived by a northbound driver as he is blocked from proceeding through the intersection by either an eastbound or westbound vehicle at the stopline). These blocking utilization ratios are then substituted into Equations 4 and 5 to yield updated values of the average service times on each approach. The procedure is then iterated with these new values of average service times until equilibrium occurs. The initial assumed values of the service times are bounded by t_m and T_c because these are the service times at zero and maximum conflicting flow, respectively. The iterative process converges quite rapidly, especially if an appropriate search technique is used in the iterations (it is virtually instantaneous in the current version of the program, which is written in TRUE BASIC on an Apple Macintosh 512).

Having obtained stable values of the average service rates on each approach, the calculation of the variance of the service rates is relatively straightforward. Bearing in mind that the service time distribution is bimodal (only values of t_m and T_c are possible), then knowing the average service time, the variance can be calculated as

$$V(s) = t_m^2 (T_c - s)/(T_c - t_m) + T_c^2 (s - t_m)/(T_c - t_m) - s^2 \quad (11)$$

The values of the average and variance of the service times on each approach can now be used in Equations 1 and 2 to obtain the delay characteristics for each approach. To obtain the total delay suffered by a vehicle in negotiating the multiway stop-sign intersection, an estimate must be made of the delay incurred when decelerating to a stop and accelerating back up to speed. It is often assumed that all vehicles will decelerate from and accelerate back up to the posted speed limit on the approach to the intersection. This acceleration–deceleration delay is a significant proportion of the total delay under low flow conditions.

APPLICATION OF THE MODEL

When using the model the user can specify the number of lanes on each approach and the total approach flow from each direction. The current version of the model, as used in the network equilibrium program, assumes no turning flows. A version that allows the user to specify individual turning flows, if such information is available, is currently under development. The model responds with estimates of average queue length on each approach, and various measures of the average delay experienced on each approach. For analysis of individual intersections, the method is quick and simple and is easily implemented on a microcomputer or a programmable calculator. The calculations could also be performed manually, although the iterative calculations could become tedious.

When using the model within an equilibrium assignment network program, it is necessary to identify the links in the network that represent the conflicting approaches for the approach in question. The flows on these links can be read from file, together with the number of lanes on each approach. The delay on the approach in question can then be calculated. To obtain an estimate of the deceleration–acceleration delay, it is necessary to find the link feeding into the origin of the approach in question. From this link the current speed of traffic on the approach can be read, modified to account for the current link flow. The deceleration–acceleration delay from this speed can then be calculated, and added to the intersection delay to obtain the total delay on this approach link.

The model has been applied to a range of general situations to demonstrate the effects of changing the approach flow, the conflicting flows, the flow from the opposite direction, and the number of lanes on the approach. The effects of changing the flow on the northbound approach on the average system delay experienced on the northbound approach (not including the acceleration–deceleration delays) are shown in Figure 3, for a range of conflicting flows on the eastbound and westbound approaches. In these calculations, it is assumed that the flows are equal on the eastbound and westbound approaches, and that the flow on the southbound approach is equal to the flow on the northbound approach. It can be seen that the capacity of the northbound approach is inversely related to the flows on the east–west approach. Thus, at zero east–west flow, the capacity of the northbound approach is 900 vph [in agreement with Herbert's conclusion *(4)*]. As flow on the east–west approach increases, the capacity of the northbound approach decreases. When the eastbound and westbound flows are 475 vph, the northbound capacity is also 475 vph. This is the 50/50 demand split noted by Herbert with the maximum intersection capacity of 1,900 vph (= 4 × 475). As the northbound flow increases with constant east–west flow, the system delay (the time from joining the queue to leaving the stopline) increases in a manner typical of most traffic links. As with all delay models based on queueing theory principles, care should be taken when interpreting the delays predicted when the flow approaches the capacity. Because of the extreme sensitivity of delay to changes in flow in this region, the delay values predicted should be used only in a diagnostic fashion and should not be interpreted literally.

Using the results shown in Figure 3, Table 2 (comparable to that shown in the 1985 HCM) here has been derived. The capacity obtained under various demand splits is shown. It can be seen that there is approximate agreement between Tables 1 and 2 for those demand splits that are common to both. It can also be seen from Table 2 that the minimum intersection capac-

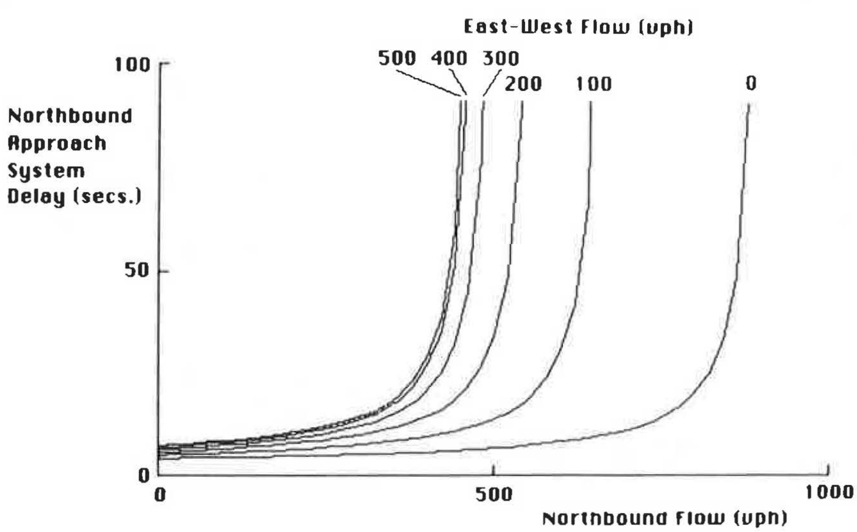

FIGURE 3 System delay as a function of approach flow and conflicting flow (two lane by two lane).

TABLE 2 CAPACITY OF A TWO-BY-TWO LANE FOUR-WAY STOP INTERSECTION AS PREDICTED BY DELAY MODEL

Demand Split	Capacity (vph)
50/50	1,900
55/45	1,760
60/40	1,650
65/35	1,600
70/30	1,560
80/20	1,520
90/10	1,570
100/0	1,800

ity occurs when the demand split is 80/20. Thus multiway stop signs operate best when there is either balanced flow on each approach or no flow on one approach.

Changing the number of approach lanes provides different values of system delay as shown in Figure 4 (compared to Figure 3). In this example there are two approach lanes on both the north–south and east–west approaches. As expected, the capacities are increased and the delays are decreased when the number of lanes is increased. Such increased capacities could be achieved in practice by means of flaring of the intersection approaches such that there are an increased number of lanes only in the vicinity of the intersection.

Increasing the southbound flow also has a small effect on the delays suffered by the northbound flows, as shown in Figure 5. Note that for each band of curves, the highest curve corresponds to a southbound flow of 600 vph, and the lowest curve corresponds to a southbound flow of zero vehicles per hour. In this example, with one approach lane from each direction, the eastbound and westbound flows have been varied from 100 vph to 300 vph. The southbound flow has been varied from 0 to 600 vph and the effect on northbound delay as the northbound flow varies is shown. Clearly, as the southbound flow increases, the northbound delay increases marginally. The reason for this increase in delay is that as the southbound flow increases, there is less opportunity for east–west traffic to cross the intersection

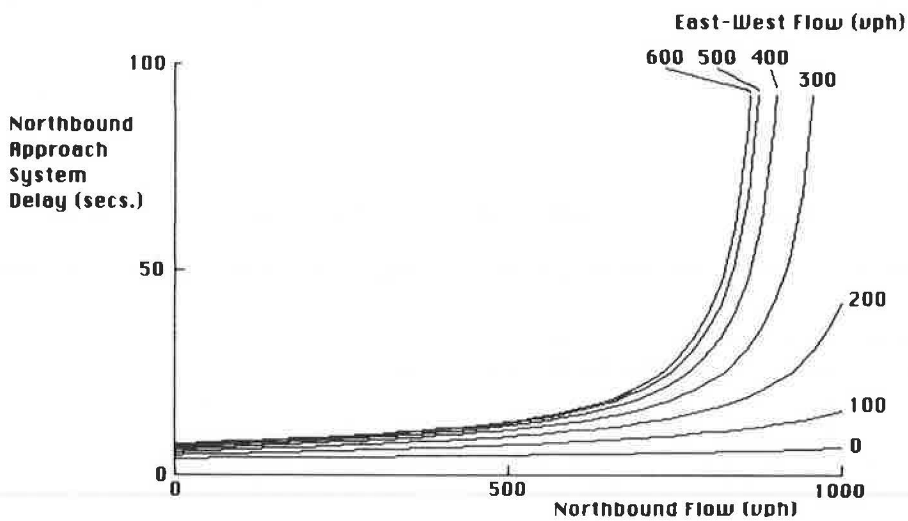

FIGURE 4 System delay as a function of approach flow and conflicting flow (four lane by four lane).

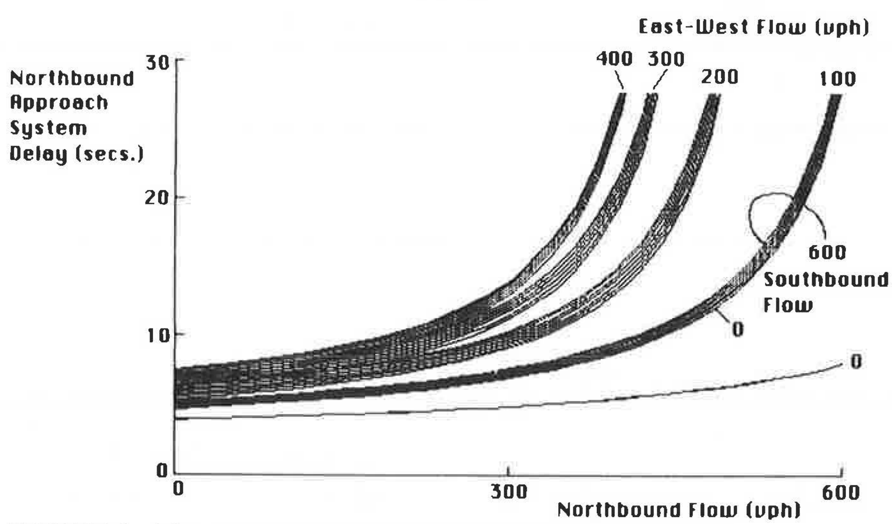

FIGURE 5 The effect of southbound flow on northbound delay.

unimpeded. Therefore, queues on the east–west approaches increase, which provides less opportunity for the northbound traffic to proceed unimpeded. Thus the northbound delays increase. However, the magnitude of this tertiary effect of southbound traffic on northbound delay is generally small and can be ignored in most cases, especially where the conflicting flows are either very high or very low.

A further validation of the results produced by the model involves estimation of the delays incurred for the flows and demand splits quoted in the 1985 HCM as giving rise to a Level of Service C. Part of the HCM table is reproduced in Table 3, together with the delays predicted by the model for each of the approaches. Note that the flow combinations quoted in the HCM consistently give rise to a total delay on the more heavily trafficked approach of approximately 25 sec (including acceleration–deceleration delay to and from 30 mph at a rate of 3 mph/sec), although the model predicts that this delay falls slightly with increasing demand split for the two-lane situation. On the basis of the average delays recommended by the *Manual on Uniform Traffic Control Devices* (MUTCD) *(10)*, and also suggested by Lee and Savur *(3)* and Sutaria and Haynes *(11)*, it would appear that a desirable minimum operating condition for a four-way stop intersection would be the boundary between Levels of Service B and C, with an average delay of 30 sec. It would appear from the preceding results that the Level of Service C traffic flows given in the 1985 HCM are generally consistent with the results of the delay model, but that slightly higher flows could be tolerated at four-way stop intersections before average delays rise to the recommended value of 30 sec for Level of Service C.

TABLE 3 TOTAL INTERSECTION DELAYS PREDICTED BY DELAY MODEL FOR LEVEL OF SERVICE C FLOW COMBINATIONS AS QUOTED BY THE 1985 HCM

Demand	Two-by-Two Lanes		Four-by-Four Lanes	
Split	Total Flow	Delays	Total Flow	Delays
50/50	1,200	24.4/24.4	2,200	26.9/26.9
55/45	1,140	24.3/23.1	2,070	27.2/22.4
60/40	1,080	23.9/22.2	1,970	27.4/21.6
65/35	1,010	22.7/20.0	1,880	27.1/21.1
70/30	960	21.9/19.8	1,820	26.9/20.8

NOTE: The delays given refer to total delays (including acceleration and deceleration delays) on the north–south and east–west approaches, respectively.

A comparison with the flow–delay curve produced by Lee and Savur *(3)* provides a final check on the results produced by the delay model. A graph displaying the results of their simulation modeling is reproduced in Figure 6. It can be seen that average queue delay—which in queueing theory terminology is actually system delay and not queue delay because it includes the time spent in service at the stopline—has been plotted against the total flow entering the intersection. The distributional split across the approaches is not specified, although note from Figure 3 that similar total flows will give rise to different average delays, depending on the distributional split. For comparison, therefore, it is assumed that the total traffic flow is split equally across all four approaches. The results generated from the delay model for these conditions (from Figure 4) are superimposed on Lee and Savur's results in Figure 7. It can be seen that there is an alarming difference between the two sets of results, with the delay model predicting substantially lower delays. Closer examination of Lee and Savur's results, however, indicates a probable error in their graph. From the shape of the graph, it appears that the capacity of the intersection is a total flow of approximately 1,900 vph. Compared with Herbert's *(4)* results, this is substantially lower than the capacity of 3,600 vph for a four-lane by four-lane four-way stop intersection, and in fact is equal to the capacity of a two-lane by two-lane intersection. Based on the fact that the results have been plotted for a two-lane by two-lane four-way stop intersection, the results generated from the delay model for these conditions (from Figure 3) are superimposed on Lee and Savur's results in Figure 8. With this change in interpretation, it can be seen that the results produced by the delay model are very consistent with those produced by the TEXAS simulation

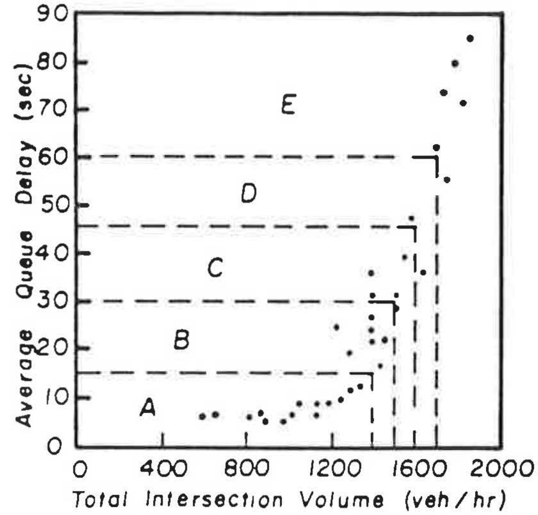

FIGURE 6 Delay estimates from TEXAS simulation model.

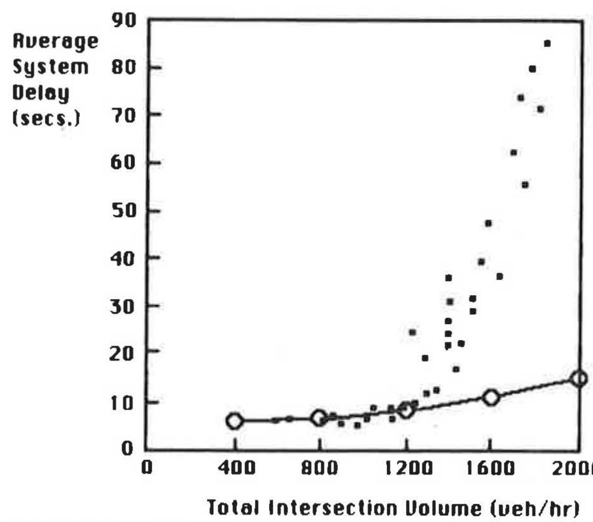

FIGURE 7 Comparison of Lee and Savur simulation results with delay model results (four lane by four lane).

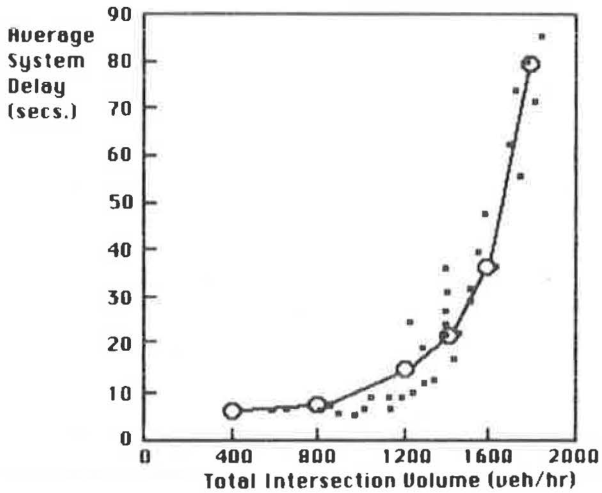

FIGURE 8 Comparison of Lee and Savur simulation results with delay model results (two lane by two lane).

model, and that the delay model is reliable for use under other conditions.

CONCLUSION

Development of a queueing model to predict delays at multiway stop-sign intersections has been described. The paper draws on previously reported empirical observations to provide values of critical input parameters, and uses these within the framework of an M/G/1 queueing model to predict delays. It is shown that delays at a multiway stop-sign intersection are the result of a set of complex interactions between the flows on all approaches to the intersection. It is also shown that there are primary, secondary, and tertiary influences on the delays experienced on an approach; namely, the flow on that approach, the flows on conflicting approaches, and the flows on opposite approaches.

In comparison with previously quoted results for multiway stop-sign intersections, the model shows reasonably good agreement in terms of capacities and levels of service for various demand splits. It is also able to reproduce results obtained from a validated, discrete-event simulation model. What the queueing model adds, however, is the ability to predict levels of performance over a much wider range of operating conditions, without the need for detailed simulations.

The model that has been developed can be used to ascertain the delay characteristics of a specific multiway stop-sign intersection, or can be used as a subroutine to calculate delays at multiway stop signs within the framework of a network assignment model. Research is continuing on the extension of the model to handle turning flow input to account for the fact that turning vehicles hinder and are hindered less than vehicles proceeding straight through the intersection. With this refinement, it is expected that the average delays will be reduced for each value of total entering flow.

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