

A Behavioral Approach to Risk Estimation of Rear-End Collisions at Signalized Intersections

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A conceptual approach to estimating the risk of rear-end collisions at a signalized intersection is presented. It is argued that the creation of a large optlon zone increases the range of the indecision zone, the direction implication of which is an increase in the risk of rear-end collisions. With the aid of field data collected for two warning intervals (3 and 6 sec) before the red light, a large optlon zone is shown to increase the variance underlying the stopping probability curve, and thus to determine a larger range for the indecision zone. Data from urban intersections support the basic argument that a long warning period causes a significant increase in the number of rear-end collisions.

At present, a general consensus appears to exist in the literature about the effect of traffic signals on rear-end collisions. Most of the researchers apparently have concluded that signalizing an intersection significantly increases the number of rear-end collisions. For example, in a sample of 34 urban intersections Hakkert and Mahalel (1) found that after the introduction of a traffic signal control, the annual number of rear-end collisions increased from 33 to 77. In a similar study of 31 intersections in Milwaukee, Short et al. (2) found an increase of 37 percent in the number of such accidents. King and Goldblatt (3) observed the same phenomenon of increased rear-end collisions in a statistical analysis of U.S. accident data nationwide.

In addition to the fact that the number of rear-end accidents increases after the introduction of traffic signals at an intersection, it is typical that the highest number of accidents at signalized intersections are rear-end collisions. A statistical analysis conducted by the author of almost all signalized intersections in Israel (600) indicated that, over a 2-year period (1983 to 1985), about 39 percent of all accidents were rear-end collisions, compared with about 27 percent that were right-angle collisions. A similar result was identified by Hanna et al. (4) in a study of signalized-intersection accidents in rural communities in Virginia. They found that 43 percent of all accidents were rear-end collisions and 37 percent were right-angle collisions.

In spite of the fact that rear-end collisions at signalized intersections are significantly more frequent than right-angle collisions, the former have not received much attention either in the literature or in practice. The reason for the interest in right-angle collisions is their relatively high severity. The gen-

eral belief is that an improvement in existing procedures defining the change interval will gradually reduce the number of right-angle collisions. The term "change interval" means the sequence of intervals at a signalized intersection that occurs from the moment the green light ends for one direction and a green light begins for the conflicting approach. The change interval might consist of various combinations and proportions of yellow and red for the conflicting approaches. Despite the consensus referred to at the outset, much of the research to date that is concerned with accidents at signalized intersections concentrates on proper design procedures for the change interval to reduce the number and severity of right-angle collisions.

Because of the large number of rear-end collisions, and in spite of their relatively low severity, the authors believe this type of accident deserves more attention. The purpose of this paper is to present a conceptual approach to estimating the risk of rear-end collisions at a signalized intersection. This work is based on a behavioral analysis of drivers approaching an intersection when the yellow light appears at the end of the continuous green light. An analysis will be made of the influence of the length of the warning interval—the interval between the continuous green and the continuous red—on the probability of a rear-end collision. The theoretical analysis is then supported by field observations carried out under controlled conditions at several signalized intersections.

RISK-GENERATING PROCESS OF A REAR-END COLLISION

Most rear-end collisions at a signalized intersection occur when two successive drivers approaching the intersection make conflicting decisions when the yellow light appears. A high risk of a rear-end collision will exist if the first driver decides to stop while the second one wants to cross the intersection. When the collision actually occurs, it is reasonable to assume that the second driver did not anticipate the stopping decision of the driver in front, and thus could not react in time to prevent the accident. The highest probability of a rear-end collision exists when the probability of two successive drivers reaching conflicting decisions about whether to cross or stop is the highest.

The probability for conflicting decisions is a function of the distance or time of the two drivers from the intersection when the yellow light appears. This probability of conflicting decisions can be derived from a stopping probability function that describes the probability of stopping when the yellow light

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appears as a function of the distance from the intersection.

Let $P(x)$ be the probability of stopping when a driver is at distance x from the intersection when the green phase ends. The probability of deciding to cross the intersection will then be $1 - P(x)$. Note that this function (Figure 1a) represents a realization of Bernoulli trials carried out at various distances from the stop line the moment the green light ends. The probability of these trials changes as a function of distance; the probability of stopping is high when the driver is relatively far from the intersection, and low when the driver is close.

The probability of two drivers' reaching conflicting decisions about whether to pass or stop will be highest when the expression $P(x) * [1 - P(x)]$ obtains the maximum value. This happens when $P(x) = 0.5$. Figure 1b shows this probability, the value of which becomes lower as the distance to the stop line increases or decreases.

The zone around the point at which the stopping probability has a value of 0.5 is the zone in which it is most difficult for the driver to reach a decision on the proper action when the green light ends.

In practice (5-7), it is customary to describe the area between the 10th and 90th percentiles of the stopping probability function as an indecision zone. An example of the implementation of the concept of the indecision zone is found in a work by Parsonson (6). He suggested placing in this zone a detector loop whose purpose would be to prevent, in unsaturated cycles, a situation in which a driver is caught in the indecision zone at the beginning of the yellow light.

A necessary condition for the occurrence of a rear-end collision is the presence of vehicles in the intersection approach

when the yellow light appears. The probability of a rear-end collision increases when the number of vehicles in the indecision zone increases. The actual number of rear-end accidents is thus a function of two factors:

1. Traffic volume—the larger the volume of vehicles in the approach to the intersection, the higher the probability that vehicles will be located in the indecision zone when the green light ends.
2. The range of the indecision zone—the larger the indecision zone, the higher the probability that vehicles will be located in the zone when the green light ends; the range of the indecision zone depends on the value of the variance of the random variable that generated the stopping probability function.

The characteristics of this random variable and the stopping probability function is discussed in the section entitled The Relation Between the Option and Indecision Zones. At this point, it will be sufficient to show the effect of this variance. Figure 2 shows two stopping probability functions that differ in their underlying variance. It is easy to see that the larger this variance, the larger the indecision zone.

The behavioral aspects of the causes of rear-end collisions have been discussed, and now deterministic normative methods to analyze the intersection-approach problem will be discussed next.

DILEMMA AND OPTION ZONES

In many studies (8-11) concerned with the events occurring in the approach to a signalized intersection, the phenomena are analyzed through the use of dilemma and option zones. These zones are defined by equations that are based on the normative behavior of a reasonable driver when the yellow light flashes.

Drivers who are located in the dilemma zone at the end of

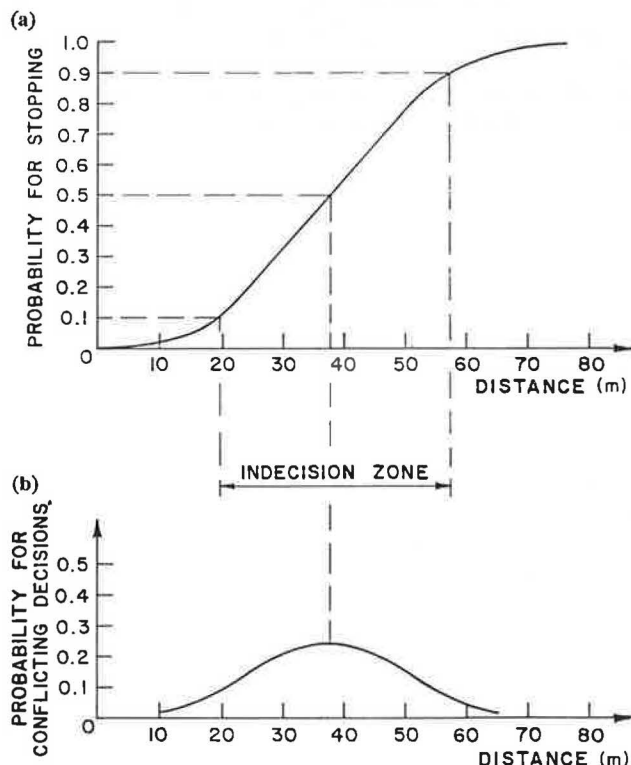


FIGURE 1 Hypothetical stopping probability function and the probability for conflicting decisions.

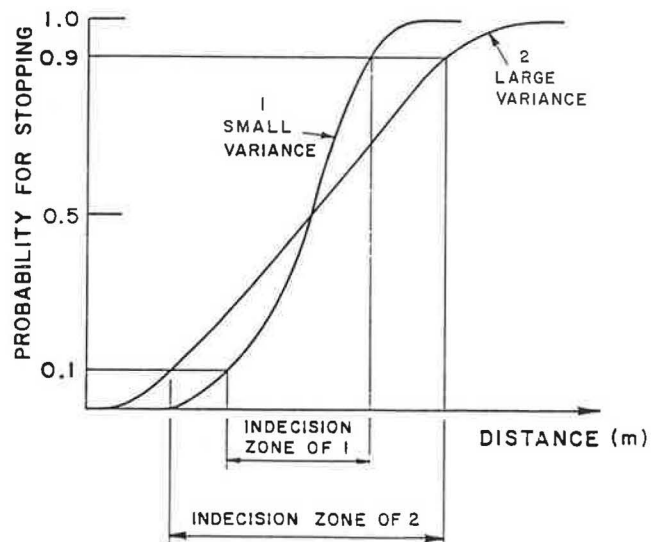


FIGURE 2 Two hypothetical stopping probability functions in which $P(x) = 0.5$ occurs at the same x and that have different underlying variances.

the green light can neither stop their vehicles before the stop line nor cross the line before the light turns red. Drivers who are in the option zone when the light turns yellow can either stop their vehicles at the stop line or cross it before the light turns red. The ability of a driver to cross the stop line or to stop before it is based on deterministic normative values. It is usually assumed that deceleration takes place at a rate of about 10 ft/sec^2 and that when an attempt is made to cross the intersection, the driver will continue at a constant speed or accelerate at a rate of 5 ft/sec^2 (9). Figure 3 shows the shape of the dilemma and option zones as a function of approach speed.

The importance of the definition of dilemma and option zones lies in a normative ability to analyze and judge various actions taken by drivers at an intersection approach. For example, May defined a risk-measurement factor based on the events occurring in dilemma and option zones (9). It is important to realize, however, that these zones describe, under normative deterministic assumptions, what a driver can do in each zone. They do not describe what a driver will actually do, not even in the stochastic sense. Thus, it can be concluded that the dilemma and option zones are tools of diagnostics or analysis; they are not, and cannot describe, the actual behavior of drivers.

In many of the studies carried out following the work by Gazis et al. (8), special emphasis was placed on reducing the size of the dilemma zones. The motivation behind this objective was to lessen the risk of right-angle collisions. The manifestation of this school of thought is the *Proposed Recommended Practice for Determining Vehicle Change Interval* by ITE (12). In these guidelines, the proposed speed approach for determining the length of the yellow light is the 85th percentile of the actual speed distribution or of the posted speed limit. This recommendation indicates that the tendency is to use a relatively high approach speed to reduce the size of the dilemma zone. By doing so, there is a smaller chance that a driver who is unable to stop before the stop line when the red signal lights up will eventually cross during the red light. This, of course, is in line with legal attitudes as expressed in traffic laws.

The direct implication of determining the length of the yellow light according to the relatively fast drivers is to create a large option zone for the slower drivers. This option zone provides slow drivers a relaxed decision situation because whether they decide to stop or cross the intersection, they can do so within the legal time frame. Although this situation may be desired by the individual driver, it has serious implications at the system level.

The option zone, by definition, is an area in which either decision—to stop or to cross—is legitimate; thus, a high proportion of conflicting decisions may be expected by the various drivers located in this zone. The high proportion of possible conflicting decisions by itself creates a high potential for rear-end collisions. To demonstrate this contention, imagine that a stop sign is considered by some drivers to be a recommendation to stop and by others a recommendation to cross the intersection. This situation, by its very nature, will create conflicts and, thus, rear-end collisions. The hypothetical situation is analogous to the interpretation of the option zone advanced here.

In the next sections is an analysis, through field data, of whether increasing the option zone influences the indecision zone as defined in this paper.

RELATION BETWEEN THE OPTION AND INDECISION ZONES

The implications of increasing the size of the option zone on the size of the indecision zone were analyzed under controlled conditions at four urban signalized intersections in Tel Aviv. The events at those intersections were twice recorded on film with a cine camera—once with a small option zone and once with a larger zone. At each intersection, the results of the experiment were first recorded in the situation in which signals always operated (either with or without flashing green). The mode of operation at these intersections was changed to the other mode. Appropriate announcements were made on the

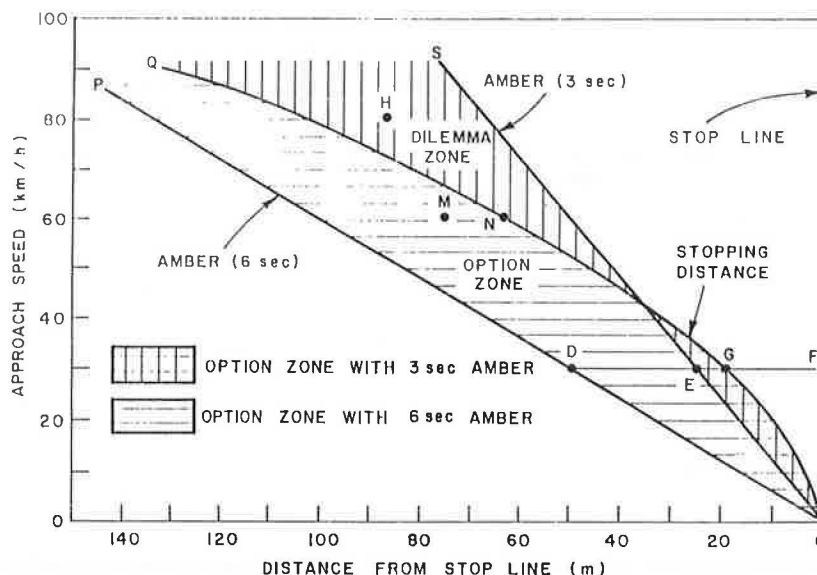


FIGURE 3 Options and dilemma zones for two warning periods.

radio, and the second set of experiments in the new mode of operation was made 1 to 2 months later. Thus, it can be assumed that drivers adjusted to the new operation mode of signals. A detailed description of the data is given by Becker (13).

The increase in the option zone was achieved, not by lengthening the yellow light, but by substituting the last 3 sec of the green light with a flashing green. The flashing green was not new to the Israeli driver because it had been used in Israel for years at most interurban signalized intersections and at those urban intersections that had high approach speeds. Thus, it can be assumed, as was done in this study, that the flashing green is perceived by the Israeli driver mostly as an extension of the

warning period, to indicate the close appearance of the red light. As shown in Figure 3, the increase in the warning period significantly increases the option zone.

Data Description

The basic characteristics of the four intersections included in the sample are given in Table 1. The size of the sample is the number of vehicles at the end of the green light that were actually exposed to a stopping or crossing decision. The sample does not include vehicles forced to stop by vehicles in front.

The first three intersections may be seen to be characterized

TABLE 1 BASIC DATA OF THE FOUR INTERSECTIONS INCLUDED IN THE STUDY

Intersection No.	Characteristic	Warning Period ¹	
		3 sec.	6 sec.
1	No. of cycles	75	67
	Sample size (veh) ²	52	47
	Volume (VPH)	320	330
	Average speed (km/h) ³	31.7	37.3
	Speed variance	76	147
2	No. of cycles	80	83
	Sample size (veh)	256	341
	Volume (VPH)	950	865
	Average speed (km/h)	22.1	21.4
	Speed variance	95	72
3	No. of cycles	73	68
	Sample size (veh)	255	239
	Volume (VPH)	1317	1404
	Average speed (km/h)	37.1	34.7
	Speed variance	98	111
4	No. of cycles	42	47
	Sample size (veh)	60	131
	Volume (VPH)	Missing Data	1065
	Average speed (km/h)	59.4	63.0
	Speed variance	156	85

¹ The 6 sec. warning period is composed of 3 sec. flashing green and and 3 sec. yellow.

² The sample size refers to the number of vehicles at the end of the green. The vehicles preceded by stopped vehicles are not included.

³ Speed of vehicles at the end of the continuous green.

by a low approach speed (20 to 40 km/h), and the fourth by a higher approach speed (60 km/h). The data were collected in two situations:

1. *Three-second warning*: a yellow light appeared for 3 sec after the continuous green light and was followed by the red light.

2. *Six-second warning*: a flashing green light of 3-sec duration appeared after the continuous green, followed by a yellow light for 3 sec, and then the red light. All together, the warning period lasted for 6 sec.

For each vehicle, the data included its position at the end of the continuous green light, its speed, and its deceleration rate if it stopped.

The Model

As mentioned previously, the indecision zone is derived from the stopping probability function, which was estimated according to Sheffi and Mahmassani (7). According to that model, a variable T is defined as the driver's perceived time to the stop line at the beginning of the warning period (the end of the continuous green). Because of differences in driver perception, T was assumed to be a random variable with normal distribution; that is,

$$T \sim N(t, \sigma_T^2)$$

In addition, it was assumed that if T is less than a critical value, T_{cr} , a driver would decide to cross the intersection; otherwise, he would decide to stop. T_{cr} was also assumed to be a random variable:

$$T_{cr} \sim N(t_{cr}, \sigma_{cr}^2)$$

Consequently, the probability for stopping at the end of the green light is

$$P_{STOP}(T) = P(T_{cr} \leq T) = \Phi\left(\frac{t - t_{cr}}{\sigma}\right)$$

where $\sigma = \sqrt{\sigma_T^2 + \sigma_{cr}^2 - 2\sigma_{T,cr}}$ and $\Phi(\cdot)$ denotes the standard cumulative normal function.

The two parameters of the model (t_{cr} , σ) were estimated with a program called CHOMP (Choice Modeling Program), which is used to estimate multinomial probit models.

Results

The expected value of the critical distance (t_{cr}) defines the distance from the intersection in which 50 percent of the drivers at the beginning of the warning period will eventually stop and 50 percent will cross. The majority of drivers located between the stop line and the expected value of the critical distance at the beginning of the warning period (yellow or flashing green) will decide to cross; at longer distances than the

critical, most will decide to stop. It is reasonable to assume that the expected value of the critical distance will be between the stopping distance and the distance a vehicle can travel during the warning period.

From the results given in Table 2, it can be seen that in all the samples, the influence of lengthening the warning period is expressed in longer expected critical distances (t_{cr}). The warning period in the present study was changed from 3 to 6 sec, so a 3 sec change in t_{cr} demonstrates that, in terms of expected time from the intersection, the stopping decisions of drivers undergo no alteration. In other words, even in cases of a longer warning period, it is the beginning of the red light that determines the behavior of drivers rather than the length of the warning period.

The actual increase in the length of the warning period was 3 sec. However, from Table 2 it appears that the increase in t_{cr} was not always exactly 3 sec. When the increase in t_{cr} is less than 3 sec there will be a higher number of stopping decisions as compared with an increase of exactly 3 sec. This means that there exists a group of drivers who, when at the same location, decide to cross with short warning intervals, but stop with long warning intervals when the solid green ends. In cases in which the change in t_{cr} is greater than 3 sec, there exists a group of drivers who could stop with a short warning interval, but a long warning period causes this group to cross.

At Intersections 1 and 2 (see Table 2), the change in the expected critical distance is very close to 3 sec (2.85 and 3.19 sec, respectively). At Intersection 3, the change is 3.96 sec, and at Intersection 4, 2.18 sec. Considering the limitations of the sample size and the small number of intersections, it may be concluded that, in terms of expected critical distance, a significant change in driver behavior does not occur from lengthening the warning period.

The variance underlying the stopping probability function increased significantly at the first three intersections; at Intersection 4, the variance changed only slightly (Table 2). For example, the variance at Intersection 1 increased from 0.31 to 6.14. The direct implication of the increased variances is an increase in the range of the indecision zone. As can be seen, this increase is about 90 to 350 percent. For example, at Intersection 1, the range of the indecision zone increased from 1.42 to 6.34 sec. The stopping probability curves of Olson and Rothery (14), which were estimated during two yellow durations, also show a tendency for long-range indecision zones during a long yellow light.

Despite the fact that the range of the indecision zone did not change at one intersection only, it appears that this result is not random. As previously mentioned, Intersection 4 is characterized by a relatively high approach speed. Stopping distances, therefore, are longer here than at the other intersections. This means that drivers who can stop are situated farther from the intersection. It is reasonable to assume that a driver's temptation to cross when at a short distance from the stop line is higher than when at a long distance; thus, a driver is more likely to stop at option zones with high-speed approaches than at option zones with low approach speeds. This finding is related to the hypothesis advanced by Mahalel and Zaidel (15) that drivers' stopping decisions are more strongly influenced by their distance from the stop line than by their approach speeds. At low approach speeds, drivers are likely not to stop even if

TABLE 2 INDECISION ZONE BOUNDARIES FOR TWO WARNING INTERVALS

Intersection	Warning Interval ¹ (sec)	\hat{t}_{cr} (sec)	$\hat{g}_{0.9}$	Indecision Zone Boundaries ²		
				Inner boundary (sec)	Outer boundary (sec)	Total length (sec)
1	3	2.42	0.31	1.71	3.13	1.42
	6	5.27	6.14	2.10	8.44	6.34
2	3	1.45	0.87	0.26	2.64	2.38
	6	4.64	3.16	2.36	6.91	4.55
3	3	3.12	0.53	2.19	4.05	1.86
	6	7.08	4.98	4.22	9.94	5.72
4	3	4.46	1.39	2.95	5.97	3.02
	6	6.64	1.42	5.11	8.16	3.05

¹ A warning interval of 3 sec. consists of 3 sec. amber. A warning interval of 6 sec. consists of 3 sec. flashing green and 3 sec. amber.

² The inner and outer boundaries are the values of the 10th and 90th percentiles respectively of the stopping probability function.

they can; at high approach speeds, whenever drivers can stop they do so with high probability. Evidence that drivers' decisions reflect a higher sensitivity to distance than to speed may also be found in Chang et al. (16).

DISCUSSION OF FINDINGS

The relationship between the option zone and the range of the indecision zone has been discussed in this paper. Empirical evidence demonstrated that with a low approach speed, the resulting increased option zone causes a significant increase in the indecision zone.

The calculations of conflicting decisions were made with the assumption of independency behavior between consecutive drivers. This simplified assumption ignores the possibility of dependency as a result of car-following behavior. However, it is logical to assume that the monotone relationships between the stopping probability and rear-end accidents will exist, even if dependency will be taken into consideration in a more sophisticated model.

In an analysis of the risk of rear-end collisions (see section entitled Risk-Generating Process of a Rear-End Collision), it was assumed that this risk might increase as a result of an increase in the range of the indecision zone. This hypothesis thus indicates the possibility that the addition of a flashing green light as an extra warning period might increase the number of rear-end collisions. Various studies (17-19) have revealed that the number of rear-end collisions at urban inter-

sections with a flashing green signal is significantly higher than at other signalized intersections. This important finding corroborates the hypothesis regarding the relationship between the range of the indecision zone and the risk of rear-end collisions.

The operational implication of this relationship is that additional research should be conducted to find the pattern of warning intervals that will minimize the range of the indecision zone. In other words, ways should be found to shape the stopping probability function to be as close as possible to a step function. In that way, the number of rear-end collisions might decrease.

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