The Effect of Contact Area Shape and Pressure Distribution on Multilayer Systems Response

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Numerical analysis of stresses, strains, and displacements in multilayer systems for any shape of contact area and pressure distribution is presented. The scheme developed for microcomputers is made of two programs: one for evaluating the stresses and deformations under a point load and the second for performing the numerical integration of the point load solution. Results obtained by using the scheme for the uniformly distributed pressure, Boussinesq, and three-layer system cases were compared to those of analytical and other numerical solutions. They were revealed to be in excellent agreement. Results for a rectangular contact area and nonuniform pressure distribution are presented. The effect of the contact area and pressure distribution shapes on the design strains is illustrated and discussed. The scheme presented is preferable to the 3-D finite element analyses. It dispenses with the use of mainframe computers and is highly accurate.

In flexible pavement design the tire contact area is assumed circular and the pressure uniformly distributed. However, there are experimental results that show that the contact area resembles either the circular, elliptical, or even rectangular shape, and the contact pressure distribution is either uniform, parabolic, or irregular depending on the characteristics of the tire, the inflation pressure, and the load *(1, 2)*. It is generally assumed that the difference between pavement response under real loading conditions and under uniform pressure distributed over a circular area is small. This assumption seems adequate for flexible pavement design. Yet, in analyzing trial tests it may be more appropriate to use the real loading conditions for comparing computed and measured stresses and deformations *(3, 4)*. In these cases, the existing multilayer computer programs [BISTRO *(3)* and BISAR *(4)*] were used together with superposition of circular uniform loads to approximately simulate the real loading conditions. In other cases *(5)*, the 3-D finite element computer program was used to calculate strains in a multilayer system. The limitations of both approaches either in representing the real loading conditions or in computer frame size are obvious. As the loads and tire pressures on modern vehicles are increasing, there is an evident need for a scheme for computing the pavement response under these new configurations.

A numerical approach for computing stresses and deformations in elastic multilayer systems for any shape of contact area and pressure distribution is presented. The scheme, developed for microcomputers, includes computer programs for evaluating the stresses and deformations in an elastic multilayer system under the action of a concentrated force and for performing numerical integration over the given contact area.

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The paper includes

- A brief description of the theoretical background of the programs,
- A verification of the scheme by comparing the results with other well known solutions, and
- A comparison of the results of analyses using circular and rectangular contact areas and uniform and irregular pressure distributions.

THEORETICAL BACKGROUND

Layered Elastic Systems

The mathematical background of the analysis of stresses and displacements in layered elastic systems was presented by Schiffman *(6)*. The surface loading conditions are those of the axisymmetric case and include the point load condition relevant to the present study.

The computer program developed by Uzan *(7)* for unidirectional horizontal and vertical uniformly distributed loads over a circular area and for different interlayer friction conditions (in a range between fully adhesive and completely frictionless) was modified to include the concentrated force. Near the load, the stresses and deflection tend toward infinity.

The results obtained using the computer program for the Boussinesq case (i.e., a point load applied at the surface of a homogeneous, isotropic, elastic half-space) were compared with the rigorous analytical solutions. They were revealed to be in excellent agreement. The computer program can be used for the numerical integration over the given contact area in calculating stresses and strains in layered elastic systems.

Numerical Integration of the Point Load Solution

The contact area is subdivided into surface elements, with given pressure magnitude at the nodal points. Figure 1 shows a circular area, subdivided into isoparametric elements, with 4 to 6 nodes. Due to symmetry of the circular contact area, only a half-circle, which represents a radius vector, is used to compute stresses and deformations along the x-axis. The integration over the surface element is carried out using the 4 × 4 Gauss quadrature scheme:

$$u_{im} = a_m \sum_{j=1}^{4} \sum_{k=1}^{4} w_j \, w_k \, p_{jk} \, u_{ijk}$$

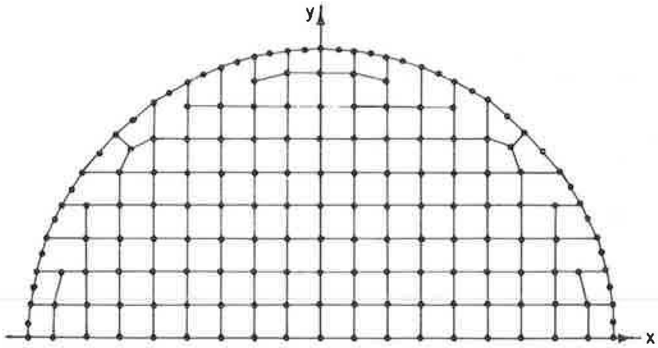

FIGURE 1 Breakdown mesh of a circular area.

where

- u_{im} = ith component of the displacement or stress vector at point (x, y) due to the given pressure distribution over element m.
- a_m = area of element M.
- w_j, w_k = Gauss weighting factors corresponding to the point of integration (x_j, y_k) inside element m.
- p_{jk} = pressure evaluated at the integration point (x_j, y_k) using the given values of the pressure at the nodal points, and up to the second-order interpolation polynomial associated with the isoparametric element type.
- u_{ijk} = ith component of the displacement or stress vector at point (x, y) due to a unit force applied at the integration point (x_j, y_k).

Summing the contributions of all elements (from $m = 1$ to m = number of elements) leads to the final result of stress, strain, or displacement components. The term u_{ijk} is evaluated numerically by Lagrange interpolation, from tabulated values obtained using the computer code for layered elastic systems with point force loading. Therefore, the numerical integration scheme requires the following steps:

1. Define the layered system, that is, layer thicknesses, elastic parameters (modulus of elasticity and Poisson's ratio), and interface conditions.
2. Define the depth where stresses and displacements are to be evaluated.
3. Run the layered elastic computer program for a unit point load, the given depth, and a number of radial distances. The values of the stresses and displacements obtained are tabulated to permit interpolation in the numerical integration.
4. Define the contact area and subdivide it into isoparametric elements. The size and shape of the element are dictated by the pressure distribution, the regularity of the limits of the contact area, and the regularity of the function u_{ijk}.
5. Run the numerical integration program to get stresses, strains, and displacements for various positions (x, y), at the given depth of the layered system.

The following paragraph show the accuracy of the results obtained with the computer programs.

VERIFICATION

Stresses, strains, and displacements were computed for the following conditions using the procedure developed:

1. One- and three-layer systems. The one-layer system corresponds to the Boussinesq case. The three-layer system variables were (a) layer thicknesses of h_1 = 1.5, 2, 3, and 4 in. and h_2 = 8 in.; (b) moduli of elasticity of E_1 = 400 ksi, E_2 = 60 ksi, and E_3 = 6 ksi; (c) Poisson ratios of μ_1 = 0.35, μ_2 = 0.40, and μ_3 = 0.45, where the indices refer to the layer number sequence. The three-layer systems chosen for the verification are those used by Chen et al. *(5)*.
2. Uniform pressure applied over a circular area. The breakdown mesh of the contact area is shown in Figure 1.
3. Two depths—at the surface and at the bottom of the asphaltic layer in the three-layer system.

The results were compared to those of the analytical or other numerical integration solutions. They were revealed to be in close agreement, with a maximum difference of 0.3 percent. The error associated with most 3-D finite element procedures is expected to be much higher than 0.3 percent. However, the accuracy and efficiency of computations are achieved at the expense of restricting the analysis to linear systems only.

ILLUSTRATIVE RESULTS

The tensile strains at the bottom of the asphalt concrete (AC) layer and the compressive strains at the top of the subgrade were computed for the following loading conditions and layered systems:

Loading Conditions

1. Single-wheel load—4,500 lb.
2. Various contact area shapes and pressure distributions—(a) 75 psi uniformly distributed on a circular area (Figure 1); (b) 63.775 psi uniformly distributed on a rectangular area; and (c) nonuniform pressure distribution on a rectangular area, as reported by Chen et al. *(5)* and shown in Figure 2.

Layered Systems

Three-layer systems as described in the previous section were treated.

The computation results are shown in Figures 3 and 4. It is seen that

1. The maximum tensile strains of the rectangular contact area are similar for the uniformly and nonuniformly applied pressure, except for the thin AC layer (h_1 = 1.5 in.). The maximum strain is found to be under the center of the loading area except for the thin AC layer. In the latter case, the strain reaches its highest value along the transverse (y) direction in Figure 2, close to the high-pressure values.

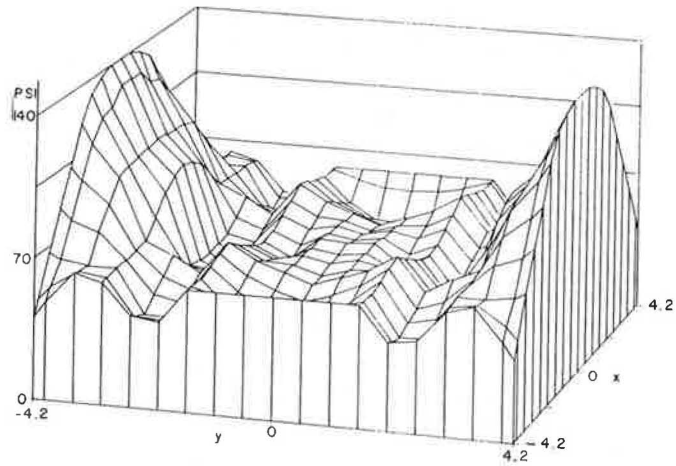

FIGURE 2 3–D pressure distribution on a rectangular contact area.

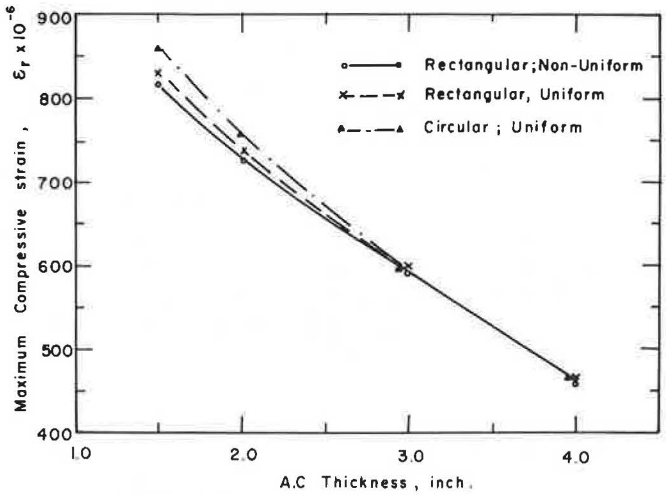

FIGURE 4 Maximum compressive strains at the top of a subgrade.

2. The tensile strains are in general higher for the circular contact area and uniformly distributed pressure than for the rectangular contact area, in cases of both uniform and non-uniform pressure distribution. In the case of a thin AC layer, the particular pressure distribution used in the analysis led to high tensile strains similar to those obtained under the circular area. For the 4-in. thickness, the effect of the contact area shape is smaller than for the 2- and 3-in. thicknesses of AC.

3. The compressive strains at the top of the subgrade are only slightly affected by the contact area shape and pressure distribution. The maximum values for all layered systems were obtained under the center of the loading area.

4. The results suggest that the effect of contact area shape and pressure distribution on the pavement response depends on the depth at which it is computed. This effect is less pronounced as the depth increases.

5. The assumption of circular contact area and uniformly distributed pressure is slightly conservative, and seems adequate for design purposes in the case of thick AC layers and pavements.

The results were compared to those presented by Chen et al. *(5)* using a 3-D finite element program. Both sets of results were obtained for the same multilayer system, wheel load, area shape, and pressure distribution. The tensile strains were similar in both analyses, but the compressive strains of the 3-D finite element were up to 40 percent lower than those of the technique described in the paper. A computation error of that magnitude can lead to an overestimation of the pavement life by a factor of 10.

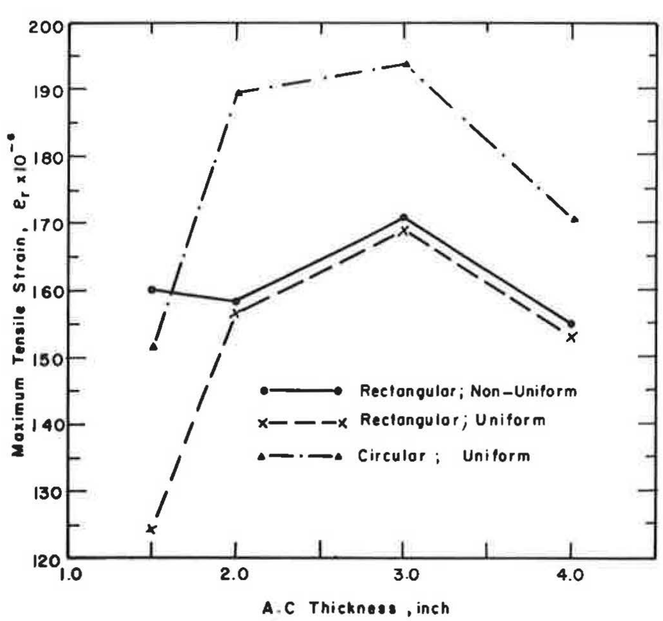

FIGURE 3 Maximum tensile strains at the bottom of the AC layer.

CONCLUSIONS

The computer programs described in this paper can be used for a wide range of problems, including (a) any contact area shape, (b) any pressure distribution, (c) vertical and horizontal loads, and (d) interface conditions varying from rough to frictionless *(7)*.

There are some limitations that should be stressed. In some cases of very thick layers, the solution may not converge. When the pavement response is computed at the surface, the results can be obtained at up to a certain distance from the point force.

The computer programs can be run on microcomputers with 256 kB memory such as the IBM PC or the Digital Rainbow 100+. The programs are written in Fortran and can be used for any pavement or geotechnical purposes. They are very accurate, and superior to the 3-D finite element programs in both accuracy and ease of use.

The theoretical background of two computer programs for analyzing multilayered linear elastic systems, under non-uniformly distributed load over any given contact area shape, has been presented. The programs, written for microcomputers, dispense with the use of mainframe computers.

The results obtained using the programs are found in close agreement with other analytical or numerical integration solutions. They are more accurate than the 3-D finite elements and other approximations. Illustrative results of the effect of the load distribution and contact area shape are presented.

The computer programs are useful to practicing and research engineers who have restricted or no access to mainframe computers. The programs may be used for analyzing problems involving uncommon load distributions and contact area shapes in both pavement and geotechnical fields.

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