# Nonlinear Analysis of Highway Bridges

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**A nonlinear procedure is developed for the derivation of the load-deflection relationship for highway bridges. The approach is based on the finite difference method. The algorithm is described. A closed-form expression is developed to model behavior of a girder section. The procedure is demonstrated on a composite steel girder bridge.**

There is a growing need for more accurate methods of bridge evaluation. Live-load spectra have changed, with amounts of load usually increased. Bridges are subjected to deterioration. On the other hand, costs of repair or strengthening are often prohibitively high. Therefore, a tool to reveal the actual strength of a bridge and to predict its serviceable life time is useful.

A procedure was developed for the flexural and torsional analysis of simply supported highway bridges. The structure is modeled as an orthotropic plate. The finite difference method is used to calculate the nonlinear bridge responses.

The objective of the paper is to present the developed procedure. The general steps and formulas are described; the approach is also demonstrated on a composite steel girder bridge. The method is simple to use and requires less computing time than the FEM or grillage method.

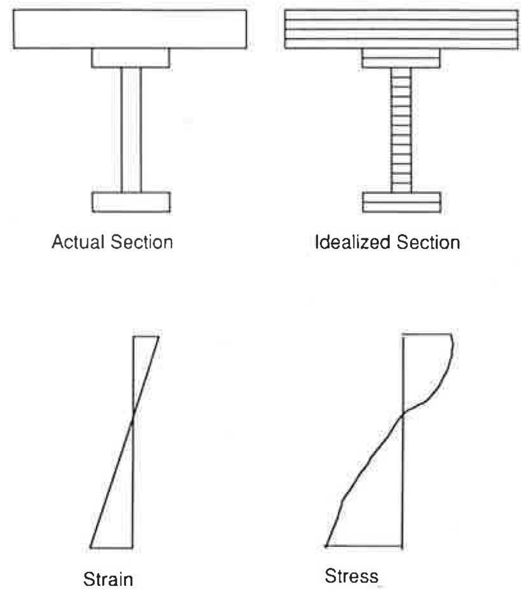


**FIGURE 1 Typical composite section and strain diagram.**

## SECTION ANALYSIS

The purpose of the section analysis is to develop force-deformation relationships for the bridge elements considered. In particular, moment-curvature ($M$–$\phi$) and torque-twist relationships are considered.

The $M$–$\phi$ curve can be determined using the computer program developed by Tantawi *(1)*. In his approach, the section is idealized as a set of uniform layers, as shown in Figure 1. Strain is increased gradually in increments. At each strain level, the corresponding moment is calculated using nonlinear stress-strain relationships for materials such as steel and concrete.

To simplify the calculations, a closed-form expression was derived to represent $M$–$\phi$ curves for the slab and composite girder section:

$$\phi = M/EI_e + C_1(M/M_y)^{C_2} \tag{1}$$

where

$EI_e$ = elastic bending rigidity,
$M$ = applied moment, and
$M_y$ = yielding moment.

$C_1$ and $C_2$ are constants determined by solving the following two equations:

$$\phi_y = M_y/EI_e + C_1\,(M_y/M_y)^{C_2} \tag{2}$$

$$\phi_u = M_u/EI_e + C_1\,(M_u/M_y)^{C_2} \tag{3}$$

The resulting $C_1$ and $C_2$ are

$$C_1 = \phi_y - M_y/E_{Ie} \tag{4}$$

$$C_2 = \ln[(\phi_u EI_e - M_u)/(\phi_y EI_e - M_y)]/\ln(M_u/M_y) \tag{5}$$

in which

$M_u$ = ultimate moment,
$\phi_y$ = curvature at yielding, and
$\phi_u$ = curvature corresponding to ultimate strength.

For composite girders, $C_2$ ranges from 20 to 24 and $C_1$ from 25 to 28 × $10^{-5}$ ft$^{-1}$. For slabs, $C_2$ is about 22 to 26 and $C_1$ 27 to 30 × $10^{-5}$ ft$^{-1}$.

An $M$–$\phi$ curve for a typical composite girder is presented in Figure 2. For comparison, an $M$–$\phi$ curve calculated using Tantawi's program *(1)* is also shown in Figure 2. A torsion-twist curve for the same composite girder is shown in Figure 3.

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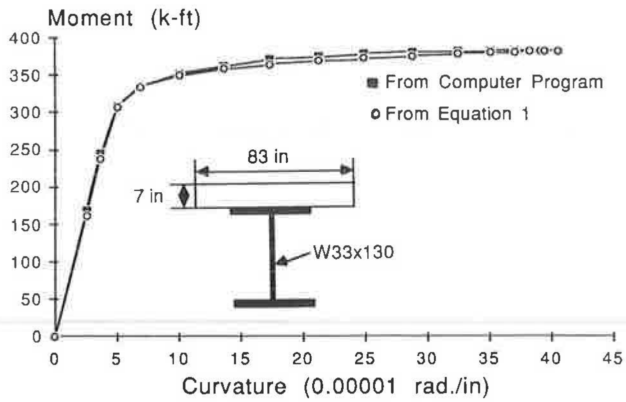


**FIGURE 2** $M$–$\phi$ **curve for a typical composite section.**

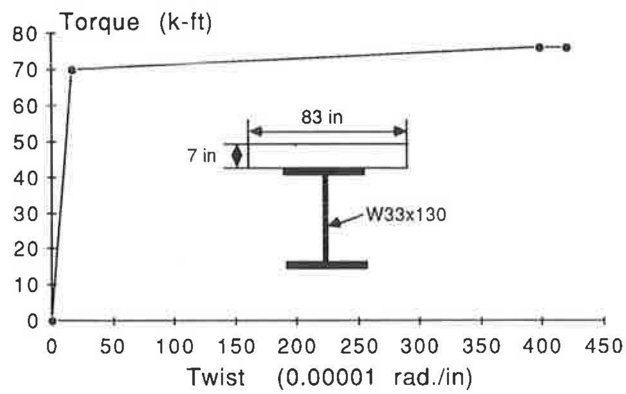


**FIGURE 3 Torsion-twist curve for a typical composite section.**

## FINITE DIFFERENCE ANALYSIS

Based on the assumptions made by Heins and Yoo *(2)* and Heins and Kuo *(3)*, the system of girders, diaphragms, and slab is represented by an orthotropic plate. Segments of the longitudinal and transverse members and slab are shown in Figure 4. The corresponding plate elements are also shown in Figure 4. In Figure 4, $m_x$, $m_{xy}$ and $q_x$ are the bending moment, torsional moment, and shear, respectively, exerted on the transverse member and slab; $m_y$, $m_{yx}$ and $q_y$ are the bending moment, torsional moment, and shear, respectively, exerted on the longitudinal member and slab. The equilibrium differential equation for the plate element is

$$-P(x, y) = \partial^2 M_x/\partial x^2 + \partial^2 M_{xy}/\partial x \partial y + \partial^2 M_{yx}/\partial y \partial x + \partial^2 M_y/\partial y^2 \quad (6)$$

where

$M_x$ = the bending moment per unit width of the section in $x$ direction,
$M_y$ = the bending moment per unit width of the section in $y$ direction,
$M_{xy}$ = the twisting moment per unit width of the section in $x$ direction,
$M_{yx}$ = the twisting moment per unit width of the section in $y$ direction, and
$P(x, y)$ = the applied load per unit area.

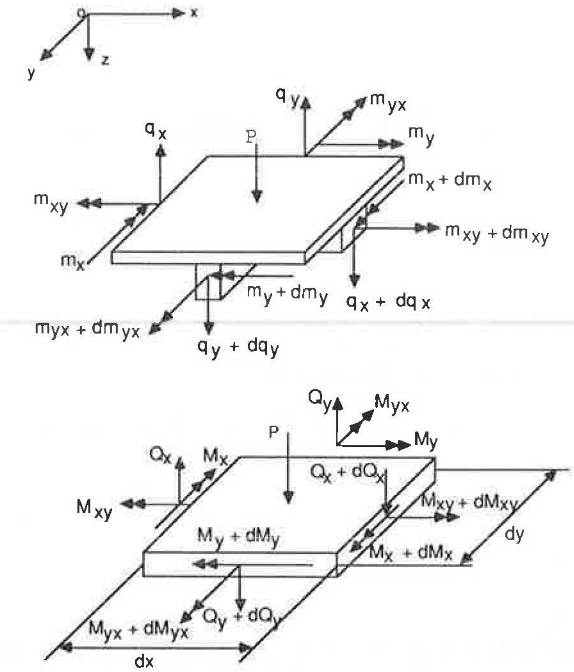


**FIGURE 4 Orthotropic plate element.**

The total load per plate element $P_T$ can be divided into three components exerted on the girder, diaphragm, and slab.

A system of differential equations was developed that expresses the compatibility conditions in force and displacement equations between plate elements. This system can be expressed in the following matrix form:

$$[K] \{w\} = \{P\} \quad (7)$$

where

$[K]$ = stiffness matrix,
$\{w\}$ = vector of grid point deflections, and
$\{P\}$ = vector of loads.

From these equations, the unknown grid point deflections $\{w\}$ were calculated. Then moments ($M_x$, $M_y$, $M_{xy}$, $M_{yx}$) and shear forces ($Q_y$, $Q_x$) were derived from the following equations:

$$M_x = -D_x \partial^2 w/\partial^2 x \quad (8)$$

$$M_y = -D_y \partial^2 w/\partial^2 y \quad (9)$$

$$M_{xy} = -D_{xy} \partial^2 w/\partial x \partial y \quad (10)$$

$$M_{yx} = --D_{yx} \partial^2 w/\partial y \partial x \quad (11)$$

$$Q_y = \partial M_{xy}/\partial x + \partial M_y/\partial y \quad (12)$$

$$Q_x = \partial M_{yx}/\partial y + \partial M_x/\partial x \quad (13)$$

where the various order derivatives can also be evaluated using the finite difference method.

In the finite difference analysis, special mesh patterns were developed that allow for a considerable reduction of computing time. The details of these patterns are given by Zhou and Nowak (unpublished).

## BRIDGE STRENGTH EVALUATION

Based on the finite difference formulation, a procedure was developed to evaluate the ultimate capacity of bridge structures. The evaluation involves a considerable nonlinear structural analysis. The available procedures, such as the Newton-Raphson algorithm and the modified Newton-Raphson algorithm or incremental method *(4)*, require excessive computational effort. Therefore, a special computer time saving procedure was developed for this study.

The developed incremental-iterative approach includes seven major steps. First, elastic stiffness is used to evaluate bridge responses under dead loads. Then truck load is increased by increments. The incremental load $\Delta P_j$ is determined as a function of the initial load increment $\Delta P_0$ and number of iterations $n_{j-1}$ in the previous load increment,

$$\Delta P_j = \Delta P_0 / n_{j-1} \tag{14}$$

Using Equation 1, the tangent stiffness matrix is formed, corresponding to the deflections $\{w_j\}$ due to truck load level $P_j$. The tangent stiffness $K_t(w_j)$ is defined in Figure 5.

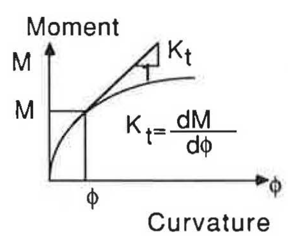


**FIGURE 5 Evaluation of tangent stiffness.**

Iterations are carried out to compute deflections and curvatures corresponding to the increased load level using the finite difference formulas

$$\{w_{i+1}\} = [K_t(w_j)]^{-1}\{\delta_i\} + \{w_i\} \tag{15}$$

where $\{\delta_i\}$ is the unbalanced force resulting in the previous iteration.

Then the secant stiffness corresponding to deflections $\{w_{i+1}\}$, $K_s(w_{i+1})$, is derived, as shown in Figure 6. The unbalanced force resulting in this iteration is evaluated, see Figure 7 as

$$\{\delta_{i+1}\} = \{P_j\} + \{\Delta P_j\} - [K_s(w_{i+1})]\{w_{i+1}\} \tag{16}$$

If the unbalanced force is not close to zero, the next iteration is carried out. The tangent stiffness, as in the first iteration, is used. This process significantly reduces computation time. The calculations are continued until convergence criteria are satisfied and the unbalanced force is close to zero.

After each load increment, failure criteria are checked. The calculations are terminated when the permanent deflection exceeds 1 percent of span length or concrete crush occurred. Otherwise another cycle of iteration is carried out using a next load increment. The algorithm is presented in Figure 8.

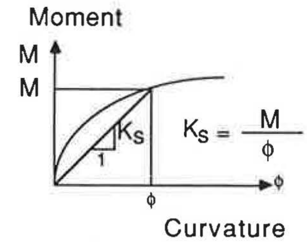


**FIGURE 6 Evaluation of secant stiffness.**

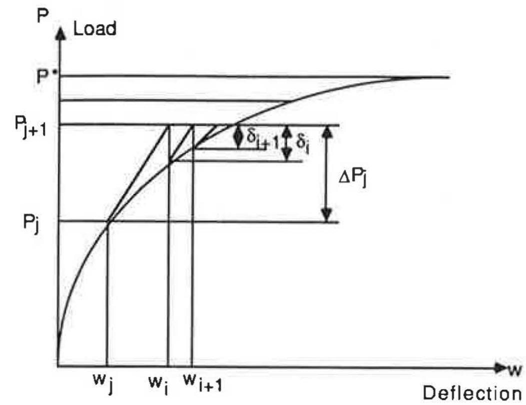


**FIGURE 7 Solution process using incremental-iterative method.**

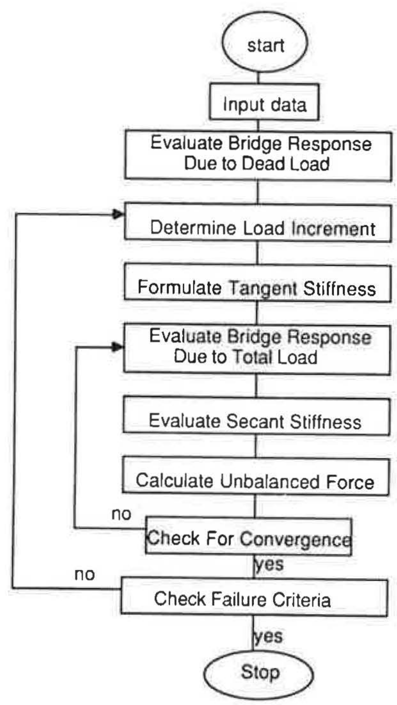


**FIGURE 8 Flowchart for incremental-iterative method.**

## NUMERICAL EXAMPLE

The developed procedure is demonstrated on a composite steel girder bridge designed according to AASHTO guidelines *(5)*. The span is 60 ft; the cross section is shown in Figure 9.

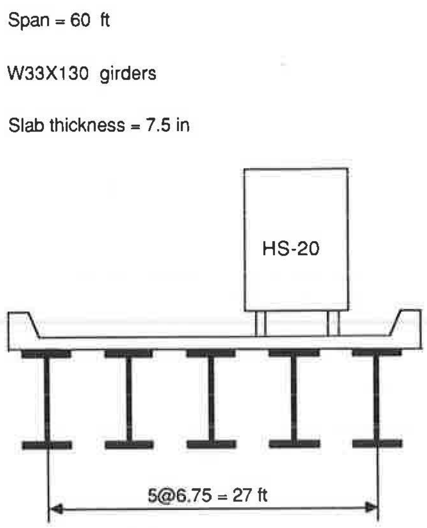


**FIGURE 9 Cross section for a typical composite girder bridge.**

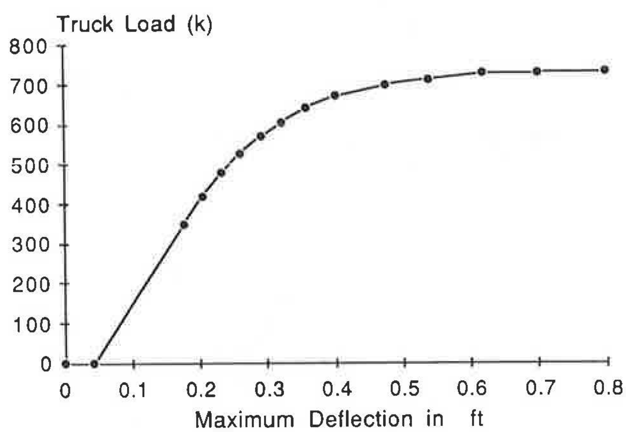


**FIGURE 10 Load-deflection curve for a typical composite girder bridge.**

AASHTO *(5)* truck HS–20 wheel configurations and load proportions were used, with the transverse position as shown in Figure 9. The curve for the resulting live load of total truck weight versus deflection is presented in Figure 10.

## CONCLUSIONS

A numerical procedure was developed for nonlinear analysis of girder bridges. The approach based on the finite difference method allowed for evaluation of the ultimate strength of the structure.

A closed-formula expression was developed to model $M$–$\phi$ curves for composite girder or slab sections. The formula permitted considerable computing time saving compared to other available methods.

A load-deflection curve was calculated for a typical composite steel girder bridge to demonstrate the developed procedure.

## ACKNOWLEDGMENTS

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