A Methodology for Feeder-Bus Network Design

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The U.S. transit industry faces financial difficulties. Among the strategies suggested for improving transit financial conditions, the development of better-integrated intermodal systems has the advantage of potentially achieving both cost reduction and improved service. A network optimization methodology for the design of an integrated feeder-bus–rail rapid transit system is presented. The Feeder-Bus Network Design Problem (FBNDP) is defined as that of designing a set of feeder-bus routes and determining the frequency on each route so as to minimize operator and user costs. The FBNDP is first considered under many-to-one demand and its formulation is discussed as a mathematical programming problem. Then the generalization of the formulation to the many-to-many demand pattern is reviewed. The FBNDP is a larger and complex routing-type problem that can be solved only heuristically. A heuristic method that generalizes the savings approach to consider operating frequency is presented. The analysis presented illustrates the capabilities of the proposed model as a strategic planning tool for feeder-bus network design. It indicates that changes that increase the relative weight of operator cost often result in feeder-bus networks with less circuitous routes operated at lower frequencies, whereas changes that increase the relative weight of user cost result in feeder routes operated at higher frequencies. The solutions provided by the proposed model have been tested and found superior to manually designed networks, particularly under variable demand.

The U.S. transit industry faces financial difficulties *(1)*. Rising costs and shrinking resources are the main causes for these difficult financial conditions. From 1960 to 1983, the annual urban transit operating costs rose by more than $6.7 billion. Only a small declining fraction of these operating expenditures were covered by farebox revenues *(2, 3)*. In recent years, the uncertain financial conditions of the transit industry have been made more acute by the reductions in federal funding for mass transit. Since FY 1981, total federal funding available for mass transit has declined by 28 percent *(2)*. In view of declining federal assistance, transit agencies can expect to face growing pressure to reduce deficits.

Several strategies have been suggested for improving the financial conditions of transit agencies: (a) new funding schemes to raise subsidies from local or state governments, or both; (b) new fare structures to increase revenues; (c) improvements in service quality to attract ridership; (d) reduction in the commitment to peak-period services ("shedding the peak"); (e) reduction or elimination of services to low-density areas; (f) use of more cost-effective technologies; (g) privatization of transit services; and (h) development of better-integrated intermodal systems.

Each of these strategies has its shortcomings. New funding schemes may be infeasible under the current political climate, because they would require governments to increase taxes for mass transit. New fare structures based on distance or time, or both, may increase revenue (depending on demand elasticities) with a loss of patronage to the automobile mode. Such a shift would increase the need for investments in highway facilities and would have negative environmental impacts. Service improvements would result in higher transit operating costs, which most likely would not be fully compensated for by increased revenues. The logic behind the strategy of shedding the peak is that a substantial proportion of transit investment is needed only for peak-period service. If private operators could share the burden of these services, it would reduce the capital requirements of transit agencies. The success of the strategy of shedding the peak as well as that of privatization depends on the willingness of the private sector to enter the transit industry, which, so far, has been limited.

The strategy of reducing (or eliminating) services to low-density areas may cause some captive riders to lose their mobility. Public transit operators may be reluctant to consider this strategy *(4)*. The employment of more cost-effective transportation technologies, such as vanpools, carpools, paratransit, and taxi, has in some cases reduced the cost of transit services. However, these technologies are suitable primarily for low-density areas and are usable only as components of an integrated intermodal transit system.

The development of better-integrated intermodal transit systems can achieve both reduction in cost and improvement in service quality. Better integration can reduce transit cost by eliminating duplications and employing the most cost-effective mode in each segment of the system. An improvement in service quality would result from better coverage and reduced access cost, fewer transfers, and shorter travel times. In turn, improved service would increase revenues to provide further deficit reduction. One type of integration with specific additional advantages involves the employment of bus transportation as access mode to a rail rapid transit system. First, a well-integrated feeder-bus–rail rapid transit system reduces parking requirements at the rail stations, thereby reducing the rail system's capital cost. Second, a well-integrated feeder-bus–rail system may attract automobile trips (primarily work trips), thereby increasing the economic viability of the rail system.

The potential for improving the financial condition of transit by designing an integrated feeder-bus–rail rapid transit system has been recognized for some time *(5–8)*. The integration of

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feeder bus and rail rapid transit has been suggested as one of the most promising future directions for public transit in large U.S. cities. In the last 15 years, several new rail rapid transit systems have been constructed in large U.S. cities (San Francisco; Atlanta; Washington, D.C.; Buffalo; Miami). The bus and rail rapid transit systems in most of these cities are not well integrated *(9)*. A common practice in designing feeder-bus networks has been to turn the buses back at the nearest rail stations. The duplication of bus service along rail lines is also a common phenomenon *(9)*.

There is currently no methodology for designing an integrated feeder-bus–rail transit network. Most of the existing work on transit network design has focused on single-mode networks *(10–12)*. Often the focus has been on individual components of the network-design problem such as route structure *(13–15)*, service frequency *(16, 17)*, and station spacing *(18, 19)*. Results are presented of a recent study in which a network optimization methodology was developed for designing an integrated feeder-bus–rail transit system. The proposed methodology focuses on the network design elements of the integration problem while including related operational elements such as service frequency.

THE FEEDER-BUS NETWORK DESIGN PROBLEM

Integrated feeder-bus–rail transit systems have a variety of design components, which may include the network structure of the rail and bus systems and the levels of service on each component of the system. However, the basic decisions regarding the structure of the rail network, such as the locations of rail lines and rail stations, are based on projected land use and are not greatly affected by decisions regarding the feeder-bus system. The integration problem can therefore be viewed as one of designing a feeder-bus system that can access an existing rail network, which is defined as the Feeder-Bus Network Design Problem (FBNDP). Specifically, the FBNDP is the problem of designing a set of feeder-bus routes and determining the service frequency on each route so as to minimize the sum of operator and user costs.

The proposed methodology represents the FBNDP as a network optimization problem. The network includes two types of nodes—rail nodes and bus nodes—that represent rail stations and bus stops, respectively. Similarly, rail links represent rail line segments, whereas bus links represent feeder-bus route segments. The demand is assumed to be concentrated at nodes and the temporal distribution of demand is not represented. This representation of demand is common to network models, primarily those dealing with strategic problems such as network design and location. In the FBNDP, the demand can be viewed as hourly averages for a given time period, for example, peak period or off-peak period.

The FBNDP can be viewed as the problem of achieving the optimal balance between operator cost and user cost. Operator cost includes the capital cost associated with the fleet and variable costs, which are related to vehicle hours of travel. In the context of the FBNDP, user cost includes the time costs of access, wait, and riding. Because the design of the feeder-bus system determines the riding time in the rail system, riding time includes in-vehicle time on both bus and rail.

Two different demand patterns are considered—many-to-one and many-to-many. Many-to-one (M-to-1) refers to a demand pattern with multiple origins and a single destination. Peak-period work trips to and from the central business district (CBD) may exhibit this pattern. Many-to-many (M-to-M) demand refers to a pattern with multiple origins and destinations. Clearly, nonwork trips would likely follow this pattern.

M-to-1 FBNDP

In the development of a network optimization model for the M-to-1 FBNDP, the following assumptions are made:

1. Each bus stop is served by one feeder-bus route.
2. Each bus route is linked to exactly one rail station.
3. Buses have standard capacity and operating speed.

The first assumption may appear restrictive. However, it is valid for a system serving M-to-1 demand. When all the passengers have a common destination, it is unnecessary to have multiple bus routes serving the same bus stop. This assumption is relaxed in the case of the M-to-M demand pattern. The second assumption implies that buses are not allowed to travel along rail lines. This assumption is consistent with one of the basic purposes of integration discussed earlier—the elimination of duplicate services. The third assumption is consistent with the common practice of operating fixed-route transit service with the same type of vehicle. It is also a widely accepted assumption in mathematical models for routing-type problems, which significantly reduces the complexity of the models.

Figure 1 presents a feeder-bus–rail transit system with five rail lines serving a single destination. Also shown are the feeder-bus routes for two of the rail lines. The feeder-bus–rail system shown in Figure 1 can be represented as a spanning-tree network (Figure 2) in which the destination is the root, the rail stations are first-level nodes, and the bus stops are higher-level nodes. Figure 2 constitutes a conceptual representation of an integrated feeder-bus–rail system, to be used in the formulation of a network optimization model for the FBNDP. The costs associated with the first-level links (rail links) represent riding-time costs in the rail system. Those associated with the higher-level links (bus links) represent bus operating costs and passenger riding-time costs. Those associated with the bus nodes represent passenger wait-time costs.

Given the association between a feeder-bus–rail system as shown in Figure 1 and the corresponding spanning-tree network of Figure 2, the optimal solution to the FBNDP under an M-to-1 demand pattern would be obtained by finding the spanning-tree network that minimizes the sum of operator and user costs. The problem of finding the minimum-cost spanning-tree network can be formulated as a mathematical programming model *(20)*. The structure of the model is as follows:

Minimize (rail riding cost + bus operator cost + bus user cost) subject to logical route constraints, route capacity constraints, fleet size constraint, and route length constraints.

There are five types of logical route constraints. The first places each bus node on a single feeder-bus route. The second

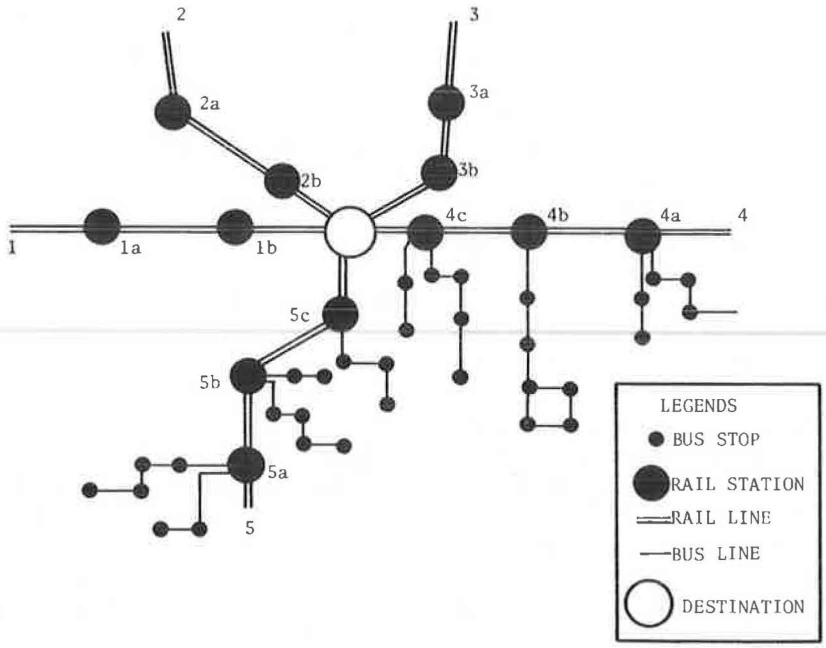

FIGURE 1 Feeder-bus–rail transit system for the case of many-to-one demand.

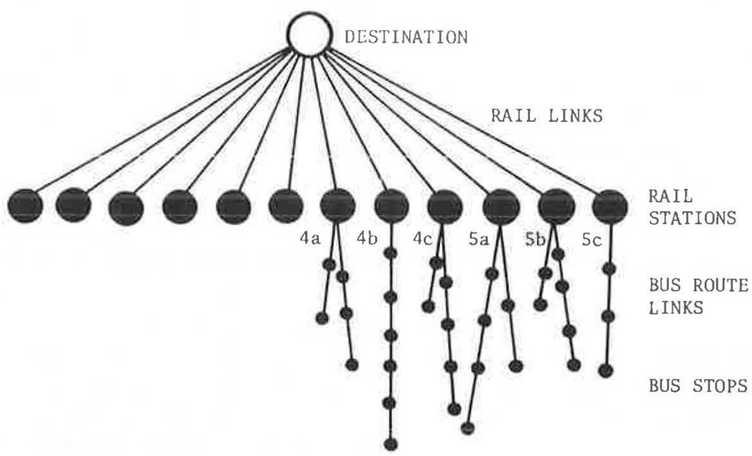

FIGURE 2 Spanning-tree network representation of the feeder-bus–rail transit system for the case of many-to-one demand.

ensures that each bus route is linked to a single rail node. The third is a route continuity constraint, which states that a route that enters a bus node must leave that node. The fourth ensures that every route is linked to a rail node. This is a "subtour elimination" constraint, which appears in mathematical models for the well-known Traveling Salesman Problem (TSP). The route capacity constraints ensure that the demand on any route does not exceed the capacity. It should be noted that unlike existing vehicle routing problems, in the FBNDP route capacity is not an input parameter, because route frequency is a decision variable. The fleet size constraint ensures that the total seat-hours offered on the feeder-bus system does not exceed the available seat-hours. Finally, the route length constraints ensure that the length of any route does not exceed the prespecified maximum route length.

The mathematical programming model for the M-to-1 FBNDP is a large and difficult vehicle-routing type model. There are two elements that contribute to the added complexity of this model relative to mathematical models for existing vehicle-routing problems. First, the objective function in the FBNDP model is nonlinear. Second, the FBNDP includes an additional decision variable—service frequency. The proposed solution method for the FBNDP is discussed next.

M-to-M FBNDP

The proposed mathematical model for the M-to-1 FBNDP can be generalized to the M-to-M demand pattern. The M-to-M FBNDP differs from the M-to-1 FBNDP in that the set of destinations includes the entire set of rail stations. In the M-to-M FBNDP, the demand at each bus stop is a multidimensional quantity. Clearly, the M-to-M FBNDP is not simply a sum of M-to-1 FBNDPs, because under an M-to-M demand pattern a single feeder-bus route usually serves demands to multiple destinations. The problem under the M-to-M demand pattern appears significantly more difficult. First, the design of the feeder-bus network should take into account not only the linkings to alternative rail stations, but also alternative connections to rail lines. Depending on which rail line is chosen for connection, passengers may or may not have to transfer between rail lines. Second, the optimal feeder-bus network may include some bus stops on more than a single feeder-bus route. This results in a significantly more complex feeder-bus network.

Interestingly, with relatively minor modifications the conceptual representation of Figure 2 is applicable to the M-to-M FBNDP. Figure 3 shows a feeder-bus–rail transit system with five destinations, and Figure 4 shows the spanning-tree network representation of the system. Unlike the spanning tree of Figure 2, that in Figure 4 includes multiple rail links between each rail node and the dummy node. These links represent the travel from the associated rail station to all other destinations. In the M-to-1 FBNDP there is only a single destination and therefore only a single link is needed to represent the travel from any given rail station to the destination. Unlike Figure 2, the tree network representation in Figure 4 does not provide a suitable representation of the M-to-1 FBNDP for a network optimization model, because a bus node in Figure 4 does not uniquely identify the destination of demand.

To formulate a mathematical model for the M-to-M FBNDP, the network representation of Figure 4 needs to be modified so that a single destination is associated with each bus node. This is done by splitting each bus node into multiple subnodes (one for each destination). The locations of the subnodes are the same as that of the original bus node. However, each subnode

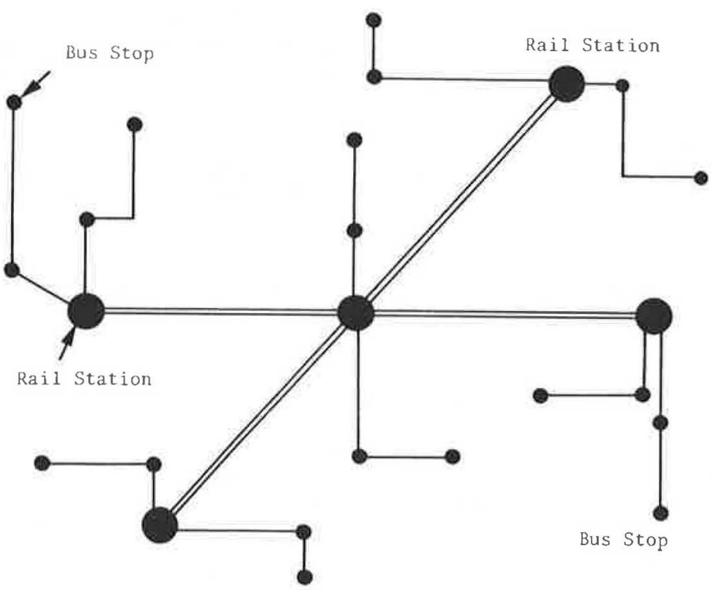

FIGURE 3 Feeder-bus–rail transit system for the case of many-to-many demand.

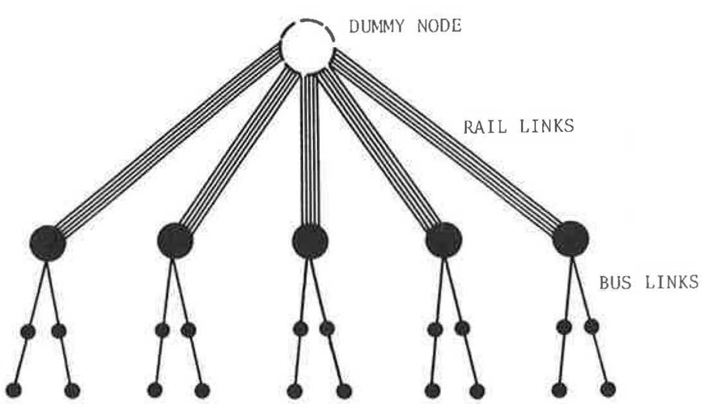

FIGURE 4 Spanning-tree network representation of the feeder-bus–rail transit system for the case of many-to-many demand.

represents the demand to a single destination. With this transformation, the spanning-tree representation of Figure 4 becomes similar to that of Figure 2. The only difference is the multiple rail links of Figure 4. However, this does not represent any conceptual difficulty, because the flow on the rail link connected to any given rail node is completely defined by the allocation of bus subnodes to that rail node. The spanning-tree representation of Figure 4 (with the foregoing transformation) represents the M-to-M FBNDP as an M-to-1 FBNDP. The mathematical model for M-to-1 FBNDP can therefore be used to represent the M-to-M case.

As stated, the proposed mathematical model includes a fleet size constraint. As such it represents the FBNDP in the context of short-term planning. In the long term, the fleet size can be adjusted to changes in the system. With a relatively minor modification, the proposed model can be adapted to represent the long-term FBNDP. In the long-term model, the fleet size constraint is removed and a fleet size cost component is added to the objective function. This additional cost component represents the depreciation cost associated with the fleet, based on an assumed unit depreciation cost per bus hours used.

SOLUTION ALGORITHM

As stated earlier, the FBNDP is a large and complex routing-type problem. Routing models with linear objective functions can be solved optimally only for very small test networks *(21)*. Under the M-to-1 demand pattern, for a system with 5 rail stations, 50 bus stops, and 6 feeder-bus routes, the proposed mathematical model would include 1,756 variables and more than 10^{15} constraints. Clearly such a model can be solved only heuristically.

The spanning-tree networks of Figures 2 and 4 are similar to the network representation of a single-echelon physical distribution system. On the basis of this similarity, one can notice the similarities between the FBNDP and the Multi-Depot Vehicle-Routing Problem (MDVRP), which is the problem of designing a set of delivery routes from several depots to a large number of demand points so as to minimize the total route distance. The similarity between the two problems is useful to the development of a heuristic method for the FBNDP, because it may allow the use of certain concepts of existing heuristics for the MDVRP. However, the FBNDP differs from the MDVRP in several basic elements: (a) although the objective in the MDVRP is to minimize operator cost, the FBNDP objective includes both operator and user costs; and (b) in the MDVRP the operating frequency is predetermined, whereas in the FBNDP operating frequency is a decision variable.

A detailed description of the proposed heuristic method for the FBNDP is beyond the scope of this paper and can be found elsewhere *(20)*. Only a summary of the basic elements is provided here. The variables, parameters, and measures used in the discussion of the heuristic method are defined in Table 1. The method consists of an initial algorithm and two improvement procedures. The initial algorithm generates an initial feasible feeder-bus network and determines the frequency on each feeder-bus route. Subsequently, the initial network is improved by these procedures.

The initial algorithm uses the sequential savings approach, which was used in previous algorithms for the vehicle-routing problem *(22)*. It starts by computing the cost of a direct route

TABLE 1 VARIABLES, PARAMETERS, AND MEASURES FOR HEURISTIC METHOD

Symbol	Units	Description
q_i	Pass./Hour	Average demand per hour at bus stop i
f_{ij}	Veh./Hour	Bus operating frequency of direct route from stop i to station j
l_{ij}	Miles	Distance from stop i to station j
C_{sj}	$/Passenger	Unit rail cost from station j to destination s
λ_0	$/Veh.-Mile	Unit bus operating cost
λ_r	$/Pass.-Hour	Value of passenger riding time
λ_w	$/Pass.-Hour	Value of passenger wait time
U	Miles/Hour	Average bus operating speed
L_k	Miles	Length of route segment k
Q_k	Pass./Hour	Demand on route segment k
TC_i^j	$/Hour	Total cost of direct route from stop i to station j
$SAVTC_i^k$	$/Hour	Saving from including stop i in route segment k

from each bus node to its nearest rail node as given by Equation 1. The four terms of Equation 1 represent rail riding cost, bus operating cost, bus passenger riding cost, and bus passenger waiting cost, respectively. The cost of wait time at rail stations is assumed to be relatively small and is not represented in Equation 1.

$$TC_i^j = C_{sj}q_i + 2 \cdot \lambda_0 f_{ij} \cdot l_{ij} + (\lambda_r/U) \cdot l_{ij} \cdot q_i + \lambda_w q_i/2f_{ij} \quad (1)$$

Equation 1 provides the direct-route cost as a function of operating frequency. To obtain the minimum direct-route cost, the first-order conditions are used to obtain the optimal frequency, as given by Equation 2.

$$f_{ij}^* = 0.5(\lambda_w q_i/\lambda_0 l_{ij})^{1/2} \quad (2)$$

From Equation 2, it can be seen that the optimal frequency is that which results in equality between operator cost and passenger wait-time cost. Substituting Equation 2 in Equation 1, the minimum cost of a direct route is obtained as follows:

$$TC_i^j = C_{sj}q_i + 2(\lambda_0\lambda_w l_{ij} q_i)^{1/2} + (\lambda_r/U) \cdot l_{ij}q_i \quad (3)$$

The second and third components of Equation 3 represent the direct-route cost associated with the feeder-bus system. The algorithm initiates a route by selecting the unassigned bus node with the largest feeder-bus component of direct-route cost. The route is then expanded by including unassigned bus nodes based on the criterion of maximum savings. The savings from including unassigned bus node i in route k is estimated by Equation 4:

$$SAVTC_i^k = TC_i^j - \{C_{sj}q_i + 2(\lambda_0\lambda_w)^{1/2}[(L_k^a Q_k^a)^{1/2} - (L_k Q_k)^{1/2}] + (\lambda_r/2U)(L_k^a Q_k^a - L_k Q_k)\} \quad (4)$$

where L_k^a *and* Q_k^a are the route length and demand after the inclusion of bus node i.

Equation 4 provides a correct measure of the savings in transportation cost from including bus node i in route k only if the rail station that minimizes the direct cost for i is the same as that to which route k is linked. If this is not the case, Equation 4 needs to be modified. This is done by using the "modified distance" concept proposed by Tillman and Cain *(23)*. The initial algorithm continues to expand the emerging route until there is no cost savings from any further expansion or until any savings would result in a violation of the constraint on route length. If the emerging route cannot be expanded, a new route is initiated.

The improvement procedures attempt to correct two limitations of the initial algorithm. First, since the initial algorithm builds routes sequentially, the order of bus nodes on any given route may be suboptimal. Any reduction in route length would reduce operator and user riding costs. Therefore, the first improvement procedure attempts to reduce the length of each route by solving the TSP. This is done using the 2-optimal procedures proposed by Lin *(24)*. The second limitation of the initial algorithm is that a bus node assigned in an early stage is not reassigned at a later stage as new routes are developed. The second improvement procedure attempts to correct this problem by displacing a bus node from its current route to another route if it results in a reduction of total cost.

ANALYSIS

The purpose of the following analysis is twofold. First, it is shown that the proposed heuristic method provides reasonable feeder-bus networks. At this point the solutions provided by the method relative to optimality cannot be evaluated. To do so would require the optimal solution for one or more test problems or a "good" lower bound on the optimal solution. Given the complexity of the FBNDP and the computational requirement of the mathematical programming model, neither of these can be accomplished at this stage. There is currently no adequate method for deriving useful lower bounds for vehicle-routing problems. The validation of the proposed heuristic method for the FBNDP was done by first comparing various measures of the network structure with those observed in real-life transit systems. Then the results provided by the proposed model were evaluated by comparing them with those developed manually by five transportation planners. The overall results of this comparison show that the proposed model designs bus networks that are superior to manually developed networks, even for small problems. The advantage of the proposed model is found to be most significant when there are differences in the demand generated at various bus stops. It is important to state that the heuristic model can be used to provide an initial feeder-bus network that may be improved incrementally by an experienced transit planner.

The second and perhaps more important purpose of this analysis is to show the capabilities of the proposed model as a strategic planning tool for the design of a feeder-bus network. In this function, the model is used for answering "what if" questions, that is, for providing responses to changes in the system. The changes to be considered represent new transit design and operating policies as well as changes in the environment in which the transit agency operates, that is, changes that are beyond the control of the agency. These changes are represented by several test cases. The responses of the model are evaluated by comparing the network structure under each test case with that obtained for the base case.

A base case is defined in which the demand is constant over the entire service area. This avoids the irregularities in the solution that would be caused by variable demand and enables the inspection and identification of the changes in network structure that take place under the new conditions. The assessment of the model's responses is done based on systemwide measures such as vehicle miles of travel, passenger miles of travel, operator cost, user cost, and others. In each test case the changes in systemwide measures to be expected under that scenario can be specified. The model's responses are then evaluated relative to the expected changes.

The test problem for this analysis includes 55 bus stops and 4 rail stations, covering a service area of 2×2.5 mi^2. The test problem was designed so that the bus stop density is 11 stops per square mile with demand density of 2,200 passengers per square mile. Such a demand density is typical for urban areas *(25)*. Table 2 provides the bus stop locations and demands for the base case. The locations of the rail stations are given in

TABLE 2 BUS STOP LOCATIONS AND DEMANDS

Bus Stop No.	X-Coordinate[a]	Y-Coordinate[a]	Demand (trips/hour)
1	30	234	200
2	62	235	200
3	119	250	200
4	182	249	200
5	134	228	200
6	163	230	200
7	115	222	200
8	87	215	200
9	24	203	200
10	60	193	200
11	125	197	200
12	150	210	200
13	183	196	200
14	108	186	200
15	85	177	200
16	37	169	200
17	130	173	200
18	185	164	200
19	12	163	200
20	67	153	200
21	105	157	200
22	123	152	200
23	32	133	200
24	55	135	200
25	73	135	200
26	89	144	200
27	142	137	200
28	161	143	200
29	18	107	200
30	46	107	200
31	107	115	200
32	147	117	200
33	172	124	200
34	31	95	200
35	91	103	200
36	113	99	200
37	13	80	200
38	66	87	200
39	83	83	200
40	141	92	200
41	167	97	200
42	67	65	200
43	122	75	200
44	150	67	200
45	177	68	200
46	95	59	200
47	17	47	200
48	47	43	200
49	130	48	200
50	71	35	200
51	108	33	200
52	169	35	200
53	13	25	200
54	35	17	200
55	63	7	200

[a] The distances are specified in hundreds of miles.

TABLE 3 RAIL STATION LOCATIONS

Rail Station No.	X-Coordinate[a]	Y-Coordinate[a]
56 (Destination)	42	72
57	78	116
58	123	137
59	160	178

[a] The distances are specified in hundreds of miles.

Table 3. Rail station 56 represents the destination under the M-to-1 demand pattern. The parameters for the base case are given in Table 4. The bus operating cost is based on a cost per seat mile of $0.06 and bus capacity of 50 seats *(26)*. Typical values from the literature are used for riding-time and wait-time costs as well as for bus capacity and operating speed.

Table 5 provides the base-case solution, which is illustrated in Figure 5. The feeder-bus network includes 16 routes with service frequency ranging from 16 to almost 26 trips per hour (headways in the range of 2.3 to 3.75 min). The average headway is 2.8 min and the total route distance per square mile is 3.0. These values are within the range observed in real-life transit systems. The base-case solution includes 703 vehicle-mi of travel and 7,926 passenger-mi of travel and provides a system cost of $6,000/hr, consisting of 10 percent rail cost, 35 percent bus operating cost, and 55 percent bus user cost.

The analysis includes the following test cases:

1. Change in design objective,
2. Introduction of demand variability,
3. Change in vehicle capacity,
4. Change in labor and fuel cost,
5. Opening of a new rail station, and
6. Closing of a rail station.

In the first test case the objective function is defined as that of minimizing bus user cost. The purpose is not to suggest the minimization of user cost as an appropriate objective but to evaluate the model's response to changes in the design objective. Under the new objective, a reduction in bus user cost and an increase in bus operator cost should be expected relative to the base-case network. The second test case introduces variable demand. Using Monte Carlo simulation, the demand at bus stops is generated from a uniform distribution with a mean of 200 passengers per hour. Under variable demand a feeder-bus network with larger differences in route length and route frequency should be expected. In the third test case, a change is considered in the type of vehicle used; a standard bus with seating capacity of 50 is changed to an articulated bus with a seating capacity of 95. This change in vehicle type reduces operating cost per seat mile because of higher labor productivity. Assuming that labor cost constitutes 60 percent of total operating cost and fuel and maintenance cost represents 40 percent, the operating cost per vehicle mile for an articulated bus is estimated at $3.30. As vehicle capacity increases, a feeder-bus network with fewer but longer routes, operated at lower frequencies, should be expected. The combined effect of longer routes and lower frequencies should result in higher user costs. The change in total vehicle miles of travel and consequently in total operator cost depends on the spatial distribution of demand and cannot be determined a priori.

The operating and maintenance (O&M) cost of a typical transit service consists of 60 percent labor cost, 25 percent fuel cost, and 15 percent administrative cost *(27)*. In the fourth test an increase of 40 percent in both labor and fuel costs is considered. This translates into an increase of $1.02 per vehicle mile. With an increase in operating cost per vehicle mile, operators can be expected to restructure the bus network to reduce vehicle miles of travel. This can be achieved by a

TABLE 4 BASE-CASE PARAMETERS

Descriptions	Units	Value
Bus Operating Cost	\$/veh.-mile	3.0
Rail User Cost	\$/pass.-mile	0.15
Riding Time Cost	\$/pass.-hr.	4.0
Waiting Time Cost	\$/pass.-hr.	8.0
Max. Allowable Route Length	mile	2.5
Bus Capacity	seat	50
Bus Operating Speed	mile/hr.	20
Maximum Available Seat-Hours	seat-hrs.	5500

TABLE 5 MODEL SOLUTION FOR THE BASE CASE

Route No.	Route Structure	Route Demand (passengers)	Route Length (miles)	Route Frequency (trips/hr.)
1	1 2 10 24 57	800	1.62	18.14
2	51 46 42 38 56	800	1.08	22.22
3	12 13 18 59	600	0.97	20.31
4	9 16 57	400	1.03	16.09
5	4 6 59	400	0.79	18.37
6	41 32 58	400	0.59	21.26
7	21 26 58	400	0.56	21.82
8	3 5 7 11 17 59	1000	1.29	22.73
9	40 39 35 57	600	0.99	20.10
10	19 23 30 29 34 56	1000	1.37	22.06
11	8 14 15 20 25 57	1000	1.30	22.65
12	52 45 44 49 50 56	1000	1.96	20.00
13	43 36 31 58	600	0.70	23.90
14	55 54 48 56	600	0.88	21.32
15	33 28 27 22 58	800	0.81	25.66
16	53 47 37 56	600	0.85	21.69

Systemwide Performance Measures

NR = 16	FS = 54	TPM = 7926	BWC = 2075
AF = 21.1	TRL = 16.8	TSC = 6033	BRC = 1255
RSH = 2706	TVM = 703	RC = 592	BUC = 3330
			BOC = 2110

NR = no. of routes
AF = average frequency (trips/hr.)
RSH = required seat-hrs. (seat-hrs.)
FS = fleet size (no. of vehicles)
TRL = total route length (miles)
TVM = total veh.-miles (veh.-miles)
TPM = total pass.-miles (pass.-miles)
BWC = waiting cost for bus (\$/hr.)
RC = rail cost (\$/hr.)
TSC = total system cost (\$/hr.)
BRC = riding cost on bus (\$/hr.)
BUC = bus user cost (\$/hr.)
BOC = bus operating cost (\$/hr.)

reduction in total route length, reduction in operating frequencies, or both. The last two test cases consider changes in the rail network. First, adding a new rail station would improve the accessibility to the rail system and should be expected to reduce both operator and user costs for the feeder-bus system. These cost reductions would result from a reduction in total route length. The closing of a rail station should be expected to have the opposite effect, that is, an increase in vehicle miles of travel and passenger miles of travel. It would increase travel time in the rail system for passengers who previously boarded at the closed station.

The results of the analysis are summarized in Tables 6 and 7. Table 6 shows the responses of the model in terms of systemwide operation measures, whereas Table 7 provides the re-

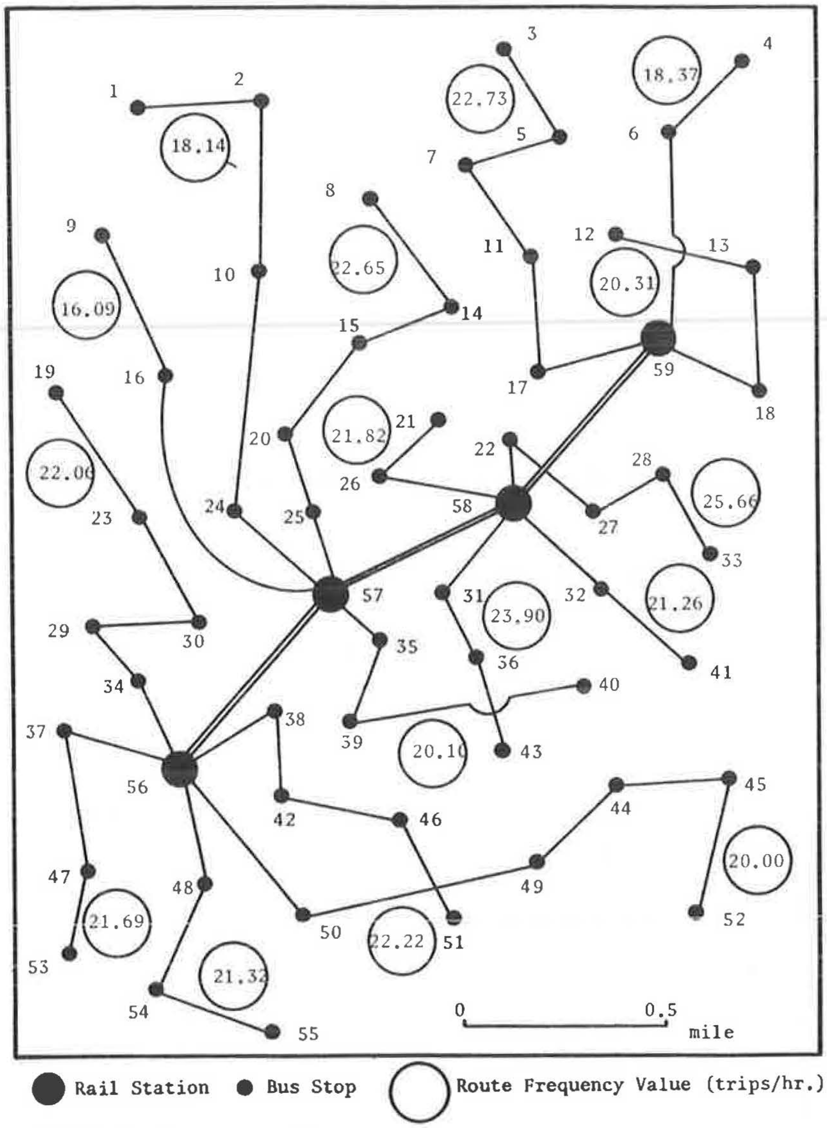

FIGURE 5 Base-case solution.

TABLE 6 SYSTEMWIDE OPERATION MEASURES

Case	Number of Routes	Fleet Size	Average Frequency	% change	Total Vehicle -Miles	% change	Total Passenger Miles	% change
Base	16	54	21.1	---	703	---	7926	--
Min. Bus User Cost	15	59	22.6	+7.1	761	+8.3	7654	-3.4
Variable Demand	15	53	20.4	-3.3	687	-2.3	7840	-1.1
Inc. in Veh. Capacity	15	50	20.3	-3.8	652	-7.3	7832	-1.2
Inc. in Optr. Cost	20	54	21.0	-0.5	700	-0.4	6952	-12.3
Add a Rail Station	15	52	21.7	+2.8	682	-3.0	7548	-4.8
Close a Station	14	54	21.4	+1.4	707	+0.6	8482	+7.0
M-to-M Demand	15	54	20.8	-1.4	704	+0.2	7950	+0.3

TABLE 7 SYSTEMWIDE COST COMPONENTS

Case	Rail Cost	% change	Bus User Cost	% change	Bus Optr. Cost	% change	Total System Cost	% change
Base	592	--	3330	--	2110	--	6033	--
Min. Bus User Cost	822	+38.9	3186	-4.3	2283	+8.2	6291	+4.3
Variable Demand	777	+31.3	3314	-0.5	2060	-2.4	6151	+2.0
Inc. in Veh.Capacity	727	+22.8	3387	+1.7	2152	+2.0	6266	+3.9
Inc. in Optr. Cost	1110	+87.5	3129	-6.0	2104	-0.3	6343	+5.1
Add a Rail Station	755	+27.5	3283	-1.4	2046	-3.0	6084	+0.9
Close a Station	673	+13.7	3440	+3.3	2120	+0.5	6234	+3.3
M-to-M Analysis	973	+64.4	3396	+2.0	2112	+0.1	6481	+7.4

TABLE 8 SOLUTION FOR M-TO-M DEMAND PATTERN

Route No.	Route Structure	Route Demand (passengers)	Route Length (miles)	Route Frequency (trips/hr.)
1	54 53 47 37 34 56	1000	1.28	22.83
2	55 50 48 56	600	0.84	21.84
3	19 23 29 30 57	800	1.27	20.51
4	9 16 24 57	600	1.05	19.55
5	1 10 20 25 57	700	1.30	18.94
6	43 39 35 57	500	0.80	20.45
7	51 46 42 38 57	800	1.11	21.91
8	2 10 15 26 58	700	1.40	18.28
9	7 8 14 21 58	800	1.21	21.02
10	3 5 12 11 17 22 58	1100	1.41	22.84
11	4 6 12 59	500	0.84	19.89
12	13 18 28 27 59	800	1.29	20.36
13	41 33 32 58	600	0.85	21.74
14	52 45 44 40 58	800	1.36	19.80
15	49 43 36 31 58	700	0.98	21.81

Systemwide Performance Measures

NR = 15	FS = 54	TPM = 7850	BWC = 2112
AF = 20.8	TRL = 17.0	TSC = 6481	BRC = 1285
RSH = 2707	TVM = 704	RC = 973	BUC = 3396
			BOC = 2112

NR = no. of routes
RSH = required seat-hrs. (seat-hrs.)
TRL = total route length (miles)
TPM = total pass.-miles (pass.-miles)
RC = rail cost ($/hr.)
BRC = riding cost on bus ($/hr.)
BOC = bus operating cost ($/hr.)
AF = average frequency (trips/hr.)
FS = fleet size (no. of vehicles)
TVM = total veh.-miles (veh.-miles)
BWC = waiting cost for bus ($/hr.)
TSC = total system cost ($/hr.)
BUC = bus user cost ($/hr.)

sponses in terms of cost measures. In all the cases, the proposed model provides consistent and reasonable responses to the various "what if" questions. Under the objective of minimizing bus user cost, user cost has decreased by 4.3 percent. However, this required an increase of 8.2 percent in bus operator cost and 38.9 percent in rail user cost, with a net effect of 4.3 percent increase in total system cost. The variability of demand increased the variations in route length and operating frequencies. The operating frequencies under variable demand range between 11.6 and 28.4 buses per hour compared with a range of 16.1 to 25.7 in the base-case network. The increase in bus capacity resulted in a reduction in the number of routes and

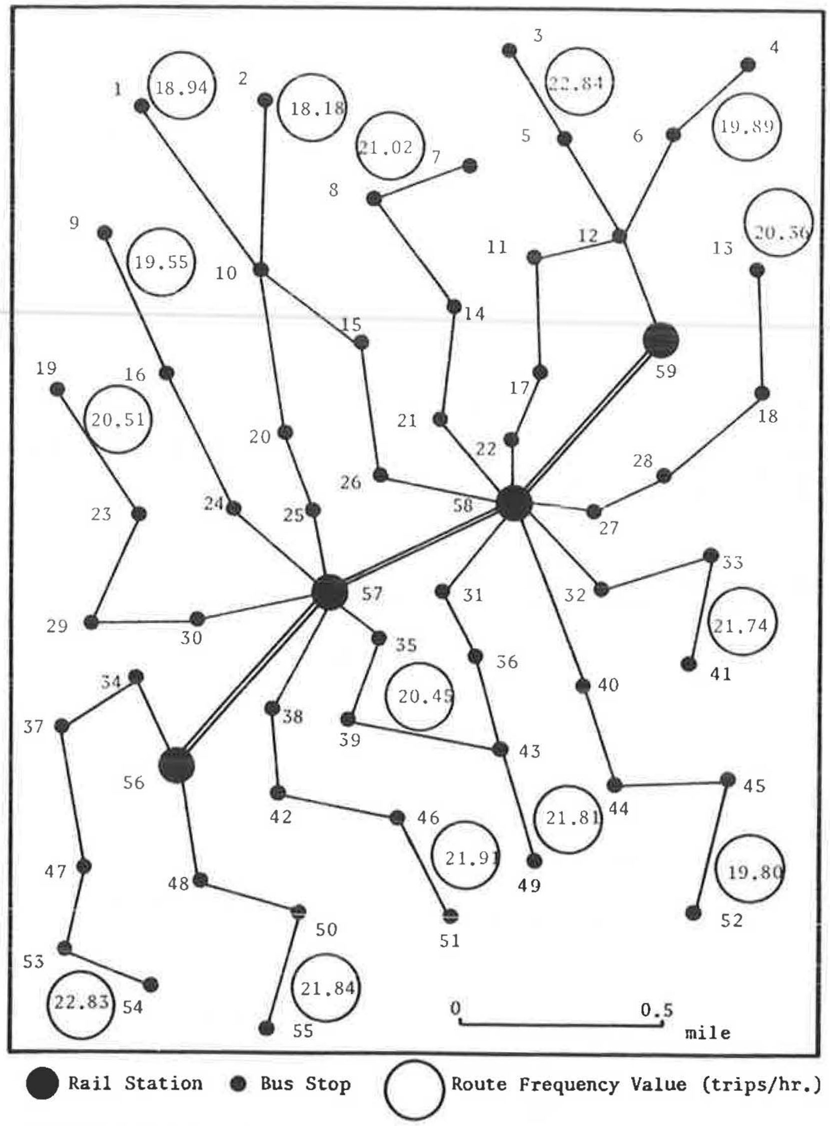

FIGURE 6 Solution for M-to-M demand.

increase in average route length, with an associated decrease in average operating frequency from 21.1 trips per hour to 20.3 trips per hour. The decrease in operating frequencies increased total user cost and resulted in a 3.9 percent increase in total system cost. The increase in labor and fuel costs results in a marginal reduction in bus vehicle miles of travel and in a significant increase in rail passenger miles. The opening of a new rail station decreases both vehicle miles and passenger miles of travel in the bus system. Consequently, it results in the expected reduction in both bus operator and user costs. The closing of a rail station has the opposite effect.

In the last part of the analysis the FBNDP is considered under the M-to-M demand pattern. This part of the analysis is used to evaluate the model's responses to changes in demand pattern. The demand at each bus stop is divided equally among the four destinations (rail stations). As discussed earlier, under the M-to-M demand pattern a bus stop may be served by multiple feeder-bus routes. Consequently, a bus network with more circuitous routes, greater total route length, higher vehicle miles of travel (therefore higher operator cost), higher total passenger miles of travel, and lower operating frequencies should be expected. The combined effect of lower frequencies and higher passenger miles of travel would increase bus user cost.

Table 8 shows the solution for the M-to-M base case, which is illustrated in Figure 6. It can be noted that some of the bus stops are served by multiple routes. The results of Table 6 show the expected reduction in average operating frequency as well as the increases in total vehicle miles and passenger miles of travel relative to the base-case network under the M-to-1 demand pattern. Table 7 shows the increases expected in bus operator cost and bus user cost.

SUMMARY AND CONCLUSION

The U.S. transit industry faces financial difficulties. Among the strategies suggested for improving the financial conditions of transit agencies, the development of better-integrated intermodal systems has the advantage of potentially achieving both cost reduction and improvement in service quality. A network-optimization based methodology for the design of an integrated feeder-bus–rail transit system has been presented. The FBNDP is defined as that of designing a set of feeder-bus routes and determining the service frequency on each route so as to minimize the sum of operator and user costs. The FBNDP is considered under two demand patterns—M-to-1 and M-to-M. A conceptual representation of the M-to-1 FBNDP as a spanning-tree network is presented. Based on this representation, the M-to-1 FBNDP is formulated as a mathematical programming problem. It is shown that the spanning-tree network representation can be generalized to the M-to-M FBNDP. Consequently, the FBNDP under the M-to-M demand pattern can be solved using a model for the M-to-1 case.

The FBNDP is a large and complex routing problem that can be solved only heuristically. The proposed heuristic method for the FBNDP is based on the savings approach while generalizing it to represent operating frequency. A comparison shows that the solutions provided by the proposed model were generally superior to manually designed networks. The advantage of the model's solution is found to increase under variable demand. The proposed model may be viewed as a way of deriving an initial feeder-bus network that may be improved incrementally by an experienced transit planner. The proposed heuristic method is not limited to small problems and can efficiently construct networks of reasonable size. It is shown to be capable of providing consistent and reasonable answers to "what if" questions.

As stated earlier, the FBNDP can be viewed as the problem of achieving the optimal balance between operator and user costs. This analysis indicates (see Table 6) that changes that increase the relative weight of operator cost result in a feeder-bus network with lower total vehicle miles of travel, achieved by operating more but less circuitous routes at lower frequencies. Changes that increase the relative weight of user cost result in feeder-bus networks with higher vehicle miles of travel. The reduction in user cost is often achieved by operating fewer but more circuitous routes at higher frequencies, thereby reducing wait time at bus stops. This operating strategy reduces user cost because unit wait-time cost is higher than the unit cost of riding time.

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