Computational Experience with a Convergent Algorithm for the Simultaneous Prediction of Transportation Equilibrium

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A report is given of the computational experience with a globally convergent algorithm [Shortest Path to the most Needy Destination (SPND)] that predicts trip generation, trip distribution, modal split, and traffic assignment simultaneously on a Simultaneous Transportation Equilibrium Model, developed by Safwat and Magnanti, when it is applied to analyze intercity passenger travel in Egypt. A good convergence criterion, known a priori to be zero at equilibrium, was found on the basis of the solution procedure itself. In order to achieve an accuracy of about 1 percent within the optimum value of the objective function, the CPU time on a VAX-11 VMS computer was 379 sec for a network with 24 origins, 552 origin-destination pairs, 152 nodes, and 224 links. The SPND algorithm is expected to perform better in applications involving the usual urban traffic congestion in contrast to the "fictitous severe congestion" caused by the existence of fleet capacity constraints on the Egyptian intercity system. A companion paper by Safwat in this Record addresses the behavioral aspects of the application.

Safwat and Magnanti *(1)* developed a combined trip-generation, trip-distribution, modal-split, and trip-assignment model that can predict demand and performance levels on large-scale transportation networks simultaneously, that is, a Simultaneous Transportation Equilibrium Model (STEM). The STEM is formulated as an equivalent convex optimization program (ECP) that is solved by a globally convergent algorithm [Shortest Path to the most Needy Destination (SPND)].

The STEM methodology is intended to achieve a practical compromise between behavioral and computational aspects of modeling transportation systems. The model was applied to analyze intercity passenger travel in Egypt.

In a companion paper in this Record, Safwat addresses the behavioral aspects of the application. In this paper, the major objective, however, is to assess the computational experience with the SPND algorithm when applied to the Egyptian intercity system.

In the next section, a brief description of the STEM methodology is provided. This is followed by a summary of the major aspects of modeling the Egyptian intercity passenger transport system. Computational issues are discussed next. This includes two major issues: the convergence criterion and computational efficiency. The last section contains a summary and conclusions.

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STEM METHODOLOGY

A brief description is given of a STEM, an ECP, and an algorithm (SPND) for solving the ECP in order to predict equilibrium on the STEM. For a detailed description, the reader is referred to the paper by Safwat and Magnanti *(1)*.

A STEM

In this subsection, a STEM that describes user travel behavior in response to system performance on a transportation network is presented as follows:

$$G_i = \alpha_i S_i + E_i \quad \text{for all } i \in I \tag{1}$$

$$S_i = \max[0, \ln \sum_{j \in D_i} \exp(-\theta_i U_{ij} + A_j)] \quad \text{for all } i \in I \tag{2}$$

$$T_{ij} = G_i \frac{\exp(-\theta_i U_{ij} + A_j)}{\sum_{k \in D_i} \exp(-\theta_i U_{ik} + A_k)} \quad \text{for all } ij \in R \tag{3}$$

$$\left.\begin{aligned} C_p &= U_{ij} \quad \text{if } H_p > 0 \\ C_p &\geq U_{ij} \quad \text{if } H_p = 0 \end{aligned}\right\} \text{for all } p \in P \tag{4}$$

$$C_p = \sum_{a \in A} \delta_{ap} C_a(F_a) \quad \text{for all } p \in P \tag{5}$$

In this model, the demand variables are as follows:

- G_i = number of trips generated from origin i,
- T_{ij} = number of trips distributed from origin i to destination j,
- H_p = number of trips via path p from any given origin i to any given destination j, and
- F_a = number of trips using link a.

The performance variables are as follows:

- S_i = accessibility variable that measures the expected maximum utility of travel on the transport system as perceived from origin i,
- U_{ij} = average minimum "perceived" cost of travel from i to j,
- C_p = average cost of travel via path p from any given i to any given j, and

C_a = average cost of travel on link a expressed as a function of the number of trips (F_a) on that link.

The remaining quantities are as follows:

E_i = composite measure of the effect that the socioeconomic variables, which are exogenous to the transport system, have on trip generation from origin i;

A_j = composite measure of the effect that the socioeconomic variables, which are exogenous to the transport system, have on trip attraction at destination j;

α_i = parameter that measures the additional number of trips that would be generated from any given origin i if the expected maximum utility of travel, as perceived by travelers at i, increased by unity;

θ_i = parameter that measures the sensitivity of travelers at any given origin i to changes in system performance between any given origin-destination (O-D) pair $ij : j \varepsilon D_i$;

$$\delta_{ap} = \begin{cases} 1 & \text{if link } a \text{ belongs to path } p \\ 0 & \text{otherwise} \end{cases}$$

and the defined sets are as follows:

I = set of origins,
R = set of O-D pairs ij,
P = set of simple paths in the network, and
D_i = set of destinations accessible from origin i.

The basic assumptions of this STEM may be summarized as follows:

1. Trip generation (G_i) is given by any general function as long as it is linearly dependent on the system's performance through an accessibility measure (S_i) based on the random utility theory of travel behavior (i.e., the expected maximum utility of travel).

2. Trip distribution (T_{ij}) is given by a logit model whose measured utility functions include the average minimum perceived travel costs (U_{ij} for all $j \varepsilon D_i$) as variables with a linear parameter θ_i.

3. Modal split and trip assignment are simultaneously user optimized. Note that the STEM framework allows for the modal split to be given by a logit model or to be (together with trip assignment) system optimized *(2)*.

An ECP

Consider the following optimization problem:

Minimize

$$Z(S, T, H) = J(S) + \psi(T) + \phi(H)$$

subject to

$$\sum_{j \varepsilon D_i} T_{ij} = \alpha S_i + E_i \quad \text{for all } i \varepsilon I \tag{6}$$

$$\sum_{p \varepsilon P_{ij}} H_p = T_{ij} \quad \text{for all } ij \varepsilon R \tag{7}$$

$$\left.\begin{array}{ll} S_i \geq 0 & \text{for all } i \varepsilon I \\ T_{ij} \geq 0 & \text{for all } ij \varepsilon R \\ H_p \geq 0 & \text{for all } p \varepsilon P \end{array}\right\} \tag{8}$$

where

$$J(S) = \sum_{i \varepsilon I} \frac{1}{\theta_i} [\alpha_i S_i^2 + \alpha_i S_i - (\alpha_i S_i + E_i) \times \ln(\alpha_i S_i + E_i)]$$

$$\psi(T) = \sum_{i \varepsilon I} \frac{1}{\theta_i} \sum_{j \varepsilon D_i} (T_{ij} \ln T_{ij} - A_j T_{ij} - T_{ij})$$

$$\phi(H) = \sum_{a \varepsilon A} \int_0^{F_a} C_a(w)\, dw$$

$$F_a = \sum_p \delta_{ap} H_p \quad \text{for all } a \varepsilon A \tag{9}$$

Constraints 6 and 7 are flow conservation equations on the transport network stating (a) that the number of trips distributed from a given origin to all possible destinations must equal the total number generated from that origin, and (b) that the number of trips on all paths joining a given O-D pair must equal the total number distributed from that origin to that destination. Constraints 8 state, as postulated earlier, that all the decision variables must be nonnegative. Expressions 9 define the link-path incidence relationships, stating that the flow on a given link equals the sum of flows on all paths sharing that link.

The objective function Z has three sets of terms. The last of these, $\phi(H)$, corresponds to the familiar transformation introduced by Beckman et al. *(3)*. The second set of terms, $\psi(T)$, is similar to those used by Evans *(4)* and by Florian and Nguyen *(5)*, as well as in other related models. The first set of terms, $J(S)$, was introduced by Safwat and Magnanti *(1)*, who proved that under mild monotonicity assumptions on performance functions and nonnegativity and inequality assumptions on demand parameters (i.e., $\theta_i > 0$, $E_i > \alpha_i > 0$ for all $i \varepsilon I$), the ECP has a unique solution that is equivalent to equilibrium on the STEM.

Algorithm for Predicting Equilibrium on the STEM (SPND)

In this subsection, an algorithm (SPND) for solving the ECP to predict equilibrium on the STEM is introduced. The (SPND) algorithm belongs essentially to the class of feasible-direction methods.

At any given iteration r, the method involves two main steps. The first step determines a direction for improvement, d^r. The second step determines an optimum step size, λ^*, along that direction. The current solution, X^r, is then updated, $X^{r+1} = X^r + \lambda^* .d^r$, and the process is repeated until a convergence criterion is met.

In accordance with the Frank-Wolf method *(6)*, the feasible direction d^r is determined as follows:

$$d^r = Y^r - X^r$$

where X^r is the given current solution (S^r, T^r, F^r) and Y^r is the solution of the following linearized subproblem (LP1):

Minimize $Z_L^r(Y) = \nabla Z(X^r)Y$ subject to Equations 6, 7, 8, and 9. The steps of the (SPND) algorithm to determine a feasible direction d^r at iteration r are as follows:

Step 1. Update link costs by calculating $C_a^r = C_a(F_a^r)$ for all $a \in A$. Set $i = 1$ in an ordered set of origins I.

Step 2. Find the shortest path tree from i to all $j \in D_i$. Let U_{ij}^r be the cost on the shortest path from i to j.

Step 3. Calculate $w_{ij}^r = 1/\theta_i (\ln T_{ij}^r - A_j) + U_{ij}^r$ for all $j \in D_i$.

Step 4. Determine j^* satisfying $w_{ij}^{r*} = \min_{j \in D_i} \{w_{ij}^r\}$.

Step 5. Calculate $C_i^r = 1/\theta_i [S_i^r - \ln (\alpha_i S_i^r + E_i)]$.

Step 6. Set $i \leftarrow i + 1$. If $i \in I$, go to Step 2. Otherwise, continue.

Step 7. Find an optimum solution to LP1 and a feasible direction d^r as described in the following.

The optimum solution to LP1 is the vector $y^r = (\hat{S}^r, \hat{T}^r, \hat{H}^r)$, given by

$$\hat{S}_i^r = \begin{cases} 0 \text{ if } C_i^r + w_{ij^*} \geq 0 \\ (M_i - E_i)/\alpha_i \text{ otherwise} \end{cases} \quad \text{for all } i \in I$$

Hence

$$\hat{G}_i^r = \alpha_i \hat{S}_i^r + E_i \quad \text{for all } i \in I$$

$$\hat{T}_{ij}^r = \begin{cases} \hat{G}_i^r & \text{if } j = j^* \varepsilon D_i \\ 0 \text{ otherwise} \end{cases} \quad \text{for all } ij \in R$$

$$\hat{H}_p^r = \begin{cases} \hat{G}_i^r & \text{if } p = p_{ij}^* \in P_{ij^*} \\ 0 \text{ otherwise} \end{cases} \quad \text{for all } p \in p_{ij} \text{ and } ij \in R$$

where M_i is the maximum trip generation from i (assuming zero travel cost everywhere in the system).

The path flows can be decomposed into link flows using the link-path incidence relationship as follows:

$$\hat{F}_a^r = \begin{cases} \sum_i \delta_{ap^*} \hat{G}_i^r \text{ if link } a \text{ belongs to path } p^* \text{ between } i \text{ and } j^* \\ 0 \text{ otherwise} \end{cases}$$

Hence, the feasible direction at iteration r is the vector $\mathbf{d}^r$ with the following components:

$\mathbf{d}_i^r = \hat{S}_i^r - S_i^r$ for all $i \in I$

$\mathbf{d}_{ij}^r = \hat{T}_j^r - T_{ij}^r$ for all $ij \in R$

$\mathbf{d}_a^r = \hat{F}_a^r - F_a^r$ for all $a \in A$

The main computational effort in this direction-finding algorithm is finding the set of shortest paths from all origins to all destinations in Step 2, which is identical to that of the traffic assignment problem with fixed demand. The additional calculations in Steps 3–5 are insignificant compared with those in Step 2. Step 7 just loads the shortest paths with the total demand to the most "needy" destinations, which involves even less effort than the all-or-nothing loading procedure. This procedure is referred to as the shortest path to the most needy destination (SPND) algorithm as dictated by its direction-finding step.

In the following two sections, the computational issues involved in the application of STEM to intercity passenger travel in Egypt are addressed. In the next section, the major aspects related to modeling intercity passenger travel in Egypt are introduced. Then the computational results of the application are analyzed.

MODELING THE EGYPTIAN INTERCITY PASSENGER SYSTEM

Intercity passengers in Egypt utilize two major networks: highway and railway. On highways, passengers may select among private automobiles, taxis, and buses. On railways they may select among diesel, express, and local trains. Furthermore, passengers may be divided into three types according to whether their income is high, middle, or low. For more details on this topic, the reader is referred to papers by Safwat *(2, 7)*. This application focused on one passenger type (the low-income group) and considered taxis, buses, express trains, and local trains as the feasible modes for this passenger type.

The usual link congestion problems encountered in urban travel are insignificant on the Egyptian intercity system. Instead the system is congested because of its fleet capacities. Though these fleet capacity constraints are essentially "hard" constraints, the modeling approach was to treat them as a congestion term added to the link cost functions in a way that drives the user cost to a very high value whenever flows exceed fleet capacity. That is, for any given link, the fleet capacity cost (FCC) may be expressed as follows:

$$\text{FCC} = \delta(\text{flow/fleet capacity})^{\beta}$$

where δ and β are link congestion parameters, assumed equal to 0.1 and 20, respectively. These assumptions, particularly $\beta = 20$, give very steep cost functions. Consequently, more iterations would be required than are customary to achieve any given level of accuracy. Similar approximations to deal with hard link capacities have been suggested by several researchers [Daganzo *(8)* and Hearn *(9)*].

To model the demand functions, a logit trip-distribution model based on observed data was calibrated. Trip generation was assumed to have a minimum E_i of 90 percent of observed values. The maximum trip-generation M_i was calculated assuming that transportation costs are zero everywhere in the system. This choice certainly gives a sufficiently large value, which may be large enough to significantly reduce the step size between successive iterations and hence to adversely affect the performance of the algorithm. A better choice of M_i would be to assume that transportation costs are at their minimum values corresponding to zero flows on the system.

The application included four network sizes (see Table 1) ranging from 90 nodes and 224 links to 152 nodes and 534 links. All networks had 24 origins and 552 origin-destination pairs. Each of these four networks is essentially a composed

TABLE 1 MAJOR CHARACTERISTICS OF THE FOUR PROBLEMS IN THE ANALYSIS

		Network Size				
Name	Description	No. of Origins	No. of O-D Pairs	No. of Nodes	No. of Links	Fleet Capacity
NET1	Express and local	24	552	90	224	Severely constrained
NET2	Express, local, and normal bus	24	552	125	394	Less constrained
NET3	Express, local, and normal bus and taxi	24	555	152	534	Not constrained
NET4	Express (doubled), local, and normal bus and taxi	24	552	152	534	More relaxed than NET3

multimodal network consisting of individual modal networks connected through a set of loading and unloading links at different zonal centroids. This network representation allows modal split and trip assignment to be performed simultaneously.

COMPUTATIONAL RESULTS

In this section, the application is assessed from the computational point of view. The major issues considered are related to the convergence criterion and computational efficiency.

Convergence Criterion

Several convergence measures were tested to find the best criterion for stopping the algorithm when the current solution is sufficiently close to the exact equilibrium. Many of the measures tested were essentially comparisons between the results of the last two iterations. It was obvious that there is a strong correlation between these criteria and the optimum step size, λ^r, at any given iteration r. Figure 1 shows the step size λ at each iteration of the SPND algorithm for the first 200 iterations. It is clear that using any criterion related to λ may cause the procedure to stop prematurely (e.g., at iteration 13).

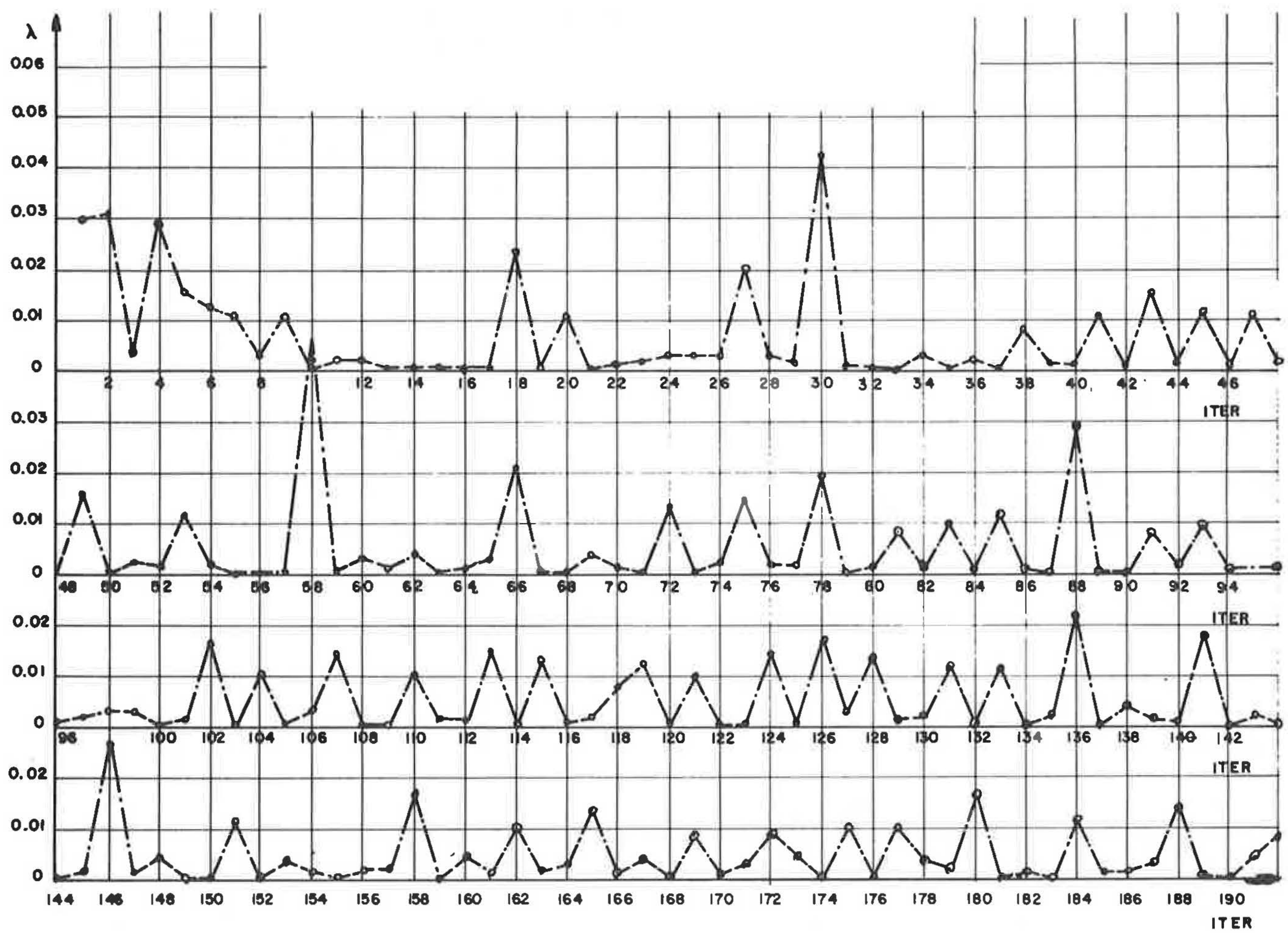

FIGURE 1 Step size (λ) versus number of iterations (*ITER*) for express and local trains, bus, and taxi.

The value of the objective function Z, itself, is monotonically decreasing in the SPND algorithm and could have been a good criterion except that its optimum value is not known a priori.

An apparent criterion (which was found to be the best one) was one based on the procedure itself. As discussed earlier, the direction for trip generation at each iteration is determined on the basis of the following calculation:

$$U_i^r = \frac{1}{\theta_i}[S_i^r - \ln(\alpha_i S_i^r + E_i)] + \frac{1}{\theta_i}[\ln T_{ij}^r{}_* - A_{j*}] + U_{ij*}^r \quad \text{for all } i$$

where $j*$ is the most needy destination in the set D_i at iteration r. The value of U_i^r may be interpreted as the marginal cost of generating an additional trip from origin i going through the shortest path to the most needy destination $j*$. If U_i^r is negative, the current level of demand generated at i may be increased, and vice versa. Hence at equilibrium, U_i^* should satisfy the following conditions:

1. $U_i^* = 0$ if $E_i < G_i^* < M_i$,
2. $U_i^* \geq 0$ if $E_i = G_i^*$,
3. $U_i^* \leq 0$ If $G_i^* = M_i$.
3. $U_i^* \leq 0$ if $G_i^* = M_i$.

Let

$$\text{ERMSE} = \left(\sum_i U_i^r\right)^{1/2}$$

where ERMSE is the equilibrium root-mean-square (rms) error and the summation is over all i such that $E_i < G_i^r < M_i$. (This assumption may not represent any limitation, because M_i can always be selected to ensure the strict inequality. Also, if the problem is defined such that there is always at least one "attractive" destination for any given origin i, then $S_i^r > 0$ and hence $G_i^r > E_i$ for all i and r.)

Then at equilibrium ERMSE is 0 and hence can be used as a "good" convergence criterion for the SPND algorithm. Figure 2 shows the performance of the ERMSE measure for the four problems included in the analysis (see Table 1). It is obvious that the ERMSE has the desirable properties of a good convergence criterion. Why it has a slow convergence toward its optimum value (i.e., zero) is explained next.

Computational Efficiency

Computational efficiency depends on several factors, such as network size, fleet capacity constraints, initialization, steepness of cost functions, parameters of demand functions, and the nature of the algorithm itself.

It turns out that the existence of fleet capacity constraints (FCC), which is essentially a special feature of the Egyptian system, has had an adverse influence on the computational performance of the SPND algorithm.

First, in order to obtain a reasonable initial feasible solution, the existence of FCC required performance of an incremental traffic assignment process in the initialization step of the SPND algorithm. (A description of the modified initialization step is given in the next section.) Second, in order to maintain reasonable feasibility as the algorithm proceeds, the step size in the one-dimensional search at any given iteration had to be restricted in a similar manner to that suggested by Daganzo *(8)*. Results by Hearn and Ribera *(10)* ensure convergence of the modified procedure. Third, the FCC term in the user cost function produces steep cost functions. These three major implications of the FCC have resulted in additional computational efforts for the SPND algorithm, in the sense of increasing the number of iterations and the CPU time for initialization. At any given iteration, however, the method is extremely efficient, as indicated earlier.

Different components of the computer CPU time (in seconds) on VAX-11/VMS for all four problems considered in analysis are shown in Table 2. The CPU time for a typical iteration varied between 1.57 and 3.32 sec (excluding input-output time). Total CPU time for 100 iterations varied between 216 and 379 sec (including input-output time and initialization).

At the 100th iteration the objective function was approximately within 1 percent of its optimum value. Figure 3 shows the value of the objective function at different iterations of the algorithm. The tailing-off phenomenon of the SPND algorithm (a well-known characteristic of the Frank-Wolf method) is evident from Figure 3. The decrease in the objective value during the first 100 iterations was 5 to 6 times that of the following 100 iterations. Also, for problems that are more relaxed in terms of fleet capacities, convergence is relatively faster. This confirms earlier comments on the influence of fleet capacity on computational efficiency. Nevertheless, in view of the foregoing computational results, the SPND algorithm appears to be reasonably efficient for analyzing large-scale systems.

Initialization Process and One-Dimensional Modified Search Procedures

Step 0—Initialization

Step 0.1: Assume that the network is empty and calculate minimum link perceived costs; that is, set $F^o = 0$ and calculate $C^o = C_a(0)$, for all $a \in A$. Set $i = 1$ in an ordered set of origins I.

Step 0.2: Find the shortest tree from i to all other destinations. That is, U_{ij}^o for all $j \in D_i$. Set $j = 1$ in an ordered set of O-D pairs D_i.

Step 0.3: Calculate initial trip generation and trip distribution as follows:

$$G_i^o = E_i$$

$$T_{ij}^o = G_i^o \frac{\exp(-\theta u_{ij}^o + A_j)}{\sum_{k \in D_i} \exp(-\theta u_{ik}^o + A_k)} \quad \text{for all } j \in D_i$$

Then set $i \leftarrow i + 1$; if $i \in I$, go to Step 0.2; otherwise, set i and j = 1, and continue.

Step 0.4: Determine the increment ΔT_{ij}^o to be assigned to the shortest path, p^o, from i to j such that the fleet capacity on any

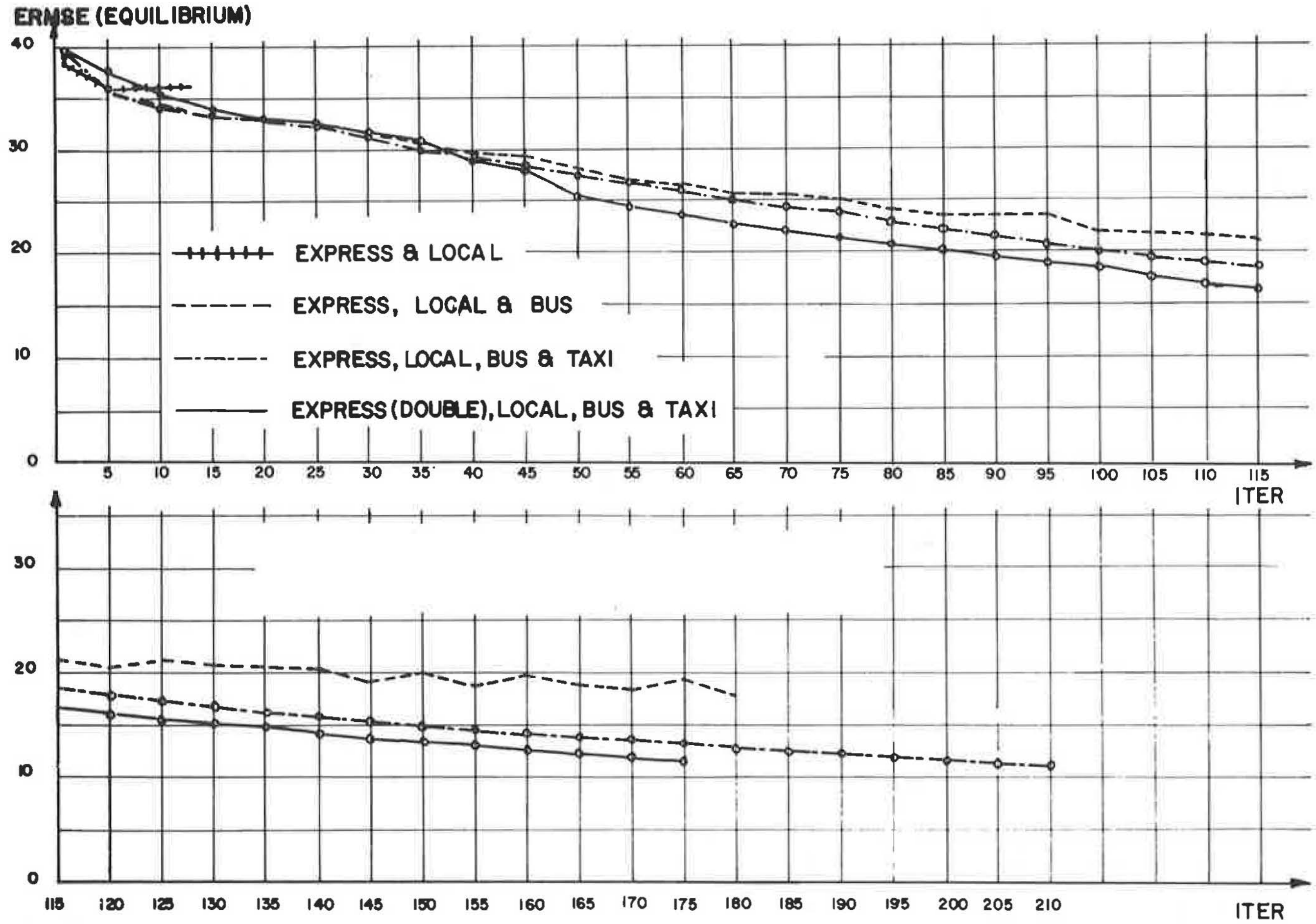

FIGURE 2 Equilibrium (*ERMSE*) versus number of iterations (*ITER*).

TABLE 2 COMPUTER CPU TIME ON VAX-11/VMS

CPU Time Components	Time (sec) by Problem Name NET1	NET2	NET3	NET4
Initialization	11.5	68	20.2	19.4
Typical iteration				
Direction finding	1.37	1.95	2.57	2.57
One-dimensional search	0.2 ~ 0.5	0.3 ~ 0.5	0.38 ~ 0.75	0.38 ~ 0.75
Convergence test, intermediate output	0.26	0.35	0.38	0.38
Total per iteration	1.98	2.7	3.5	3.5
Final output	6.13	8.	8.85	8.85
Total CPU time (for 100 iterations)	215.63	346.	379.	378.2

link on p^o may not be exceeded by more than 20 percent. That is,

$$(\text{CAPACITY})_a \leftarrow 1.2 * (\text{CAPACITY})_a \quad \text{for all } a \; \varepsilon \; p^o$$

$$(\text{CAPACITY})_p^o = \min_{a \varepsilon p^o} \; (\text{CAPACITY})_a$$

$$\Delta T_{ij}^o = \min[T_{ij}^o, (\text{CAPACITY})_p^o]$$

Step 0.5: Assign the increment ΔT_{ij}^o and update link fleet capacities and flows. That is,

$$F_a^o = \begin{cases} f_a^o + \Delta T_{ij}^o & \text{if } a \; \varepsilon \; p^o \\ F_a^o & \text{otherwise} \end{cases}$$

$$(\text{CAPACITY})_a \leftarrow (\text{CAPACITY})_a - \Delta T_{ij}^o \quad \text{for all } a \; \varepsilon \; p^o$$

$$T_{ij}^o \leftarrow T_{ij}^o - \Delta T_{ij}^o$$

Then set $j \leftarrow j + 1$; if $j \; \varepsilon \; D_i$, go to Step 0.4; otherwise, continue.

Step 0.6: Update link costs and shortest trees. That is,

$$C_a^o = C_a(F_a^o) \quad \text{for all } a \; \varepsilon \; A$$

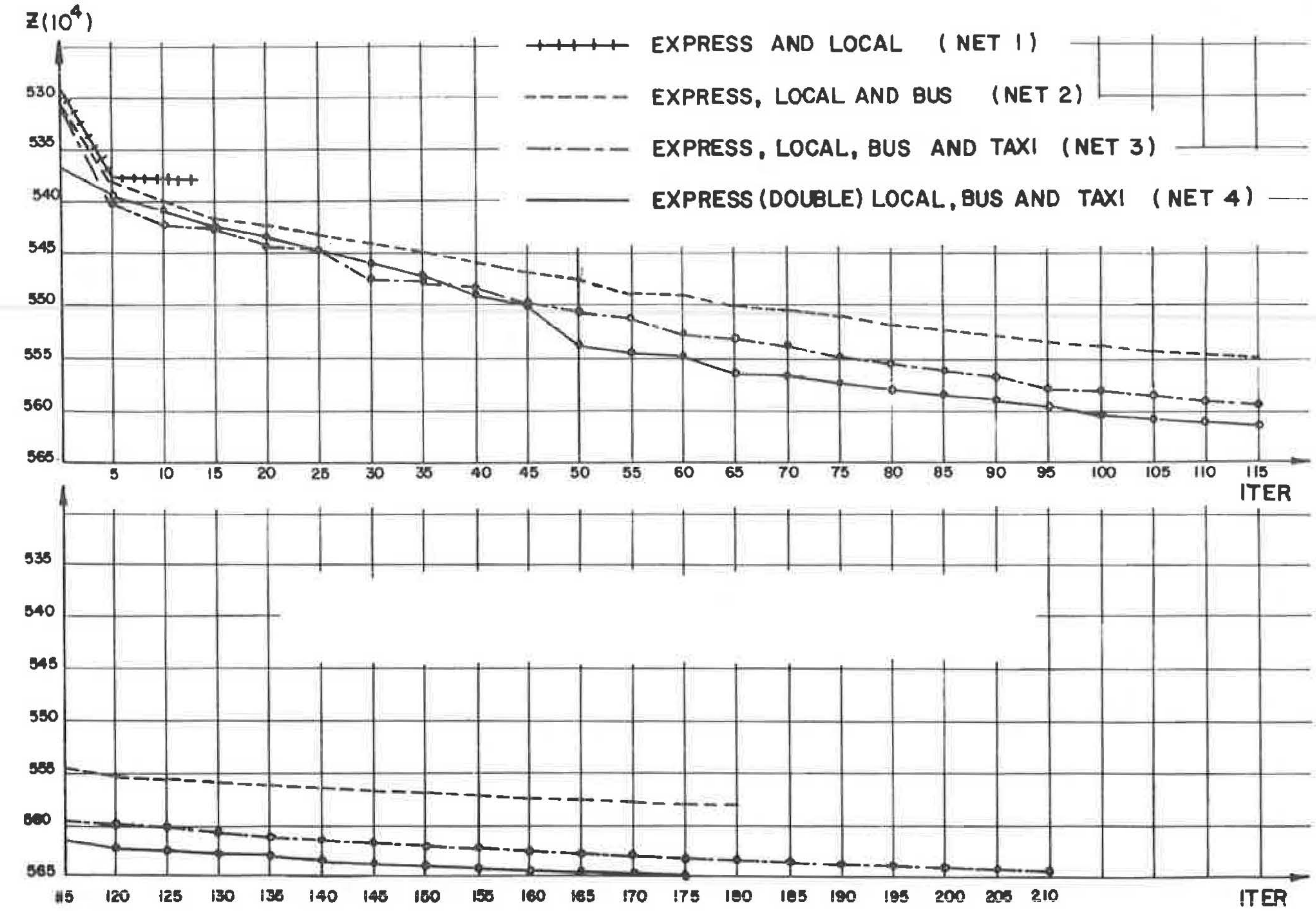

FIGURE 3 Objective function (Z) versus number of iterations (ITER).

Then set $i \leftarrow i + 1$; if $i \varepsilon I$, find the shortest tree, and go to Step 0.4; otherwise, continue.

Step 0.7: Check for termination. That is, if $T^o_{ij} = 0$ for all ij, then stop; an initial feasible solution is obtained.

If $T^o_{ij} \neq 0$ for some O-D pairs but has been constant for the last two iterations, then stop; an initial feasible solution cannot be obtained. Otherwise, set $i = 1$ and $j = 1$, and go to Step 0.4.

As far as the one-dimensional search is concerned, the step size is essentially restricted in such a way to maintain reasonable feasibility of the solution as the search proceeds. The idea has been suggested before by Daganzo *(8)*. The following modifications are formally introduced:

Step 2—One-Dimensional Search

Step 2.1: Calculate maximum step size, λ_{max}, as follows:

$$\lambda_{max} = \min \left[1, \min_{d_a > 0} \frac{(\text{CAPACITY})_a - F_a}{d_a} \right]$$

where d_a is the descent direction on link a.

$$(\text{CAPACITY})_a \leftarrow 1.3 * (\text{CAPACITY})_a$$

Step 2.2: Minimize $Z(\lambda)$ subject to $0 \leq \lambda \leq \lambda_{max}$.

In order for the modified procedure to converge, Daganzo *(8)* invokes a strong assumption that is not satisfied in this case, namely, the link cost is required to approach infinity as the link flow approaches its capacity. Hearn and Ribera *(10)* proved convergence of this modified procedure under a weaker and more natural assumption. They required that whenever the flow approaches capacity, the link cost be sufficiently large that the integral $\phi(H^o)$ in the objective function at the initial solution be strictly less than that $\phi(H^c)$ when the flows are at their capacities. This assumption is satisfied in the modified procedure because the flows in any initial solution cannot exceed more than 20 percent of capacities, whereas in subsequent iterations the flows can reach up to 30 percent more than capacities. The corresponding costs are magnified with a power of 20, implying that the value of $\phi(H^c)$ where the flows are at their relaxed capacities should always be strictly greater than that of $\phi(H^o)$ at the initial solution.

SUMMARY AND CONCLUSIONS

Modeling transportation systems must invariably balance behavioral richness and computational tractability. Safwat and Magnanti *(1)* developed a combined trip-generation, trip-distribution, modal-split, and trip-assignment model that can predict demand and performance levels on large-scale transportation networks simultaneously—a STEM methodology, which is intended to achieve a practical compromise between behavioral and computational aspects of modeling transportation systems.

In order to assess its applicability, the STEM was applied to analyze intercity passenger travel in Egypt. In a companion paper in this Record, Safwat addresses the behavioral aspects of the application. In this paper, the major objective was to report the computational experience with the SPND algorithm concerns were to suggest a convergence criterion for the algorithm and to assess its computational efficiency.

The major conclusions of this paper may be summarized as follows:

1. A good convergence criterion based on the solution procedure itself was found. As the algorithm proceeds, the convergence measure gets closer to its optimum value at equilibrium, which is known a priori to be zero.
2. As far as the computational efficiency of the SPND algorithm is concerned, the computer CPU time required to achieve an accuracy of about +1 percent within the optimum value of the objective function varied between 216 and 379 sec on a VAX-11/VMS minicomputer depending on the network size, which varied from 24 origins, 552 O-D pairs, 90 nodes, and 224 links to 24 origins, 552 O-D pairs, 152 nodes, and 534 links. The algorithm is expected to perform better in applications involving the usual urban traffic congestion in contrast to the fictitious severe congestion caused by the existence of fleet capacity constraints on the Egyptian system.

Safwat and Walton *(11)* applied the STEM to urban travel in Austin, Texas. The Austin network consisted of 520 zones, 19,214 O-D pairs, 7,096 links and 2,137 nodes. The computer CPU time on an IBM 4381 was 430 sec for a typical iteration. A modification of the SPND algorithm (though consistently converged to the unique equilibrium solution, it does not yet have a formal proof of global convergence) arrived at a reasonably accurate solution in only 10 iterations.

An extended version of the STEM was included in a comprehensive intercity transportation planning methodology in Egypt [see paper by Moavenzadeh et al. *(12)*]. Several case studies involving multimodal transportation of passengers and freight in Egypt have been completed *(13)*.

Further research in relation to the STEM methodology should include more applications, particularly in the urban context, as well as more refinement of the model assumptions and computational procedures.

ACKNOWLEDGMENTS

This work was funded by the U.S. Agency for International Development through the Technology Adaptation Program at Massachusetts Institute of Technology. Support for writing of the paper was provided by the Department of Civil Engineering, Michigan Technological University, Houghton, Michigan.

The author would like to thank three anonymous referees and Yosef Sheffi, Massachusetts Institute of Technology and chairman of the TRB Task Force on Transportation Supply Analysis, for their invaluable comments on earlier versions of the paper.

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