Sight Distance Relationships Involving Horizontal Curves

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Recent AASHTO design policy developments and research have increased needed stopping sight distance lengths on horizontal curves. The mathematical methodology recommended by AASHTO for calculating needed sight clearances across the inside of horizontal curves exists only for the case where the curve is longer than the needed sight distance. There is no explicit exact solution available for the case where the sight distance is longer than the curve, a situation expected to happen more frequently as sight distance needs are increased. This research provides exact solutions for the case where the sight distance is greater than the curve length for a moving observer. It relates the available sight distance; horizontal curve geometric elements; and location of observer, road object, and clearances to an obstacle to vision located inside the driving path of the observer. The lesser clearances required for these cases are often as much as 50 percent smaller than those obtained using the longer curve case. User-friendly design procedures and aids using these results have been developed and are presented in the paper. They are easily programmable, and can, hence, serve as a part in a decision support system. The methodology should be valuable in evaluation of safety and operation of critical highway locations.

Stopping sight distance *SSD*, as defined by AASHTO in 1940 and later *(1–4)*, is the minimum sight distance that allows a vehicle traveling at or near design speed to stop just before reaching an object in its path. It is one of the most fundamental criteria controlling the geometric alignment of roads and streets. If this criterion is to be met, the available sight distance *S* must exceed the needed *SSD*. *S* is related to the horizontal alignment by the design parameters of the curve, the position of the driver and the road object, and the location of sight-limiting obstacles inside the curve. These obstacles to vision are created by cut slopes, foliage, and structures of all types. The other sight distances used in street and highway design, passing and decision sight distances, use *S* measured in almost the same way, although the determinations of the needed values result from different models of driver behavior.

This paper is a major extension of stopping sight distance research conducted in the University of Michigan Transportation Research Institute (UMTRI), completed for the NCHRP *(5, 6)*. It presents many new results involving the line of sight, travel path, distance of an obstacle to vision from the travel path, and effects of lateral observer eye position and road object position on sight distance availability. For reference purposes, it also restates some earlier findings.

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Sight distance availability on horizontal curves is developed by considering geometric and trigonometric relationships involving the elements shown, for example, in Figure 1.

Table 1 presents the effect of different *SSD* values on *M*, the maximum lateral required clearance, for three representative long rural curves. It compares the *M* values for the AASHTO *SSD* recommendations from the current 1984 AASHTO policy with the values recommended by UMTRI *(2, 4, 5)*. Clearances as great as twice those of the current policy value are found. In addition, Neuman et al. *(7)* have shown the need for even greater values than those shown in the table for horizontal curves where the stopping distance is increased because of the lesser friction available for stopping because of lateral stability friction requirements. They cite a curve of high design speed for which the needed clearance is more than tripled.

AASHTO presents the maximum clearance requirement *M* for *SSD* sight obstructions inside horizontal curves only for the case $S \leq L$, where *L* is the length of the curve, and both observer and road object are on the curve *(4)*. For the other case $S > L$ for which the needed clearance is less, no mathematical relationship is presented. AASHTO recognizes that the needed clearance is less when the observer or road object is near the ends of the curve *(4*, p. 247). For both of these situations, AASHTO recommends use of straightforward graphical procedures with scaled plan sheets or of Raymond's empirically developed curves, particularly where spiral transitions are being used *(8)*.

This research was undertaken to derive relations involving *M* and *S* for all known practical cases. A search of the literature revealed no closed form solutions except for some of the simplest approximating cases.

Figure 2 shows the way in which the maximum needed clearance varies on and near the curve for the 60-mph design speed curve for the two *SSD* values *S* = 850 ft (256.0 m) and *S* = 650 ft (198.1 m) used in Table 1. It is similar to the *SSD* profile presented by Neuman et al. *(7)*. When the observer is on the tangent within a distance *S* from the point of curvature (*PC*) of a long curve, the largest value of *m* needed to have a clear line of sight varies nonlinearly from 0 (at a point *S* in advance of the *PC*) to *M* (when the observer is at the *PC* or beyond it on the curve). All points on the curve closer than *S*/2 to the *PC* also need a clearance less than *M*. In the paper *(7)*, derivations are given and findings are presented for the following:

1. Relationships involving the geometric design parameters of the curve *R* (hence *D*), *L*, *I*, the lateral clearance *m*, and the available sight distance *SD* as measured along a straight line from the observer to the road object that touches the obstacle to vision.

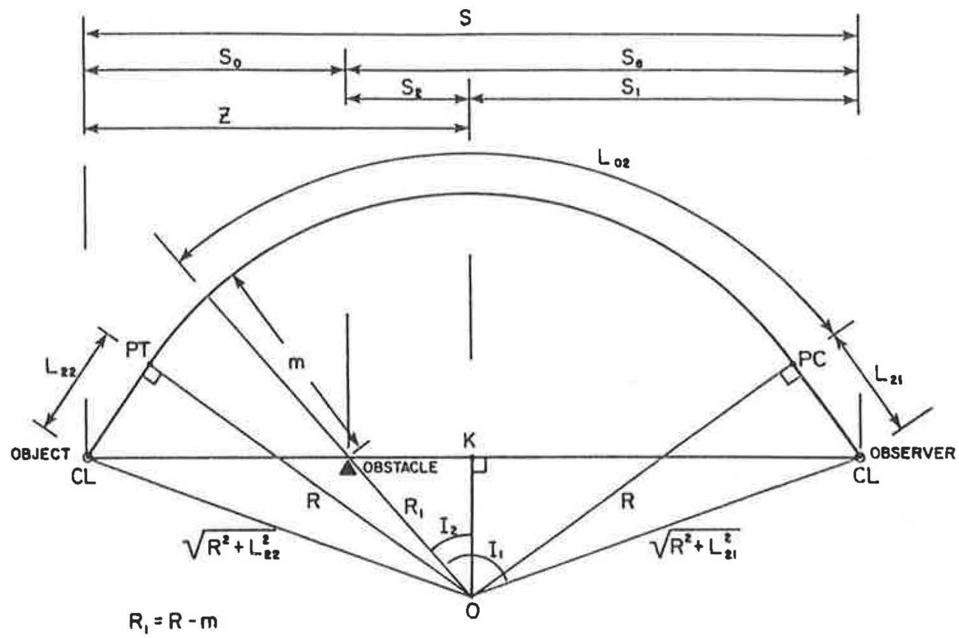

FIGURE 1 Case 4: both observer and road object off the horizontal curve.

2. Relationships as the preceding but with the available sight distance S measured along the road from the observer to the road object.
3. Relationships involving the maximum value of the lateral clearance m^* for a specified curve and S.
4. Chord approximations for some of the exact solutions developed.
5. A recommended curve design and existing curve evaluation procedure including graphical design aids.

GEOMETRIC RELATIONSHIPS

Figure 1 shows the geometric relationships among the locations of the observer, obstacle to vision, and road object. As the observer moves along the road centerline, the farthest point at which a road object can be seen also changes, and the magnitude of the available sight distance change clearly depends on the curve geometry properties. The geometrical relationships among the factors affecting this sight distance variation for horizontal curves consisted of four cases as follows:

Case 1: Observer and object in the horizontal curve.
Case 2: Observer before the *PC* and object in the horizontal curve.
Case 3: Observer in the curve and object beyond the point of tangency *PT* of the curve.
Case 4: Observer before the *PC* and object beyond the *PT*. *PC* and *PT* are interchangeable in these cases.

Geometrical relationships are derived for these cases for a single obstacle to vision inside the curve, and the results are measured along the line of sight *SD*. The derivation is divided into three parts as follows:

TABLE 1 EFFECT OF STOPPING SIGHT DISTANCE ON REQUIRED CLEARANCE

	Design Speed and Typical Degree of Curve					
	50 MPH 8° (81 kmh)		60 MPH 5° (97 kmh)		70 MPH 4° (113 kmh)	
	SSD ft (meters)	M ft (meters)	SSD ft (meters)	M ft (meters)	SSD ft (meters)	M ft (meters)
AASHTO 1984 Upper	475 (144.8)	40 (12.2)	650 (198.1)	47 (14.3)	850 (259.1)	64 (19.5)
UMTRI Upper	560 (170.7)	56 (17.1)	850 (259.1)	80 (24.4)	1200 (365.8)	128 (39.0)

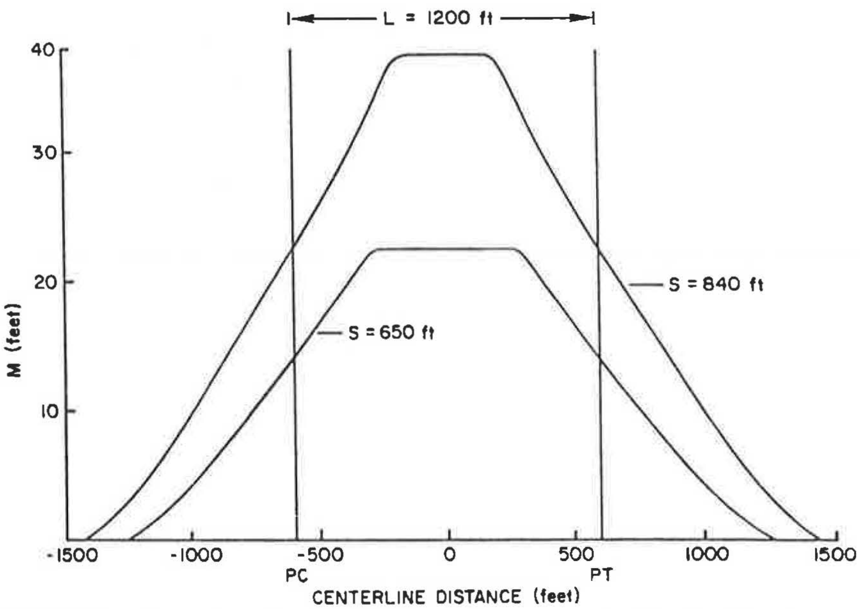

FIGURE 2 Examples of horizontal curve clearance needed for V = 60 mph, D = 5°, R = 1,126 ft, L = 1,200 ft, and I = 60°.

1. Derivation of the line-of-sight distance component SD_e from the observer to the vision obstacle.
2. Derivation of the line-of-sight distance component SD_0 from the vision obstacle to the road object.
3. Obtaining the total line-of-sight distance $SD = SD_e + SD_0$.

Derivations for the four cases follow:

Case 1: Observer and Road Object in the Horizontal Curve

Case 1 can be obtained from Figure 1 by setting both L_{22} and L_{21} equal to zero. The notation used in the derivation is also shown in that figure. The vision obstacle is located inside the curve with radius R at a distance m from the centerline (CL) of the lane. Observer and object are both assumed to be located on the CL.

Derivation of SD_e

Using Figure 1, with $L_{22} = 0$ and $L_{21} = 0$, the sight distance component SD_e from the observer to the vision obstacle is

$$SD_e = SD_1 + SD_2 = R \sin (I_1 - I_2) + (R - m) \sin (I_2) \quad (1)$$

where

$$I_1 = 180(L_{01}/\pi R) \quad (2)$$

in which L_{01} is the distance from the observer to the station of the vision obstacle along the CL, as shown. I_2 can be obtained from Figure 1 using the trigonometric relationship for h, the shortest distance from the center point of the curve perpendicular to the line of sight, setting the expressions for h equal, and solving for I_2. Then

$$I_2 = \tan^{-1} (\{R[1 - \cos (I_1)] - m\}/[R \sin (I_1)]) \quad (3)$$

Derivation of SD_0

Using the results just obtained and Figure 1, the sight distance component SD_0 from the vision obstacle to the road object is

$$SD_0 = SD_1 - SD_2 = R \sin (I_1 - I_2) - (R - m) \sin (I_2) \quad (4)$$

Derivation of SD

Using the results just obtained and Figure 1, the total sight distance SD, for the case when both observer and road object are in the horizontal curve, is

$$SD = SD_e + SD_0 = 2SD_1 = 2R \sin (I_1 - I_2) \quad (5)$$

where I_1 and I_2 are as derived in Equations 2 and 3.

Case 2: Observer Before the *PC* and Road Object in the Horizontal Curve

Case 2 and the notation used are shown in Figure 1 when setting only L_{22} equal to zero. The derivation follows the previous three steps.

Derivation of SD_e

The sight distance component SD_e from the observer to the obstacle derived from Figure 1 for $L_{22} = 0$ is

$$SD_e = SD_1 + SD_2 = \sin (I_1 - I_2) (R^2 + L_{21}^2)^{1/2} + (R - m) \sin (I_2) \quad (6)$$

From trigonometry,

$$I_1 = 180(L_{02}/\pi R) + \tan^{-1}(L_{21}/R) \quad (7)$$

I_2 can be obtained from Figure 1 using h:

$$I_2 = \tan^{-1} \{[R - m - (R^2 + L_{21}^2)^{1/2} \cos (I_i)]/[(R^2 + L_{21}^2)^{1/2} \sin (I_1)]\} \quad (8)$$

Derivation of SD_0

The available sight distance component SD_0 from the obstacle to vision to the road object is obtained using Figure 1 with $L_{22} = 0$. Thus,

$$SD_0 = Z - SD_2 = [(R^2 + L_{21}^2) \sin^2 (I_1 - I_2) - L_{21}^2]^{1/2} - (R - m) \sin (I_2) \quad (9)$$

Derivation of SD

The available total sight distance SD for the case where the observer is on the tangent in advance of the PC or beyond the PT and the road object is in the curve is developed using the results obtained in Equations 6 and 9.

$$SD = SD_e + SD_0 = (R^2 + L_{21}^2)^{1/2} \sin (I_1 - I_2) + [(R^2 + L_{21}^2) \sin^2 (I_1 - I_2) - L_{21}^2]^{1/2} \quad (10)$$

where I_1 and I_2 are as given in Equations 7 and 8. When $L_{21} = 0$, Equation 10 reduces to the sight distance formula derived in Case 1.

Case 3: Observer In the Curve and Road Object on the Tangent Beyond the *PT* of the Curve

Case 3 can be obtained from Figure 1 by setting $L_{21} = 0$. The notation used is from Figure 1. The derivation is similar to that for Case 2 with only the locations of the observer and the object being interchanged.

Derivation of SD_e

The sight distance component SD_e from the observer to the obstacle to vision is

$$SD_e = SD_1 + SD_2 = R \sin (I_1 - I_2) + (R - m) \sin (I_2) \quad (11)$$

where

$$I_1 = 180(L_{01}/\pi R) \quad (12)$$

$$I_2 = \tan^{-1} \left(\{R[1 - \cos (I_1)] - m\}/[R \sin (I_1)] \right) \quad (13)$$

Derivation of SD_0

The sight distance component SD_0 from the obstacle to vision to the road object is

$$SD_0 = Z - SD_2 = [R^2 \sin^2 (I_1 - I_2) + L_{22}^2]^{1/2} - (R - m) \sin (I_2) \quad (14)$$

Derivation of SD

The total available sight distance SD, when the observer is in the curve and the road object is beyond the PT, is obtained using the results obtained in Equations 11 and 14. Thus,

$$SD = SD_e + SD_0 = R \sin (I_1 - I_2) + [R^2 \sin^2 (I_1 - I_2) + L_{22}^2]^{1/2} \quad (15)$$

where I_1 and I_2 are as given in Equations 12 and 13.

When $L_{22} = 0$, Equation 26 reduces to the available sight distance formula for Case 1.

Case 4: Observer Before the *PC* and Road Object Beyond the *PT*

In this case, both the observer and the road object are on the tangents of the horizontal curve, and hence L_{22} and L_{21} are both strictly greater than zero. Case 4 and the notation used in the text are shown in Figure 1. The derivation is similar to those described previously.

Derivation of SD_e

The sight distance component SD_e from the observer to the vision obstacle is obtained from Figure 1. Thus,

$$SD_e = SD_1 + SD_2 = (R^2 + L_{21}^2)^{1/2} \sin (I_1 - I_2) + (R - m) \sin (I_2) \quad (16)$$

where

$$I_1 = 180 (L_{02}/\pi R) + \tan^{-1} (L_{21}/R) \quad (17)$$

$$I_2 = \tan^{-1} \{[R - m - (R^2 + L_{21}^2)^{1/2} \cos (I_1)]/[(R^2 + L_{21}^2)^{1/2} \sin (I_1)]\} \quad (18)$$

L_{02} is the distance from the PC to the station along the path of travel at the radial location of the obstacle.

Derivation of SD_0

The sight distance component SD_0 from the obstacle to vision to the road object is obtained from Figure 1. Thus,

$$SD_0 = Z - SD_2 = [L_{22}^2 - L_{21}^2 + (R^2 + L_{21}^2) \sin^2 (I_1 - I_2)]^{1/2} - (R - m) \sin (I_2) \quad (19)$$

where L_{21} is as defined earlier, L_{22} is the distance from the PT to the road object, and I_1 and I_2 are as given in Equations 17 and 18.

Derivation of SD

The total available sight distance SD, for the case in which neither observer nor object are on the curve, is

$$SD = SD_e + SD_0 = (R^2 + L_{21}^2)^{1/2} \sin (I_1 - I_2) + [L_{22}^2 - L_{21}^2 + (R^2 + L_{21}^2) \sin^2 (I_1 - I_2)]^{1/2} \quad (20)$$

By setting $L_{22} = 0$, one obtains Case 2; by setting $L_{21} = 0$, Case 3; and by setting both L_{21} and $L_{22} = 0$, Case 1. Hence, Equations 20 covers all cases for all locations of observer, obstacle to vision, and object on road.

The relationship for the line-of-sight distances *SD* can be transformed to the sight distances along the observer's travel path. These sight distance formulas are presented in Tables 2 and 3 for all four cases. L_{22} is the distance between the observer and the *PC* when the observer is on the tangent before the *PC*. L_{22} is the distance between the *PT* and road object when it is on the tangent. These distances are shown in Figure 1. Further, in Tables 2 and 3 the various central angles of the line of sight used in the derivations, as shown in Figure 1, are labeled I_1 through I_4. In Table 3, L_{01} represents the distance from the observer to the centerline station of the obstacle to vision when the observer is in the curve, and L_{02} is the distance from the *PC* to the vision obstacle station when the observer is at or before the *PC*.

The distance traveled by the vehicle along the curve is considered as the sight distance in horizontal curve analysis because it is the vehicle stopping distance. In the following, results for *SD* are transformed to results for *S*, the travel path.

Case 1: Observer and Road Object in the Horizontal Curve

Using the notation of Figure 1 with $L_{22} = 0$ and $L_{21} = 0$, the sight distance *S* along the path traveled becomes

$$S = (I_1 - I_2)(\pi R/90) \tag{21}$$

where I_1 and I_2 are from Equations 2 and 3.

Case 2: Observer Before the *PC* and Road Object in the Horizontal Curve

The notation of Figure 1, with $L_{22} = 0$, is used. S_1, S_2, and S_3 denote the sight distance components along the path traveled. S_1 consists of a sight distance component in the curve S_{1c} and on the tangent S_{1t}, $S_1 = S_{1c} + S_{1t}$. Each sight distance component can be obtained using the geometry of Figure 2. Hence,

$$S_1 = S_{1c} + S_{1t} = (I_1 - I_2 - I_3)(\pi R/180) + L_{21}$$

$$S_2 = I_2(\pi R/180)$$

$$S_3 = (I_5 - I_2)(\pi R/180) = [(I_1 - I_2) - I_2](\pi R/180)$$

The total available sight distance is then

$$S = S_1 + S_2 + S_3 = L_{21} + [2(I_1 - I_2) - I_3](\pi R/180) \tag{22}$$

where I_1 and I_2 are from Equations 7 and 8, and

$$I_3 = \tan^{-1}(L_{21}/R) \tag{23}$$

TABLE 2 SUMMARY OF SIGHT DISTANCE FORMULAS FOR HORIZONTAL CURVES ALONG THE PATH TRAVELED

Case	Location of Observer/Object	Sight Distance Formula*
1	Curve/Curve	$S=(I_1-I_2)(\pi R)/90$
2	Tangent/Curve	$S=[2(I_1-I_2)-I_3](\pi R)/180+L_{21}$
3	Curve/Tangent	$S=[2(I_1-I_2)-I_4](\pi R)/180+L_{22}$
4	Tangent/Tangent	$S=[2(I_1-I_2)-I_3-I_4](\pi R)/180$ $+L_{21}+L_{22}$ General Sight Distance Formula for Horizontal Curves

* L_{21}, L_{22} = The distance between the observer and PC, and PT and object respectively for observer before the PC and object beyond the PT, as shown in Figure 2.

I_i for i=1 to 4 = Central angle of the line of sight used in the derivations as shown in Figure 2.

TABLE 3 SUMMARY OF EQUATIONS FOR I_1, I_2, I_3, AND I_4

Case	Eq. Number	Equation
1	4	$I_1 = 180(L_{01})/(\pi R)$
	8	$I_2 = \tan^{-1}\{[R(1-\cos(I_1))-m]/[R \sin(I_1)]\}$
2	13	$I_1 = 180(L_{02})/(\pi R) + \tan^{-1}(L_{21}/R)$
	17	$I_2 = \tan^{-1}\{[R-m-(R^2+L_{21}^2)^{1/2} \cos(I_1)]/[(R^2+L_{21}^2)^{1/2}\sin(I_1)]\}$
	37	$I_3 = \tan^{-1}(L_{21}/R)$
3	4	$I_1 = 180(L_{01})/(\pi R)$
	8	$I_2 = \tan^{-1}\{[R(1-\cos(I_1))-m]/[R \sin(I_1)]\}$
	39	$I_4 = \tan^{-1}(L_{22}/R)$
4	13	$I_1 = 180(L_{02})/(\pi R) + \tan^{-1}(L_{21}/R)$
	17	$I_2 = \tan^{-1}\{[R-m-(R^2+L_{21}^2)^{1/2} \cos(I_1)]/[(R^2+L_{21}^2)^{1/2}\sin(I_1)]\}$
	37	$I_3 = \tan^{-1}(L_{21}/R)$
	39	$I_4 = \tan^{-1}(L_{22}/R)$

Case 3: Observer in the Curve and Road Object Beyond the *PT* of the Curve

In notation of Figure 1 with $L_{21} = 0$, S_1, S_2, and S_3 again denote the sight distance components along the travel path. S_3 consists of sight distance components S_{3c} in the curve and S_{3t} on the tangent; $S_3 = S_{3c} + S_{3t}$. Each sight distance component can be obtained using the geometry of Figure 1. Hence,

$$S_1 = (I_1 - I_2)\ (\pi R/180)$$

$$S_2 = I_2\ (\pi R/180)$$

$$S_3 = S_{3c} + S_{3t} = (I_5 - I_4 - I_2)\ (\pi R/180) + L_{22}$$

where $I_5 = I_1 - I_2$.

The total available sight distance then becomes

$$S = S_1 + S_2 + S_3 = [2(I_1 - I_2) - I_4]\ (\pi R/180) + L_{22} \quad (24)$$

where I_1 and I_2 are from Equations 12 and 13, and

$$I_4 = \tan^{-1}\ (L_{22}/R) \quad (25)$$

Case 4: Observer Before the *PC* and Road Object Beyond the *PT*

In the notation of Figure 1, S_1, S_2, and S_3 again represent the sight distance components along the path traveled. S_1 and S_3

consist of sight distance components S_{1c} and S_{3c} in the curve and S_{1t} and S_{3t} on the tangent. $S_1 = S_{1c} + S_{1t}$; $S_3 = S_{3c} + S_{3t}$.

$$S_1 = S_{1c} + S_{1t} = (I_1 - I_2 - I_3)(\pi R/180) + L_{21}$$

$$S_2 = I_2 (\pi R/180)$$

$$S_3 = S_{3c} + S_{3t} = (I_5 - I_4 - I_2)(\pi R/180) + L_{22}$$

where $I_5 = I_1 - I_2$.

The total available sight distance then becomes

$$S = S_1 + S_2 + S_3 = L_{21} + [2(I_1 - I_2) - I_3 - I_4](\pi R/180) + L_{22} \quad (26)$$

where I_1, I_2, I_3, and I_4 are from Equations 41, 42, 23, and 25, respectively.

Equation 26 is the general sight distance formula for horizontal curves for given travel path distances. It covers all cases of available sight distance with respect to locations of observer, vision obstacle, and road object. It is accordingly named the "general horizontal curve sight distance formula." With this relationship, S can be calculated for any combination of the locations of observer and vision obstacle in a horizontal curve.

Methodology and Nomograph for S and m

A calculation procedure was developed to determine available sight distances on horizontal curves using this general sight distance formula. It is given for the two possible observer position cases and the two road object position cases. The two observer cases are (a) observer on the tangent before the *PC* or beyond the *PT*, and (b) observer in the curve or at the *PC* or *PT*. For these locations, the following two positions of the road object are (c) road object on the tangent beyond the *PT* or *PC*, and (d) road object in the curve or at the *PT* or *PC*.

The calculation procedure involves the following five steps:

1. Determine central angles I_1 and I_2.
2. If Case a applies, determine central angle I_3.
3. Determine central angle I_{ob}.
4. Determine the location of the road object with respect to the *PT* (Case c or d).
5. Determine the total available sight distance S.

To calculate the available sight distance S, the geometric characteristics of the horizontal curve R (or D), L, I, and m, the radial distance of the vision obstacle from the observer's path, must be known or measured. Similarly when a required value of S is given, m needed for the curve with given R and L can be easily calculated. In a typical geometric design application, the required curve radius R can be found when m, S, and I are given.

Lateral Clearance m

Relationships for lateral clearance m for the four cases of Table 2 are now derived using Equations 3 and 8 and the results are summarized in Table 4.

Case 1: Observer and Road Object in the Horizontal Curve

From Equation 3, m becomes

$$m = R[1 - \cos(I_1) - \tan(I_2)\sin(I_1)] \quad (27)$$

where $I_1 = 180(L_{01}/\pi R)$, L_{01} is the distance from the observer to the station of the obstacle along the *CL*, and I_2 can be obtained from Equation 21:

$$I_2 = I_1 - (90S/\pi R) \quad (28)$$

Case 2: Observer Before the *PC* and Road Object in the Horizontal Curve

Solving m from Equation 8, one obtains

$$m = R - (R^2 + L_{21}^2)^{1/2} [\cos(I_1) + \tan(I_2)\sin(I_1)] \quad (29)$$

where

$$I_1 = 180(L_{02}/\pi R) + \tan^{-1}(L_{21}/R)$$

$$I_2 = I_1 - 90(S - L_{21}/\pi R) - I_3/2;\ I_3 = \tan^{-1}(L_{21}/R) \quad (30)$$

Case 3: Observer in the Curve and Road Object Beyond the *PT* of the Curve

From Equation 3, m becomes

$$m = R[1 - \cos(I_1) - \tan(I_2)\sin(I_1)]$$

where I_1 is as in Case 1, and I_2 can be obtained from Equation 24.

$$I_2 = I_1 - 90(S - L_{22}/\pi R) - I_4/2 \quad (31)$$

where $I_4 = \tan^{-1}(L_{22}/R)$

Case 4: Observer Before the *PC* and Road Object Beyond the *PT*

Solving for m from Equation 8, one obtains

$$m = R - (R^2 + L_{21}^2)^{1/2} [\cos(I_1) + \tan(I_2)\sin(I_1)]$$

where

$$I_1 = 180(L_{02}/\pi R) + \tan^{-1}(L_{21}/R)$$

$$I_2 = I_1 - 90(S - L_{21} - L_{22}/\pi R) - I_3/2 - I_4/2 \quad (32)$$

with I_3 as in Case 2 and I_4 as in Case 3.

Critical Lateral Clearance m^*

Every curve has a largest critical value of m, here called m^*. Where $S < L$, this value is M as presented by AASHTO for two *SSD* values (*4*, p. 244–245). Where the needed S is greater

TABLE 4 SUMMARY OF EQUATIONS FOR LATERAL CLEARANCE *m*

	Equation
Case 1 and Case 3	$m=R[1-\cos(I_1)-\tan(I_2)\sin(I_1)]$ where: Case 1: $I_1=180(L_{01})/(\pi R)$ $I_2=I_1-(90\ S)/(\pi R)$ Case 3: $I_1=180(L_{01})/(\pi R)$ $I_2=I_1-90(S-L_{22})/(\pi R)-I_4/2$ $I_4=\tan^{-1}(L_{22}/R)$
Case 2 and Case 4	$m=R-(R^2+L_{21}{}^2)^{1/2}[\cos(I_1)+\tan(I_2)\sin(I_1)]$ where: Case 2: $I_1=180(L_{02})/(\pi R)+\tan^{-1}(L_{21}/R)$ $I_2=I_1-90(S-L_{21})/(\pi R)-I_3/2$ $I_3=\tan^{-1}(L_{21}/R)$ Case 4: $I_1=180(L_{02})/(\pi R)+\tan^{-1}(L_{21}/R)$ $I_2=I_1-90(S-L_{21}-L_{22})/(\pi R)-I_3/2-I_4/2$ $I_3=\tan^{-1}(L_{21}/R)$ $I_4=\tan^{-1}(L_{22}/R)$

than the curve length L, m^* for that curve can be determined using the nomograph of Figure 3, which was prepared as a design aid to relate S to m^* when $S > L$. In addition, Figure 3 can be used to determine the critical value of any of the four independent parameters m^*, R (or D), I, and S, when the other three are given. L is not an independent parameter, being fully specified by I and R.

The formulas for m^* are summarized in Column 2 of Table 5 for the following three cases:

1. $S < L, I^* < I$.
2. $S = L, I^* = I$.
3. $S > L, I^* > I$.

Cases 1 and 2 are covered in the 1984 AASHTO policy guide *(4)*; Figure 4 can be used for Case 3.

Table 5 presents both the exact and chord approximation results for m^* and M. In Table 5, the central angle of the line of sight corresponding to total sight distance S is denoted by I^*. These chord approximations were shown to have an error of less than 0.5 ft for all curves with radii in excess of 400 ft. The error overstates the needed clearance.

The three cases $S < L$, $S = L$, and $S > L$ for chord approximation are as follows:

1. $S < L$

From Figure 3(a), by the right triangle relationships,

$$(c/2)^2 + M^2 = (S/2)^2 \quad (33)$$

$$(c/2)^2 + (R - M)^2 = R^2 \quad (34)$$

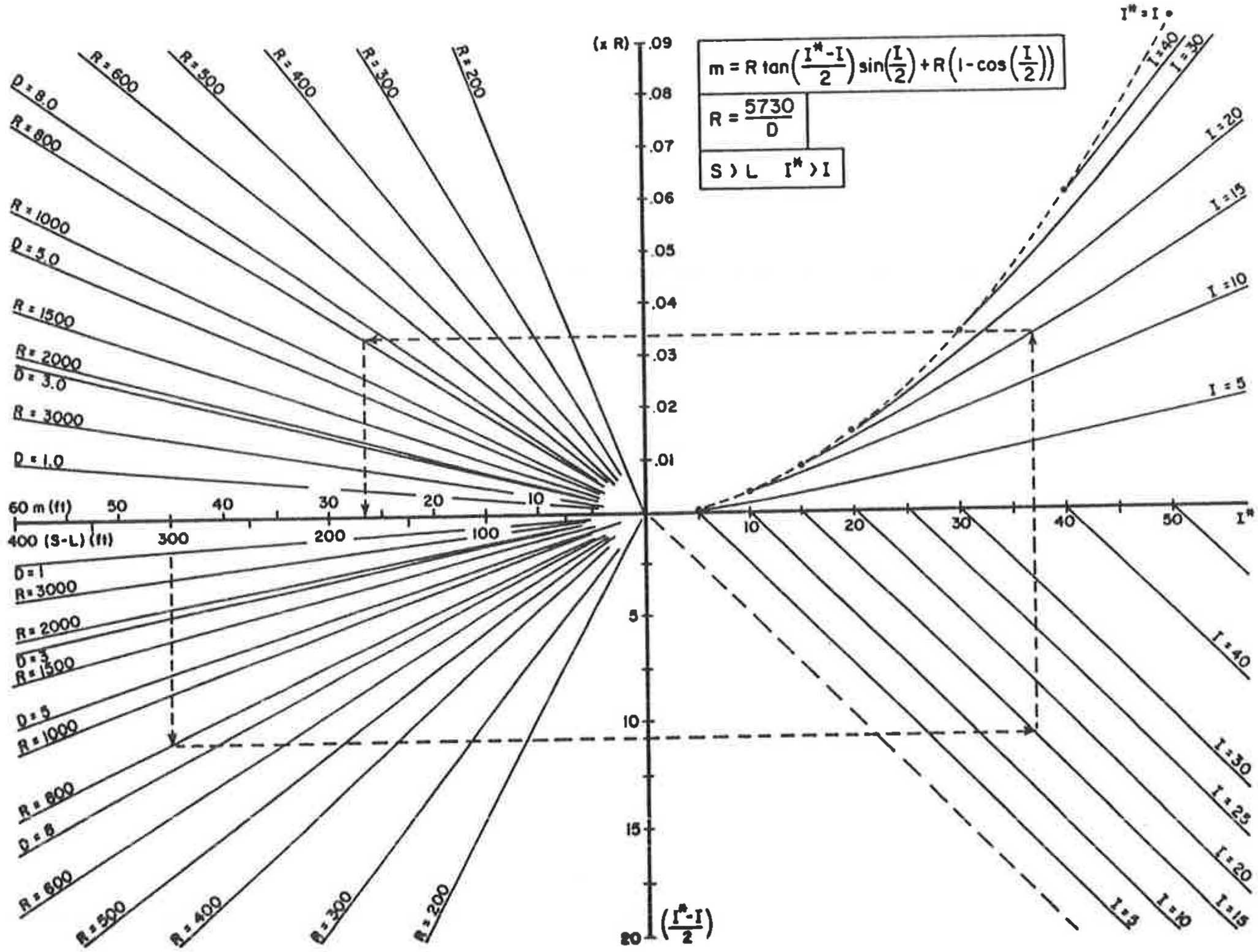

FIGURE 3 Critical lateral clearance m^* as a function of I, R, L, and S.

Solving for $c/2$ and substituting in Equation 34,

$$M = S^2/8R \tag{35}$$

2. $S = L$

From Figure 3(b) and the previous case,

$$O = S^2/8R = L^2/8R \tag{36}$$

3. $S > L$

From Figure 3(c),

$$S = L + 2T \tag{37}$$

$$(S/2)^2 = K^2 + m^2,\ K^2 + (R - m)^2 = E^2, \text{ and } T^2 + R^2 = E^2 \tag{38}$$

TABLE 5 CRITICAL HORIZONTAL CURVE CLEARANCES

Case (1)	Exact Solution (2)	Chord Approximation (3)
S<L	$M = R(1-\cos(I^*/2))$	$M = S^2/(8R)$
S=L	$M = R(1-\cos(I^*/2))$	$M = L^2/(8R)$
S>L	$m^* = R\sin(I/2)\tan[(I^*-I)/2] + R(1-\cos(I/2))$	$m^* = L(2S-L)/(8R)$

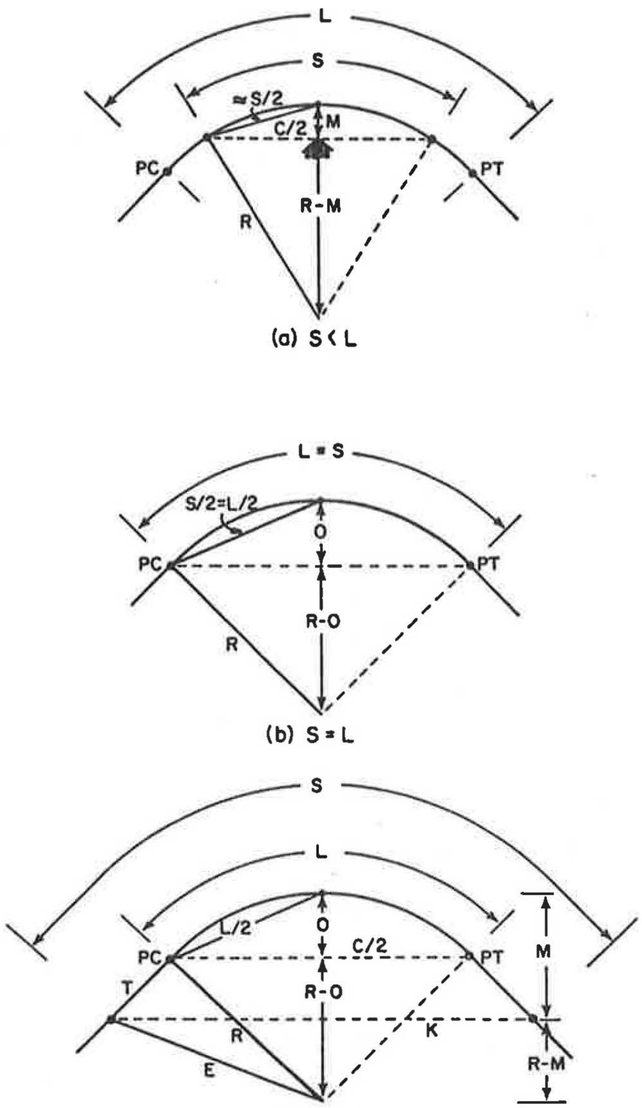

FIGURE 4 Chord approximations for M for horizontal curves: (a) $S < L$, (b) $S = L$, and (c) $S > L$.

where m is the maximum value for this case.

Eliminating K, E, and T from Equation 38,

$$m = L(2S - L)/8R \tag{39}$$

When $S = L$, Equation 36 can be obtained from Equation 35 or 38.

For the case $S > L$, one can compare the result with the case $S < L$ by use of the ratio

$$m/M = [L(2S - L)/8R]/[S^2/8R)] = (L/S)(2 - L/S) \tag{40}$$

For example, consider a 2° curve [R of 2,865 ft (873.3 m)], 900 ft (274.3 m) long with a required S = 1,150 ft (350.5 m). Then

$$M = 1{,}150^2/[(8)(2{,}865)] = 58 \text{ ft } (17.7 \text{ m})$$

$$m/M = (900/1{,}150)[2 - (900/1{,}150)] = 0.953$$

$$m = (0.953)(58) = 55 \text{ ft } (16.8 \text{ m})$$

The elasticity of the offset M with respect to S for $S < L$ is 2, indicating that a 1 percent increase in sight distance increases M by 2 percent. In the case $S > L$, the change is always greater than 1 percent.

Figure 5 shows the reduction in m^* as a percentage of M for various ratios of sight distance to curve length. This curve was developed from the chord approximation. For example, on short curves where S is twice the curve length, m^* is only 75 percent of M.

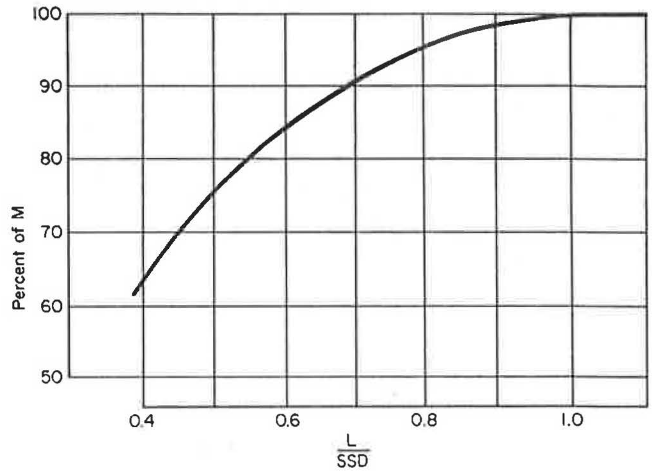

FIGURE 5 $S > L$: maximum obstacle clearance m^* as a percentage of M.

STEP 1: Determine I_1 and I_2.

Case A: Observer before the PC ($L_{21} > 0$):

$$I_1 = 180(L_{02}/\pi R) + \tan^{-1}(L_{21}/R) \tag{41}$$

$$I_2 = \tan^{-1}\{[R - m - (R^2 + L_{21}^2)^{1/2}\cos(I_1)]/[(R^2 + L_{21}^2)^{1/2} \sin(I_1)]\} \tag{42}$$

Case B: Observer in the curve ($L_{21} = 0$, $I_3 = 0$) or at the PC or PT:

$$I_1 = 180\,(L_{01}/\pi R) \tag{43}$$

$$I_2 = \tan^{-1}\left(\{R[1 - \cos(I_1)] - m\}/R\sin(I_1)\right) \tag{44}$$

Go to Step 2.

STEP 2: If Case A holds, then determine I_3; otherwise go to Step 3.

$$I_3 = \tan^{-1}(L_{21}/R)$$

Go to Step 3.

STEP 3: Calculate I_{ob}.

Case A: Observer before the PC ($L_{21} > 0$):

$$I_{ob} = (180L/\pi R) + \tan^{-1}(L_{21}/R) \tag{45}$$

Case B: Observer in the curve ($L_{21} = 0$, $I_3 = 0$) or at the PC or PT:

$$I_{ob} = 180(L_{ob}/\pi R) \tag{46}$$

Go to Step 4.

STEP 4: Test location of road object with respect to PT.

Object is beyond the PT if and only if $I_{ob} < 2(I_1 - I_2)$.

Object is at the PT if and only if $I_{ob} = 2(I_1 - I_2)$.

Object is in the curve if and only if $I_{ob} > 2(I_1 - I_2)$.

Go to Step 5.

STEP 5: Calculate total sight distance S using the general sight distance formula of Tables 2 and 3.

If Case A: Set L_{21} equal to the distance from the observer to the PC. If object is beyond the PT, then calculate L_{22} and I_4.

$$L_{22} = [R \cos(I') - h]/\sin(I') \qquad (47)$$

$$I_4 = \tan^{-1}(L_{22}/R)$$

where $I' = I_{ob} - (I_1 - I_2)$ and $h = (R - m)\cos(I_2)$.

I_{ob} is obtained from Equation 45, I_1 and I_2 from Equations 41 and 42. Otherwise, set $L_{22} = 0$ and obtain I_1 and I_2 from Equations 41 and 42.

If Case B: Set $L_{21} = 0$. If object is beyond the PT, then calculate L_{22} and I_4:

$$L_{22} = [R \cos(I') - H]/\sin(I') \qquad (48)$$

$$I_4 = \tan^{-1}(L_{22}/R) \qquad (25)$$

where $I' = I_{ob} - (I_1 - I_2)$ and $h = (R - m)\cos(I_2)$.

I_{ob} is obtained from Equation 46, I_1 and I_2 from Equations 43 and 44. If the object is not beyond the PT, set $L_{22} = 0$ and obtain I_1 and I_2 from Equations 43 and 44.

Calculation Procedure for Critical Lateral Clearance Values

The three cases $S < L$, $S = L$, and $S > L$ are considered.

CASE 1: S < L and I* < I

Calculate m^* using the following formulas or use AASHTO (*4*, pp. 244–245):

$$m^* = R[1 - \cos(I^*/2)]$$

$$S = (\pi R i^*)/180 \quad \text{for } I^* < I$$

$$R_1 = R - m^*$$

Then

$$m^* = R[1 - \cos(90S/\pi R)] \qquad (49)$$

Maximum lateral clearance m^* takes a constant value in a section of length a and starts at distance b measured from the start of the curve, where $a = \pi R(I - I^*)/180$ and $b = \pi R I^*/360$. The starting and end points of the section with maximum lateral clearance are connected to points located at the required sight distance before and after the curve using transition curves. For the transition curve from R = infinity to $R = R_1$, the distance from the baseline at R_1 is $h = R - R_1 \cos(I^*/2)$; or in general ($I \neq I^*$), the distance from the baseline at R_1 is $h = R - R_1 \cos(I/2)$.

CASE 2: S = L *and* I* = I

Calculate m^* using the following formulas or AASHTO (*4*, pp. 244–245):

$$m^* = R[1 - \cos(I/2)] \qquad (50)$$

$$S = \pi R I/180 \quad \text{for } I^* = I$$

$$R_1 = R - m^*$$

The maximum value for m occurs at the midpoint of the curve. This point is connected to points located at the sight distance before and after the curve using transition curves. For the transition curve from R = infinity to $R = R_1$, the distance from the baseline at R_1 is $h = R - R_1 \cos(I/2)$.

CASE 3: S > L *and* I* > I

Calculate m_{max} using the following formulas or use Figure 4.

$$m_{max} = R \tan[(I^* - I)/2] \sin(I/2) + R[1 - \cos(I/2)] \qquad (51)$$

$$S = S_s + \pi R I/180 \quad \text{for } I^* > I$$

where S_s is the sight distance component on the tangent $R_1 = R - m_{max}$. The maximum value for m occurs at the midpoint of the curve. This point is connected to points located at the required sight distance before and after the curve using transition curves. For the transition curve from R = infinity to $R = R_1$, the distance from the base line at R_1 is $h = R - R_1 \cos(I/2)$.

Example of the use of Figure 4 (*5, 6*):

1. Determine required sight distance S. Check that $I^* > I$ and $S > L$.
2. Enter the diagram with the difference $S - L$ (in feet) between the sight distance and the curve length and read for appropriate curve radius R or curvature D the $(I^* - I)/2$ value.
3. With this $(I^* - I)/2$ value, enter the right bottom part of the figure and using the central angle I of the curve read for the radius R the critical lateral clearance m^* (ft), or read as the maximum m value the scale with m expressed as a fraction of the radius R and multiply this m value by radius R to obtain the critical value of m (in feet).

For example, if $S - L = 300$ ft (91.4 m), $R = 800$ ft (243.8 m), $I = 15°$; follow the dashed line and read $m^* = 0.033R$ or $m^* =$ 26.5 ft (8.1 m).

Effect of Spiral Transition

The spiral transition curve affects the need for clearance on the inside of the path of travel when the driver is on the tangent or spiral portion of his path. The magnitude of this effect was developed in the UMTRI research for a range of typical AASHTO transition curve designs for speeds from 50 to 70 mph (80.5 to 112.6 km/hr) *(5)*. The maximum effect on *m* was about 3 ft (0.9 m). It is suggested that this value be used in such situations.

Eye and Object Location

Current design practice generally places both the observer and the road object on the centerline of the inside lane *(4)*. For a case and sensitivity study of the effect of placing these elements at another pair of lane locations, the findings were that *S* is not affected significantly by such changes, and further, that even this small effect decreases with increasing *S (5)*.

DISCUSSION

It is believed that the results of this research will be of value to practicing design and traffic engineers concerned with achieving adequate stopping sight distances at horizontal curves in new design and in adjusting speeds or clearances on existing roads. These models can easily be merged in a sight distance design and evaluation system incorporating results developed for vertical curve sight distance, as described in previous research undertaken at the University of Michigan *(6)*. It is no longer necessary to rely on graphical techniques executed on plan sheets or to use values for an inappropriate case that has been shown to be excessive.

For passing and decision sight distance analyses in which needed values of *S* may be twice as great as those found in stopping sight distance applications, the usefulness of these relationships should be even greater because these sight distances exceed the length of the horizontal curves in almost all cases, and significant reductions in roadside clearing expense and possibly even land acquisition or sight easement costs could be achieved.

The researchers have compared current and possible stopping sight distance policy values with information on the geometric characteristics of many rural highway curves and have found that smaller clearances are needed than those given by the AASHTO-treated case at more than half of the curves. This strongly supports the early use of this labor saving technique.

SUMMARY

This research was initiated because of a concern with the needed large increases in roadside clearances to obstacles limiting sight distance on the inside of horizontal curves that result from increases in recently adopted AASHTO stopping sight distance policies and from changes recommended in recent NCHRP research. AASHTO does not provide mathematical solutions for this type of design problem in which the sight distance is longer than the curve. It was also found that by ignoring the greater available sight distance when the driver or road object is off the curve, current AASHTO methodology for calculating the relationships involving sight distance, curve parameters, and clearance inside the curve leads to the selection of excessive needed clearance distances.

Several relationships were derived and brought together with earlier work that provide analytic, closed-form, and easily usable tools for the designer concerned with maintaining or achieving adequate clearances to sight obstacles based on sight distance needs and considerations.

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