Analysis and Design of Weight-Distance Taxation

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In this paper, the concept of highway weight-distance taxation is discussed from the revenue-cost equity point of view. This discussion is followed by an analysis of a number of weight-distance taxation schemes using a linear programming technique. It is demonstrated that this linear programming technique can be effectively adopted for the design of rate schedules for a weight-distance tax. Full-scale analyses based on the actual data of a state highway cost allocation study are presented for illustrative purposes.

Most states today are still adhering to the traditional two-tier highway user tax system first enacted in the early 1920s. This two-tier system consists of a first structure of vehicle registration fees and a second structure of fuel taxes. Unfortunately, as revealed by cost allocation studies conducted in many states, the two-tier system is not an equitable tax structure because it does not impose tax charges on different vehicle classes according to their respective cost responsibilities.

The inefficiency and inadequacy of the two-tier system becomes more obvious as the problem of raising sufficient funds to meet highway needs becomes increasingly critical at all levels of government. A number of studies *(1–3)* have revealed that, although fuel consumption increases with vehicle size and weight, it does not increase proportionately to cost responsibility. Fuel taxes therefore do not adequately reflect the cost responsibility of vehicles of different sizes and weights. In addition, it also creates inequity between high-fuel-efficient and low-fuel-efficient vehicles of the same weight. Vehicle registration fees, on the other hand, fail to produce equity between low-annual-mileage and high-annual-mileage vehicles.

Recognizing the inherent weaknesses of the traditional two-tier tax system, it is not surprising to find that the net result of equity analysis on such taxing schemes typically is one that shows large combination trucks being heavily subsidized by light trucks and passenger cars. As an illustration, Table 1 presents the results of some recently conducted cost allocation studies.

In the search for an equitable tax structure that would relate more closely to the cost responsibilities of various vehicle classes, an increasing number of states are now considering imposing a weight-distance tax on heavy trucks. In this paper, a mathematical programming approach to examine how a weight-distance tax could improve the equity of a highway user tax structure is described. It is also shown that the technique can be used effectively to design the rate schedule of a weight-distance tax.

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THEORY OF WEIGHT-DISTANCE TAXATION

The term weight-distance tax is not uniquely defined in the literature. It is generally referred to as a tax levied on a vehicle on the basis of its gross weight and the distance it travels within a given state over a given period of time.

An equitable highway user tax structure should accordingly reflect the costs of a highway program that are caused by each individual vehicle class. The basis of a weight-distance tax is that many highway expenditures are related to vehicle-miles of travel (VMT), and many others are related to operating axle weights. By varying user tax charges in direct proportion to VMT, inequity between low- and high-annual-mileage vehicles can be avoided. In addition, inequity between light and heavy vehicles could be reduced by having tax rates graduated in accordance with vehicle or axle weights.

Mathematically, the weight-distance tax relationship may be expressed as the following general equation:

$$T_{ij} = R_j(W_{ij})M_{ij} \quad (i = 1, 2, \ldots, n_j; \; j = 1, 2, \ldots, N) \quad (1)$$

where

T_{ij} = required tax payment in dollars per annum by vehicle i of vehicle class j;
W_{ij} = weight of vehicle i of vehicle class j;
$R_j(W_{ij})$ = tax schedule rate in cents per vehicle mile of travel for vehicle class j, expressed as a function of W_{ij};
M_{ij} = annual VMT by vehicle i of vehicle class j;
N = total number of vehicle classes; and
n_j = number of weight groups in vehicle class j.

As can be seen from the expression in Equation 1, the determination of the rate schedule relationship $R_j(W_{ij})$ is the most important aspect of weight-distance tax design. It has a direct bearing on whether a weight-distance rate schedule would yield an equitable tax structure. Before the relationship $R_j(W_{ij})$ could be derived, two important factors must first be defined. These two factors are the vehicle weight W_{ij} and vehicle classification represented by the variables n_j and N. A discussion of the significance of these two factors follows.

TABLE 1 RESULTS OF SOME RECENT COST ALLOCATION STUDIES *(4)*

(USER REVENUE/COST-RESPONSIBILITY) RATIOS

Cost Allocation Study	Passenger Cars	Single Unit Trucks	Combination Trucks
Florida (1979)	1.04	0.91	0.51(*)
Georgia (1979)	1.03	0.66	0.44(*)
Oregon (1980)	1.00	1.25	0.92
Colorado (1981)	1.22	1.24	0.56
Kentucky (1982)	1.57	--	0.57(**)
Maryland (1982)	1.17	0.83	0.56
Connecticut (1982)	1.11	1.61	0.63
Ohio (1982)	0.90	2.25	0.35
Wisconsin (1982)	0.94	1.40	0.89
Maine (1982)	1.02	1.16	0.97
N. Carolina (1983)	0.96	2.14	0.78
Federal (1982)	1.10	1.50	0.60
Indiana (1984)	1.24	1.13	0.62

(*) for trucks with 5 or more axles
(**) for all trucks

The Weight Factor *W*

It is a fundamental fact that in highway cost allocation, vehicle responsibilities of pavement costs, including construction, maintenance, rehabilitation, and reconstruction costs, are closely related to the actual vehicle axle loads applied on the pavements. It is therefore obvious that the most equitable weight to be used in the weight-distance relationship in Equation 1 is the operating weight of the vehicle concerned.

In reality, a given vehicle would carry different loads in different trips. Ideally, a vehicle i of vehicle class j should make a tax payment computed in the following manner:

$$T_{ij} = \sum_{t=1}^{k} R_j(W_{ij})_t(M_{ij})_t \qquad (i = 1, 2, \ldots, n_j; \; j = 1, 2, \ldots, N) \tag{2}$$

where

k = total number of time periods in a year, assuming the weight carried by vehicle i during each period is constant;
$(W_{ij})_t$ = operating weight of vehicle i during time period t; and
$(M_{ij})_t$ = miles traveled by vehicle i of vehicle class j in time period t.

T_{ij}, $R_j(W_{ij})$, n_j, and N are as defined in Equation 1.

Unfortunately, it is impractical to document such detailed records for every vehicle on the road. As a result, virtually all weight-distance taxation schemes that are in operation in various states today are based on gross registered vehicle weights. In other words, the tax charges in these taxation schemes are calculated by the following equation:

$$T_{ij} = R'_j(W'_{ij})M_{ij} \qquad (i = 1, 2, \ldots, n_j; \; j = 1, 2, \ldots, N) \tag{3}$$

where

W'_{ij} = registered gross vehicle weight of vehicle i of vehicle class j; and
$R'_j(W'_{ij})$ = tax schedule rate in cents per vehicle-mile of travel for vehicle class j, expressed as a function of W_j.

T_{ij}, M_{ij}, n_j, and N are as defined in Equation 1.

Theoretically, the relationship in Equation 3 would be valid if there existed an exact transformation between operating vehicle weight W and registered gross vehicle weight W'. Symbolically, this means that Equation 3 is true if the following relationships hold:

$$W' = f(W) \tag{4}$$

and

$$R'(W') = R(W) \quad (5)$$

Both the 1984 Indiana *(4)* and the 1983 Wisconsin *(5)* cost allocation studies attempted to develop the relationship in Equation 4 by means of establishing correspondence matrices relating vehicle registered weights and operating weights. The results of these two studies indicated that there was a definite relationship between the two weights, although it was not a one-to-one correspondence relationship as depicted schematically in Figure 1.

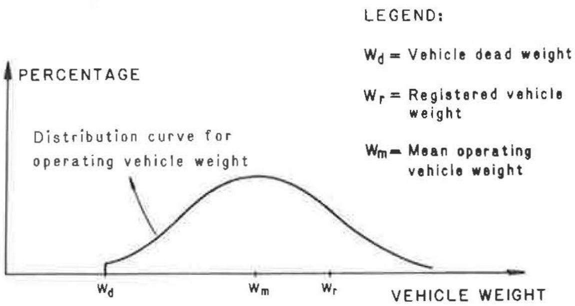

FIGURE 1 Schematic diagram showing the relationship between registered gross vehicle weight and operating vehicle weight.

Because of the variability of operating weights for a given registered vehicle weight, $R'(W')$ is only an approximation of the true relationship $R(W)$. This approximation introduces an inequity between heavily loaded and lightly loaded vehicles within a registered vehicle class. This inequity may in fact be quite small if the average load per vehicle-mile carried by each vehicle in the vehicle class does not vary over a wide range. A vehicle registration system with a detailed vehicle classification based on vehicle axle configuration and size would also help to reduce such inequity.

Vehicle Classification Factors n_j and N

The expression in Equation 1 assumes a discrete stepwise relationship for rate schedule $R(W)$. A continuous $R(W)$ function, as shown in Figure 2(a), is a more accurate representation of its true relationship. Figure 2(b) shows the discretized form of rate schedule function $R(W)$ commonly adopted in practice. The number of weight groups n_j in each vehicle class should be sufficiently large to avoid creating objectionable inequity between vehicles of different weights. A common range of weight increment is between 2,000 to 5,000 lb.

There are two features in Figure 2(a) that deserve to be mentioned. First, for a given vehicle operating weight, there can be N numbers of tax rate depending on the axle configuration of the vehicle concerned. In general, the higher the ESAL value of the vehicle, the higher the rate. This is simply a reflection of the results of a typical cost allocation study.

Second, the greater the value of the number N, the more equitable the taxation scheme would be. This is a logical conclusion because, if vehicle classes 1 and 2 in Figure 2(a) are combined into one vehicle class, the resulting rate schedule would not equitably represent the true responsibilities of two vehicles originally classified under Classes 1 and 2. The net result would be an overpayment of tax by vehicle Class 1, and an underpayment by vehicle Class 2.

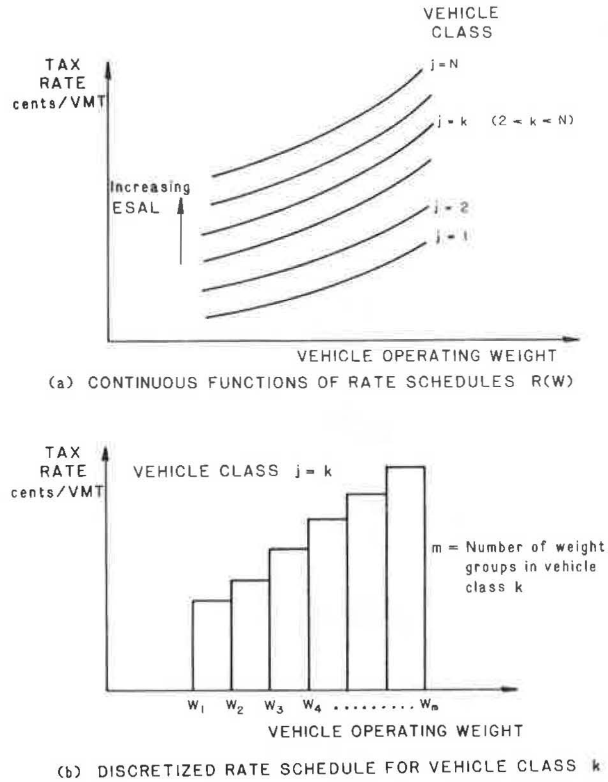

FIGURE 2 Schematic representation of continuous and discretized rate schedule function $R(W)$.

Weight-Distance Tax Design Procedure

There is no known recommended design procedure in the literature for a weight-distance tax. However, based on the discussion in the preceding paragraphs, a logical design procedure for a reasonably equitable weight-distance taxation scheme may consist of the following steps:

1. Define vehicle classes in terms of axle configuration and subclasses in accordance with vehicle operating weight.
2. Perform a cost allocation analysis to determine the cost responsibility of each vehicle class in terms of its operating weight.
3. Design a tax rate schedule on the basis of the operating weight classification and associated number of vehicle-miles of travel to satisfy highway funding requirements.
4. Develop correspondence matrices between operating vehicle weights and registered vehicle weights for all vehicle classes.
5. Transform operating weight-based tax rate schedule in Step 3 into a registered weight-based tax rate schedule.

The remaining sections of this paper will be devoted to illustrating how a linear programming technique could be used for the design and analysis of weight-distance taxation schemes. That is, the following analysis will center on Step 3 of the procedure.

Vehicle Class	Vehicle Type	Axles Configuration	Total No. of Axles	Number of Single Axles	Number of Tandem Axles
3	Single-Unit Truck		2	2	
6	Single-Unit Truck		3	1	1
7	Combination Trucks		3	3	
9	Single-Unit Truck		4	2	1
10	Combination Trucks		4	2	1
11	Combination Trucks		4	2	1
12	Combination Trucks		5	1	2

FIGURE 3 Axle configuration characteristics of vehicle classes.

LINEAR PROGRAMMING FORMULATION OF WEIGHT-DISTANCE TAX PROBLEM

In the weight-distance tax problem, the objective is to maximize equity between the cost responsibility and revenue payment of each vehicle weight subgroup within each vehicle class.

To be completely equitable, a weight-distance tax structure must satisfy the following fundamental requirements:

1. Completeness requirement. A system equity constraint that requires that highway expenditures be entirely financed by highway users. This constraint conforms to the highway financing policies in most states.
2. Rationality requirement. A vehicle class and subclass weight group equity constraint that states that each highway user group should pay its fair share of cost responsibility.
3. Compatibility requirement. This requirement specifies that tax rates be graduated in accordance with vehicle weights so as to be compatible with the cost responsibility concept depicted in Figure 2(a).

These three requirements are essential for the establishment of an equitable tax structure. They serve as a useful guide for the formulation of constraint equations in the weight-distance tax problem.

The mathematical formulation for a typical weight-distance taxation scheme with a discretized rate schedule as shown in Figure 2(b) may be expressed in terms of minimizing an objective function as follows:

$$\text{I. Minimize} \sum_{j=1}^{N} \sum_{i=1}^{n_j} [Z_{ij}] \qquad (i = 1, 2, \ldots, n_j; \; j = 1, 2, \ldots, N) \tag{6}$$

or

$$\text{II. Minimize (maximum } [Z_{ij}]) \qquad (i = 1, 2, \ldots, n_j; \; j = 1, 2, \ldots, N) \tag{7}$$

TABLE 2 VEHICLE CLASS AND WEIGHT GROUP CLASSIFICATION

Veh Class	Sub-Group	Gross Operating Weight in Pounds	Veh Class	Sub-Group	Gross Operating Weight in Pounds
3	1	< 20,000	11	1	< 30,000
3	2	20,000 - 25,000	11	2	30,000 - 35,000
3	3	> 25,000	11	3	35,000 - 40,000
			11	4	40,000 - 45,000
6	1	< 20,000	11	5	45,000 - 50,000
6	2	20,000 - 25,000	11	6	> 50,000
6	3	25,000 - 30,000			
6	4	30,000 - 35,000	12	1	< 30,000
6	5	> 35,000	12	2	30,000 - 35,000
			12	3	35,000 - 40,000
7	1	< 30,000	12	4	40,000 - 45,000
7	2	30,000 - 35,000	12	5	45,000 - 50,000
7	3	> 35,000	12	6	50,000 - 55,000
			12	7	55,000 - 60,000
9	1	< 20,000	12	8	60,000 - 65,000
9	2	20,000 - 25,000	12	9	65,000 - 70,000
9	3	> 25,000	12	10	> 70,000
10	1	< 30,000			
10	2	> 30,000			

TABLE 3 1983 ANNUAL VMT OF VEHICLE CLASSES AND WEIGHT GROUPS

Veh Class	Sub-Group	VMT in Millions Veh Class	VMT in Millions Sub-Group	Veh Class	Sub-Group	VMT in Millions Veh Class	VMT in Millions Sub-Group
3	1	1014.2	926.3	11	1	266.7	106.2
3	2		139.9	11	2		59.0
3	3		48.0	11	3		30.1
				11	4		22.9
6	1		108.4	11	5		25.8
6	2		61.0	11	6		22.7
6	3		19.6				
6	4		25.8	12	1	2472.5	451.0
6	5		53.5	12	2		310.2
				12	3		124.8
7	1	75.8	36.7	12	4		132.1
7	2		19.6	12	5		174.8
7	3		19.5	12	6		135.1
				12	7		170.0
9	1	71.6	3.6	12	8		214.9
9	2		22.9	12	9		237.9
9	3		45.1	12	10		521.7
10	1	15.7	7.4				
10	2		8.3				

subject to

1. Completeness constraint

$$\sum_{j=1}^{N} \sum_{i=1}^{n_j} (V_{ij}X_{ij} + F_{ij}) \geq \left(\sum_{j=1}^{N} \sum_{i=1}^{n_j} C_{ij} \right) R \qquad (8)$$

2. Rationality constraints

$$C_{ij}R - V_{ij}X_{ij} - F_{ij} = D_{ij} \qquad (i = 1, 2, \ldots, n_j; \; j = 1, 2, \ldots, N) \qquad (9)$$

or

TABLE 4 VEHICLE CLASS AND WEIGHT GROUP COST RESPONSIBILITIES FOR 1983

Veh Class	Sub-Group	% Responsibility Veh Class	% Responsibility Sub-Group	Veh Class	Sub-Group	% Responsibility Veh Class	% Responsibility Sub-Group
3	1	6.766	4.618	11	1		0.518
3	2		1.568	11	2		0.515
3	3		0.580	11	3		0.325
				11	4		0.294
6	1	2.604	0.628	11	5		0.410
6	2		0.408	11	6		0.463
6	3		0.209				
6	4		0.364	12	1	31.258	1.350
6	5		0.995	12	2		0.981
				12	3		0.495
7	1	0.974	0.262	12	4		0.674
7	2		0.293	12	5		1.193
7	3		0.419	12	6		1.095
				12	7		2.304
9	1	1.087	0.036	12	8		4.006
9	2		0.310	12	9		5.316
9	3		0.759	12	10		13.844
10	1	0.106	0.046				
10	2		0.060				

TABLE 5 FEATURES OF WEIGHT-DISTANCE TAX STRUCTURES ANALYZED

Tax Structure	Truck Registration Requirements: By Vehicle Type*	Truck Registration Requirements: By Axle Configuration	Truck Operating Weight Information: Gross Veh. Wt.	Truck Operating Weight Information: Axle Weight
A	No	No	Yes	No
B	Yes	No	Yes	No
C	Yes	Yes	Yes	No

*Vehicle type refers to classification of trucks by single-unit and combination trucks.

$$\left(\frac{C_{ij}}{V_{ij}}\right)R - X_{ij} - \frac{F_{ij}}{V_{ij}} = Z_{ij} \quad (i = 1, 2, \ldots, n_j; \; j = 1, 2, \ldots, N) \tag{10}$$

3. Compatibility constraints

$$X_{ij} \geq X_{(i-1)j} \quad (i = 2, 3, \ldots, n_j; j = 1, 2, \ldots, N) \tag{11}$$

4. Nonnegativity constraints

$$X_{ij} \geq 0 \quad (i = 1, 2, \ldots, n_j; j = 1, 2, \ldots, N) \tag{12}$$

where

D_{ij} = difference between the tax payment and cost responsibility of vehicle weight group i in vehicle class j;
V_{ij} = total annual vehicle-miles of travel of all vehicles in weight group i of vehicle class j;
Z_{ij} = D_{ij}/V_{ij}, the difference between the tax payment per vehicle-mile and cost responsibility per vehicle-mile of vehicle weight group i of vehicle class j;
n_j = number of vehicle weight groups in vehicle class j;
N = total number of vehicle classes;
X_{ij} = tax rate in cents per mile for each vehicle in weight group i of vehicle class j;
F_{ij} = user charges, other than weight-distance tax, paid by weight group i of vehicle class j;
R = total revenue needed for highway funding requirements; and
C_{ij} = percent cost responsibility for weight group i of vehicle class j.

TABLE 6 RATE SCHEDULE PATTERN FOR TAX STRUCTURE *A*

Tax Rate	Single-Unit Truck: Class 3 Wt. Group	Single-Unit Truck: Class 6 Wt. Group	Single-Unit Truck: Class 9 Wt. Group	Combination Truck: Class 7 Wt. Group	Combination Truck: Class 10 Wt. Group	Combination Truck: Class 11 Wt. Group	Combination Truck: Class 12 Wt. Group
X_1	1	1	1				
X_2	2	2	2				
X_3	3	3	3	1	1	1	1
X_4		4		2	2	2	2
X_5		5		3		3	3
X_6						4	4
X_7						5	5
X_8						6	6
X_9							7
X_{10}							8
X_{11}							9
X_{12}							10

TABLE 7 RATE SCHEDULE PATTERN FOR TAX STRUCTURE *B*

	Single-Unit Truck			Combination Truck			
Tax Rate	Class 3 Wt. Group	Class 6 Wt. Group	Class 9 Wt. Group	Class 7 Wt. Group	Class 10 Wt. Group	Class 11 Wt. Group	Class 12 Wt. Group
X_{a1}	1	1	1				
X_{a2}	2	2	2				
X_{a3}	3	3	3				
X_{a4}		4					
X_{a5}		5					
X_{b1}				1	1	1	1
X_{b2}				2	2	2	2
X_{b3}				3		3	3
X_{b4}						4	4
X_{b5}							5
X_{b6}							6
X_{b7}							7
X_{b8}							8
X_{b9}							9
X_{b10}							10

Equations 6 and 7 present two possible objective functions that could be used for determining the decision variables X_{ij}. Objective Function I minimizes the total inequity systemwide, desirable from the system operator's (for example, a state highway agency's or a state government's) point of view. On the other hand, Objective Function II minimizes the inequity of the individual vehicle weight group with the largest inequity. The latter objective function tends to spread out inequity amounts evenly to all vehicle classes and weight groups. Objective Function II therefore is more likely to produce a tax schedule acceptable to individual users, whereas Objective Function I may have more appeal to large groups of users.

Equation 8 provides the constraint that the computed tax schedule would at least produce the needed revenue R for the intended highway program. Equation 9 is a set of equations that calculate the inequity amounts for all vehicle weight groups. This equation, however, is not suitable for use directly in the linear programming formulation because it tends to create inequity between low- and high-VMT vehicle groups. More weight would be placed on high-VMT vehicle groups in the optimization process. A more appropriate set of rationality constraints is presented in Equation 10, in which the inequity between tax payment and cost responsibility per vehicle-mile of travel is considered. The inequality constraints in Equation 11 ensure that the compatibility requirements are satisfied by the derived rate schedule. The nonnegativity constraints in Equation 12 require that all tax rates derived be nonnegative.

NUMERICAL ILLUSTRATIVE EXAMPLES

Numerical examples are presented in this section to examine a number of weight-distance taxation schemes using the proposed linear programming technique. These analyses are based on the state highway data of Indiana and the results of the 1984 Indiana Highway Cost Allocation Study *(4)*.

For clarity in presentation and to highlight the salient features of the weight-distance taxation schemes considered, tax revenues other than weight-distance tax are excluded from these examples. That is, all the F_{ij} values in Equations 8 and 9 are set to zero. The procedure described in this paper, however, is still applicable to cases for which the F_{ij} values are not zero. Also, out of the original 14 vehicle classes used in the Indiana cost allocation study, only 7 major truck classes are included in the present analysis. The axle configuration characteristics of these 7 truck classes are shown in Figure 3. Each of these truck classes consists of a number of weight groups as defined in Table 2.

Other data required for the analysis are the annual VMT value and cost responsibility factor for each of the vehicle weight groups presented in Table 2. These data are presented in Tables 3 and 4, respectively, for 1983.

Weight-Distance Tax Structures Considered

Three weight-distance tax structures are analyzed to study the relative merit of each design on the basis of equity consideration. The main features of these three tax structures are presented in Table 5. Tax Structure *A* has a tax rate graduated on the basis of gross operating vehicle weight, regardless of vehicle type (single-unit truck versus combination truck) and axle configuration. For instance, as presented in Table 6 the same rate X_3 would be charged to vehicles with weight between 30,000 and 35,000 lb, irrespective of whether they belong to Class 6 single-unit truck, Class 12 combination truck, or other

TABLE 8 RATE SCHEDULE PATTERN FOR TAX STRUCTURE *C*

	Single-Unit Truck			Combination Truck			
Tax Rate	Class 3 Wt. Group	Class 6 Wt. Group	Class 9 Wt. Group	Class 7 Wt. Group	Class 10 Wt. Group	Class 11 Wt. Group	Class 12 Wt. Group
X_{c1}	1						
X_{c2}	2						
X_{c3}	3						
X_{d1}		1					
X_{d2}		2					
X_{d3}		3					
X_{d4}		4					
X_{d5}		5					
X_{e1}			1				
X_{e2}			2				
X_{e3}			3				
X_{f1}				1			
X_{f2}				2			
X_{f3}				3			
X_{g1}					1		
X_{g2}					2		
X_{h1}						1	
X_{h2}						2	
X_{h3}						3	
X_{h4}						4	
X_{h5}						5	
X_{h6}						6	
X_{i1}							1
X_{i2}							2
X_{i3}							3
X_{i4}							4
X_{i5}							5
X_{i6}							6
X_{i7}							7
X_{i8}							8
X_{i9}							9
X_{i10}							10

truck class. Most of the weight-distance tax structures currently in use in various states [for instance, Oregon *(2)*, Arizona *(6)*, and Arkansas *(7)*] may be classified as Tax Structure *A*.

A refinement to Tax Structure *A* may be made by having different sets of rate schedules for single-unit and combination trucks. This arrangement is represented by Tax Structure *B* given in Table 5 and its rate schedule pattern as presented in Table 7. Another refinement is to provide a separate set of rate schedules for each vehicle class on the basis of vehicle axle configuration. This gives rise to Tax Structure *C* given in Table 5 and the rate schedule pattern presented in Table 8.

An inspection of Table 5 suggests that a further refinement is possible by differentiating the weight groups in Tax Structure *C* by axle weight distribution. An analysis performed in connection with the Indiana cost allocation study *(4)* found, however, that for vehicles with the same axle configuration and gross

TABLE 9 WEIGHT-DISTANCE TAX SCHEDULES BY LINEAR PROGRAMMING DESIGN

Vehicle Type	Vehicle Class	Weight Group	Design I Rate Schedule (Cents/VMT)			Design II Rate Schedule (Cents/VMT)		
			Tax Structure A	Tax Structure B	Tax Structure C	Tax Structure A	Tax Structure B	Tax Structure C
Single-Unit Trucks	3	1	2.500	2.789	2.797	2.500	2.797	2.797
		2	3.345	5.607	5.607	3.345	5.042	5.607
		3	3.345	6.042	6.042	4.052	6.736	6.042
	6	1	2.500	2.789	2.907	2.500	2.797	2.907
		2	3.345	5.607	3.344	3.345	5.042	3.344
		3	3.345	6.042	5.332	4.052	6.736	5.332
		4	3.632	7.055	7.054	4.052	8.649	7.054
		5	5.417	9.299	9.299	6.362	9.299	9.299
	9	1	2.500	2.789	2.500	2.500	2.797	2.500
		2	3.345	5.607	6.739	3.345	5.042	6.739
		3	3.345	6.042	8.433	4.052	6.736	8.433
Combination Trucks	7	1	3.345	2.444	3.570	4.052	2.444	3.570
		2	3.632	3.632	7.475	4.052	3.632	7.475
		3	5.417	3.632	10.744	6.362	6.362	10.744
	10	1	3.345	2.444	3.108	4.052	2.444	3.108
		2	3.632	3.632	3.632	4.052	3.632	3.632
	11	1	3.345	2.444	2.443	4.052	2.444	2.443
		2	3.632	3.632	4.634	4.052	3.632	4.634
		3	5.417	3.632	5.417	6.362	6.362	5.417
		4	5.417	3.632	6.391	6.362	6.392	6.391
		5	5.417	3.632	7.946	6.362	6.392	7.946
		6	5.417	4.056	10.198	6.362	7.569	10.198
	12	1	3.345	2.444	1.497	4.052	2.444	1.497
		2	3.632	3.632	1.582	4.052	3.632	1.582
		3	5.417	3.632	1.980	6.362	6.362	1.980
		4	5.417	3.632	2.553	6.362	6.362	2.553
		5	5.417	3.632	3.409	6.362	6.392	3.409
		6	5.417	4.056	4.056	6.362	7.569	4.056
		7	6.777	6.777	6.777	6.777	7.569	6.777
		8	9.317	9.317	9.317	9.317	8.879	9.317
		9	11.152	11.168	11.168	9.317	8.879	11.168
		10	11.152	11.768	13.261	9.317	8.879	13.261

Note: 1. Design I refers to linear programming design with objective function (I) in Equation (6). Design II refers to linear programming design with objective function (II) in Equation (7).
2. Each set of rate schedule is designed to yield $226.6 million revenue from the seven truck classes.

TABLE 10 REVENUE/COST RATIOS BY TRUCK TYPE FOR DIFFERENT WEIGHT-DISTANCE TAX STRUCTURES

Vehicle Type	Tax Structure *A*	*B*	*C*
Objective Function I of Equation 6			
Single-unit trucks	0.737	1.000	1.000
Combination trucks	1.109	1.000	1.000
All trucks	1.000	1.000	1.000
Objective Function II of Equation 7			
Single-unit trucks	0.764	1.000	1.000
Combination trucks	1.070	1.000	1.000
All trucks	1.000	1.000	1.000

weight, there were insignificant variations in the pattern of percent axle load distribution. Such a refinement, which would involve large registration administrative costs and enforcement effort, but with negligible improvement in the equity of the tax system, is not practically justifiable and is therefore not included in the present analysis.

Results of Linear Programming Analyses

The total highway expenditure in Indiana in fiscal year 1983 was about $570 million. In order to produce a realistic rate schedule, the needed highway fund value is set to be $500 million in the present examples. The assumption is, as stated

TABLE 11 REVENUE/COST RATIOS OF WEIGHT-DISTANCE TAX SCHEDULES DESIGNED WITH OBJECTIVE FUNCTION I OF EQUATION 6

Vehicle Type	Vehicle Class	Weight Group	Tax Structure A	Tax Structure B	Tax Structure C
Single-Unit Trucks	3	1	0.894	0.997	1.000
		2	0.596	1.000	1.000
		3	0.554 (0.796)	1.000 (0.998)	1.000 (1.000)
	6	1	0.860	0.959	1.000
		2	1.000	1.678	1.000
		3	0.627	1.133	1.000
		4	0.515	1.000	1.000
		5	0.582 (0.709)	1.000 (1.107)	1.000 (1.000)
	9	1	1.000	1.115	1.000
		2	0.492	0.832	1.000
		3	0.397 (0.435)	0.716 (0.756)	1.000 (1.000)
Combination Trucks	7	1	0.937	0.685	1.000
		2	0.486	0.486	1.000
		3	0.504 (0.615)	0.338 (0.476)	1.000 (1.000)
	10	1	1.076	0.786	1.000
		2	1.000 (1.035)	1.000 (0.910)	1.000 (1.000)
	11	1	1.368	1.000	1.000
		2	0.832	0.832	1.000
		3	1.000	0.671	1.000
		4	0.848	0.568	1.000
		5	0.682	0.457	1.000
		6	0.531 (0.887)	0.398 (0.676)	1.000 (1.000)
	12	1	2.232	1.633	1.000
		2	2.294	2.294	1.000
		3	2.732	1.835	1.000
		4	2.123	1.423	1.000
		5	1.590	1.065	1.000
		6	1.335	1.000	1.000
		7	1.000	1.000	1.000
		8	1.000	1.000	1.000
		9	1.000	1.000	1.000
		10	0.841 (1.109)	0.887 (1.043)	1.000 (1.000)

Note: Values in parentheses refer to revenue/cost ratios of vehicle classes.

TABLE 12 REVENUE/COST RATIOS OF WEIGHT-DISTANCE TAX SCHEDULES DESIGNED WITH OBJECTIVE FUNCTION II OF EQUATION 7

Vehicle Type	Vehicle Class	Weight Group	Tax Structure A	Tax Structure B	Tax Structure C
Single-Unit Trucks	3	1	0.894	1.000	1.000
		2	0.597	0.899	1.000
		3	0.671 (0.806)	1.115 (0.986)	1.000 (1.000)
	6	1	0.860	0.962	1.000
		2	1.000	1.508	1.000
		3	0.760	1.264	1.000
		4	0.574	1.226	1.000
		5	0.684 (0.767)	1.000 (1.123)	1.000 (1.000)
	9	1	1.000	1.119	1.000
		2	0.496	0.748	1.000
		3	0.480 (0.494)	0.799 (0.790)	1.000 (1.000)
Combination Trucks	7	1	1.135	0.685	1.000
		2	0.542	0.486	1.000
		3	0.592 (0.723)	0.592 (0.585)	1.000 (1.000)
	10	1	1.304	0.786	1.000
		2	1.116 (1.200)	1.000 (0.910)	1.000 (1.000)
	11	1	1.658	1.000	1.000
		2	0.928	0.832	1.000
		3	1.175	1.175	1.000
		4	0.995	1.000	1.000
		5	0.801	0.804	1.000
		6	0.604 (1.043)	0.742 (0.911)	1.000 (1.000)
	12	1	2.707	1.633	1.000
		2	2.561	2.296	1.000
		3	3.213	3.213	1.000
		4	2.492	2.504	1.000
		5	1.866	1.875	1.000
		6	1.569	1.866	1.000
		7	1.000	1.117	1.000
		8	1.00	0.953	1.000
		9	0.834	0.795	1.000
		10	0.703 (1.083)	0.670 (1.021)	1.000 (1.000)

Note: Values in parentheses refer to revenue/cost ratios of vehicle classes.

earlier, that all other forms of tax revenue are negligible. That is, $F_{ij} = 0$ and $R = \$500,000,000$ in Equation 8. The values of coefficients V_{ij} and C_{ij} in Equations 8 and 9 are given in Tables 3 and 4, respectively.

The derived weight-distance tax schedules for Tax Structures *A*, *B*, and *C* are summarized in Table 9. Each tax structure is analyzed and designed under the two different objective functions shown in Equations 6 and 7. From Table 4, the seven truck classes have a combined cost responsibility of 45.22 percent. This result means that each set of rate schedules in Table 9 would produce a total revenue of $226.6 million from these truck classes.

The equity of each tax structure was assessed by examining the revenue/cost ratios of various vehicle classes. A revenue/cost ratio of a vehicle class is computed by dividing the percent revenue contribution of the vehicle class by its percent cost responsibility. A revenue/cost ratio of 1 implies perfect equity for the vehicle class as a whole. A revenue/cost ratio with a value less than 1 means that the vehicle class underpays its fair share of cost responsibility; whereas a value greater than 1 means overpayment. Likewise, revenue/cost ratios for vehicle weight groups are computed and interpreted in the same manner. The computed revenue/cost ratios by vehicle type, vehicle class, and weight groups are presented in Tables 10, 11, and 12, respectively.

Summary of Analysis Findings

Based on the linear programming results in Tables 9 through 12, the major findings of these analyses are summarized as follows.

1. Table 10 shows that in Tax Structure *A* in which the same rate schedule is applied to both single-unit and combination trucks, a cross-subsidization exists between the two vehicle types. Combination trucks as a group would be overcharged. This result should be within expectation because a combination truck that has more axles and hence a lower ESAL value than a single-unit truck of the same operating weight would have a lower cost responsibility per vehicle-mile of travel. It is therefore inequitable to charge both trucks with a single tax rate.
2. Table 10 also shows that Tax Structure *B*, which provides a separate rate schedule for single-unit and combination trucks, is effective in eliminating cross-subsidization between the two truck types. However, an inspection of Tables 11 and 12 reveals that both Tax Structures *A* and *B* are highly inequitable among vehicle classes within a vehicle type and among vehicle weight groups within a vehicle class.
3. The revenue/cost ratio results in Tables 11 and 12 indicate that Tax Structure *C* is successful in achieving equity for both vehicle types and all vehicle classes and weight groups considered in the analysis. This equity suggests that, by adopting a vehicle axle-configuration classification such as the one used in Tax Structure *C*, a highly equitable tax schedule scheme could be attained.
4. The rate schedules in Table 9 indicate that considerable differences in tax rates for some vehicle weight groups may be obtained by using different objective functions in linear programming formulations. With regard to equity, the revenue/cost ratio values in Tables 10, 11, and 12 do not show significant differences. These results, however, tend to suggest that a slightly more equitable scheme may be achieved by using Objective Function II of Equation 7 in which the maximum Z_{ij} value is minimized. In the case of Tax Structure *C*, which achieves perfect equity for all weight groups, both objective functions in Equations 6 and 7 give the same results.

CONCLUSION

In this paper, the theory of weight-distance taxation demonstrates that a linear programming technique can be used to analyze the equity of a weight-distance tax structure. Linear programming is also shown to be a useful tool for the deign of rate schedules for such a taxation scheme.

An analysis based on the Indiana highway system revealed that a weight-distance tax structure that relies on a single set of rate schedules for all vehicle types is unlikely to produce an equitable taxation scheme. Based on the findings of this study, it is recommended that a registration system based on vehicle axle configuration be adopted. Such a registration system would provide a sound framework for establishing an equitable weight-distance tax structure.

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