A Conceptual Framework for the Development of Performance-Related Materials and Construction Specifications

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In this paper, pavement design and performance concepts that provide a systematic basis for the development of specifications for materials and construction (M&C) factors are identified. It is assumed that the conceptual framework for specifications development includes eight sets of relationships among the process variables and nine sets of inputs or outputs for the relationships. Independent variables are selected that have predictable effects on performance-related output variables. From these independent variables, variables (EPF) appearing explicitly in prediction functions are selected and subdivided into traffic factors, environmental factors, and pavement structure factors. EPF variables can be replaced by surrogate variables (SPF) when M&C control for these secondary variables is easier to provide. Other secondary variables are the control factors (CF), which have predictable effects on EPF or SPF variables. EPF variables related to the M&C process are denoted MCF. In general, a stochastic prediction model consists of a prediction function that may be completely known from mechanistic considerations; that may be partially known except for undetermined constants; or that may be assumed to be a linear combination of linear, curvilinear, and interaction effects (with undetermined constants) among the independent variables. When the effect of error variables is added to the prediction function, the prediction model can be compared to the experimental design, and analyses of variance and covariance and regression analysis can be applied. General forms of prediction equations for stress and distress, stress-load equivalence relationships, traffic prediction relationships, relationships among M&C specification factors, and performance-cost relationships are presented. Pavement design criteria and M&C specification factors are added as the initial conditions for the definition of a pavement design for a given pavement requirement. It is assumed that the M&C specifications will be derived in every case from computational algorithms, though development of these algorithms requires results from future research.

The purpose of this paper is to identify and bring together pavement design and performance concepts that can provide a systematic basis for the development of performance-related specifications for materials and construction (M&C) factors. The concepts form a framework in which the elements are interrelated and support the development of performance-related specifications. Essential inputs and outputs for algorithms for the development of such specifications are described in detail.

Motivation and support for preparation of the paper has come from the author’s participation as a panel member for two NCHRP projects. Incorporation of pavement design and performance concepts as framework elements stems from the author’s involvement in the recent revision of the AASHTO Guide for Design of Pavement Structures (1-3), and in the development of research plans (4) for long-term pavement performance studies in the Strategic Highway Research Program (SHRP).

The scope of the paper is implied by the overview presented in the next section. Subsequent sections contain details for pavement design and performance variables and prediction equations. Criteria and general considerations for specification algorithms are given in the final sections. Specific algorithms and their M&C applications require new research outputs of the types expected from NCHRP and related FHWA and SHRP projects.

OVERVIEW

Performance-related specifications for pavement M&C must be developed in the context of the pavement design-performance process, which has three phases: design, construction, and service. Process activities are grouped under general headings within each phase in the following way:

1. Design phase activities
   1.1 Acquisition of field and laboratory data on local M&C capabilities and alternatives.
   1.2 Selection of design period, distress and performance criteria, reliability factor, and prediction equations or algorithms.
   1.3 Prediction of design period traffic and performance of alternative pavement designs.
   1.4 Specification of levels and tolerances for M&C design and control factors.
   1.5 Economic evaluation of alternative designs and specifications, and selection of optimum design.

2. Construction phase activities
   2.1 Development of M&C control plans, acceptance plans, and payment schedules.
   2.2 Acquisition, preparation, evaluation, and control of construction materials.
   2.3 Construction and control of design levels and tolerances for M&C factors.
   2.4 Implementation of acceptance plans and payment schedules.
3. Service phase activities
3.1 Treatment of pavement by traffic and environmental factors.
3.2 Routine maintenance.
3.3 Monitoring of treatment factors and pavement condition.
3.4 Evaluation of pavement performance and prediction variances.
3.5 Feedback to design phase and preparations for next stage of pavement life cycle.

It is assumed that the framework for specifications development incorporates relationships among the process variables. The eight sets RI-R8 of relationships identified for this purpose are contained within the ovals shown in Figure 1. Input and output variables for the relationships are shown in Boxes A-I. The abbreviations M, E, or A alongside input lines indicate the classes of the methods (mechanistic, empirical, or algebraic, respectively) that may be used to establish the relationships, as indicated in the figure legend.

Virtually all the relationships among the process variables are stochastic and therefore have statistical uncertainties associated with those of the inputs and outputs. The magnitudes of the uncertainties, generally expressed as error variances or standard deviations, provide objective bases for determining the relative importance of various M&C factors and for deriving realistic tolerance limits for the control of each M&C factor.

Box A contains the distress prediction factors, the primary independent variables of the process, which fall in three categories. Category A1 variables are pavement structure factors that describe the physical nature of the pavement materials, pavement layers, and construction procedures. Category A2 variables describe environmental conditions such as moisture and temperature regimes, roadbed conditions such as soil strength and swell propensity, and roadside conditions such as shoulder design or drainage conditions. Category A3 variables include traffic factors such as axle load distribution and frequency, and annual changes for any of the traffic factors.

Box A contains those independent variables that have a predictable effect on one or more of the performance-related output variables in Boxes B, C, and F. For the purposes of this paper, an independent variable has a predictable effect on a dependent variable if and only if empirical evidence has demonstrated that sufficiently large changes in the independent variable produce statistically significant changes in the dependent variable. The necessary evidence can be supplied by a designed experiment in which (a) the prediction factor is controlled at two or more levels that span its range of interest, (b) all other prediction factors are controlled at fixed levels, and (c) all remaining independent variables operate at chance levels. If the experimental results show that the effect of a given factor on one or more performance-related dependent variables is statistically significant, then the effect is predictable and the factor is performance-related.

This definition of a performance-related prediction factor implies that a relationship exists between the factor and one or more of the variables in Boxes B, C, and F, and that the relationship shows how much change to expect in the dependent variables for a given change in the prediction factor. Mathematically, this dependence implies that the dependent variable is an algebraic function of the prediction factor and that the partial derivative of the function with respect to the prediction factor is not identically zero over the factor's entire range.

The major dependent variables of the process are shown in Boxes B, C, and F, which are outputs of Relationships RI,
R2, R4, and R6, respectively. Stress indicators in Box B represent pavement responses to single applications of particular stress levels created by some combination of structural, environmental, and loading factors. Stress indicators are generally functions of strains or deflections.

Distress indicators in Box C represent various kinds of pavement responses that exist after repeated applications of one or more stress levels. Distress indicators pertain to singular modes of distress such as cracking, rutting, or faulting, or composite modes of distress such as roughness or serviceability loss.

Box F contains performance indicators for the amount of acceptable service that the pavement provides before any particular life-cycle phase is terminated. Performance indicators may be expressed in terms of the length of time or the number of load applications carried before one or more distress indicators reach an unacceptable (terminal) level. Examples are the number of years before thermal cracking reaches an unacceptable level or the total number of equivalent axle loads carried before the present serviceability index (PSI) reaches a given value. Thus, the performance indicators in Box F are functions of one or more distress indicators in Box C. Because each distress indicator in Box C is a function of one or more stress indicators in Box B, all three sets of dependent variables in Boxes B, C, and F are performance-related. The variables in Boxes B, C, and F therefore represent the set of performance-related output variables of the design-performance process except for costs.

A relationship is not complete unless it also shows how much variation in the dependent variable is not attributable to changes in any prediction factor, that is, how much unpredictable variation or error is to be expected for the dependent variable. Knowledge of both predictable and unpredictable variation is essential to the development of performance-related specifications for M&C factors.

Relationships shown in the middle column of Figure 1 are for the prediction of stress indicators (R1), distress indicators (R2 and R4), and performance indicators (R6). It is assumed (a) that these relationships have engineering credibility and validity for the prediction of variables in Boxes B, C, and F from the prediction factors in Box A, (b) that the prediction factors in each relationship are expressed explicitly, and (c) that each relationship is accompanied by adequate information on the uncertainties of its predictions, that is, the prediction errors.

Relationships R1 for the prediction of stress indicators may be derived empirically from designed experiments, derived theoretically by the incorporation of mechanistic principles within established equations or algorithms, or obtained by some combination of these two approaches. Relationships R1 can only predict the pavement response (e.g., strain) to a given level of a single-stress application, and therefore can provide only an initial step towards performance prediction.

Relationships R2 for prediction of distress indicators are extensions of Relationships R1 for repeated applications of fixed stress levels. Empirical derivations of Relationships R2 necessarily involve full-scale pavement studies wherein all levels of the distress prediction factors remain fixed under repeated load applications. This situation was approximated at the AASHO Road Test, at least for those time periods during which environmental factors remained relatively constant.

Mechanistic-empirical derivations for Relationships R2 begin with Relationships R1 and associated prediction factors, then require field observations of the cumulative effects of repeated stress applications at each of several stress levels. For cases in which Relationships R1 do not alone produce satisfactory distress predictions, an additional step is to include additional factors from Box A that do not appear in Relationships R1. Empirical aspects of derivations of Relationships R2 are essentially calibrations of Relationships R1 to include conditions that cannot be accounted for through mechanistic considerations, including repeated stress applications.

Relationships R2 provide a necessary and sufficient basis for the algebraic derivation of stress-load equivalence relationships (R3). For a given distress indicator, these relationships show how many stress applications at any particular level produce the same amount of distress as an equivalent number of standard stress applications. Ratios of the two applications numbers produce equivalence factors (in Box D) that can be evaluated for particular combinations of the stress prediction factors, that is, structural, environmental, and traffic factors. For fixed levels of the structural and environmental factors, the equivalence ratios reduce to load equivalence factors.

Extensive tables of load equivalence factors derived from AASHO Road Test relationships are given in the revised AASHTO Design Guide (1). Relationships R2 behind these factors are for the prediction of PSI loss from structural and load factors. Standard stress levels for these factors are for 18,000-lb equivalent single-axle loads (ESALs) at varying levels of surfacing thickness for rigid pavements, or for varying structural numbers for flexible pavements. All are specific to the AASHO Road Test environment over a 2-year period of traffic operations.

Relationships R4 are for the prediction of distress indicators when the pavement receives an accumulation of mixed-stress applications, as in normal highway experience. These relationships may be derived empirically through statistically designed field studies of existing pavements, or algebraically through the supplementation of Relationships R2 with load equivalence factors that convert all load applications to standard loading conditions. An obvious problem with the second approach is that the resulting Relationships R4 are strictly valid only for those distress indicators for which the equivalence factors have been derived.

Relationships R6 for performance prediction may be derived algebraically from Relationships R4 or empirically from designed field studies of the primary dependent variables of which are performance indicators (in Box F). The difference between Relationships R4 and R6 is that the latter incorporate design criteria from Box E. Three types of design criteria are shown. Category E1 criteria include designer-selected stress and distress indicators, prediction equations, and levels of terminal distress. Criteria E2 include a designer-selected reliability level, an assumed process standard deviation, and a
resultant reliability factor. The performance equation of Relationships \( R6 \) must also provide Criteria \( E3 \) for a designer-selected design period and for a design period traffic prediction produced by Relationships \( R5 \) among traffic factors in Box \( A3 \). The product of the traffic prediction and the reliability factor produces a design requirement, namely, the predicted number of equivalent axle load applications that the pavement will receive during its performance period.

After years of highway research, some of the Relationships \( R1-R6 \) have been determined, but many of the reported relationships are tenuous or incomplete, and many potential prediction factor effects remain to be studied. Because these relationships are basic to the systematic development of performance-related specifications, near-term efforts should be made (a) to assemble and evaluate all reported Relationships \( R1-R6 \), and (b) to design and perform new studies that may be needed to fill important gaps in the available relationships, particularly with respect to their prediction variances. Many existing gaps may become filled through prospective SHRP studies.

The steps in M&C specification development are as follows:

1. Identification of all independent variables that appear explicitly in one of the Relationships \( R1, R2, R4, \) or \( R6 \) as prediction factors for stress, distress, or performance. All such factors belong in Box \( A \) by definition.
2. Identification of all independent variables that are candidates for M&C specifications, that is, that are likely to require control in the course of materials testing and pavement construction. Each such M&C specifications factor belongs to one or another of the three categories in Box \( G \).

   Category \( G1 \) contains all explicit prediction factors \( EPF \) in Box \( A \) that are also candidates for M&C control. Examples include layer thicknesses.

   Category \( G2 \) contains all surrogate prediction factors \( SPF \) that can be used for the (indirect) control of one or more EPF. Surrogate factors are generally needed to control those EPF whose evaluation is more difficult and costly than can be afforded. Examples include the use of California bearing ratio (CBR) as a surrogate for roadbed modulus, or the use of compressive strength as a surrogate for portland cement concrete (PCC) flexural strength. Box \( G2 \) should contain all surrogates the credibility and validity of which have been established through experience. (Some EPF require no surrogates; several competing SPF exist for some EPF; no surrogates exist for some EPF; and some SPF are surrogates for more than one EPF.)

   Category \( G3 \) contains auxiliary control factors \( CF \) that are neither EPF nor SPF. The roles of these factors are (a) to enhance the control of one or more EPF or SPF, or (b) to enhance one or more of the construction procedures. CF having Role (a) include slump as a partial control for PCC strength, asphalt content for enhancement of asphalt concrete (AC) stiffness control, and roadbed density for indirect control of subgrade strength. An example of Role (b) is moisture control for enhancement of roadbed compaction. Box \( G3 \) should contain all CF that do not appear in Box \( G1 \) or \( G2 \), but that have been found useful in the past experience of pavement designers, materials experts, and road builders.

3. Determination of which SPF and CF are truly performance-related and to what degree. More specifically, this step is the determination of the degree to which each candidate SPF is related to one or more EPF, and the degree to which each candidate CF is related to one or more EPF or SPF. All such secondary performance relationships of the conceptual framework are shown collectively as Relationships \( R7 \) in Figure 1.

Relationships \( R7 \) are prediction equations for predicting EPF from one or more of the SPF or CF, and for predicting SPF from one or more CF. All Relationships \( R7 \) are derived empirically from statistically designed experiments that make it possible to estimate the relative predictive value of each SPF and CF, and to quantify the prediction uncertainty associated with each relationship. As for the primary relationships, many Relationships \( R7 \) have been developed and reported in the highway research literature.

Gaps and deficiencies existing in Relationships \( R7 \) should be ameliorated in the future through design and implementation of new laboratory or field studies as needed.

When the set of secondary relationships is complete, it will be possible to evaluate the prediction utility of all SPF relative to the EPF, and of all CF relative to both the EPF and the SPF. The SPF and CF that have little or no predictive value cannot be said to be performance-related. Although some of these factors might be used for M&C control, their specifications cannot be based on statistical reasoning and are not performance-related in the context of this paper.

4. The last oval (\( R8 \)) in Figure 1 contains algorithms for the derivation of M&C specifications for any specific design requirement. It is assumed that the output specifications in Box \( I \) include (a) target or design levels for relevant M&C factors in Box \( G \), (b) tolerance or acceptance levels for each M&C factor whose target level has been specified, (c) acceptance plans for certain M&C factors, and (d) payment schedules associated with as-constructed levels of the acceptance plan variables.

Inputs to the algorithms include (a) the (primary) distress and performance relationships (\( R4 \) and \( R6 \)), including design criteria from Box \( E \), (b) all M&C factors in Box \( G \) that are to be controlled, and (c) the (secondary) relationships (\( R7 \)) that relate SPF and CF to the EPF that appear in the primary relationships.

For a given design requirement, for example, a set of design values for performance indicators, the relationships input to the algorithms (\( R8 \)) provide a necessary and sufficient basis for the deterministic specification of target design levels for all M&C factors. In general, however, the relationships are satisfied by a large number of alternative sets of M&C factor levels, although some alternative levels can be excluded for M&C factors if the algorithms (\( R8 \)) include criteria for the selection of optimal alternatives.

It is assumed that each input relationship (\( R4, R6, \) or \( R7 \)) includes a measure (e.g., error variance) of the prediction precision of the relationship. Thus, one criterion for the selection of design level alternatives can be the maximization...
of performance prediction precision through the selection of SPF and CF the combined relationships of which have minimum error variance for attainment of the design requirement.

A perhaps overriding criterion for selection of optimal combinations of factor levels is the minimization of overall costs associated with materials provision, construction procedures including quality control, and routine maintenance to be provided during the performance period of the pavement. It is assumed that all necessary unit costs are contained in the Box H inputs to the specifications algorithms (R8). Thus, the algorithms can produce relative costs for alternative sets of factor levels that satisfy the primary and secondary performance relationships, and can therefore reduce all possible alternatives to those having minimal costs. It follows that the algorithms must incorporate a third criterion, precision, for weighing costs against benefits, to ensure that factor level specifications that produce minimal costs are also the combinations that lead to maximum precision.

The objective specification of tolerance or acceptance levels for M&C factors is more complex than the specification of target design levels for the factors, and should be based on the stochastic properties of the R4, R6, and R7 prediction equations. One approach to the derivation of tolerance levels is to assume for each factor the coefficients of variation that are expected in the M&C process under different degrees of control that range from very loose to very tight. For example, the coefficient of variation for subgrade strength in roadbed construction might be 100 percent if little or no control is exercised, but as low as 10 percent under very tight control. Box H contains relative unit costs for the various levels of control of all M&C factors.

From the variance components of the prediction equations to be discussed later, it is possible to estimate the amount of performance variance that is associated with different levels of the factor variances or coefficients of variation. When combined with control costs, these estimates provide a basis for assessing the benefits and costs of any particular set of tolerance specifications, where benefits are relative to the prediction precision implied by particular levels of control. Using this approach and the specifications algorithms (R8), many alternative sets of tolerance levels can be examined and those that are optimal with respect to objective criteria can be determined.

INTRODUCTION AND CLASSIFICATION OF PERFORMANCE-RELATED VARIABLES

The process variables may be grouped in the following classes:

1. Primary dependent variables
   1.1 Stress indicators \( r \) (strains, deflections, etc.)
   1.2 Distress indicators \( d \)
      a. Singular distress (cracking, rutting, etc.)
      b. Composite distress (roughness, PSI loss, etc.)
   1.3 Performance indicators
      a. Fixed-stress applications \( N_p \) to \( d = d_i \)
      b. Mixed-stress applications \( W_p \) to \( d = d_i \)
      c. Performance period \( Y_T \) for \( d < d_i \)
   1.4 Cost indicators
      a. Cost components \( c \) for materials, construction, and so forth
      b. Life cycle costs \( C \) for analysis period \( Y_A \)

2. Primary stress-distress prediction factors
   2.1 Traffic factors \( T_F \)
      a. Load frequencies, distributions, growth rates, and so forth
      b. Load equivalence factors and ESAL accumulations
   2.2 Environmental factors \( E_F \)
      a. Climate
      b. Roadbed and roadside
   2.3 Pavement structure factors \( S_F \)
      a. Materials and layer properties
      b. Construction and maintenance procedures

3. Secondary stress-distress prediction factors
   3.1 M&C surrogate factors for primary prediction factors
   3.2 M&C control factors

4. Pavement design criteria
   4.1 Distress-performance criteria
      a. Distress indicators and prediction functions
      b. Terminal distress levels and performance indicators
   4.2 Reliability criteria
      a. Reliability level \( R \)
      b. Process standard deviation \( S_0 \)
      c. Reliability factor \( F_R \)
   4.3 Time and applications criteria
      a. Design period \( Y_T \)
      b. Design applications \( W_T \) = \( \dot{w}_T \times F_R \)
      c. Design period traffic \( \dot{w}_T \)

5. Uncontrolled independent variables \( z \)
   5.1 Uncontrolled deviations from specified levels
      a. Stress-distress prediction factor deviations
      b. Design criteria deviations
   5.2 All remaining uncontrolled independent variables

In the preceding expressions, the subscript \( i \) refers to a terminal condition, \( i \) to a level of fixed stress, and \( T \) to serviceable lifetime. Class 1 contains all the primary dependent variables; classes 2-5 contain the independent variables. The variables are discussed in the subsections that follow.
Primary Dependent Variables

The primary dependent variables to be observed or evaluated are either stress indicators, distress indicators, performance indicators, or cost indicators. (Indicators are results of specific measurement procedures including use of equipment for evaluating generic variables.) For example, a specific CBR test might be used as an indicator of subgrade strength, or the Thornthwaite index might be used as an indicator of climatic moisture. There are generally several alternative indicators for variables, even for specific variables such as pavement deflection under load.

Pavement stress is induced by some combination of loading and environmental conditions. Stress indicators are generally strains and deflections or functions thereof. Stress indicators are often called pavement response variables because they indicate the response of the pavement to a single stress condition or load application.

Distress indicators represent undesirable changes in the physical condition of the pavement over time, and therefore result from some combination of repeated environmental stresses, repeated load applications, or deterioration with age. Distress indicators have been developed for each singular mode of distress such as fatigue cracking, thermal cracking, rutting, faulting, joint deterioration, scaling, and raveling. Singular distress indicators provide an essential basis for the diagnosis and repair of structural conditions.

Composite distress indicators, such as longitudinal roughness or PSI loss, are essential to evaluation of the functional condition of the pavement, that is, the degree to which the pavement has begun to fail to provide a smooth and safe facility for its users. Composite distress indicators must be used for the comparison of pavements that have different singular distress modes, and for the aggregation of all distress indicators within a given pavement type. In principle, a composite distress indicator is a weighted index of all singular distress indicators that bear on the functional condition of the pavement.

It is assumed that pavement performance is defined by the amount of service to road users that the pavement provides while in an acceptable functional condition. Amount of service, in turn, is a function of the traffic carried; being in an acceptable functional condition implies that a criterion distress indicator has not reached an unacceptable terminal level in the eyes of road users. A specific performance indicator is the number of ESAL applications $W$, that the pavement has carried until its PSI loss has become $p_t - p_r$, where $p_t$ and $p_r$ are initial and terminal levels, respectively, of the PSI of the pavement.

For reasons to be discussed in a later section, a distinction is made between accumulated load applications $N$, all of which occur at the same stress level, and the number of ESAL applications $W$ that accumulate under mixed stress and load conditions, as in normal highway operations. If the rate of load accumulation is known and is not zero, then either $N$ or $W$ can be converted to the total years $Y$, of acceptable service during which $d$ remains less than $d_t$. Thus, $Y$, the performance period of the pavement, is also a primary dependent variable.

In the foregoing discussion, pavement performance is quantified in terms of functional throughput. Thus, a high-volume road that requires rehabilitation (because of use) after 5 years may have greater performance than a low-volume road that requires rehabilitation (because of age) after 20 years. Alternative performance indicators can be defined in terms of singular distress indicators $d$ and their terminal levels $d_t$, that represent needs for particular types of pavement rehabilitation. None of the distress indicators so far discussed takes into account the fact that distress generally does not increase at a constant rate over the performance period. If desired, shape parameters of the distress curve over time and applications could be used to weight the linear performance indicators that have just been defined.

Cost indicators for cost components are associated with the acquisition and processing of pavement materials, pavement construction (including testing and quality control), routine maintenance applied over the performance period, and rehabilitation steps needed before the M&C process is iterated for the next phase of the pavement life cycle. Aggregate costs for each phase lead to life cycle cost indicators, perhaps on an annual basis over an analysis period $Y_A$ of 30 years or more. Because performance indicators are measures of the benefits provided by specific pavement designs, incremental benefits are defined by additional ESAL throughputs at acceptable service levels. Combined with the performance indicators, the cost indicators provide a basis for assessment of benefits relative to costs for particular sets of pavement design specifications.

Prediction Factors for Stress and Distress

Prediction factors for stress and distress belong to primary and secondary classes. Primary factors (EPF) appear explicitly in prediction functions for stress and distress that are recognized by the pavement design community, and that can be represented by Relationships $R1$, $R2$, $R4$, and $R6$ in Figure 1.

Secondary stress and distress prediction factors for the design-performance process include accepted surrogates SPF and those CF that have demonstrable relationships with the EPF. The EPF are performance-related by definition, but the secondary factors are only indirectly related to distress or performance.

Primary Factors

The EPF have three subclasses. Traffic factors (TF) describe individual loadings that produce pavement stress and accumulated loadings that are associated with pavement distress. Examples are individual axle loads, rate of loading, load placement, average daily traffic, axle load distribution of daily traffic, traffic growth rate, axle load equivalence factors, and years of traffic. Cumulative load applications derived from the TF include the number of axle loadings $N$, at stress levels $i = 1, 2, \ldots$ and the number of ESAL applications $W$ that have accumulated at any particular time.

Environmental factors (EF) include various indicators of (a) climatic moisture, and temperature; (b) roadbed properties
of strength, roadbed moisture, and temperature conditions, and propensity for swell and frost-heave; and (c) roadside conditions such as drainage and shoulder support.

Pavement structure factors (SF) generally include (a) types and physical properties of construction materials, (b) layer properties such as thickness, strength, and load transfer capabilities, (c) construction procedures such as compaction or surfacing reinforcement, and (d) routine maintenance procedures.

Secondary Factors

Secondary prediction factors for stress and distress include SPF (Class 3.1) that do not appear among the EPF but that may be substituted for the EPF by means of known relationships. For example, if one EPF is the subgrade resilient modulus and if a particular CBR indicator is predictably related to the modulus, then CBR values can be used to determine modulus values to within a known degree of precision. Thus, if CBR is more feasible to control during the M&C process than subgrade modulus, then the substitution of variables can be made with a known sacrifice in performance prediction. The practical impact of the SPF is that to provide M&C control for the SPF may be more feasible than for their EPF counterparts; however, the effectiveness of the substitutions depends on the particular forms of relationships between the EPF and SPF.

Secondary prediction factors include the CF (Class 3.2) that are neither EPF nor SPF, but have predictable effects on one or more of the EPF or SPF. Thus, the M&C control of any CF induces some degree of control on the primary EPF to which the CF is related. It follows that such CF are performance-related to the degree that their levels influence the EPF levels, and hence the stress, distress, and performance indicator outputs. An example of a multivariate relationship is that between PCC flexural strength (a primary prediction factor) and both slump and cement content of the PCC mix (control factors).

Pavement Design Criteria

Class 4 of the performance-related variables contains pavement design criteria that are assumed or specified by the pavement designer. The three subclasses of design criteria are distress criteria (Class 3.1), reliability criteria (Class 3.2), and design period criteria (Class 3.3). No variable in this class is an M&C factor, but all have indirect effects on M&C specifications.

Specification of distress and performance criteria (Class 4.1) implies (a) selection of one or more singular or composite distress indicators, (b) selection of relationships between indicators and prediction factors, and (c) specification of a terminal distress level for each indicator.

In Class 4.2, specification of reliability criteria involves (a) selection of a reliability level \( R \), (b) assumption of a process standard deviation \( S \), and (c) calculation of a reliability factor \( F_R \). Further definitions for these three independent variables are given in a later section. (The reliability level \( R \) of the design-performance process is defined as the probability that the actual performance period of the pavement equals or exceeds the specified design period.)

Design period criteria (Class 4.3) include the design period \( Y_T \), the predicted design period traffic \( w_T \) (in ESALs), and the predicted design applications \( W_T \). Length of the design period \( Y_T \) is usually dictated by pavement management system (PMS) considerations for the various phases of the pavement life cycle. Typical design periods are in multiples of 5 years.

The predicted design period traffic \( w_T \) is derived from an equation or algorithm using estimated traffic factors. Thus, \( w_T \) is both a dependent variable relative to traffic factors and an independent variable relative to pavement design.

Design applications \( W_T \) are derived by multiplying the reliability factor \( F_R \) by the predicted design period traffic \( w_T \). The resulting value is a design requirement for the actual outcome of the performance indicator \( W_T \). If \( F_R = 1 \), design applications are equal to the predicted design period traffic, and the pavement design has a reliability level of \( R = 50 \) percent.

A number of the variables discussed thus far are shown in Figure 2. The vertical scale is for any particular distress variable \( d \) whose terminal distress level \( d \) is represented by the horizontal dashed line across the top of the figure. Two horizontal scales are shown, one for years \( Y \) and one for equivalent load applications \( W \) that have accumulated at any point in time.

The distress history for \( d \) is shown as an irregular curve that begins at zero when \( Y \) and \( W \) are zero. When \( d \) reaches \( d_T \), the performance period has ended, and the corresponding performance indicators have values \( Y_T \) and \( W_T \).

Finally, Figure 2 shows design period years \( Y_T \) and the corresponding design period traffic \( w_T \). The performance period \( Y_T \) is expected to be greater than the design period \( Y_T \) if the designer specifies a reliability level \( R \) that is greater than 50 percent. Unless the annual rate of ESAL accumulation is constant, the \( Y \) and \( W \) scales of Figure 2 are distorted relative to one another.

Uncontrolled Independent Variables

The final class (Class 5) of performance-related variables includes all independent variables that are specified neither as stress or distress prediction factors nor as pavement design criteria. In statistically designed experiments, the collective or net effect of these variables is known as experimental error. In pavement design applications, the error effects, at least in part, are responsible for prediction errors. Two subclasses have been identified. Class 5.1 represents error variances that arise because the prediction factors have uncontrolled deviations from their design levels. Class 5.2 represents the net effects on distress and performance of all remaining uncontrolled and unidentified independent variables that operate during the course of the design-performance process.

To some degree, the EPF variances (Class 5.1) may be partially controlled through the tolerance levels for M&C
factors, but any remaining error effects must be accepted as normal aspects of the overall prediction process. Although the individual effects of uncontrolled variables cannot be identified, their collective effect can be estimated through statistically designed studies. As will be discussed later, the magnitude of uncontrolled variation is an important element in the development of tolerance or acceptance limits for M&G factors.

**PRINCIPLES OF PREDICTION EQUATION DEVELOPMENT**

Because stress, distress, and performance prediction equations are important elements of the M&G specifications framework, the methods used to derive the equations are of considerable interest. In this section, many basic principles of experimental design and analysis that bear on the derivation of prediction equations, particularly with respect to the assessment of their statistical properties, are brought together.

The discussion in this section is general; particular types of performance-related prediction equations are discussed in the next section.

**Prediction Models**

Derivation of prediction equations begins with the specification of a mathematical model for an assumed relationship between a predicted dependent variable $V$ and a specific set of independent predictor variables $\{U\} = U_1, U_2, \ldots$. Let $v$ be an indicator for values of $V$ and let $\{u\} = u_1, u_2, \ldots$ be corresponding indicators for values of the predictor variables. The prediction function may be written as $f(u, c)$, where $\{c\} = c_1, c_2, \ldots$ is a set of constants (coefficients) that modify the independent variables $\{u\}$. The predicted value of $v$ is denoted $\hat{v}$, so that

$$\hat{v} = f(u, c) \quad (1)$$

It is understood that $v$ and $\hat{v}$ may be some transformation (e.g., a logarithm or power) of $V$, and that the set $\{u\}$ may include various transformations of the corresponding variables in set $\{U\}$. The functional or computational form of $f$ is completely specified, but values for some of the constants in $\{c\}$ may need to be derived from experimental data.

For a fixed set of predictor variables $\{u\}$, the functional form must show how $\hat{v}$ changes when $u_1, u_2, \ldots$ are changed...
in value. Mathematically, the change in \( \hat{v} \) due to a change in one of the \( u_i \), associated with the partial derivative of \( f \) with respect to \( u_i \), is the main effect of \( u_i \) on \( \hat{v} \). If the partial derivative is constant, the \( u_i \) effect is linear and does not interact with the effect of any other \( u_j \) (\( j \neq i \)). If the partial derivative is a (nonconstant) function of \( u_i \) alone, then \( u_i \) has a curvilinear noninteracting effect on \( \hat{v} \). If the partial derivative is also a function of some \( u_j \) (\( j \neq i \)), then \( u_i \) and \( u_j \) have an interacting effect on \( \hat{v} \). In this case, the main effect of \( u_i \) is defined by setting \( u_j = \bar{u}_j \), the mean value of \( u_j \).

These examples can obviously be extended to higher-order interactions among three or more factors and various types of nonlinear effects. If all relevant \( u \) and the functional forms of all \( u \) effects are known (e.g., from principles of pavement mechanics), it is possible to assume the correct mathematical form for \( f \). In such cases, the prediction function is said to be mechanistic. In general, however, a mechanistic derivation does not produce specific values for all constants in \( f \). Because undetermined constants must be estimated from experimental data, the ultimate function \( f \) is said to be mechanistic-empirical.

When little is known about the mathematical form of \( [u] \) effects on \( \hat{v} \), it is conventional to express \( f (u, c) \) as a linear combination of linear effects, curvilinear effects, and interacting effects of the independent variables. In this formulation, one constant in \( c \) is allocated to each effect, and the model is said to be strictly empirical.

Differences between observed values \( v \) and predicted values \( \hat{v} \) of \( V \) are prediction errors that represent the net effect on \( v \) of all independent variables that are not included in the predictor set \( [u] \). Because many of these variables are uncontrolled or unidentified (Class 5), their net effect on a particular observation \( v \) is unpredictable. If the set of all such error variables is denoted by \( z \), and their net effect on \( v \) is denoted by \( e(z) \), then a statistical model for \( v \) is

\[
\hat{v} = e(z) = f(u, c) + e(z)
\]

Thus each observation in the model of Equation 2 is the sum of the predictable net effect of \( [u] \) and the unpredictable net effect of \( z \). In later sections, it will be shown that the net error effects can be decomposed into components of assignable variation.

Although individual errors are unpredictable, the statistical distribution of \( e(z) \) for a large number of independent predictions can be characterized by the mean value of the distribution, its variance or standard deviation, and perhaps other distribution parameters. If the error distribution has mean value zero, the prediction model of Equation 1 is unbiased or valid for the conditions that produced the error distribution. The error variance \( S^2_z \) or standard deviation \( S_z \) is a measure of the precision of the prediction model of Equation 1.

If the error distribution is approximately normal, it is completely defined by its mean value and variance. Because these parameters must be estimated from experimental data, every statistical model of Equation 2 must be evaluated empirically. It is not sufficient to derive only the prediction function of Equation 1; the error distribution of \( e(z) \) in Equation 2 must also be derived so that both the validity and the precision of the prediction function can be assessed.

Finally, the prediction function of Equation 1 is said to be a linear model if it can be expressed as a linear combination of coefficients \( c \); otherwise the model is said to be nonlinear in the coefficients. The error function in Equation 2 represents additive errors. For some models, the error function may be multiplicative, generally by assumption. In this case, taking logarithms of both sides of Equation 2 produces an additive logarithmic function of the error variables \( z \).

### Experimental Designs

Over the past 50 years, a substantial body of knowledge and literature has been developed for the design and analysis of experiments (5, 6). Only the most relevant design concepts are presented herein; implementation details are omitted.

An experimental design for the development of a prediction equation is essentially a well-defined plan for assessing the effects of sets of independent variables on one or more dependent variables. The plan begins with specifications for experimental units and treatments that are applied to the units. For pavement studies, the experimental units are either pavement test sections or test specimens of pavement components. Test sections may be either portions of existing highways or specially constructed sections within a highway project or test track. Test specimens may be either samples taken from test sections or specially assembled laboratory specimens. If the experimental units are test sections, the experiment is a field study that takes place either on existing highways or on a test track such as was constructed for the AASHO Road Test at Ottawa, Illinois, during 1955-1961. If the units are test specimens, the experiment is a laboratory study because many if not all measurements will be made in a laboratory setting.

For a particular prediction equation, the given quantities for the experimental design include (a) definitions for dependent variables \( P \) and their indicators \( v \), (b) specifications for those independent variables and corresponding indicators that determine the physical makeup of the experimental units, (c) specifications for other independent variables and indicators that determine the traffic loadings and environmental treatments to which the units are subjected, (d) specification of the prediction factor effects that must be assessed, and (e) specification of the inference space within which the derived prediction equation is to be valid.

For experimental design purposes, independent variables fall into four categories: (a) multilevel factors whose values are to be varied at two or more controlled levels; (b) single-level factors that have only one fixed level throughout the experiment; (c) covariables whose values are not controlled by design but are measured in the course of the study; and (d) error variables \( z \), whose values are neither controlled nor measured, but whose net effects are assessed through experimental design and statistical analysis.

The inference space defines the time-space region within which all uncontrolled or unmeasured independent variables...
in \( z \) are permitted to affect the observed dependent variables \( v \). In essence, the error effects are those that would be observed in \( v \) if all predictor variables \( z \) were held at fixed levels throughout the experiment. Alternatively, the inference space can be viewed as the region that contains a hypothetical population of experimental units and treatments. In this view, the study units and treatments represent samples of the population. The inference space or population provides limits for the validity of the prediction equation to be derived, but only if the space has been properly classified and sampled in the experimental design.

Classification of the inference space is generally along the lines of physical categories within which different subsets of \( z \) are expected to exert effects on \( v \). For example, in field studies the inference space might be the entire United States. In this case, the inference space classes might be test sites within a highway project, projects within areas (e.g., states), and areas within regions (e.g., climatic zones). At each successive level in this hierarchy, new subsets of error effects are introduced. All subsets must be properly sampled if the prediction equation is to be valid for the entire country. For a laboratory study wherein specimens are manufactured and tested, the inference space might be restricted to the results at a single laboratory and to only the materials and testing equipment available at that laboratory. A much larger inference space would cover several sources for each type of material and several laboratories and testing procedures. An experimental design can be developed for any inference space, but only if the space has been properly specified in advance. Experimental costs may be greater for extensive inference spaces than for more confined spaces, but the additional costs must be weighed against the advantages of a wider range of validity for the results.

After all the foregoing given conditions are specified, there are two general requirements for the experimental design. The first requirement is to provide for proper observation of prediction factor effects, the second to provide for proper sampling of error effects \( e(z) \) throughout the inference space. The first requirement can be met through the use of factorial designs, the second through the use of appropriate randomization and replication procedures.

To develop a factorial design, at least two levels must be specified to cover the range of interest for every prediction factor. Three or more levels may be necessary for attribute factors that have more than two classes or for variable factors whose effects are likely to be curvilinear. If there are \( k \) prediction factors, each at two levels, then the total number of factor-level combinations is \( 2^k \). Exactly one of these combinations is used for the makeup and treatment of each experimental unit. If two or more units are used for any combination, the combination is said to be replicated. If every possible combination is used, the result is a complete factorial design. This design makes it possible to observe every possible main effect and interaction effect of the prediction factors, and more important, to separate analytically each effect from all other effects. Moreover, every observation is used to calculate every effect, a feature known as hidden replication. If the total number of factor-level combinations is large (e.g., 100), a partial or fractional factorial design may be used, but it then becomes impossible to separate certain effects from one another. In such cases, the fractional design generally produces certain confoundings of higher-order interaction effects with lower-order interaction effects.

If only the factor-level combinations that are of most interest to the experimenter are used, it is likely that a number of predictor factors will be intercorrelated, and complete separation of their effects becomes impossible. In the worst of such cases, the experimental results become useless for assessment of individual prediction factor effects.

After the factorial combinations have been specified, the next phase of experimental design is to specify the procedures that will be used to place and replicate the experimental units and treatments within the specified inference space. The first step is to classify the inference space into categories that represent various types or levels of uncontrolled variables \( z \) and their error effects \( e(z) \). As mentioned, for a field study whose inference space covers a single climatic zone, these categories might be (a) areas within the zone, (b) projects within each area, and (c) test sites within each project.

It is assumed that each experimental unit or test section occurs at a single test site, that \( n \) different factor-level combinations appear in the factorial design, and that one unit is required for each combination. Proper placement of experimental units within the inference space requires that every possible test site in the space (a climatic zone) has an equal chance of becoming the site (a test section) for a given factor-level combination. The assignment of factor-level combinations to test sites must be accomplished by randomization procedures that permit no subjectivity in test site selection. In practice, every possible test site in the space is identified by a numerical code, and code numbers are drawn by lot to determine the location of each factor-level combination in a test section. This procedure is called complete randomization of the experimental units; it provides assurance that (a) all units are subject to all error variables \( z \), (b) that the prediction factor effects are unbiased relative to error effects, and that (c) unbiased estimates can be calculated for the error effects.

The randomization procedure is equivalent to random sampling of the test site population, with one sample per factor-level combination. To observe the error effects explicitly, it is necessary to replicate at least some factor-level combinations within the inference space, that is, to draw more than one sample for some factor-level combinations. The replication must cover all classes of the inference space, because otherwise not all error effects are observed. For example, if all replications were made at the level of the test site, the replicate differences would not reflect error effects between projects, that is, effects associated with different contractors and concomitant variations in materials and construction. For the example, if each of the \( n \) factor-level combinations is to be once replicated, \( 2n \) sites must be randomly selected throughout the inference space.

Details for determining the number of replicate units and their allocation to factor-level combinations will not be discussed, but the general requirement is that the replications must provide a sufficient basis for the estimation of error variance, and therefore for the assessment of prediction equation precision.
Proper randomization and replication procedures are essential to the validity of all inferences associated with the prediction equation and its applicability to the inference space. Specification and implementation of these procedures is at least as important as specifications for the factorial design.

For some studies, restricted randomization procedures may be used as alternatives to complete randomization. In the foregoing example, it might be appropriate to select random areas within the climatic zone, then to select random projects within each area, and finally to select random test sites within each project. In this case, however, the experiment must be replicated at the area level, so that all error effects can be observed. A comprehensive discussion of randomization and replication procedures for many types of experimental designs is given by Anderson and McLean (5).

Finally, the experimental design may specify that certain prediction factors be excluded from the factorial design but evaluated for each experimental unit. Such factors are called covariables whose values are distributed throughout the inference space by the randomization procedures. Thus, in models of Equations 1 and 2, the prediction function \( f(u, c) \) contains all factors that are varied in the factorial design plus any covariables that have been designated.

**Data Analysis**

After the experimental design has been implemented, the resulting data provide corresponding values for all dependent variables \( v \) and for all independent variables \( u \) that were set forth in the experimental design. Computer software for several types of statistical analyses are available for derivation of the required prediction equations. Four such methods are as follows:

1. Analysis of variance (ANOVA),
2. Analysis of covariance,
3. Linear regression analysis, and

These analytical methods make it possible to attain the following objectives. As indicated, analyses of variance or covariance are used to attain the first three objectives; the last three objectives are attained through linear or nonlinear regression analysis.

1. Estimates of prediction factor effects (by Method 1 or 2),
2. Estimates of error effects (by Method 1 or 2),
3. Significance assessments of prediction factor effects (by Method 1 or 2),
4. Estimates of model constants (by Method 3 or 4),
5. Precision estimates for individual constants (by Method 3 or 4), and
6. Precision estimates for predictions (by Method 3 or 4).

As discussed earlier, each analysis is relative to a particular mathematical model and corresponding experimental design.

Objective 1 is to quantify the amount of variation in an observation \( v \) that is attributable to each main effect and interacting effect of the predictor variables \( u \).

Objective 2 is to quantify the amount of variation in an observation \( v \) that is attributable to error variables \( z \). If the experimental design involves restricted randomization, then Objective 2 includes the partition of total error variation into subsets that correspond to various levels of error variation (e.g., between projects and between test sites within projects).

Objective 3 is attained through statistical significance tests that compare each EPF effect with appropriate error effects. If any comparison produces a significant effect at a given probability level (e.g., 5 percent), it is inferred that the relevant EPF effect is real, that is, that the effect truly exists within the inference space of the experimental data. The selected significance level represents the magnitude of the controlled chance that the inference is erroneous.

Thus, the results show not only which factor effects are significant, but also how much of the total variation in an observation \( v \) can be attributed to factor effects \( u \) and how much can be attributed to error variables \( z \). Analyses of variance and covariance therefore set the stage for regression analyses by determining just which factors and combinations thereof are statistically significant components of the regression models.

The remaining analytical objectives begin with linear or nonlinear regression analyses that produce estimates for constants (coefficients) that modify the predictor variables (Objective 4).

In regression analyses, prediction error \( e(z) \) is decomposable into lack-of-fit error \( e'(z) \) and pure experimental error \( e''(z) \). Thus,

\[
v = \bar{v} + e(z) = f(u, c) + e'(z) + e''(z) \tag{3}\]

In essence, lack-of-fit variation represents that part of the variation in \( v \) that is neither pure experimental error nor completely explained by the particular regression model \( f(u, c) \) that has been used. If the lack-of-fit variation is not significantly greater than pure error variation, it is generally inferred that no improvements are needed for \( f(u, c) \), because further extensions of the model do not represent significant contributions from the prediction factors. If, on the other hand, the lack-of-fit variation is significant relative to pure error variation, it may or may not be possible to reduce the former through the use of alternative functions for \( f(u, c) \).

Objective 5 is to determine the precision of the estimated constants \( c \) in \( f(u, c) \), generally in terms of a confidence interval for each constant. As will be discussed, chance variations in regression coefficients have direct bearing on tolerance and acceptance limits for M&C factors that relate to corresponding coefficients in the prediction equations.

Objective 6 is to determine confidence intervals for the predictions \( v \) that are produced by the evaluated regression function. The intervals are determined by the confidence level used (e.g., 90 percent) and the variance of the overall prediction errors. If \( f(u, c) \) is a linear combination of PF effects, Objectives 5 and 6 can be attained through use of algebraic formulations that have been developed for regression models (5).
To the fullest possible extent, regression analyses should be preceded by appropriate analyses of variance or covariance, because inferences from the two types of analyses are interdependent and mutually supportive.

If the experimental design is deficient, it may not be possible to perform appropriate analyses of variance and covariance. In such cases, it is most likely that Objectives 4, 5, and 6 cannot be fully attained through regression analyses. As implied earlier, experimental design deficiencies generally arise through incomplete balance of prediction factor level combinations in the factorial design, failure to randomize experimental units properly throughout the inference space, or failure to obtain estimates of experimental error through appropriate replication of units within the inference space.

Because performance-related M&C specifications are generally relative to specific prediction equations for stress, distress, and performance, the specifications can have no more credibility and validity than are provided by the prediction equations upon which they are based. It is therefore essential that the equations be derived from statistically designed studies and from statistical analyses that produce valid results having known precision within a well-defined inference space.

PERFORMANCE-RELATED PREDICTION EQUATIONS

In this section, the relationships (R1-R8) shown in Figure 1 are discussed in detail. Relationships R1, R2, R4, and R6 represent primary equations for the prediction of pavement stress, distress, and performance. Relationships R3 for stress-load equivalence and R5 for design period traffic prediction are considered to be auxiliary to the primary relationships. Relationships R7 among M&C factors represent secondary equations for the prediction of stress, distress, and performance. All cost-benefit and optimization relationships are considered to be components of algorithms (R8) for the derivation of M&C specifications.

Stress and Distress Prediction for Fixed-Stress Levels

In Figure 1, Relationships R1 represent prediction equations for pavement response indicators when the responses are induced by a single application of a fixed-stress level. Prediction factors are either pavement structure factors SF, environmental and roadbed factors EF, or traffic factors TF (that do not include number N of stress repetitions). The general prediction equation for a response indicator may be written as

\[ r = \hat{r} + e_r(z) = f_r(SF, EF, TF', N; c) + e_r(z) \]  

(4)

where \( [c] \) is the set of all constants in \( f_r \), and \( e_r(z) \) is the net effect of all error variables that were operational when Equation 4 was derived from experimental data.

The form and contents of \( f_r \) may be derived either empirically or from some combination of mechanistic principles and empirical results. In the second case, \( f_r \) generally is not a closed mathematical form but is rather a computational algorithm that incorporates various aspects of elastic or viscoelastic layer theories, finite element methodology, or stochastic processes. Empirically derived relationships for \( f_r \) include those reported at the AASHO Road Test (7) for various types of strains and deflections within flexible and rigid test sections.

Prediction equations of Relationships R2 for distress indicators \( d_i \) after \( N_i \) repeated applications of a fixed-stress level may be written

\[ d_i = \hat{d}_i + e_d(z) = f_{d_i}(SF, EF, TF', N_i; c) + e_d(z) \]  

(5)

in which it is understood that all \( SF, EF, \) and \( TF' \) values are at constant levels throughout the \( N_i \) applications. Any particular factor-level combination in \( f_{d_i} \) determines one stress level \( f_r \) and the equation is valid for whatever range of factor-level combinations existed in the experimental design from which Equation 5 was derived. It is assumed that the error effects are independent of the stress levels represented by \( f_{d_i} \).

The form of Equation 5 represents an empirical derivation. In mechanistic-empirical derivations, one or more stress indicators \( r \) are included as distress predictors within \( f_{d_i} \). As shown in Equation 4, certain prediction factors are subsumed by \( r = f_r \). If \( SF', EF' \), and \( TF'' \) represent prediction factors in Equation 5 that remain after those in \( f_r \) of Equation 4 are used, then the mechanistic-empirical distress prediction equation may be written

\[ d_i = \hat{d}_i + e_d(z) = f_{r_i}(SF', EF', TF'', N_i; c) + e_d(z) \]  

(6)

in which it is understood that more than one response indicator may appear in \( f_{r_i} \), and that for any particular pavement section all \( N_i \) applications occur at a fixed set of levels for all factors in \( f_{r_i} \).

Except under controlled laboratory conditions, it is virtually impossible to design and perform experiments for the precise determination of Equations 5 and 6. As in the AASHO Road Test (7), field studies can be designed to control the structural and traffic factors, but to achieve constant stress levels for all applications it would be necessary to have separate test sections for each set of environmental conditions, that is, for several temperature intervals within days, and for several moisture conditions within years. Equations 5 and 6 can therefore be determined only for average stress levels and over limited ranges of variation in environmental and other factors.

Stress-Load Equivalence Relationships

Distress prediction equations for fixed-stress levels are not only difficult and costly to derive, but do not apply directly to normal highway pavements whose stress levels vary from vehicle to vehicle and during daily and seasonal variations in environmental conditions. Thus, the stress applications for an operational pavement are at mixed-stress levels. To apply fixed-stress level relationships to highway conditions, it is necessary either (a) to represent all mixed-stress levels by, for example, an average stress level and associated prediction
equation, or (b) to combine those equations that cover the expected range of stress levels.

A conventional procedure for combining fixed-stress relationships is to develop equivalence relationships (R3) that can be used to convert applications at one stress level to applications at another stress level. The basic assumption for the conversion is that for any distress indicator \( d \), the amount of distress \( d_i \) observed after \( N_i \) applications at stress level \( i \) is also observed after \( N_j \) applications at stress level \( j \). Thus, when \( d_i \) and \( d_j \) are equal, the corresponding numbers of stress applications \( N_i \) and \( N_j \) are defined to be equivalent numbers of stress applications.

Symbolically, this equivalence assumption may be expressed

\[
d_i (SF, EF, TF', N_i ; z) = d_j (SF, EF, TF', N_j ; z) \tag{7}
\]

in which \( SF, EF, \) and \( TF' \) represent those factors that have predictable effects on the distress indicator \( d \), and where \( z \) is the set of all independent variables that have unpredictable effects on \( d \). The error effects of \( z \) imply that \( N_i \) and \( N_j \) exhibit chance variations among replicated observations of \( d_i = d_j \).

More complex representations of Equation 7 may be needed for distress indicators that represent time-induced distress, or that reflect distress that is induced by changes in stress levels. Such complications are beyond the scope of this paper.

Because an unlimited number of factor level combinations can lead to a given stress level on either side of Equation 7, it is conventional to narrow the general equivalence assumption to a load equivalence assumption. If the two stress levels \( i \) and \( j \) are associated with precisely the same levels for all \( SF, EF, \) and \( TF' \), except for axle load factors \( ALF \), then Equation 7 becomes

\[
d_i (SF, EF, TF'', ALF_i, N_i ; z) = d_j (SF, EF, TF'', ALF_j, N_j ; z) \tag{8}
\]

where \( TF'' \) contains all traffic predictors except for \( ALF \) and \( N \). This reduced equivalence assumption is that for the set of fixed factors in Equation 8 the amount of distress \( d_i \) observed after \( N_i \) applications of load factors \( ALF_i \) is also observed after \( N_j \) applications of load factors \( ALF_j \). The error variation implies that neither \( N_i \) nor \( N_j \) remains exactly the same in replicated observations of \( d_i = d_j \).

A further simplification of Equation 8 is to define standard axle load applications for the right side of the equation. For historical reasons, it is conventional to restrict \( ALF \) to two factors, the axle set (e.g., single, tandem, tridem) that produces a given stress state, and the weight (e.g., 30,000 lb) of the axle set. The standard axle load \( (SAL) \) is generally taken to be an 18,000-lb single axle. If \( d_o \) denotes the amount of distress observed after \( N_o \) applications of 1 \( SAL \), then the assumption of Equation 8 may be written

\[
d_i (SF, EF, TF'', ALF_i, N_i ; z) = d_o (SF, EF, TF'', SAL, N_o ; z) \tag{9}
\]

where \( N_0 \) is the number of ESALs that are equivalent to \( N_i \) applications of the \( ALF_i \) loading condition. Alternative formulations of Equation 9 are required for distress indicators for which values depend more upon time factors than axle load and applications factors.

For whatever range of distress in which Equation 9 holds, the observed applications ratio \( N_o/N_i \) must be a function of the remaining factors. Thus

\[
N_o/N_i = Q_{d_i} (d_i, SF, EF, TF", SAL, ALF_i; z) \tag{10}
\]

where \( Q_{d_i} \) is the observed equivalence factor for converting \( N_i \) to \( N_o \) by

\[
W_i = N_i \times Q_{d_i} \tag{11}
\]

where \( W_i \) is the converted equivalent of \( N_i \).

Experimental demonstration of the assumption of Equation 9 and the ratios of Equation 10 would require a field study in which (a) test sections covered an appropriate range of controlled structural and environmental conditions, (b) traffic treatments covered an appropriate range of controlled loading conditions for each combination of \( SF \) and \( EF \), and (c) observations included the numbers of applications at each loading condition that correspond to a series of observed values for each relevant distress indicator. If one of the loading conditions is for \( SALs \), then the equivalence ratios in Equation 11 could be calculated from the experimental results. The data from such field studies could of course also be used to produce the empirical distress prediction of Equation 5 or the mechanistic-empirical prediction of Equation 6. If the distress prediction equations are available, then load equivalence relationships can be derived algebraically by equating the predicted distress \( d \) at load condition \( ALF_i \) to predicted distress \( d_o \) at the standard load condition of 1 \( SAL \). For Equation 5, the equivalence relationship is

\[
f_{d_i}(SF, EF, TF", ALF_i, N_i; c) = f_{d_o}(SF, EF, TF", SAL, W_i; c) \tag{12}
\]

For the mechanistic-empirical distress prediction of Equation 6, the equivalence relationship is

\[
f_{d_i}(\tau_i; SF, EF, TF", N_i; c) = f_{d_o}(\tau_0; SF, EF, TF", W_i; c) \tag{13}
\]

where \( \tau_i \) and \( \tau_0 \) are the response prediction functions in Equation 4. It is assumed that axle load factors \( ALF_i \) and \( SAL \) are contained within \( \tau_i \) and \( \tau_0 \), respectively.

If particular values are substituted in Equations 12 and 13 for all factors except \( ALF_i, SAL, N_i, \) and \( W_i \), the result is a formula for calculating ESALs for any particular combination of \( ALF_i \) and \( N_i \). The equivalences may of course be different for different sets of values for the other factors in the two equivalence relationships. Depending on the mathematical form of Equation 12, it may be possible to derive an explicit
load equivalence function \( LEF \) such that

\[
W_i = N_i \times LEF_{d_i}(SF', EF', TF'', ALF_i, SAL; c)
\]

(14)

If specific values are substituted for all variables in \( LEF_{d_i} \), the function becomes a load equivalence factor for converting \( N_i \) applications to \( W_i \) equivalent standard applications. Counterpart determination of load equivalence functions from the mechanistic-empirical distress prediction of Equation 6 might have the general form

\[
W_i = N_i \times LEF_{r_i}(\bar{r}, \bar{r}_0; SF', EF', TF''; c)
\]

(15)

where \( \bar{r}_i \) and \( \bar{r}_0 \) represent the prediction function \( f_i \) in Equation 4 for axle load conditions \( ALF_i \) and \( SAL \), respectively.

Load equivalence factors were derived by the U.S. Bureau of Public Roads from the AASHO Road Test prediction equations for PSI loss (7). A new and more extensive tabulation of these factors is given in the AASHTO Guide for Design of Pavement Structures (1).

A geometrical representation of load equivalence concepts is shown in Figure 3. The irregular curves are hypothetical distress histories \( d \) versus \( N \) of three pavements whose structure, environment, and traffic differ only with respect to \( ALFs. \) Points whose coordinates are \((d_i, N_i), (d_0, N_0), \) and \((d_j, N_j)\) represent, respectively, distress observations for load levels \( ALF_i \), \( SAL \), and \( ALF_j \). The regular, dashed curves represent a single distress prediction function \( d = f_d \), as in Equations 5 and 6.

The equivalence assumption for \( d \) is demonstrated by abscissas that correspond to an arbitrary distress level \( d \) that is projected horizontally by a dotted line in Figure 3. Observed applications for this distress level are \( N_i, N_0, \) and \( N_j \) for the respective histories.

Corresponding abscissas for the prediction function are denoted by \( \bar{N}_i, \bar{N}_0, \) and \( \bar{N}_j \). Ratios of \( N_0 \) to \( N_i \) and \( N_0 \) to \( N_j \) are observed and predicted equivalence factors, respectively, for converting \( N_i \) and \( \bar{N}_i \) to ESAL values of \( W_i \) and \( \bar{W}_i \). Similarly, both \( N_j \) and \( \bar{N}_j \) are multiples of \( N_0 \) and \( \bar{N}_0 \). If the equivalence factors are not constant for all distress levels, then both the \( Q \) factor in Equation 11 and the \( LEF \) factor in Equation 15 depend upon the value of \( d \).

As was previously stated, the main purpose for stress-load equivalence factors is to provide a means for combining applications of mixed-stress-load levels. Suppose, for example, that the traffic history of a pavement section comprises \( N_1 \) applications at \( ALF_1 \), \( N_2 \) applications at \( ALF_2 \), and so forth. Then an appropriate equivalence relationship can be used to convert each \( N_i \) into its ESAL equivalent \( W_i \). The sum of the equivalent applications is

\[
W = \sum W_i = \sum N_i \times LEF_i
\]

(16)

where \( LEF_i \) is that in Equation 14 or 15, evaluated for \( ALF_i \) and all remaining SF, EF, and TF.

![FIGURE 3 Load equivalence concepts.](image)
Substitution of $W$ and $SAL$ into the distress prediction equation yields $\tilde{d}_w$, the predicted distress for $W$ standard axle loads under mixed-stress applications. This procedure is shown in Figure 3 for $N'_i$, applications at $ALF_i$ and $N'_j$ applications at $ALF_j$. When $N = N'_j$, $\tilde{d}_i$ is the predicted distress from the $ALF_i$ curve. On the $SAL$ curve, this ordinate corresponds to $N_0 = W_j$. When $N = N'_j$, the predicted distress from the $ALF_i$ curve is $\tilde{d}_j$, and the same ordinate on the $SAL$ curve corresponds to $W_i$. At $W = W_i + W_j$, the $SAL$ distress prediction is $\tilde{d}_w$. Thus $\tilde{d}_w$ is the predicted distress from two mixed-stress applications, $N_i$ at $ALF_i$ and $N_j$ at $ALF_j$. Note that $\tilde{d}_w = \tilde{d}_i + \tilde{d}_j$ if and only if the distress prediction function is linear, that is, if distress is directly proportional to the number of applications at any given stress level.

An alternative procedure for combining applications from mixed-stress levels requires neither the definition of a standard stress level nor the conversion of all applications to standard applications. Instead, for example, applications at the lowest stress level are calculated for the sum of the actual applications at the second-lowest stress level. Distress at the second-lowest stress level is calculated for the sum of the actual applications at the second-lowest level plus the equivalent applications from the lowest level, and this process is continued through all remaining stress levels until distress has been predicted for the highest stress level.

The alternative procedure is shown in Figure 3, beginning with $N'_j$, applications for $ALF_j$. The predicted distress level $\tilde{d}_j$ crosses the $ALF_i$ curve where $N = N'_j$. Thus, $N'_j$ is the number of $ALF_i$ applications that are equivalent to $N'_j$ at $ALF_j$. If the $ALF_i$ curve is now entered at $N = N'_j + N'_j$, the predicted distress is again $\tilde{d}_w$ for the two sets of mixed-stress applications.

A special case of equivalence functions arises if the distress function $f_{d_i}$ in Equation 5 or $f_{d_j}$ in Equation 6 is linear with increasing number of applications $N$.

For the linear case only,

$$d_i/d_i = N_i/N_i = W_i/W_i$$

(17)

where subscript $i$ refers to the terminal distress level, and $N_i$ and $W_i$ are the terminal number of applications at $ALF_i$ and $SAL$, respectively. The ratio $d_i/d_i$ is called the distress ratio or damage ratio. The corresponding applications ratio $N_i/N_i$ has also been called a damage ratio, but is more appropriately the fraction of pavement life that has been expended at distress level $d_i$. Even this connotation is misleading for the many types of distress functions that are nonlinear.

If substitution is made for $W$ from Equation 17 into Equation 16, the result is

$$W = \sum_i (N_i/N_i) W_i$$

or

$$W_i/W_i = \sum_i (N_i/N_i)$$

(18)

When $W = W_i$, Equation 18 yields the familiar Miner relationship (3),

$$\sum_i (N_i/N_i) = 1$$

(19)

at the terminal distress condition $d = d_i$.

**Distress-Performance Prediction for Mixed-Stress Levels**

Prediction equations for distress indicators when the applications are at mixed-stress levels are shown as Relationships (R4) in Figure 1. Distress prediction equations for mixed-stress-load levels have two parts, (a) an equation for predicting distress when all applications are at the standard stress level, and (b) an equivalence relationship for converting applications at any stress level to standard applications.

Only the first part of each equation will be displayed in the remainder of this section. The mixed-stress prediction equation for any distress indicator $d$ may be written

$$d_w = \tilde{d}_w + e_d(z) = f_{d_w}(SF, EF, TF^*, W_i; c) + e_d(z)$$

(20)

The corresponding mechanistic-empirical distress prediction relationship for mixed-stress applications is

$$d_w = \tilde{d}_w + e_d(z) = f_{d_w}(\tau_0; SF', EF', TF^*, W_i; c) + e_d(z)$$

(21)

where $\tau_0$ is a response prediction function (Equation 4) for standard load conditions. If the foregoing prediction equations are derived from field studies in which every test section receives mixed-stress applications, then the study cannot of itself produce equivalence factors, and previously derived equivalence factors must be used to calculate $W_i$.

It is expected that the long-term pavement performance (LTPP) studies in the Strategic Highway Research Program (SHRP) will produce a number of distress prediction equations in terms of mixed-stress levels. It may also be that certain LTPP studies are designed for the derivation of stress-load equivalence functions.

In a previous section, pavement performance was defined to be the number of equivalent standard stress-load applications $W_i$ received by the pavement during the period that distress remains at acceptable levels ($d \leq d_i$). If $d$ represents a singular mode of distress such as fatigue cracking, then $W_i$ is an indicator of structural performance; if $d$ represents roughness or PSI loss, then $W_i$ is an indicator of functional performance.

Relationships of performance indicators to distress prediction factors (Relationships (R6) in the distress prediction relationships of Equations 20 and 21. If $\tilde{d}_w$ and $W_i$ are set equal to $d_i$ and $W_i$, respectively, it is possible at least in principle to solve either equation for $W_i$ in terms of $d_i$. The symbolic solutions are

$$\tilde{W}_i = g_{d}(SF, EF, TF^*; d_i; c)$$

(22)

for Equation 20, and

$$\tilde{W}_i = g_{\tau}(\tau_0; SF', EF', TF^*; d_i; c)$$

(23)
for Equation 21. It is generally assumed that the prediction errors for \( W_i \) are multiplicative rather than additive, and that prediction errors for \( \log W_i \) are not only additive but have frequency patterns that are well approximated by normal distributions. For these reasons, empirically derived prediction equations for performance indicators are written

\[
\log W_i = \log W_i + \delta \hat{W}(z) = G_d(SF, EF, TF'; d_i; c) + \delta_w(z) \quad (24)
\]

where \( \delta_w(z) \) is the logarithmic error \( \log W_i - \log W_i \). The logarithmic version of Equation 23 is

\[
\log W_i = \log W_i + \delta \hat{W}(z) = G_r(r_0; SF', EF', TF'; d_i; c) + \delta_w(z) \quad (25)
\]

Equations 24 and 25 may be derived directly from experimental data without first deriving the distress prediction equations 20 and 21. In such derivations, the primary dependent variable would be the performance indicator \( W_i \) itself, perhaps for several levels of \( d_i \). Constants in the derived equation generally differ from those derived indirectly through algebraic manipulation of the distress prediction functions. In the direct derivations, constants are derived to minimize some function of the performance prediction errors. The corresponding constants in indirect derivations are originally determined by minimizing some function of the distress prediction errors. These differences can be reconciled by reanalyzing the data used to originally derive the distress prediction equations.

Prediction equations for a number of distress and performance indicators are found in the literature (1-3). Unfortunately, none of the reported relationships include an adequate description of the experimental design behind the relationship, particularly with respect to the inference space of the experiment. Moreover, not much information is available on the error variances for predictions from the relationships. It is therefore difficult to infer the range of validity and degree of precision for many of the reported relationships.

Traffic Prediction Equations

Prediction of design period traffic by Relationships R5 in 1 is essential to the pavement design process and will be more fully in the next section. Because normal highway traffic includes a mixture of axle loads and other stress-related factors, the traffic prediction must be for equivalent axle loads \( W \). The symbols \( w_T \) and \( \hat{w}_T \) denote the actual and predicted equivalent standard axle load, respectively, that the to-be-designed pavement will experience during its design period of \( Y_T \) years. Lower-case type is used to distinguish the actual load \( w \) from the performance indicator \( W \). Thus, \( w_T \) is the actual number of ESALs that occur during the design period of the pavement; \( W \) is the actual number of ESALs that the pavement carries during its performance period, that is, until the distress is given by \( d_i \).

In performance prediction by Equations 24 and 25, logarithms are used for the traffic prediction relationships. The general traffic prediction equation may be written

\[
\log w_T = \log w_T + \delta w(z) = H_T(TF', LEF_d', Y_T; c) + (z) \quad (26)
\]

where the \( TF' \) include all traffic prediction factors that are needed to characterize traffic during \( Y_T \), including growth factors. The load equivalence function \( LEF_d' \) provides axle load conversion factors, and is relative to a particular distress indicator \( d \).

In general, the prediction function \( H_T \) is a mathematical expression or computational algorithm that is not derived from experimental data but stems rather from rational consequences of the definition for \( w_T \) and all factors within \( H_T \). The prediction errors \( \delta w(z) = \log w_T - \log w_T \) generally reflect uncertainties in the prediction factors within \( H_T \). Specific formulations of Equation 26 are found in the literature (1-3).

Relationships Among M&C Specifications Factors

M&C specification factors are any pavement performance-related factors whose levels are specified by design and are controlled, either directly or indirectly, during the course of materials processing and pavement construction. Relationships among such factors are represented by Relationships R7 in Figure 1.

Three classes of M&C factors are defined. The first class, of the explicit M&C prediction factors \( mcf \), contains all M&C factors that are \( SF \) or \( EF \) factors in one or another of the primary prediction equations (Relationships RI-R6) that are relevant to the design situation at hand. Each of these factors appears explicitly in one or another of the prediction equations being used for pavement design. If the prediction equations are valid for the prediction of stress, distress, and performance, then the corresponding \( mcf \) are performance-related by definition.

Certain explicit M&C specification factors may not be amenable to M&C control because their evaluation is too costly or too time-consuming. In such cases, surrogate factors with closely related values should be found. It should be possible to predict the values of each such \( mcf \) from corresponding values of its surrogate \( mcf' \) to within a known of precision. Relationships between explicit M&C specification factors and their surrogates may be written

\[
mcf = \hat{mcf} + e_f(z) = g_f(mcf'; c) + e_f(z) \quad (27)
\]

Examples of surrogate factors include the use of PCC compressive strength as a surrogate for PCC rupture modulus or the use of subgrade CBR as a surrogate for subgrade reaction modulus \( k \).

The third class of M&C specification factors consists of the auxiliary control factors \( mcf' \). These factors are not explicit factors in any primary relationship, nor are they closely
enough related to any explicit factor to qualify as surrogate factors. The purpose of the \( mcf'' \) is to enhance the control of explicit or surrogate factors, not to predict stress, distress, and performance directly.

Because a given performance requirement (e.g., for \( \log W \)) assumed to be provided by a specified set of levels for the \( mcf'' \), auxiliary control of these levels helps assure that the requirement will be met. For example, if the design specifications call for a given level of the PCC rupture modulus, there will be several auxiliary M&C control factors the control of which provides assurance that the specified PCC rupture modulus level will be attained. Such M&C specification factors might properties of the PCC mix ingredients (e.g., aggregate hardness), and properties of the PCC mix itself (e.g., water/cement ratio, slump, and perhaps even air content).

By definition, any \( mcf'' \) is performance-related if and only it has predictable, statistically significant effects on an explicit factor \( mcf \) or on a surrogate factor \( mcf' \) that is itself performance-related. In short, any performance-related \( mcf'' \) is a PF for an \( mcf \) or \( mcf' \). This means that every performance-related \( mcf'' \) appears on the right-hand side of one of the following relationships for the prediction of explicit M&C specification factors \( mcf \) and surrogate factors \( mcf' \) from auxiliary control factors \( mcf'' \).

\[
mcf = mcf' + e_f(z) = g_f(mcf''; c) + e_f(z) \quad (28)
\]

\[
mcf' = mcf'' + e_f'(z) = g_f'(mcf''; c) + e_f'(z) \quad (29)
\]

The right-hand sides of Equations 28 and 29 may contain more than one \( mcf'' \).

Equations 27-29 can be determined by statistical analyses of experimental data from designed short-term laboratory or track studies. Experimental units for some relationships may be quantities of materials or mixes. In other cases, the units be laboratory specimens or cores from test plots. Some relationships require the observation of response indicators \( r \) or within specially constructed sections, but no relationship requires long-term observations from in-service highway sections. However, long-term studies may be needed to verify relationships that have been derived in the short-term studies.

The statistical analyses show which \( mcf'' \) are in fact performance-related, and the degree to which their control induces control of the \( mcf \) and \( mcf' \).

To the fullest extent feasible, experimental designs for derivation of relationships among M&C specifications factors should incorporate all \( mcf'' \) that have significant effects on the \( mcf \) or \( mcf' \), so that all interaction effects can be studied the same experiment. Several \( mcf \) and \( mcf' \) can be observed within the same experimental design, so all relevant relationships may be derivable from only a few different experimental designs.

An illustrative experiment would consist of the observations of elastic moduli, flexural strength, fatigue life, and all surrogates for these factors in laboratory specimens that cover many combinations of levels for several different \( mcf'' \).

Although some secondary prediction relationships among M&C specification factors have been reported in the highway research literature, many gaps exist; new statistically designed studies are required to examine relevant interactions among the factors and to enable comprehensive assessment of all relevant prediction error variances.

Performance-Cost Relationships

All M&C relationships discussed thus far are direct or indirect inputs to the algorithms (R8 in Figure 1) used to specifications for all \( mcf \) relevant to any particular design and construction situation. Each prediction equation must be accompanied by its variance components that quantify the uncertainties associated with the use of the equation. It is the inputs include performance requirements determined by design criteria to be discussed in the next section.

The final set of inputs include relative unit costs that are with various M&C alternatives for materials provision, roadbed preparation, pavement construction, and construction control. The unit costs make it possible to assess total costs for alternative designs, that is, alternative sets of levels all \( mcf \), \( mcf' \), and \( mcf'' \). The unit costs should also include operating costs \( aoc \) for factors such as routine maintenance other operating costs, thus making it possible for the algorithms to compare the operating costs of alternative performance periods.

If \( W \) represents a design requirement for the performance period, the traffic prediction data make it possible to estimate the expected performance years \( Y \), during which the pavement will provide acceptable levels of service. If estimated M&C costs are denoted by \( C_{MCx} \), where \( x \) represents a particular set of specifications, and if \( C_{OPX} \) denotes total estimated operating costs during the performance period \( Y \) expected for specifications \( x \), then the total costs for the relevant life cycle phase may be written

\[
\hat{C}_x = \hat{C}_{MCx} + \hat{C}_{OPX} \quad (30)
\]

In terms of the factors that are included in specifications \( x \), the cost relationship may be written

\[
\hat{C}_x = \hat{C}_{MCx}(SF_x, EF_x, TF''_x; mcf'') + \hat{C}_{OPX}(aoc, Y) \quad (31)
\]

where \( SF \) denotes pavement structure factor levels in specifications \( x \), and so forth.

PAVEMENT DESIGN CRITERIA AND DESIGN FACTORS

This section deals with the selection of pavement design criteria (Box E in Figure 1) and the identification of M&C specifications factors (Box G in Figure 1) as the initial step in the definition of a pavement design for a given design, construction, and performance situation.
Selection of Distress and Performance Criteria

A first design step is to select the distress and performance indicators that are used to determine the design. This step includes the selection of prediction equations and equivalence relationships needed for the remaining design steps.

For each distress indicator \( d \), one or more alternative terminal distress levels \( (d_j) \) must be specified. If more than one distress indicator is used, then a multiple decision function is needed to specify the set of distress levels that collectively define the termination of the performance period.

To simplify the remaining discussion, it is assumed that the primary design criterion is an indicator of functional distress \( d \) (e.g., roughness or PSI loss) with terminal level \( d_j \) and prediction equations and associated equivalence functions in the general forms given by Equations 12 and 20. The prediction equation for the corresponding performance indicator \( W_i \) is assumed to have the general form of Equation 22.

In practical applications, the designer may of course elect to produce and compare designs that are based on various alternative distress-performance prediction equations and equivalence functions.

Identification of Prediction and Control Factors

After the selection of particular distress-performance indicators and associated prediction equations, the next design step is to identify and classify all EPF that appear explicitly in one or another of the equations. The EPF are then separated into those related to the M&C process (mcf) and those (e.g., EF or TF) that are not M&C-related.

As was discussed in a previous section, some mcf (e.g., layer thicknesses) can be controlled directly during the M&C process, but other mcf may not be amenable to M&C control.

To the fullest possible extent, surrogate factors mcf" should be defined for the mcf that do not easily lend themselves to direct control.

Finally, performance-related mcf" should be selected to provide at least partial control for the mcf and mcf". The performance-relatedness of any selected mcf" is determined by the secondary prediction relationships (R7 in Figure 1) that were previously discussed.

For M&C purposes, the outcome of the foregoing design step is a list of all factors mcf, mcf", and mcf" indicators of which are observed and controlled during the M&C process. An illustrative classification of prediction factors and M&C control factors relative to flexible and rigid pavement design equations based on PSI loss as a distress indicator appears in the AASHTO Guide for Design of Pavement Structures (1).

In addition to the indicators and prediction equations, the following list of design criteria includes reliability and performance period criteria (J) that will be discussed in the next section.

1. Indicators and prediction equations
   1.1 Distress indicator: serviceability loss ratio
      \[ q = (p_1 - p) / (p_1 - 1.5) \]

1.2 Terminal distress level: \( p = p_1; q = (p_1 - p_1) / (p_1 - 1.5) \)

1.3 Performance indicator: \( \log W_i \) (log ESALS to \( p = p_1 \) and \( q = q_1 \))

1.4 Performance prediction equations: flexible pavement (2, p. 1-6); rigid pavement (2, p. 1-7)

2. Reliability criteria
   2.1 Process standard deviation: \( S_0 \)
   2.2 Reliability level: \( R \) (normal curve abscissa \( z_R \))
   2.3 Reliability factor: \( F_R = \exp[-(z_R \times S_0)] \)

3. Design and performance periods
   3.1 Design period (years): \( Y_T \)
   3.2 Design period traffic: \( w_T \) (predicted ESALS = \( w_T \))
   3.3 Design applications: \( \hat{W}_i = F_R \times \hat{W}_i \)

The PF in the referenced design equations are as follows:

1. Pavement structure factors (SF)
   1.1 Flexible pavements
      a. AC thickness \( (D_1) \)
      b. AC strength coefficient \( (a_1) \)
      c. Base thickness \( (D_2) \)
      d. Base strength coefficient \( (a_2) \)
      e. Base drainage coefficient \( (m_2) \)
      f. Subbase thickness \( (D_3) \)
      g. Subbase strength coefficient \( (a_3) \)
      h. Subbase drainage coefficient \( (m_3) \)
      i. Structural number \( (SN = a_1 D_1 + a_2 m_2 D_2 + a_3 m_3 D_3) \)

1.2 Rigid pavements
   a. PCC thickness \( (D) \)
   b. PCC rupture modulus \( (S_u) \)
   c. PCC elastic modulus \( (E_e) \)
   d. Subbase support loss \( (L_S) \) (subsumed in \( k' \))
   e. Drainage coefficient \( (C_d) \)

2. Environmental factors (EF)
   2.1 Roadbed
      a. Effective resilience modulus \( (M_R) \)
      b. Effective subgrade modulus \( (k') \)
      c. Swell potential PSI loss \( (\Delta_{SW} \text{ PSI}) \)
      d. Frost-heave potential PSI loss \( (\Delta_{FH} \text{ PSI}) \)

2.2 Climate
   a. Moisture regime (subsumed in roadbed moduli)
   b. Freeze-thaw regime (subsumed in roadbed moduli)

The following is an example of a list of surrogate factors mcf" that might be used for certain mcf.

1. Pavement structure factors
   1.1 AC surfacing factors
      a. Stiffness-elastic modulus
      b. Stability
      c. Creep predictors
      d. Fatigue predictors
      e. Aging predictors
      f. Moisture resistance
1.2 PCC surfacing factors
   a. Compressive strength
   b. Reinforcement factors
   c. Joint factors

1.3 Base and subbase factors
   a. Elastic moduli
   b. CBR
   c. R-value
   d. Triaxial test values
   e. Stability (bituminous stability)
   f. Compressive strength (cement stability)
   g. Permeability

2. Roadbed factors
   2.1 CBR
   2.2 R-value
   2.3 Plasticity

Some mcf that can be controlled directly and possible mcf" for enhancing the control of primary surrogate factors are as follows:

1. Directly controllable factors (mcf)
   1.1 AC or PCC surface thickness (D₁ or D)
   1.2 Base or subbase thickness (D₂ or D₃)

2. Factors providing indirect control (mcf")
   2.1 AC surfacing factors
      a. Asphalt grade and source
      b. Asphalt content
      c. Penetration
      d. Viscosity
      e. Air voids
      f. Aggregate gradation
      g. Density/compaction
   2.2 PCC surfacing
      a. Water/cement ratio
      b. slump
      c. Air content
      d. Durability
      e. Aggregate gradation
      f. Bar size
      g. Steel strength
      h. Dowel properties
   2.3 Base and subbase
      a. Materials types and sources
      b. Aggregate gradation
      c. Density or compaction
   2.4 Roadbed
      a. Soil type and source
      b. Density or compaction

The preceding lists are illustrative of mcf' and mcf" that might be used in a particular design situation. However, because relevant secondary relationships (Equations 27-29) have not been assembled or derived, it is not possible to assess the performance-relatedness or control efficacy of any of these mcf' or mcf".

Specification of Design and Performance Periods

The next step in pavement design is to specify a design period, that is, the number of years Yₜ for which the designed pavement is to provide acceptable service at distress levels d < d₁. It is conventional to express Yₜ in multiples of 5 or 10 years. The selected period is generally determined by PMS considerations for the particular life cycle phase to which the design relates.

Because the actual performance period cannot be predicted with certainty, the design criteria must not only specify a value for Yₜ, but must also include a specified degree of assurance that the actual performance period Y₁ is at least equal to the design period Yₜ. The design criterion for this assurance is the design-performance reliability level R, the probability that the actual performance period of the pavement is at least as great as the selected design period. Thus,

\[ R = \text{Prob}(Y₁ ≥ Yₜ) \]  \hspace{1cm} (32)

Because the performance prediction equation is generally for the number W₁ of ESAL applications that the pavement can carry until d = d₁, the designer must express both the design and performance periods in terms of ESALs. For the design period, this conversion consists of acquiring relevant traffic data and traffic projections, then in using Equation 26 to predict the number wₜ of ESALs that will occur during the design period Yₜ. Because of the prediction error of Equation 26, the actual design period traffic wₜ will differ from the predicted traffic wₜ by an unpredictable error

\[ δ_w = \log wₜ - \log wₜ \]  \hspace{1cm} (33)

The ESAL counterpart of the actual pavement performance period Y₁ is W₁. Because of the performance prediction error, the predicted performance W₁ will differ from W₁ by an unpredictable error

\[ δ_W = \log W₁ - \log W₁ \]  \hspace{1cm} (34)

In terms of ESALs, the probability R that the performance period will not be less than the design period is equivalent to the probability that the actual pavement performance W₁ is at least as great as the actual design period traffic wₜ, that is,

\[ R = \text{Prob}(W₁ ≥ wₜ) \]  \hspace{1cm} (35)

or in logarithms,

\[ R = \text{Prob}(\log W₁ ≥ \log wₜ) \]  \hspace{1cm} (36)

Although reliability R is defined by these equations, the definition fails to indicate how the reliability level can be controlled through quantitative design criteria. An ab-
breviated discussion of reliability concepts is given in the remainder of this subsection; a full development is presented in the literature (1, 2).

The difference \( \log W_t - \log w_T \) in Equation 36 is an overall deviation the sign and magnitude of which cannot be predicted for any particular iteration of the pavement design-performance process, but the average value of which over many iterations is a function of the selected reliability level.

If the overall deviation is denoted by \( \delta_0 \), then

\[
\delta_0 = \log W_t - \log w_T
\]  

(37)

and

\[
R = \text{Prob}(\delta_0 \geq 0)
\]

(38)

where \( \delta_0 \) varies by chance among independent repetitions of the design-performance process, partly because of the traffic prediction error \( \delta_w \) and partly because of the performance prediction error \( \delta_W \). To show the relation between the overall process deviation and the two prediction errors, the quantity \( \log W_t - \log w_T \)-can be subtracted from, then added to, the right side of Equation 37 to produce

\[
\delta_0 = (\log W_t - \log w_T) - (\log W_t - \log w_T) + (\log W_t - \log w_T)
\]

(39)

Rearrangement of the terms in Equation 39 gives

\[
\delta_0 = (\log W_t - \log W_t) + (\log W_t - \log w_T) + (\log w_T - \log w_T)
\]

(40)

and substitution from Equations 33 and 34 gives

\[
\delta_0 = \delta_w + (\log W_t - \log w_T) + \delta_W
\]

(41)

If the ratio \( \hat{W}_t/\hat{w}_T \) is defined to be the reliability design factor \( F_R \), then

\[
F_R = \hat{W}_t/\hat{w}_T
\]

or

\[
\log F_R = \log \hat{W}_t - \log \hat{w}_T
\]

(42)

Substitution in Equation 41 gives

\[
\delta_0 = \delta_w + \log F_R + \delta_W
\]

(43)

and from Equation 38,

\[
R = \text{Prob}[(\delta_w + \log F_R + \delta_W) \geq 0]
\]

(44)

For a given design period traffic prediction \( \hat{w}_T \), the designer can specify a reliability factor \( F_R \) and thereby determine design applications \( W_t \) by transposing Equation 42 to give

\[
\hat{W}_t = F_R \times \hat{w}_T
\]

(45)

To derive a quantitative relationship between the reliability level and the reliability factor, assumptions must be made about the probability distribution of \( \delta_0 \). For any particular design situation, Equation 43 shows that \( \delta_0 \) is the sum of two chance errors \( \delta_w \) and \( \delta_W \), plus a fixed value for \( \log F_R \). It is conventional to assume that the frequency distributions of both prediction errors are well approximated by normal probability curves. If the two errors are independent and both prediction equations are unbiased, then the distribution means are zero and the distributions are determined by their respective variances \( S_w^2 \) and \( S_W^2 \).

With the foregoing assumptions, the probability distribution of \( \delta_0 \) is also normal and has mean value

\[
\bar{\delta}_0 = \bar{\delta}_w + \log F_R + \bar{\delta}_W = 0 + \log F_R + 0 = \log F_R
\]

(46)

Because \( \log F_R \) has no chance variation, the variance of \( \delta_0 \) is given by

\[
S_0^2 = S_w^2 + S_W^2 = S_w^2 + S_W^2
\]

(47)

\( S_0^2 \) is the process or overall variance, and \( S_o \) is the process or overall standard variation.

If \( \delta_0 \) is transformed to a standard normal variate by

\[
z = (\delta_0 - \bar{\delta}_0)/S_0
\]

(48)

and if \( z_R \) is the value of \( z \) when \( \delta_0 = 0 \), then

\[
z_R = (0 - \log F_R)/S_0 = (-\log F_R)/(S_w^2 + S_W^2)^{1/2}
\]

(49)

Finally, if \( \phi(z) \) is the standard normal curve area that lies to the right of abscissa \( z \), then

\[
\phi(z_R) = \text{Prob}(z \geq z_R) = \text{Prob}(\delta_0 \geq 0)
\]

(50)

Thus, the basic relationship that connects \( R \), \( F_R \), and \( S_0 \) is

\[
R = \phi(z_R) = \phi([-\log F_R]/S_0)
\]

(51)

An extensive tabulation of Equation 51 is given in the literature (2). All of the reliability concepts and definitions that have been presented are shown in Figure 4. A numerical example follows.

Suppose that the design-performance reliability for a given project is to be \( R = 0.80 \) percent. Then \( \phi(z_R) = 0.80 \), and normal curve area tables show that \( z_R = -0.84 \). If the overall process standard deviation is assumed to be \( S_0 = 0.40 \), then substitution in Equation 50 yields \( \log F_R = 0.36 \), or \( F_R = 2.17 \). If the traffic prediction for a 15-year design period is \( \hat{w}_T = 8 \times 10^6 \) ESALs, then the design applications requirement for the project is given by Equation 45 to be \( W_t = (2.17) \times (8 \times 10^6) = 1.74 \times 10^7 \) ESALs. Thus, the design alternatives are the various combinations of distress prediction factors for which the design equation predicts \( \log W_t \) to be 7.24.

If the designer equates design applications to the design period traffic prediction, then \( \hat{W}_t = \hat{w}_T \) and \( F_R = 1 \). Because \( \log 1 = 0 \), \( z_R = 0 \) and normal curve area tables show that \( z = 0 \)
corresponds to the 50th percentile of the standard normal distribution. From Equation 51, \( \phi(0) = 0.50 \), and \( R = 50 \) percent reliability. Thus, whenever a pavement is designed with \( W_i = \hat{w}_T \), there is a 50 percent chance that its performance period will terminate before the end of the design period.

**DERIVATION OF M&C SPECIFICATIONS**

This section presents considerations and possibilities for the pavement design steps in which (quantitative) specifications are derived for all prediction factors and control factors identified in the previous section. It is assumed that the specifications are derived from computational algorithms (\( RB \) in Figure 1).

No effort is made here to define specific algorithms because much research is still needed before development of valid, comprehensive algorithms that apply to all design, construction, and performance situations is possible.

A potential strategy for the derivation of design levels for M&C factors follows. Components of variance of distress-performance prediction equations, considered to be necessary inputs for the derivation of control factor tolerances, and
possibilities for the derivation of tolerance-acceptance levels for M&C control factors are discussed afterwards.

Specification of Design Levels for Prediction Factors

It is assumed that any valid set of design or target levels for distress-performance PF must satisfy the prediction equations selected for design purposes. For example, if the performance prediction Equation 24 is used, then the primary factor levels must satisfy an equation of the form

\[ \log W_i = \log(F_R \times \hat{w}_p) = G_d (SF, EF, TF^*; d_i; c) \] (52)

where \( F_R \) is a specified reliability factor; \( \hat{w}_p \) is the predicted design period traffic; \( SF, EF, \) and \( TF^* \) are primary prediction factors; and \( d_i \) is a specified terminal distress level.

After the PF have been classified as previously described, the primary factors are separated into two sets: non-M&C factors \( PF' \) and M&C factors \( mcf \). Thus, Equation 52 may also be written

\[ \log W_i = \log(F_R \times \hat{w}_p) = G_d (PF'; mcf; d_i; c) \] (53)

The M&C factor classification determines which factors \( mcf^* \) are controlled indirectly through the use of surrogate factors \( mcf^*' \) as well as corresponding relationships of Equation 27 between \( mcf \) and \( mcf^* \). In the present context, the relevant equations for predicting primary from surrogate factors are

\[ mcf^* = g_1 (mcf^*'; c) \] (54)

The factor classification also determines which \( mcf^* \) will be used to enhance the control of whatever levels are specified for the \( mcf \) in Equation 53 and the \( mcf^* \) surrogates in Equation 54. The secondary relationships (Equations 28 and 29) that contain performance-related \( mcf^* \) as predictors for \( mcf \) or \( mcf^* \) may be expressed

\[ mcf_1 mcf^* = g_2 (mcf^*'; c) \] (55)

Equation 55 contains all available relationships between \( mcf^* \) and relevant M&C factors in Equations 53 and 54. Equations 53-55 are inputs to design level specification algorithms for all factors \( PF', mcf, mcf', \) and \( mcf^* \).

Additional inputs for every specification algorithm factor are (a) the admissible range of design levels for a factor, and (b) all M&C and operational unit costs needed to calculate the total cost of implementing any design level within the admissible range of the factor.

For some factors (e.g., climate or roadbed soil type), the admissible range of design levels for a factor may reduce to only a single level. Constraints for the ranges of other factors (e.g., for layer thicknesses) may include maximum or minimum levels. Admissible ranges of design levels for some factors may depend on the design levels of other factors. For example, the admissible ranges for thicknesses depend on those for strength levels of successive AC pavement layers.

The design level specification algorithms must also be provided with a criterion for optimizing the overall set of factor levels with respect to costs and benefits, for example, M&C charges in dollars per ESAL of traffic during the performance period.

With access to such inputs, the design level specification algorithms determine the solutions to Equation 53 within the admissible factor ranges. For each solution, Equations 54 and 55 determine alternative combinations of surrogate and auxiliary control factor levels that correspond to the primary factor levels, and evaluate the optimization criterion for every alternative combination of primary and secondary factor levels.

After all admissible solutions for Equation 53 have been examined, the algorithm outputs will be lists of factor level combinations (i.e., pavement designs) that rank highest in terms of the optimization criterion. The designer may use additional criteria to select a final design from among the optimal alternatives.

As described, the factor levels for a given design alternative are derived deterministically in that the prediction uncertainties (error variances) are not taken into account. This deterministic approach is shown in Figure 5 for a given performance indicator \( \log W_i \), one primary factor \( mcf \), and one secondary factor \( mcf^* \) for auxiliary control of the primary factor. When all remaining \( mcf \) are at fixed levels, the graph of Equation 53 provides a turning point for determining the design level \( mcf \) of a primary factor from a specific performance requirement (\( \log W_i \)). When all remaining secondary factors are at fixed levels, the graph of Equation 54 provides a turning point for determining the secondary factor design level \( mcf^* \) that corresponds to \( mcf \).

All \( mcf \) levels implied by Figure 5 are assumed to be optimal with respect to cost-benefit criteria, but such determinations in fact are beyond the scope of this paper. Eventually, algorithms for design level specifications may incorporate stochastic properties of the prediction equations and cost-benefit relationships.

Components of Prediction Error Variances

Algorithms for specification of prediction factor tolerances will depend, at least in part, on error variance components of the primary (Equation 53) and secondary (Equations 54 and 55) prediction equations that are used to specify design factor levels. This subsection contains an abbreviated discussion of variance component concepts. A full discussion is given in the literature (2, Appendix EE).

Up to this point, the prediction errors for any particular prediction equation have generally been characterized by a single error variance \( S^2 \). For example, the performance prediction errors in Equation 23 were denoted by \( \delta \), and the variance of the \( \delta \) distribution has been denoted by \( S^2 \) in a previous section. It has also been stated that \( S^2 \) always has two components, one \( (S^2_{\text{E}}) \) for the lack of fit of the assumed model to the data that were used to derive the equation, and another that represents the net effects of all error variables that operated within the inference space of the derivation;
FIGURE 5 Specification of prediction factor levels.

because these error effects bring about dependent variable variations among replicate pavement sections, the second component of $S^2$ is the replication variance $S^2_{RP}$.

If all lack-of-fit errors and replication errors are independent, then for performance prediction errors,

$$S^2_W = S^2_{LF} + S^2_{RP} \tag{56}$$

As was indicated by a process variable of Class 4, part of any replication error can be attributed to chance deviations of the PF from their specified levels. The remaining part of each error must be attributed to the effects of all remaining error variables. If the two components of each replication error are independent and additive, then the replication variance $S^2_{RP}$ has two components $S^2_{PFD}$ and $S^2_{UEV}$, where the former represents variance from prediction factor deviations (PFD) and the latter represents variance from unidentified error variables (UEV). Thus,

$$S^2_{RP} = S^2_{PFD} + S^2_{UEV} \tag{57}$$

and from Equation 56,

$$S^2_W = S^2_{LF} + S^2_{PFD} + S^2_{UEV} \tag{58}$$

Because the number of subscript letters for any component denotes its hierarchical level, $S^2_W$ is at Level 1, $S^2_{LF}$ and $S^2_{RP}$ are at Level 2, and both $S^2_{PFD}$ and $S^2_{UEV}$ are at Level 3.

Fourth-level variance components are defined by further decomposition of $S^2_{PFD}$ into variances that can be attributed to the design level deviations of each separate prediction factor. $S^2_{PFD}$ is the part of the replication variance that can be attributed to design level deviations in the $i$th prediction factor. This component accounts for all observed replication variance when (a) there are no design level deviations in any of the remaining prediction factors, and (b) no unidentified error variables are in operation.

If the chance deviations from design levels of all factors are mutually independent and additive, then

$$S^2_{PFD} = \sum_i S^2_{PFD,i} \tag{59}$$

and the decomposition of $S^2_W$ is given by

$$S^2_W = S^2_{LF} + \sum_i S^2_{PFD,i} + S^2_{UEV} \tag{60}$$

Pavement performance values and prediction factor specifications are for entire pavement sections or highway projects, and not for subsections or subprojects. Replication variance and all components thereof thus refer to performance differences among independently constructed sections that all have the same designs and treatments.

Actual as-constructed levels of prediction factors may deviate from their specifications both within and between replicate sections. It must be assumed that either or both of the deviations between and within can contribute to performance prediction error variance. It follows that $S^2_{PFD}$ should be further separated into components between and within $S^2_{PFD,FB}$ and $S^2_{PFD,DW}$, respectively. Thus, the final decomposition of the prediction error variance may be written

$$S^2_{PFD} = S^2_{PFD,FB} + S^2_{PFD,DW} \tag{61}$$

so that

$$S^2_W = S^2_{LF} + \sum_i S^2_{PFD,FB,i} + \sum_i S^2_{PFD,DW,i} + S^2_{UEV} \tag{62}$$

Empirical derivations of estimates for all variance components in Equation 62 would require a highly controlled long-term experiment in which the mean value of every PF was controlled at three levels. One level would be a pavement design level and the other two levels would represent plus and minus mean deviations that correspond to normal highway construction. For each of the three mean levels of each factor, one test section would have to be controlled so that no within-section deviations occurred and another section would be allowed to have normal within-section variations.

Thus for $k$ prediction factors, $3 \times 2 \times k$ test sections would be required to produce estimates of the components in Equation 61. At least two replications of all factor combinations would be required to produce estimates for $S^2_{LF}$ and $S^2_{UEV}$ in Equation 62, for a minimum total of $12k$ test sections.

For practical reasons, it seems most unlikely that the required experiment will ever be performed, and even less
likely that all individual variance components can be estimated from existing materials, construction, and performance data. On the other hand, performance-related specifications for any PF should take into account not only the performance effects of specified changes in the factor level, but also the error variance contributions implied by Equation 61. Thus the algorithm for M&C specifications derivation should somehow provide for estimating the components in Equation 62 and in the absence of empirical data.

Simulation methods may be used to estimate the needed variance components. One such method is to use Monte Carlo procedures to simulate design level deviations for each prediction factor, then calculate the corresponding variance components for a given prediction equation. Together with assumed values for $S_{LF}^2$ and $S_{BEV}^2$, the simulation could produce estimates for all elements of Equation 62.

A second approach is shown by Figure 6 wherein the horizontal axis represents the range of a single PF, the vertical axis represents a particular performance indicator $\log W_i$, and the central curve represents the graph of a prediction equation,

$$\log W_i = f(\ldots, PF_1, \ldots)$$

in terms of $PF_i$ when all other prediction factors are at fixed levels.

The design level specification $\overline{PF}_i$ for $PF_i$ is the value for which the prediction equation yields $\log W_i$. The mean value of $PF_i$ over many iterations of the construction process is assumed to be $\overline{PF}_i$. Thus, $\overline{PF}_i - PF_i$ is the chance deviation between specified and as-constructed levels of $PF_i$. The dotted frequency distribution centered at $\overline{PF}_i$ is assumed to represent all possible deviations in $PF_i$ that occur in many iterations of normal construction practice. The distribution has variance $s_i^2$ and standard deviation $s_i$ in the units (e.g., inches of surfacing thickness) of $PF_i$.

If all values in the $PF_i$ distribution are projected vertically to the curve for the prediction equation, then horizontally to the vertical axis, a distribution of $\log W_i$ values is produced, as shown in Figure 6. Thus each deviation $PF_i - \overline{PF}_i$ leads to a corresponding deviation $\log W_i - \log \overline{PF}_i$ for the performance indicator. In particular, the deviation $s_i$ for $PF_i$ corresponds to the standard deviation $s_{PF_i}$ in $\log W_i$.

The tangent (dashed line) to the curve at $PF_i = \overline{PF}_i$ has slope equal to the partial derivative of $\log W_i$ when evaluated at $\overline{PF}_i$ and at the design levels of all remaining prediction factors. Approximately, the slope of the tangent is the ratio

![Figure 6](image-url)
\[ S_{PF_i}/s_i \]. If the evaluated derivative is denoted by \( \bar{f}_i \), then approximately,
\[ f_i \equiv S_{PF_i}/s_i \]
or
\[ S_{PF_i} \equiv s_i \times \bar{f}_i \]  
(63)

The derivative may thus be interpreted as the functional conversion factor for changing prediction factor \( PF_i \) units to performance indicator (\( \log W \)) units.

At least for between-section deviations in predictor factors, Equation 63 may be used to estimate the individual and collective performance effects of design factor deviations, as given by Equation 59. Because normal construction variances \( s_i \) are available for many prediction factors, use of Equation 63 can provide estimates of the relative performance effects of deviations in each prediction factor. These estimates in turn can be used as an objective basis for setting M&C tolerances for prediction factors and control factors.

More research is needed on how within-section deviations of prediction factor levels are to be accounted for in the estimation method represented by Figure 6. Further study is also needed to determine the relative efficacy of the Monte Carlo and partial derivative methods that have been discussed for the estimation of variance components.

By one method or another, the algorithm for M&C specifications derivation should produce or have access to estimates for all variance components that are associated with the prediction equations being used.

Figure 7 shows estimates that have been derived for the variance components of the flexible pavement performance prediction equation that appears in the revised AASHTO Design Guide (1). A complete tabulation of all variance components for both flexible and rigid pavements is given in Vol. 2 of the AASHTO Design Guide (2, Appendix EE). The variance of performance prediction errors (Line 1) is shown to be 0.194. The estimated performance variance that is induced by surfacing thickness variance (Line 1.1.1.1) is 0.015 or about 8 percent of the total prediction error variance. Performance variance attributable to variance in subgrade modulus (Line 1.1.1.9) is 0.023, or about 12 percent of the total prediction error variance.

**Specification of M&C Factor Tolerance and Acceptance Levels**

The final pavement design step is the specification of tolerance levels within which controlled factors are permitted to vary randomly about their design levels without cause for remedial action. For convenience, it is assumed that all tolerances are two sided, and that they may be generally represented by \( mcf \pm \Delta mcf, mcf' \pm \Delta mcf' \), and \( mcf'' \pm \Delta mcf'' \) for primary, surrogate, and auxiliary M&C factors, respectively.

Acceptance levels for M&C factors will be considered to be special cases of tolerance levels, and such that M&C suppliers and contractors may be penalized in one way or another whenever the acceptance limits are exceeded. Although acceptance and tolerance levels may be somewhat different

<table>
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</tbody>
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**FIGURE 7** Illustrative variance components of the pavement design-performance process.
for any particular factor, such differences will not be discussed herein.

As was discussed in the previous section, every M&C factor is expected to deviate unpredictably from its design level in any particular iteration of the M&C process. If the deviations are random in repeated iterations of the process, their distributions will have mean value zero, and for the corresponding types of factors will have standard deviations $s_f$, $s_{f'}$, and $s_{f''}$.

It will be assumed that the tolerance factors are specified multiples of the respective standard deviations. If the respective multiples are denoted by $af$, $af'$, and $af''$, then the primary, surrogate, and auxiliary M&C factors may be written as $mcf \pm (af \times s_f)$, $mcf' \pm (af' \times s_{f'})$, and $mcf'' \pm (af'' \times s_{f''})$, respectively.

The multipliers $af$, $af'$, and $af''$ are determined by probability levels that control the chances for erroneous rejection and erroneous acceptance, as in virtually all quality control applications of statistical methods.

These tolerances are appropriate for factors whose standard deviations are independent of their design levels. For factors whose standard deviations are generally proportional to their design levels, it is better to express tolerances in terms of coefficients of variation.

The coefficient of variation $CV$ of any variance is, by definition, the ratio of its standard deviation to its mean value. Coefficients of variation for the respective M&C factors are therefore

$$CVF = \frac{s_f}{mcf}\equiv \frac{CV}{mcf}$$

$$CVF' = \frac{s_{f'}}{mcf'}\equiv \frac{CV}{mcf'}$$

$$CVF'' = \frac{s_{f''}}{mcf''}\equiv \frac{CV}{mcf''}$$

By substitution and factoring, the tolerance limits may be written in terms of coefficients of variation as follows:

$$\frac{mcf(1 \pm af \times CVF)}{mcf'}(1 \pm af' \times CVF')$$

$$\frac{mcf''(1 \pm af'' \times CVF'')}{mcf''}$$

The magnitude of the standard deviation or coefficient of variation for any M&C factor will generally depend upon the level of effort and cost in effect for M&C control of the factor levels. If $q$ is used to denote levels of control that may range from very loose ($q > 1$) to loosener than normal, to normal ($q = 1$), to tighter than normal ($q < 1$), then $q$ can be used at least conceptually as a multiplier for the standard deviations or for the coefficients of variation in the tolerances. For example, if the coefficients of variation represent normal M&C variability for the respective factors, then tolerances for the three types of factors might be expressed as $mcf(1 + q \times af \times CVF)$, $mcf(1 + q' \times af' \times CVF')$, and $mcf''(1 + q'' \times af'' \times CVF'')$. Thus, if $q = 1$ for any factor, its tolerances refer to normal variation in the as-constructed levels of the factor. Tighter-than-normal tolerances are implied by $q < 1$, and looser-than-normal tolerances by $q > 1$.

Although it is outside the scope of this paper to propose specific procedures for derivation of factor tolerances, it appears that the $q$ concept could be related to the variance components that were discussed previously. As was shown in Figure 6, it is possible to estimate the fraction of performance variance that is induced by the individual variances of primary M&C factors $mcf$.

The estimation procedure can be extended to include variances of secondary prediction factors, as was shown in Figure 5 for secondary factor design levels. Such extensions would provide indirect estimates of the fractions of performance variance that are attributable to secondary factor variances. In principle, it would therefore be possible to assess the relative costs and benefits of various levels of control $q$ for both primary and secondary factors. For example, a relatively high fraction of performance variance may be induced by the M&C variances of some primary factors, so that relatively tight control ($q < 1$) is implied for such factors. Other factors may require only loose control ($q > 1$) because their variances have relatively small effects on performance variance. Variances shown in Figure 7 illustrate how the factors can be ranked with respect to their variance effects on performance variance.

Specified degrees of control ($q'/q''$) for secondary factors might reflect the relative amount of primary factor variance that can be attributed to secondary factor variances. At present, little is known about these variance components.

Incentives or disincentives for meeting acceptence limits for any M&C factor would presumably be based on the estimated extent of performance variation that may be induced when the factor does not meet the acceptance limits. Because performance variations can be translated to years of pavement life at assumed ESAL rates per year, it is theoretically possible to estimate probabilities for decreases in pavement life expectancy that are associated with failure to meet specified acceptance limits.

**Summary Framework for Derivation of M&C Specifications**

Figure 8 shows a summary review of the many topics that have been presented in this paper. Framework elements that were shown in Figure 1 also are shown in Figure 8, but in somewhat different groupings and levels of detail. This retrospective overview shows five subgroups of elements in a logical progression from the studies that produce primary relationships to the specification algorithms that produce quantitative specifications for M&C factors. The five subgroups are identified by linkages that contain the subgroup numbers. The essential nature of each subgroup is sketched in the following paragraphs.

**Subgroup 1**

Primary relationships for the prediction of pavement stress, distress, and performance indicators are derived in primary long-term field studies whose inputs are primary prediction factors. Certain primary studies also produce stress-load
equivalence relationships for the conversion of mixed-stress applications to standard stress-load applications. Many primary relationships now exist; improvements and extensions are expected from SHRP studies.

Subgroup 2

Design criteria that include design period traffic predictions and reliability specifications are substituted in the primary prediction equations to produce design-performance requirements for the pavement performance period.

Subgroup 3

Primary prediction factors are separated into M&C factors $mcf$ and non-M&C factors. Surrogate factors $mcf'$ may be identified for certain $mcf$, and still other M&C factors $mcf''$ are identified as auxiliary factors for the control of primary and surrogate factors.

Subgroup 4

Secondary relationships among all M&C factors are derived through short-term laboratory or field studies. The relationships include prediction equations and variance components for the prediction of primary factors from surrogate or auxiliary control factors. The strength of the relationships determines the performance-relatedness of surrogate and auxiliary factors. Many secondary relationships that are needed are expected from ongoing and future studies for the development of performance-related specifications.

Subgroup 5

Quantitative specifications are derived for all primary and secondary factors by algorithms that produce design levels and tolerance-acceptance limits for each factor. Inputs to the algorithms include performance requirements, primary and secondary prediction equations and their variance components, constraints for factor levels, and unit M&C costs for the admissible levels of all factors. The algorithms calculate alternative sets of specifications for design levels and tolerance-acceptance limits, then draw upon cost-benefit criteria to determine optimal specification alternatives for a given design and construction situation. Development of necessary and sufficient algorithms for all design situations will require new results from ongoing and future research.

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