Performance of Thin-Wall Concrete Pipe

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An analytical and experimental study was conducted of thin-wall unreinforced-concrete pipes of dimension ratios 16 through 70 under surcharge loads equivalent to 30 ft of fill. Special attention was directed towards bedding and trench properties and geometries. The advantages and efficiencies of matching the stiffnesses of the bedding with that of the soil envelope are discussed. Narrow trench widths are shown to be more efficient when trench fill is of lesser stiffness than trench walls. Recommendations for practice are made.

Concrete pipes are generally classified as rigid pipes. Thin-gauge metal pipes and plastic pipes are generally classified as flexible pipes. Each has its own strategies for design. As one criterion of performance, ring compression theory for flexible pipes anticipates a stress response to a service load. Stress is not employed as a criterion of performance in the case of ring deflection theory for flexible pipes, except as a parameter related to the possibility of a buckling failure. With the D-load method, neither is stress response to service loads a criterion for the design of rigid pipes.

In recent years, with the introduction of plastic pipe with stiffnesses between the extremes of flexible and rigid, the profession and industry have faced the difficulty of nesting a design strategy for semirigid (or semiflexible) pipe between the existing extremes. Neither design strategy may be extrapolated for application to the semirigid pipe.

As regards the use of rigid pipe theory for plastic pipes, the three-edge bearing test as a measure of performance cannot be adopted. The nonbrittle nature of plastic precludes the possibility of a 0.01-in. crack being used as a criterion (*1*). Rigid-pipe theory as a basis for semirigid pipe design has therefore been rejected.

A number of problems exist should either of the two common flexible pipe theories be adopted as a basis for the design of semirigid pipes. The ring compression theory has all the virtues of a theory rooted deeply in the principles of structural mechanics, including a stress response. The presumption of membrane action of an easily altered geometry of a thin shell under service loads rejects bending in favor of an in-plane thrust. Only a flexible pipe is flexible enough to satisfy this criterion; a semirigid pipe does not qualify.

The inconsistencies encountered in the backcalculations leading to articulation of soil stiffness E' provide the necessary empirical evidence for the rejection of the ring deflection theory as a means of predicting the performance of thin-wall concrete pipe. Gabriel and Blower (*2*) showed that the coupling of soil and structure stiffnesses in the denominator of the equation for the prediction of deflection under service load is more complicated than the arithmetic summation of the two. The extrapolation of the ring deflection method for purposes of semirigid pipe design may lead to gross errors of prediction (*3*).

The industry and profession await a consistent theory that may be applied to all classes of pipe. Loads are attracted to the stiffer elements of the composite structure. The greater the stiffness of the pipe relative to its embedment, the greater the internal force responses of thrust, moment, and shear within the pipe wall. Alternatives of bedding and trench geometries, materials, and compactions add complexity to the problem. The distribution of normal and shear pressures at the pipe-soil interface are important determinants of the mode of pipe response to service loads. The ideal always is a uniform normal pressure that precludes the excitement of flexural stresses within the wall of a circular cross section.

The larger purpose of the studies was to obtain a sense of some favored alternatives for thin-wall concrete pipes. The studies were supported by the California Department of Transportation (Caltrans) and the FHWA (*4*).

EXPERIMENTAL DESIGN

Models of structural analysis always include the assumption of reasonable correspondence with the material, geometric, and connective features of those structural elements being modeled. Computer modeling of structures composed of a multiplicity of structural elements follows the same prescription.

Computer modeling of structural composites of pipes embedded in surrounding soil media was adopted as the strategy for gaining, and maximizing, experience with thin-wall concrete pipes. Physical tests were performed to establish the parameters needed for the computer analyses and to spot check the computer studies. Surcharge loads equivalent to 30 ft of fill were superimposed on buried pipes in test frames designed and built at California State University, Sacramento.

CANDE (*5*) was selected as the program for the computer analyses. Its three levels of solution include the elasticity solution of Burns and Richard (*6*) and two finite element solutions, one of which has self-generating elements.

The experimental design was as follows:

1. Model concrete pipes of 9-in. outer diameter (OD) and wall thicknesses 1/8 in. to 1/2 in. (nominal dimension ratios

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DR = 70 to 16, correspondingly) were buried in one of two loading cells with sand of uniform material and compaction. Surcharge loadings up to 25 psi, equivalent to over 30 ft of fill, were introduced in steps of 2.5 psi. Changes in vertical and horizontal diameters were measured (Figure 1).

2. With the assumed mechanical properties of pipe material to be described later and for a range of mechanical properties of the sand fill, CANDE was run for the same surcharge loadings as were physical experiments.

3. The outcomes of the physical experiments and the computer modeling of these same experiments were compared. The mechanical properties of the sand were defined when the outcomes matched.

4. Knowledge of the mechanical properties of the sand completed the information required for subsequent computer modeling of a buried pipe in the load frames. Parametric studies of pipe thickness and bedding and trench geometry were conducted with computer models and spot checked by physical experiment.

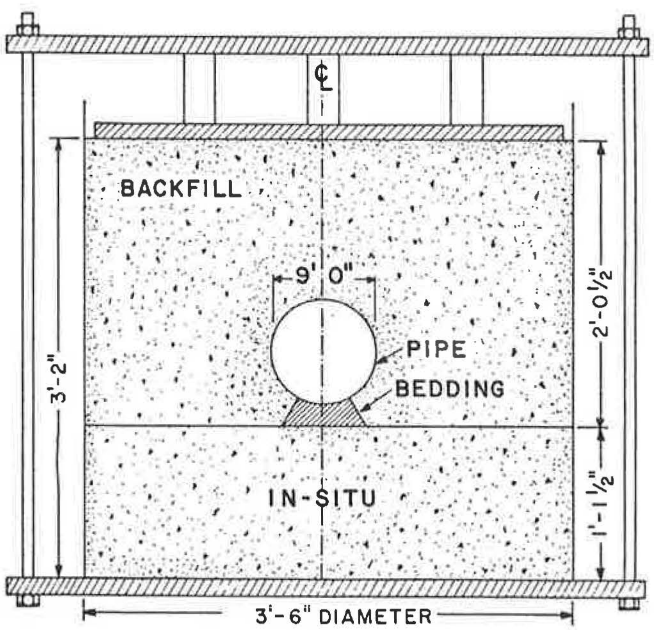
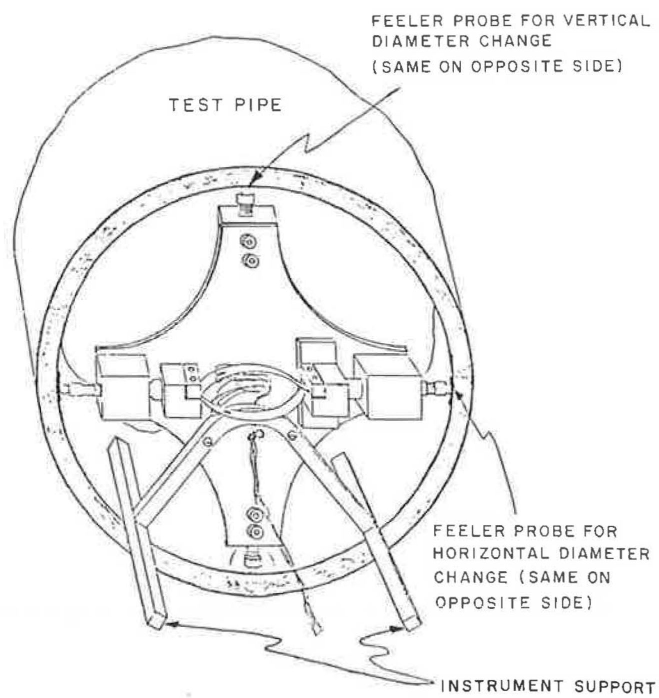


FIGURE 1 Test frame and computer model (schematic).

EQUIPMENT AND INSTRUMENTS

Each of the two load frames consists of 4-ft-high concentric sections of 42- and 48-in.-diameter (nominal) corrugated steel pipe sections forming and enclosing a nominal 3-in. concrete wall. The inside wall corrugations were filled, smoothed, and overlaid with two sheets of lubricated plastic to minimize wall friction on the boundary of the highly stressed soil. Force is transmitted to a rigid plate floating on the composite of buried pipe and surrounding soil by means of instrumented (with three strain gauges at 120° around the circumference of each rod) and calibrated tension rods. These rods, anchored below and attached above to a loading platform, are mechanically loaded by tightening nuts at the rod ends. The reactions to the motion of the loading platform deliver point loads to the rigid floating plate, positioned so as to deliver a uniformly distributed load to the entire soil surface of the soil-surface composite.

Contractions and extensions of the vertical and horizontal diameters were measured by means of a two-axis floating deformation sensor and transducer designed for continuous reading (Figure 2). The device is a pair of independent instruments with axes perpendicular to one another and mounted on a common frame. Each gauge is composed of a pair of spring steel bows of negligible bending stiffness, clamped at the ends, responding to displacements in a bending mode. Each bow has two opposing strain gauges, one centered on the convex side and the other on the concave side. The four gauges of each instrument are wired as a full bridge. Initial contact between a steel ball and a smoothened inner wall of the pipe was made with an adjusting screw at each outboard end. The instrument was able to sense motion to 0.0001 ± 0.00005 in.; its response was linear. The sensing arrangement for the vertical pipe diameter change was similar to that for the horizontal pipe diameter change.



NOTE: SENSING ARRANGEMENT FOR VERTICAL PIPE DIAMETER CHANGE SIMILAR TO THAT FOR HORIZONTAL PIPE DIAMETER CHANGE.

FIGURE 2 Two-axis deformation sensor and transducer.

CONCRETE MODULUS

The property of stiffness of a stressed structural element is a function of the material, geometry, and mode of response. Recall that the deflection of a beam is inversely proportional to its material and geometric moduli EI; the extension of a bar is inversely proportional to EA/L; and the diametral change of a ring is inversely proportional to EI/R^3; where E is the material stiffness and I, A, L, and R are geometric parameters of the element.

Whereas *I, A, L,* and *R* are well defined, *E* is difficult to evaluate when it is other than linearly elastic. The surrounding soil and the embedded pipes are both of nonlinear inelastic materials. The potential for significant error exists for the prediction of the performance of the soil-structure composite.

In Figure 3, a secant modulus E_s and a tangent modulus E_t are both shown operating at some prescribed limit of working stress f of a nonlinear strain-softening material. Because the tangent modulus, at any level of stress and at all points in the pipe, governs the deflection (the secant modulus is only a convenience for design), the assumption of modular values has significant effect on the prediction of deflection. The nonconstant stress levels within the pipe imply further variation in modulus.

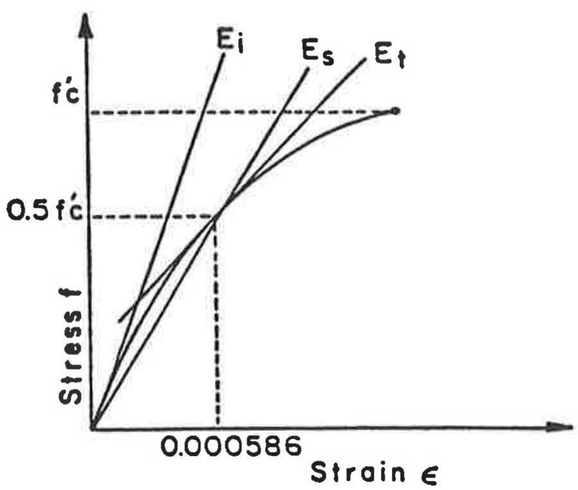


FIGURE 3 Moduli alternatives.

TANGENT MODULUS FOR CONCRETE

Obtaining the measure of tangent modulus is subject to the further disability that for soils and concrete the functional form of the curve is not generally known. The nonsmoothing nature of differentiation necessary to evaluate the tangent modulus adds further difficulty. For concrete, the properties of the stress-strain curve depend, in part, on the choice of materials, manner of preparation and casting, water content, method of curing, functional use, and rate of loading of test samples. Data developed by Ramaley and Henry (*6*) (Figure 4) suggest the appropriateness of a parabolic response between zero and a vertex of ultimate strength f'_c at a strain of 0.002. The equation for such a curve is

$$f = [1 - 250{,}000(\varepsilon - 0.002)^2]\, f'_c \qquad (1)$$

The tangent modulus at any point is

$$E_t = df/d\varepsilon = (f'_c/2)(0.002 - \varepsilon)(10^6) \qquad (2)$$

The measure of the initial modulus is

$$E_i = E(0, 0) = 1{,}000 f_c \qquad (3)$$

To judge the reasonableness of these equations, the results are compared with the estimate of secant modulus for concrete of f'_c = 5,000 psi based upon the American Concrete Institute (ACI) formula

$$E_s = 33 w^{1.5} f'_c$$

Note in Figure 3, the secant modulus lies between the tangent modulus and the initial modulus.

E_t = 3,535,000 psi (from Equation 2);
E_s = 4,074,000 psi (from ACI); and
E_i = 5,000,000 psi (from Equation 3).

The stress-strain curve described by Equation 1 was adopted.

In application, an iterative process was introduced into CANDE to establish an appropriate value for the stress-dependent tangent modulus. At each level of load, a tangent modulus was assumed. With the other parameters of the analysis held fixed, CANDE was run and the maximum stress at springline was noted. This stress was then introduced into the stress-strain law, Equation 1, and the strain was calculated. A revised tangent modulus was then computed from Equation 2, and the process was repeated until convergence, which was rapid.

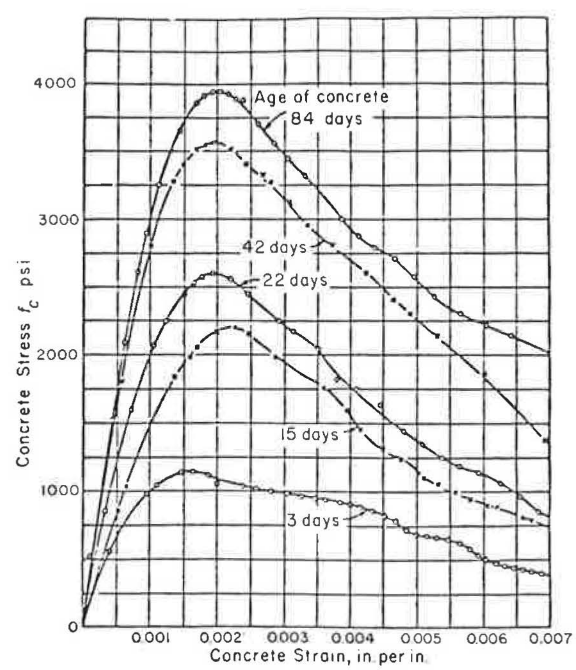


FIGURE 4 Concrete stress-strain curves from compression cylinders.

SOIL MODULUS

Pointwise definition of the mechanical properties of a solid at varying levels of stress permits linear elastic models of analysis to be used as reasonable approximations with nonlinear inelastic materials. A granular soil, however, is not a solid. Movements dissipative of the energy that deforms the soil mass inhibit it from being effectively modeled as a solid. Slippage not only has the potential to occur at the pipe-soil interface, but also at all points of contact within the granular soil mass.

In spite of this, perhaps because of the absence of alternative forms of analysis, the pretense of the solid is carried forth. If the pointwise mechanical properties of a soil were known, then an elastic solid of precisely those same properties would be expected to perform in the same manner, enabling the latter to be modeled to predict the former. The burden then becomes one of properly defining the mechanical properties of the soil.

SELECTION OF SOIL PROPERTIES

For the purpose of learning the in-place property of soil modulus, the following conditions were applied to both the computer analyses and, where appropriate, the physical tests:

1. Nine-inch pipes, of wall thickness varying from 1/8 to 1/2 in. were buried in a homogeneous soil mass.
2. The Burns and Richard (7) elasticity solution was assumed operative.
3. The height of cover above springline was twice the *OD* of the pipe.
4. The compaction and density of the sand was taken as unvarying, achieved in the physical tests by uniform free-fall deposition of the sand grains transported with the assistance of an industrial vacuum.
5. Uniform surcharge was added in increments of 2.5 psi to a maximum of 25 psi.

The following quantities were extracted from the computer analyses:

- Center-crown displacement D_v (in.)
- Crown moment M (in.-lb/in.)

Input into the computer analyses includes:

- Wall thickness t (in.)
- Average pipe radius r (in.)
- *OD* D_0 (in.)
- Presumed operating soil modulus E_s (psi)
- Acting concrete tangent modulus E_c (psi)
- Surcharge pressure p (psi)
- Poisson's ratio ν

The calculated items include

- Pipe modulus $E_p = E_c(t/r)^3/12 = (2/3)E_c/(DR)^3$ (psi)
- Dimension ratio DR
- Soil stiffness ratio $K_2 = 100E_s/(E_p + E_s)$
- Displacement ratio $\%Y = 100D_v/r$

Sample results are presented in Table 1. Note that E_s varies from 1,100 to 1,800 psi. Other results were for presumed values of soil modulus increasing with surcharge pressure (5 to 25 psi) for the following additional ranges: 2,350-3,850; 3,600-5,900; 4,850-7,950; and 6,100-10,000 psi.

TABLE 1 STUDY OF MOMENTS AND DEFLECTIONS—CANDE (LEVEL 1)

cltrns5 STUDY OF MOMENTS AND DEFLECTIONS - CANDE (LEVEL 1)

	surch press p psi	soil mod Es psi	conc mod Ec psi x10**6	pipe mod Ep psi	crown mom M in-#/in	vert disp Dv in	K1	Dv/r x10**-3	Ep/Es x10**-3	K2	slope:K1 vs Ep/Es x10**-3
t (in) = .125	5	6100	5.80	10.80	0.45	.00323	2.51	0.73	1.77	99.82	1417
r (in) = 4.438	10	7200	5.59	10.41	0.81	.0055	2.28	1.24	1.45	99.86	1590
DR = 71.01	15	8300	5.40	10.05	1.10	.0078	1.88	1.76	1.21	99.88	1556
	20	9150	5.21	9.70	1.37	.01003	1.66	2.26	1.06	99.89	1561
code:850107-151718	25	10000	5.02	9.35	1.61	.01209	1.48	2.72	0.93	99.91	1578
t (in) = .25	5	6100	5.79	90.03	3.15	.00271	21.17	0.61	14.76	98.55	1435
r (in) = 4.375	10	7200	5.58	86.76	5.79	.00508	17.59	1.14	12.05	98.81	1460
DR = 35.00	15	8300	5.40	83.97	7.93	.00709	14.97	1.60	10.12	99.00	1480
	20	9150	5.22	81.17	9.94	.00904	13.35	2.04	8.87	99.12	1505
code:850107-152051	25	10000	5.05	78.52	11.73	.01035	12.81	2.44	7.35	99.22	1530
t (in) = .375	5	6100	5.79	317.14	7.34	.00194	73.61	0.44	51.99	95.06	1416
r (in) = 4.313	10	7200	5.59	306.19	14.72	.00371	61.23	0.84	42.53	95.92	1440
DR = 23.30	15	8300	5.40	295.73	20.56	.00528	52.13	1.19	35.64	96.56	1463
	20	9150	5.22	285.92	26.10	.00692	46.47	1.54	31.25	96.97	1487
code:350107-152729	25	10000	5.04	276.06	31.15	.30828	41.30	1.97	27.61	97.31	1514

Ep = Ec*I/r**3 = (Ec*(t/r)**3)/12 = (2/3)*(Ec)*(1/DR)**3

K1 = (M/(Es*Dv*Do))*100 , where Do = outer dia = 9 in.

Dv = vert dia disp - crown/center

DR = Dimension Ratio = 2*r/t

K2 = (Es/(Ep+Es))*100

For the dense silica sand of their study, Duncan and Chang (*8*) assumed a value for Poisson's ratio of 0.3 for all values of a full range of confining pressure. The silica sand of this study is also very dense; a relative density of 98 percent was determined by the Caltrans Transportation Laboratory. Based upon the comparisons of the sands, a value for Poisson's ratio of 0.3 was adopted for all values of confining pressure.

Within the bounds of the noted presumed operating values of soil moduli, there is assumed to exist one set of modular values that closely represents the conditions operating in the test frames of the physical experiments. The outcomes of the experiments are sets of vertical and horizontal diameter changes for increasing surcharge pressures. The plots of deflection versus stiffness ratio (Figure 5) are a result of the computer analyses. Introduced into these charts are the measured vertical diameter deflections of the physical experiments. For example, the vertical deflection, when the surcharge pressure is increased from 10 to 15 psi in the physical experiment, should be laid out on a template to the same scale as the appropriate chart and then placed vertically so that it is precisely intercepted at its endpoints by the 10- and 15-psi lines drawn on the chart. Where the extension of this line intersects the horizontal axis, the relative stiffness K_2 compatible with the performance of the pipe-soil structure of the physical test is evaluated. With E_p known, E_s is evaluated. That process, extended to the full range of testing, leads to a calculation of acting soil moduli. With modification reflecting smoothing of results and the adoption of a midrange of values between the last two of those developed by analysis, the assigned values of soil moduli for varying surcharge pressures are noted in the following table.

Surcharge (psi)	*Acting Soil Modulus (psi)*	*Assigned Soil Modulus (psi)*
5.0	5,150	5,090
7.5	5,900	5,570
10.0	6,920	6,040
15.0	8,100	6,930
20.0	8,200	7,670
25.0	—	8,350

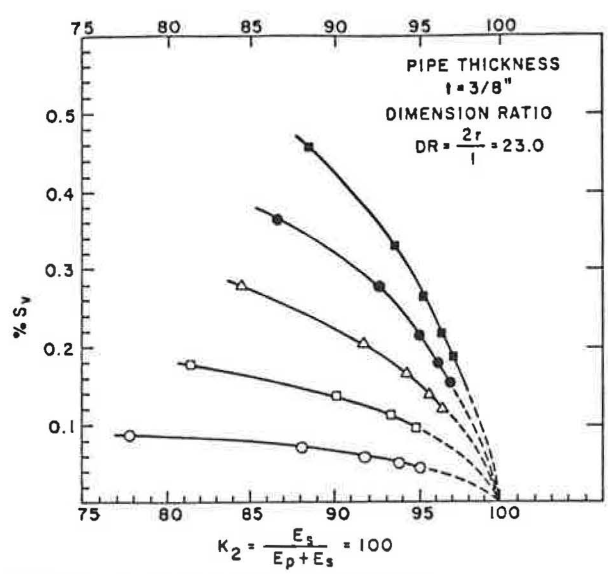


FIGURE 5 Deflection versus stiffness ratio.

BEDDING STUDY

Practice holds that the D-load strength requirement for a rigid concrete pipe, as determined by the three-edge bearing test (*1*), increases as the quality of bedding falls from Class A to Class D. A concrete cradle, bedding of the highest quality, may be required to longitudinally support a pipe over incompetent or irregular ground supports. The cradle shape is intended to distribute the reaction across the pipe-bedding interface.

A study was conducted on the influence of bedding on the performance of 9-in.-*OD* pipes of varying wall thickness (also, varying stiffness) interacting with bedding of varying stiffness including cradle spans of *OD*/2 and flat bedding. Figure 6 shows the geometric layout of the computer model using CANDE, Level 3. Note that 12 pipe elements make up the half-circle. Input to CANDE includes the measures of pipe and soil stiffnesses previously discussed. A shell structure is known to be sensitive to sharp changes in curvature and sharp changes in loading, in that each gives rise to large moments and the associated flexural stresses. Because the curvature of a circular pipe is constant, a preferred design would be one in which the loading around the pipe circumference would also be constant. The study established an understanding of the relationships between relative stiffnesses of pipe, bedding, and surrounding soil, and resulting interface loads, moments, and stresses.

Analyses were made for the embankment condition of 9-in.-*OD* pipes of wall thickness 1/8, 1/4, and 1/2 in. (of

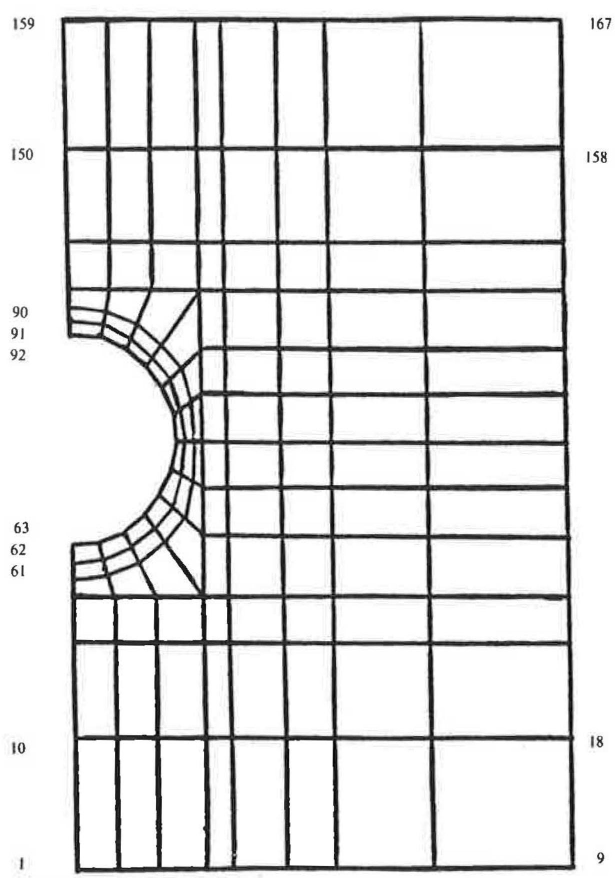


FIGURE 6 Finite element network nodal points.

DR = 70, 34, and 16, respectively); overburden pressures 5 to 25 psi; and bedding stiffness 1,000 to 4,090,000 psi.

For shaped bedding and surcharge loads of 25 psi, Figures 7 and 8 show the normal pressure at the interface and bending moment in the wall. All data pertained to an embankment condition with bedding of stiffnesses 1,000, 8,000, 512,000, and 4,090,000 psi. A plot of maximum tensile stress at any circumferential location along the pipe versus bedding stiffness for varying wall thicknesses is shown in Figure 9. Results for less pressures are closely proportional to those of 25 psi. A comparison of normal pressure and bending moment for shaped and flat bedding, of stiffnesses closely matched to the backfill soil, is shown on Figure 10. Results and inferences follow.

1. For the case of shaped bedding, for pipes of all dimension ratios and for all levels of surcharge pressure, the preferred condition of the smoothest distribution of normal pressure occurs when the stiffness of the bedding and the stiffness of the backfill are closely matched (8,000 and 8,350 psi, respectively, for the study of this report).
2. Shaped bedding of lesser stiffness than the backfill (e.g., urethane foam of stiffness 1,000 psi) results in a less smooth distribution of normal pressure around the pipe than the case of nearly matched stiffnesses.
3. As the shaped bedding stiffnesses increase beyond the favored condition of matched with the backfill, the departure from the preferred smooth distribution of normal pressure becomes more significant. With the higher stiffness shaped bedding ($E_s > E_p$), separation occurs between the pipe and the bedding except at the two endpoints of the bedding, due to the ovality of the pipe cross section. The unyielding endpoints of stiff shaped bedding introduce the undesirable condition of point loads.
4. Higher moments, at the edge of the bedding, always attend the less uniform loads.
5. For the condition where the bedding stiffness nearly matches the backfill stiffness, flat bedding with pockets of softer fill at the 5 and 7 o'clock regions due to poor compaction results in less uniform pressure and higher moments than shaped bedding.

TRENCH STUDY

One design strategy for minimizing loads attracted to a buried conduit is to place the conduit in a trench and backfill with a material less stiff than the soil forming the walls of the trench. The trench width is the variable of interest for the

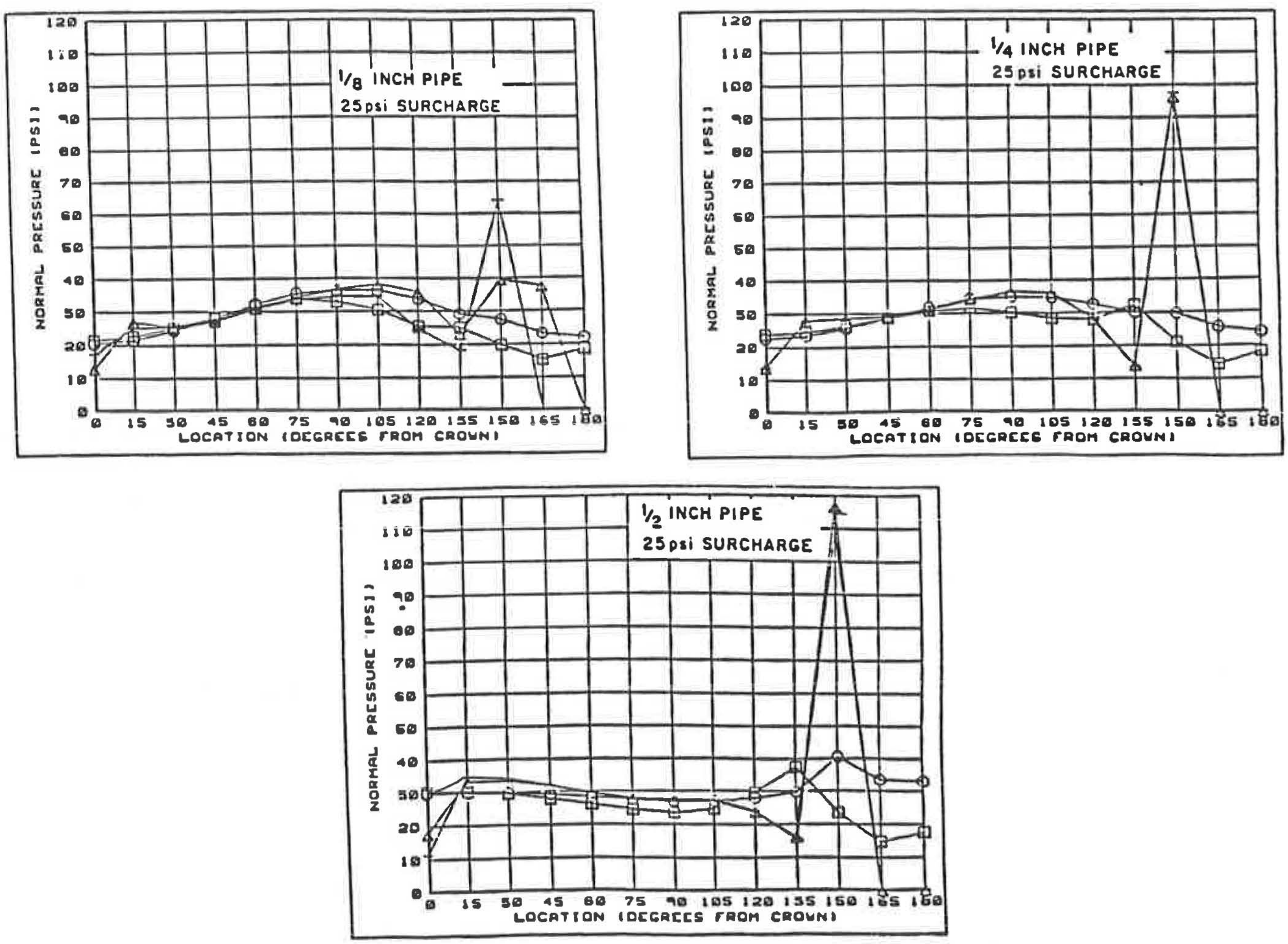


FIGURE 7 Normal pressure versus location on pipe.

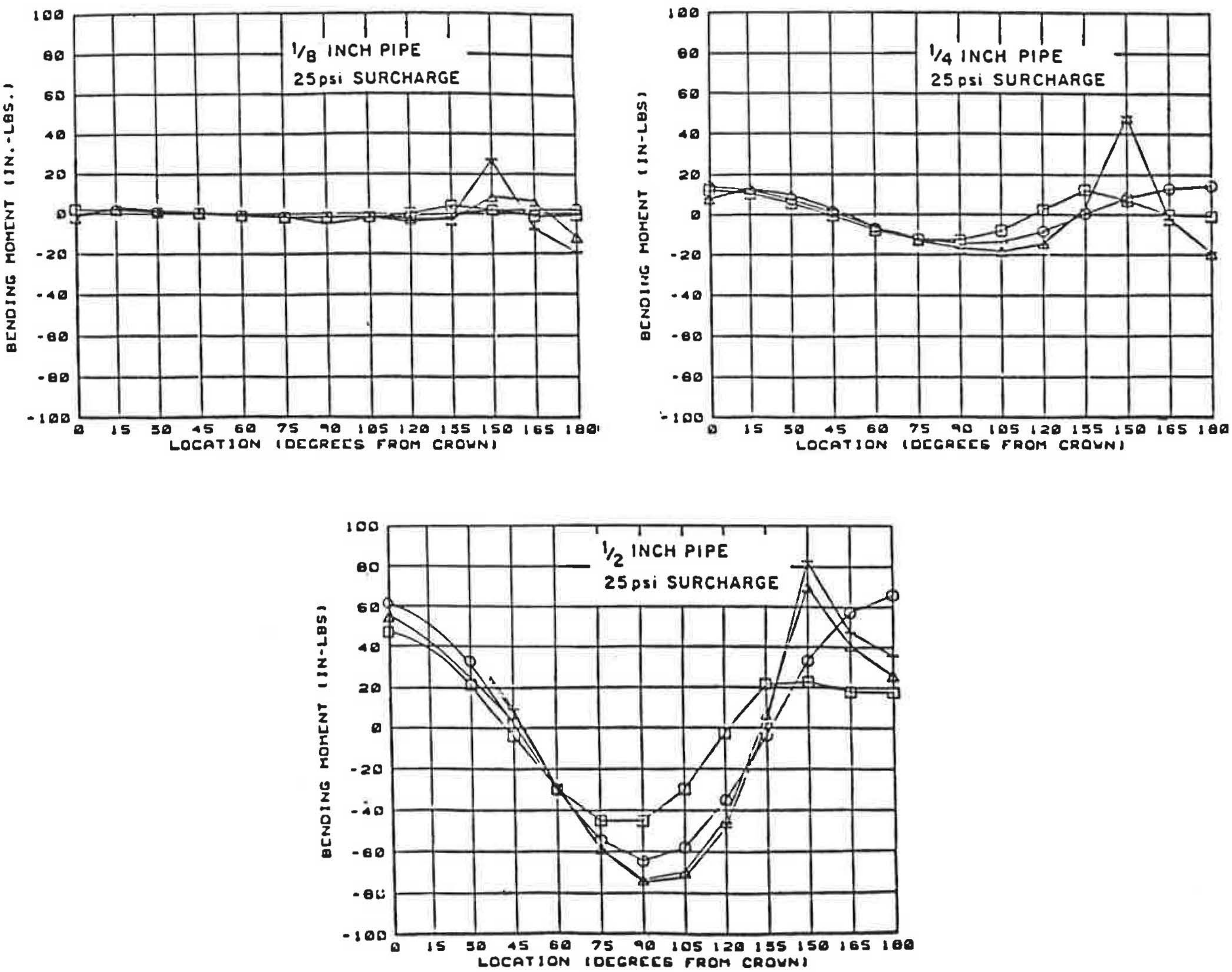


FIGURE 8 Maximum bending moment versus location on pipe.

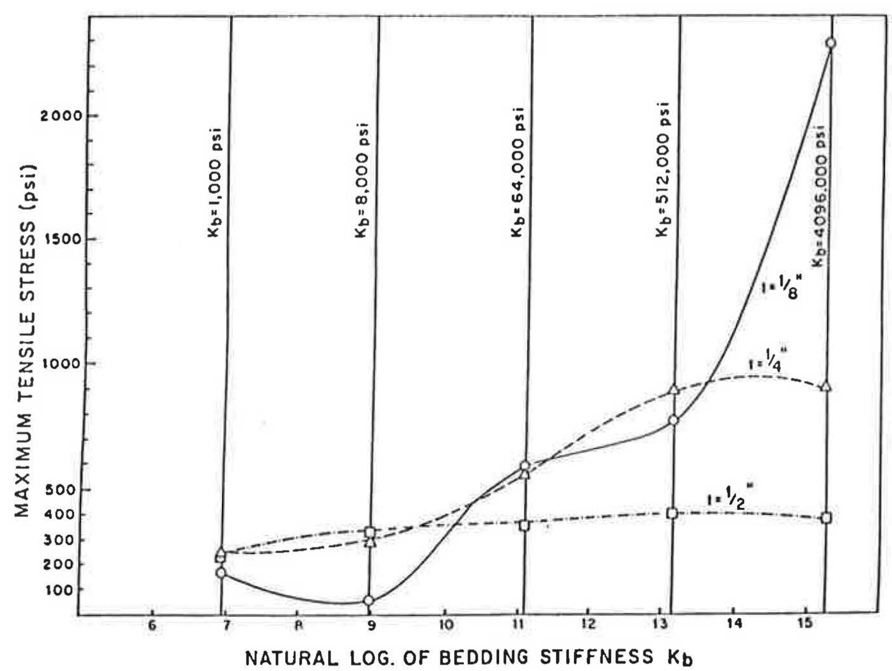


FIGURE 9 Maximum tensile stress versus bedding stiffness.

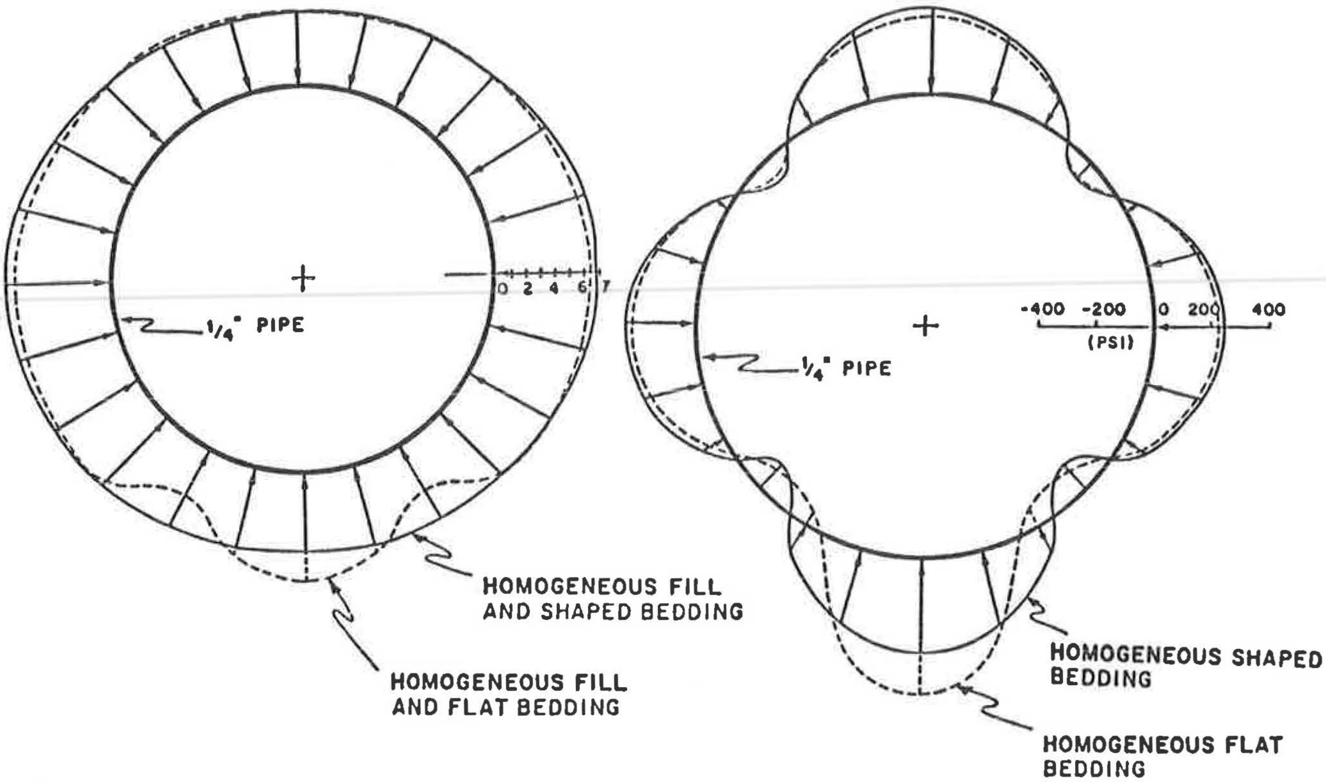


FIGURE 10 (a) Normal pressure diagram for pipe on shaped and flat beddings, and (b) Tensile stress diagram for 1/4-in. pipe on shaped and flat beddings.

studies reported herein. The support at the invert was always taken as shaped bedding with its stiffness nearly matching that of the side walls. The thickness of the pipe wall (a measure of the stiffness of the pipe) was varied. The measure of performance is the maximum tensile stress in the wall, chosen because of the brittle fracture characteristics of unreinforced concrete, and arrived at by the algebraic sum of the thrust and bending stresses. CANDE, Level 3, was the instrument of analysis with spot checks by physical testing.

The results point to the incompleteness of the notion that reducing the portion of the load attracted to the buried conduit by means of less competent trench fill is in general a preferred design. Again, as it was with the bedding study, the character (distribution) of the loading may dominate its performance. Analyses were conducted for the trench condition with one diameter (9 in.) of trench fill cover over the crown of the pipe. Pipe wall thicknesses were 1/8, 1/4, and 1/2 in. (*DR* = 70, 34, and 16, respectively). Trench widths were 1.25 × *OD*, 1.50 × *OD*, and 2.0 × *OD*, respectively. Stiffness of the trench wall, shaped bedding, and the material beyond was 5,090 psi; stiffness of the trench fill was 1,000 psi. Surcharge pressure was 5 psi. See Figures 11 and 12 for the following results and inferences.

1. The 1/8-in. pipe developed less tensile stress, both in magnitude and extent around the pipe for all conditions of trench width, than the thicker 1/4- and 1/2-in. pipes. A thinner, more compliant pipe more readily alters its geometry under load, thereby reducing load and moment.

2. The narrowest trench, 1.25 × *OD*, always presented the most favorable performance, independent of the wall thickness. The greater likelihood of the effective development of passive pressure in a narrow trench favors the development of a more uniform normal pressure at the soil-pipe interface. Less bending is implied.

3. The level of tensile stress was always significant enough to require tensile reinforcement.

In this study, it was not possible to get the pipe to survive the full 5 psi of surcharge load in the physical test for the noted conditions of bedding, backfill, and trench geometry. This is consistent with the analytical predictions of tensile stress on the unreinforced concrete sections.

IMPLICATIONS FOR THIN-WALL CONCRETE PIPE

The work of this study has shown that thin-wall concrete pipe can survive loadings that would unlikely ever be predicted by the usual design practice. Because thin-wall concrete pipe may be expected to fail quite early in its loading history when subjected to that test, D-load rigid-pipe theory will, most likely, not be extended to thin-wall concrete pipes.

Should thin-wall concrete pipe ever become thin enough to be judged semirigid, as is the case for some of the sections considered in this study, and for reasons previously stated, it is unlikely that the flexible pipe theories of ring compression and ring deflection will be successfully extended to include such pipes.

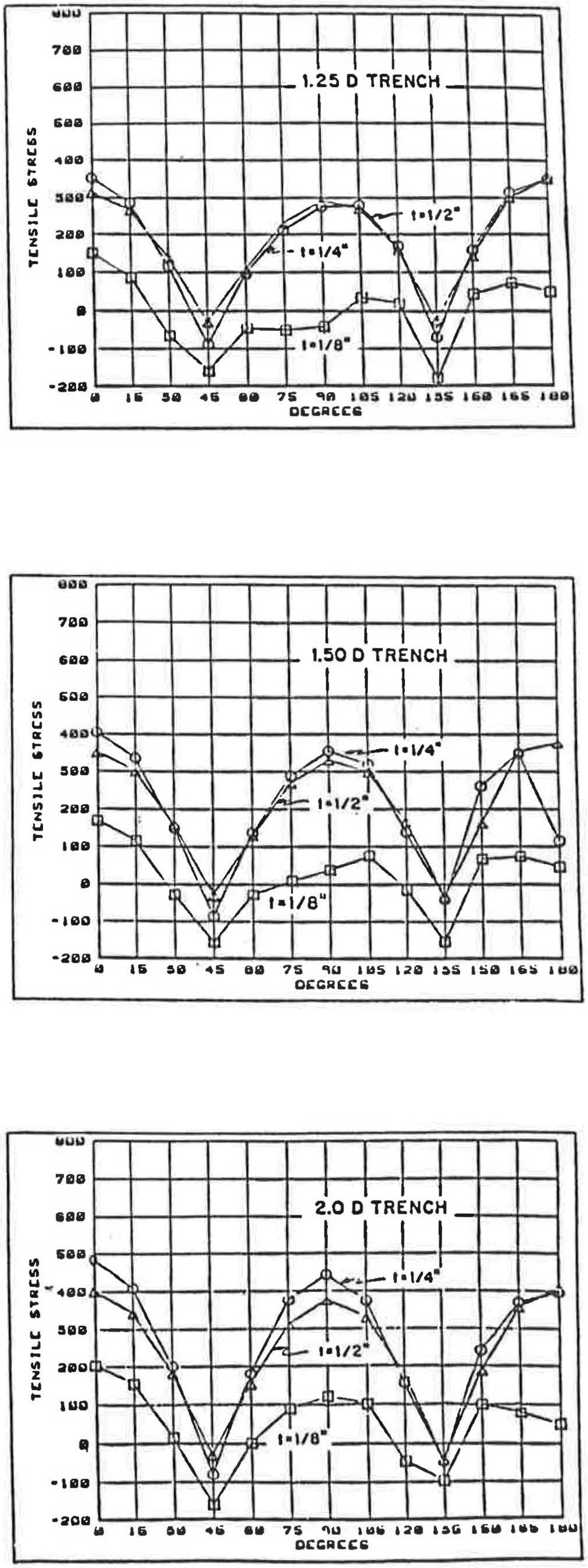


FIGURE 11 Maximum tensile stress versus location on pipe as function of trench diameter.

Note: D = Outer diameter. Surcharge = 5 psi.

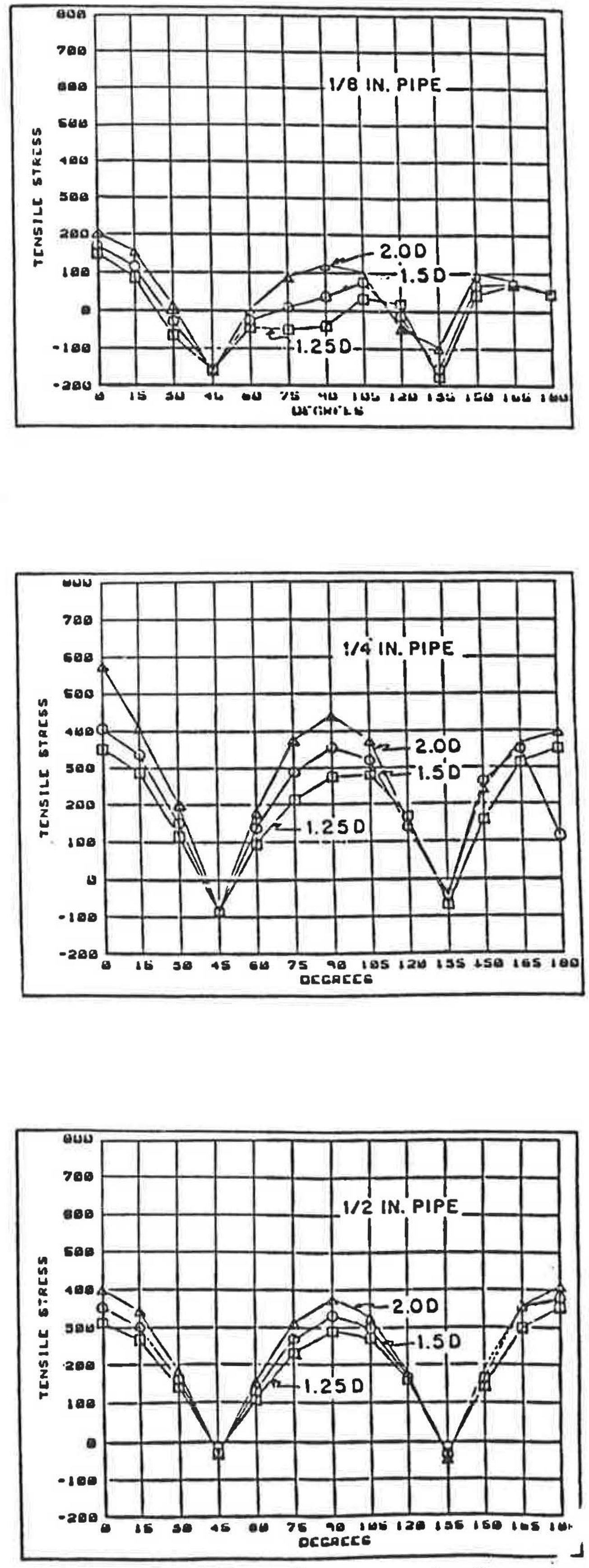


FIGURE 12 Maximum tensile stress versus location on pipe as function of pipe thickness.

Note: D = Outer diameter. Surcharge = 5 psi.

The industry and the profession await a consistent theory for semirigid pipe. In principle, this theory should be broad enough to include pipes of all materials and pipes of all stiffnesses (including both flexible and rigid) interacting with the surrounding soil matrix. The computational power to achieve this end is in place.

CONCLUSIONS AND RECOMMENDATIONS

The following apply to all concrete pipes:

1. Shaped bedding (supporting a surface of approximately $OD/2$) that is of greater stiffness than that of the surrounding soil is likely to require greater pipe strengths than shaped bedding of stiffness matching that of the surrounding soil. Such shaped-concrete beddings should be rejected for any purpose other than longitudinal support of the pipe, if required.
2. Shaped beddings more compliant than the surrounding soil offer little advantage, and possibly some disadvantage, in the performance of an embedded pipe.
3. The most favorable bedding is one that is shaped and has a stiffness approximating that of the soil envelope around the pipe. Standards ought to reflect this principle.
4. Narrow trenches, backfilled with material less stiff than the outer soil envelope, perform more efficiently than wider trenches with the same trench fill. Standards should specify a maximum trench width, compatible with reasonable construction practice, rather than a minimum trench width as is sometimes the case.

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