Passive Lateral Earth Pressure Development Behind Rigid Walls

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An analytical solution procedure is described to estimate the developed passive lateral earth pressures behind a vertical rigid retaining wall rotating about its toe or top into a mass of cohesionless soil. Various stages of wall rotation, from an at-rest state to an initial passive state to a full passive state, are considered in the analysis. A condition of failure defined by a modified Mohr-Coulomb criterion and equilibrium conditions are used to obtain the necessary equations for solution. The development of friction along the wall surface at various stages of wall rotation is also taken into account in the analysis. Finally, the results predicted by the developed method of analysis are compared with those obtained from the experimental model tests on loose and dense sand. The comparisons show good agreements at various stages of wall movement.

The estimation and prediction of the lateral earth pressure development have been among the most important aspects in geotechnical engineering. The development of active lateral earth pressures in particular has received a considerable amount of attention, because a majority of retaining structures are designed based on the active lateral earth pressures due to the tendency of outward movement. However, design of many geotechnical structures requires consideration of passive lateral earth pressures. Several analytical and experimental studies have been made in the past to investigate the magnitude and distribution of passive lateral earth pressures developed behind the retaining walls (*1-5*). These studies have been helpful for understanding the mechanism of the development of passive lateral earth pressures. However, most of the analytical studies fail to provide adequate comparisons with experimental model test results. This failure may be in part due to the uncertainties associated with the variations of the soil strength and the wall friction with respect to the magnitude of wall movement. A need for an analytical solution that takes into account the variation of material properties at various stages of wall movement therefore has been realized.

This paper presents an analytical solution method that describes the transition of the passive lateral earth pressures from an at-rest state to an initial passive state to a full passive state behind a vertical rigid wall rotating about its toe or top into a mass of cohesionless soil. The at-rest state is defined as a stage of no wall movement. The initial passive state refers to a stage of wall rotation when only the soil element either at the top or at the toe of the wall, depending upon the location of rotation center, experiences a sufficient amount of deformation (limiting deformation) to achieve a limiting passive condition. The full passive state occurs when the entire zone of soil elements from the top to the toe of the wall is in the limiting passive condition.

When the retaining wall rotates about its toe, the initial passive state will be developed initially at the top of the wall, whereas the wall rotation about its top produces the initial passive state at the toe of the wall first. The original concept of this approach and the theoretical formulation and the numerical procedures applied to the solution of active lateral earth pressure development have already been described by the authors (*6*).

The analytical method developed has been applied to estimate the passive lateral earth pressures behind a rigid retaining wall experiencing rotations about its toe and top at various stages of rotation. The results are compared with those of experimental model tests and show in general good agreements, indicating the effectiveness of the proposed method of analysis.

THEORETICAL BACKGROUND

Full details of the formulation have been presented by Bang and Kim (*6*) previously; therefore only a brief summary of the principal features is given in the following paragraphs.

The fundamental equations governing the behavior of the system are those for two-dimensional plane-strain equilibrium (*7*).

$$\partial\sigma_x/\partial x + \partial\tau_{xz}/\partial z = 0$$

$$\partial\tau_{xz}/\partial x + \partial\sigma_z/\partial z = \gamma \qquad (1)$$

where γ indicates the unit weight of the soil. The center of the coordinate is located at the top of the retaining wall with x and z axes being taken positive toward the backfill and downward, respectively. It is obvious from Equation 1 that an additional equation is necessary to solve for the three unknown stresses. Using any familiar failure criterion for this purpose will lead to the solution at the limiting state, that is, at failure. Sokolovskii (*8*) solved this problem with a Mohr-Coulomb failure criterion in 1965. In this paper, however, to describe the transition of the passive stresses from the at-rest to the full passive state (the limiting state), a relationship

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between the major and minor principal stresses has been assumed:

$$\text{Principal stress ratio} = (1 - \sin\psi)/(1 + \sin\psi) \qquad (2)$$

In Equation 2, the angle ψ describes the slope of the line tangent to the stress Mohr's circle (Figure 1). Note that if the angle ψ equals $-\phi$ (the internal soil friction angle), Equation 2 reduces to Rankine's passive lateral earth pressure expression.

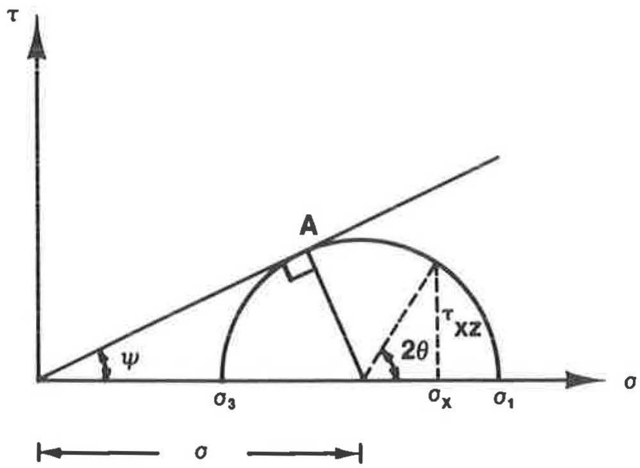

FIGURE 1 Mohr-Coulomb stress relationship.

The angle ψ is assumed to vary in magnitude from ϕ_0 to $-\phi$, where ϕ_0 indicates the inclination angle relating σ_1 and σ_3 in the at-rest state. Note that a negative magnitude of friction angle is used for the purpose of developing a general formulation, that is, a positive friction angle describes the transition of active lateral pressures. The value of ϕ_0 can be easily obtained if the at-rest lateral earth pressure coefficient K_0 is known.

$$\phi_0 = \sin^{-1}[(1 - K_0)/(1 + K_0)] \qquad (3)$$

By varying the angle ψ from ϕ_0 to $-\phi$, Equation 2 could represent both the at-rest and full passive states. However, when the wall experiences movements other than translational, the resulting rotation may produce different stress ratios at different depths. In other words, a portion of the backfill soil may experience deformations exceeding the limiting value, whereas the remaining portion may not. The former case would then achieve the $\psi = -\phi$ state and the latter case the $\phi_0 > \psi > -\phi$ state. Therefore, the angle ψ describing the relationship between the major and minor principal stresses may have to be described as a function of the depth z.

Based on this assumption and Mohr's circle relationship, three unknown stresses can be expressed as

$$\sigma_x = \sigma[1 + \sin\psi(z)\cos 2\theta]$$
$$\sigma_z = \sigma[1 - \sin\psi(z)\cos 2\theta]$$
$$\tau_{xz} = \sigma\sin\psi(z)\sin 2\theta \qquad (4)$$

where

σ = $(\sigma_1 + \sigma_3)/2$ at any depth z, and
θ = rotation angle from the x-axis to the direction of σ_1 measured clockwise positive (Figure 2).

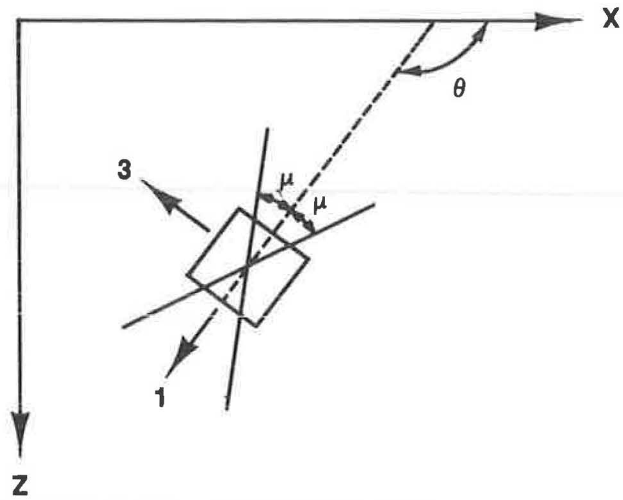

FIGURE 2 Orientation of pseudoslip lines.

Substitution of Equations 4 into Equations 1 leads to the following pair of differential equations (*6*):

$$d\sigma + 2\sigma\tan\psi(z)d\theta = \gamma\,(dz + \tan\psi(z)dx) + \sigma\,[\partial\psi(z)/\partial z]\,dx \qquad (5)$$

$$d\sigma - 2\sigma\tan\psi(z)d\theta = \gamma\,(dz - \tan\psi(z)dx) - \sigma\,[\partial\psi(z)/\partial z\,]dx \qquad (6)$$

Equations 5 and 6 contain the four unknowns x, z, σ, and θ. Additional two equations for the necessary solutions can be obtained from the geometry of the slope of pseudoslip lines as shown in Figure 2. As can be seen from the figure, the slopes can be expressed as

$$dz/dx = \tan(\theta \pm \mu) \qquad (7)$$

where μ indicates a rotation angle from the direction of σ_1 to the pseudoslip lines. It is obvious from the Mohr-Coulomb stress circle relationship (Figure 1) that

$$\mu = \pi/4 - \psi(z)/2 \qquad (8)$$

Equations 5-8 can be solved simultaneously with appropriate boundary conditions. A backward finite difference method has been applied for the solution. The resulting expressions are as follows:

$$z_{i,j} - z_{i-1,j} = (x_{i,j} - x_{i-1,j})\tan(\theta_{i-1,j} - \mu_{i-1,j}) \qquad (9)$$

$$z_{i,j} - z_{i,j-1} = (x_{i,j} - x_{i,j-1})\tan(\theta_{i,j-1} + \mu_{i,j-1}) \qquad (10)$$

$$(\sigma_{i,j} - \sigma_{i-1,j}) - 2\sigma_{i-1,j}(\theta_{i,j} - \theta_{i-1,j})\tan\psi_{i-1,j}$$
$$= \gamma[(z_{i,j} - z_{i-1,j}) - (x_{i,j} - x_{i-1,j})\tan\psi_{i-1,j}]$$
$$- \sigma_{i-1,j}(x_{i,j} - x_{i-1,j})\partial\psi/\partial z|_{i-1,j} \qquad (11)$$

$$(\sigma_{i,j} - \sigma_{i,j-1}) + 2\sigma_{i,j-1}(\theta_{i,j} - \theta_{i,j-1})\tan\psi_{i,j-1}$$
$$= \gamma[(z_{i,j} - z_{i,j-1}) + (x_{i,j} - x_{i,j-1})\tan\psi_{i,j-1}]$$
$$+ \sigma_{i,j-1}(x_{i,j} - x_{i,j-1})\partial\psi/\partial z\,|_{i,j-1} \quad (12)$$

Equations 9-12 completely describe recurrence formulas for the determination of the pseudoslip line coordinates $x_{i,j}$ and $z_{i,j}$, the pseudoslip line slope ($\theta_{i,j} \pm \mu_{i,j}$), and the associated average stress ($\sigma_{i,j}$) in terms of previous values at coordinates i-1,j and i,j-1. The solution process starts from the backfill ground surface whose coordinates and stress values are known and proceeds to the back face of the wall (*8*).

However, the detailed solution steps require a description of the function $\psi(z)$ and its derivative, which define the transition of lateral earth pressures from the at-rest to the full passive state. As discussed before, the function varies from ϕ_0 in the at-rest state to $-\phi$ in the full passive state. The variation between these two extreme values is assumed to be as follows:

1. Rotation about the toe. Let β denote the stage of wall rotation so that $\beta = 0$ for the at-rest state, $\beta = 1.0$ for the initial passive state, and $\beta = 2.0$ for the full passive state. In other words, for values of β between 0 and 1.0, transition from an at-rest to an initial passive state is described with β directly proportional to the deformation of the wall, that is, in the elastic range.

Values of β between 1.0 and 2.0 describe the transition from an initial passive to a full passive state, that is, in the elastoplastic range. Figure 3 shows the schematic variation of $\psi(z)$. At $\beta = 1.0$, the variation of $\psi(z)$ is assumed to be $\psi(z) = -\phi$ at $z = 0$ and $\psi(z) = \phi_0$ at $z = H$, because by definition the initial passive state describes a stage of wall rotation when only the soil element at $z = 0$ reaches a limiting passive condition. The original concept of this approach was first proposed by Dubrova as reported by Harr (*9*) in his method of redistribution of pressures.

The variations of $\psi(z)$ at various values of β assumed in the analysis are shown in Figure 3. They can be expressed, for $0 \leq \beta \leq 1.0$, as

$$\psi(z) = \phi_0 - (\phi + \phi_0)(1 - z/H)\beta$$

$$\partial\psi(z)/\partial z = \beta(\phi + \phi_0)/H \quad (13)$$

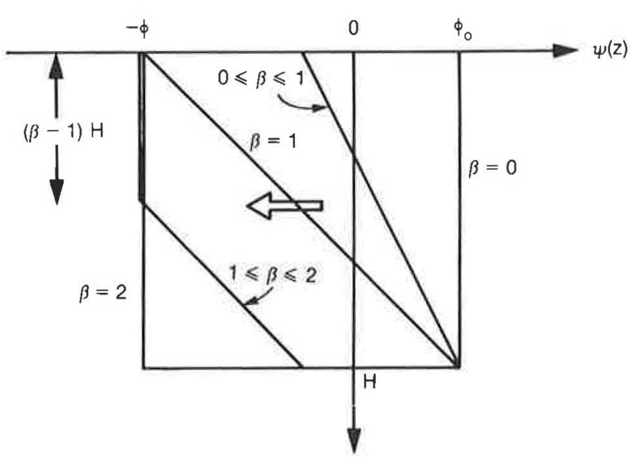

FIGURE 3 Variation of ψ (z), toe rotation.

For $1.0 < \beta \leq 2.0$, within a zone already in the limiting passive condition $[0 \leq z \leq (\beta - 1)H]$,

$$\psi(z) = -\phi$$

$$\partial\psi(z)/\partial z = 0 \quad (14)$$

Within a zone not yet in the limiting passive condition $[(\beta - 1)H \leq z \leq H]$,

$$\psi(z) = \phi_0 - (\phi + \phi_0)(\beta - z/H)$$

$$\partial\psi(z)/\partial z = (\phi + \phi_0)/H \quad (15)$$

2. Rotation about the top. The description of ψ variation in this mode of retaining wall movement remains the same as for the case of toe rotation, except the initial passive state occurs at the toe of the retaining wall. Applying the same logic, one obtains for $0 \leq \beta \leq 1.0$,

$$\psi(z) = \phi_0 - (\phi + \phi_0)z/H\beta$$

$$\partial\psi(z)/\partial z = -\beta(\phi + \phi_0)/H \quad (16)$$

For $1.0 < \beta \leq 2.0$, within a zone already in limiting passive condition $[(2 - \beta)H < z \leq H]$,

$$\psi(z) = -\phi$$

$$\partial\psi(z)/\partial z = 0 \quad (17)$$

Within a zone yet to reach the limiting passive condition $[0 \leq z \leq (2 - \beta)H]$,

$$\psi(z) = \phi_0 - (\phi + \phi_0)(\beta - 1 + z/H)$$

$$\partial\psi(z)/\partial z = -(\phi + \phi_0)/H \quad (18)$$

The variation of the angle $\psi(z)$ in this mode of retraining wall movement is schematically illustrated in Figure 4.

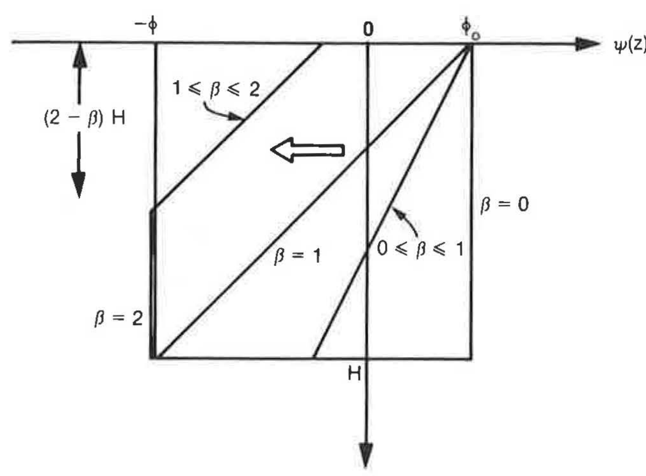

FIGURE 4 Variation of ψ (z), top rotation.

COMPARISON WITH MODEL TESTS

The results from the proposed method of analysis were compared with those from model test results reported by Narain et al. (*4*). The height of the wall was 1.5 ft and hand tamping was used to obtain the desirable soil densities. The tests were performed on dry Ranipur sand using a model wall made of steel. Included were two types of passive wall movement, rotations about the toe and the top. The normal pressures developed on the wall at different displacements were measured using three soil pressure transducers, located at depths of 0.33, 0.88, and 1.33 ft from the top of the backfill. The displacements of the wall shown in Figures 5-8 were measured at its midheight.

Figures 5-8 show the detailed comparisons of predicted and measured passive lateral earth pressures on the wall at various stages of wall rotation. The internal friction angle ϕ of sand and the wall friction angle δ reported by Narain et al. (*4*) with other pertinent soil properties used in the analysis are also included in the figures. The values of initial at-rest lateral earth pressure coefficient K_0 were obtained from the measured earth pressure distributions at rest, which were essentially constant with depth. The unit weights γ of the sand, however, were backcalculated from the relationship between the soil density and its angle of internal friction reported by Sherif et al. (*10*) (because Narain et al. did not report these properties). The values of β indicating the various stages of wall rotation as shown in the figures were obtained from limiting deformations defining the passive state. The limiting deformations were calculated by assuming that at β of 1.8 or greater the passive earth pressures were close to the largest measured values.

As shown in Figures 5 and 6, in general the agreements are good for the cases when the wall rotates about its toe with loose or dense sand backfill, except near the toe of the wall with relatively large deformations. When the wall experiences a rotation about its top, the comparison shows good agreement for loose sand (Figure 7) but not as good for dense sand (Figure 8), particularly at the pressure cell located near the midheight. It is highly unlikely that, as the middle pressure cell measurements in Figure 8 indicate, the passive pressure decreases as the wall rotation increases. Overall, the calculated lateral earth pressures predict measured values reasonably well, considering the uncertain variations in measurements. It is also noted that when the wall rotates about its toe, parabolically-shaped pressure distributions are obtained during transition periods from both analytical and experimental results (Figures 5 and 6). Similar observations have been made during the study of active lateral earth pressure transition, supporting many previous researchers' findings, both analytical and experimental, that suggest a similarly shaped pressure distribution.

CONCLUSIONS

A numerical solution method has been developed to estimate the magnitude and distribution of the passive lateral earth

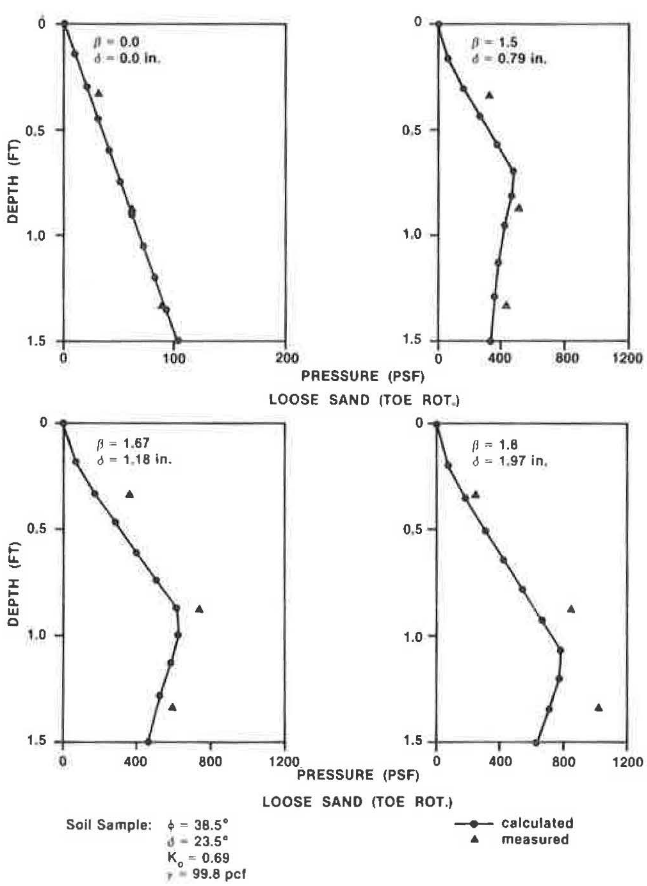

FIGURE 5 Model test comparison for loose sand, toe rotation.

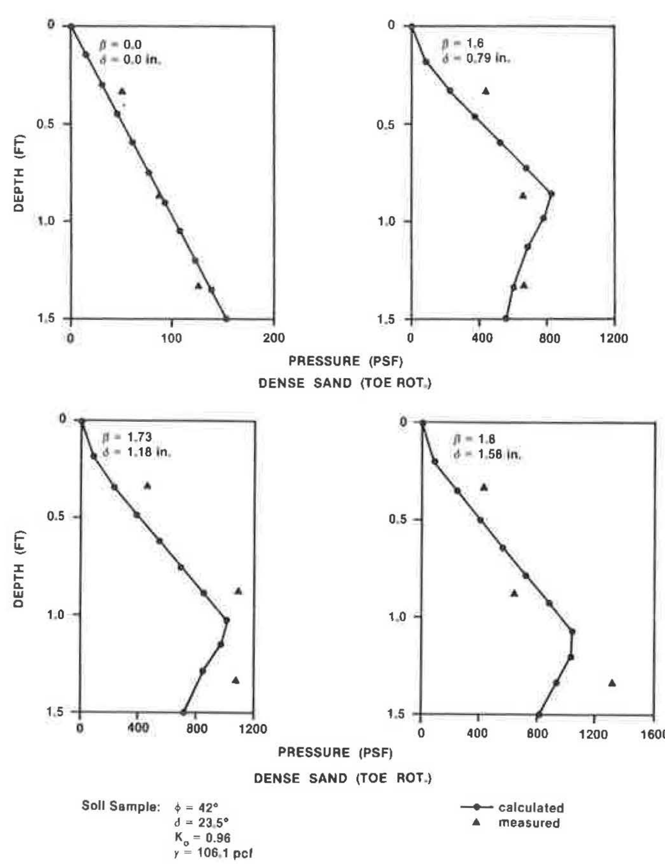

FIGURE 6 Model test comparison for dense sand, toe rotation.

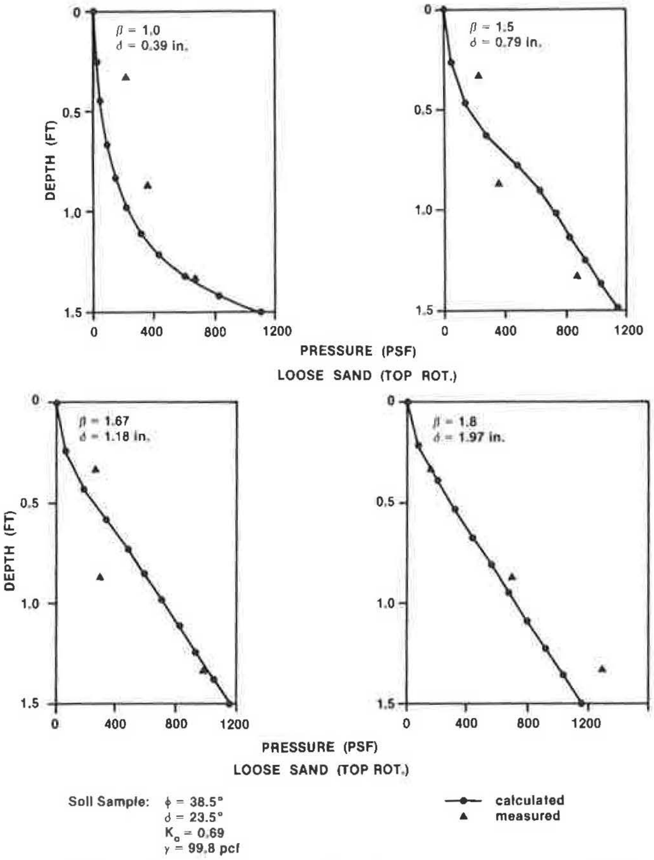

FIGURE 7 Model test comparison for loose sand, top rotation.

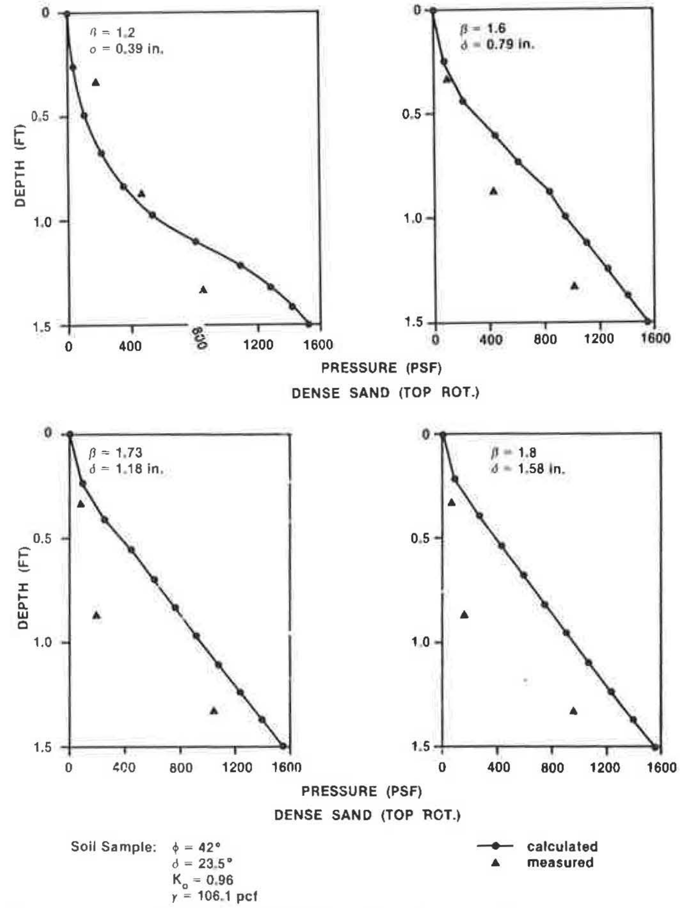

FIGURE 8 Model test comparison for dense sand, top rotation.

pressures behind a vertical rigid wall supporting cohesionless backfill soil. Included are various stages of wall rotation about either the top or the toe. The developed method is capable of predicting the transition of the passive lateral earth pressures, starting from the at-rest state associated with no wall movement to the initial-passive to the full-passive state when the entire soil mass is in limiting equilibrium state. Comparisons with several experimental model test results have also been made and good agreements are observed.

The proposed solution method can be further improved without difficulty to take into consideration the depth-dependent strength and material characteristics, the sloping backfill, and the layered soil deposits. It can also be expanded to analyze the transition of lateral earth pressures associated with various types of wall movement, including the translation and the rotation about the midheight under an active or passive condition.

The developed solution method includes many assumptions; namely, the limiting deformation to achieve a passive state, the validity of Mohr-Coulomb failure criterion, and the relationship between major and minor principal stresses. These assumptions should therefore be studied further, as additional experimental data become available, so that the true behavior of the lateral earth pressure transition can be modeled effectively. The effects of various parameters defining the system can then be analyzed in detail through an analytical parametric study.

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