Coefficient of Subgrade Reaction for Footings on Desert Sands

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The selection of the coefficient of subgrade reaction K for the design of rafts, beams, and rigid pavements is not satisfactory because of the wide range of values recommended in the literature for each soil type and density. The results of recently conducted field footing tests at seven sites in Kuwait are analyzed for the purpose of relating the vertical subgrade coefficient K to the standard penetration test values for desert sands. The footings were square plain concrete, cast directly on the ground surface and ranged in size from 0.25 m to 1 m. Several variables were investigated including footing size, soil properties, footing depth, and material. A simple relationship was found between K and the standard penetration test values that takes into account the effect of the footing size or the width of the loaded area. Test results are compared with other plate load tests on granular soils in different areas around the world, to establish confidence in the derived correlation and to explain the causes of variation in different granular deposits.

Continuous beams, raft foundations, and rigid pavements are frequently analyzed by the elementary theory of beams on elastic foundations (1). This theory is based on the Winkler's hypothesis that the contact pressure, q, at any point is proportional to the deflection, S, at the same point. The constant of proportionality, K, is called the coefficient of subgrade reaction. This coefficient is not actually constant but depends on the type of soil, the stress level, and the shape, dimensions, and rigidity of the footing. Theoretical investigations (2) indicate, however, that with proper selection of the values of the coefficient of subgrade reaction, reasonable accuracy may be expected with the use of the Winkler model.

For design purposes, the choice falls between this approach, followed by relatively simple computations, or the assessment of more fundamental material properties such as the soil modulus E and Poisson's ratio v, accompanied by finite element calculations applied to the continuum representation of the problem (3). Because of the cost of the latter approach and its present unsuitability for design processes, many design engineers choose the former method, with, however, an increasing use of computers to perform the detailed calculations.

Since the comprehensive work undertaken by Terzaghi (4), little has been done to determine the coefficient of vertical subgrade reaction on a rational basis. The range of vlues recommended by Terzaghi for a square plate 0.3 m * 0.3 m (1 ft 1 ft) resting on dry or moist sand are 6–18 MN/m³ (20–60 ton/ft³ for loose sand, 18-90 MN/m³ (60–300 ton/ft³) for medium sand, and 90–300 MN/m³ (300–1,000 ton/ft³ for dense sand. Terzaghi further proposed using values of 12, 39, and 150 MN/m³ (40, 130, and 500 ton/ft³) for loose,

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medium, and dense sand, respectively, with the corresponding reduced values of 7.5, 24, and 90 MN/m³ (25, 80, and 300 ton/ ft^3) for submerged sand. Scott (3) found that these values could be obtained from the relationship $K = 4 N \text{ ton/ft}^3$, where N is the standard penetration test (SPT) value, and recommended that they be increased by 50 percent since the Terzaghi values based on bearing plate data are conservative.

Bazaraa and Howeedy (1) analyzed the results of 83 plate load tests on dry, medium-to-fine compacted sand fill at Brownville, Nebraska, and correlated the coefficient of subgrade reaction with the corresponding values of the relative density, D_r , determined for the sand at both sides of the plate. The same workers analyzed the results of 60 plate load tests at different sites in the United States, the United Kingdom, Australia, Czechoslovakia, Jamaica, and Hong Kong, where granular deposits were encountered. The value of the coefficient of subgrade reaction for these tests was correlated with SPT Nvalues for each deposit. This was the first attempt to provide a simple and rational method for the selection of the coefficient of subgrade reaction.

Analyzed in this paper are the results of a recent field footing testing program carried out by Allison (5) at seven sites in Kuwait. The aim is to establish a simple correlation between the coefficient of vertical subgrade reaction and the SPT N values for foundations on calcareous desert sand. This is to provide accurate values for easy and convenient use by design engineers. Several variables are investigated herein including the effect of footing size, footing depth, and material. The results are compared with the preceding work (1) on different granular deposits around the world.

TESTING PROGRAM

The program carried out by Allison (5) included four series of tests conducted to establish the following: I, effect of footing size; II, effect of footing depth; III, influence of footing material; and IV, effect of soil properties.

Test series I–III were conducted at one site called KISR. Series IV was performed at the same site and at six others. Figure 1 shows the locations of test sites in Kuwait, which is situated at the tip of the Persian Gulf. SPTs were performed at each site and soil samples were taken for identification and analysis. An evaluation of the soil properties was made in the same locality. Ismael



FIGURE 1 Location plan showing test sites.

The tests were carried out using a loaded tank transporter to provide 445 kN (50 ton) of load. The test procedure was in accordance with ASTM standard D1194-72. Loads were applied to each footing in cumulative equal increments of 45 kN (5 ton) using a 445-kN (50-ton) hydraulic jack acting through a series of machined steel plates of gradually increasing diameter. This is to spread the load evenly onto the footings, which were of concrete cast directly on the ground surface. Each load increment was maintained for a time interval of not less than 15 min and until the vertical movement had ceased. The loads were not cycled during the tests.

Settlement measurements were made using mechanical dial gauges bearing on a plate fixed to the base of the jack. Failure was not reached in any of the tests because the ultimate bearing capacity was not investigated herein; however, tests were carried out beyond the permissible settlement of footings under load.

SOIL CONDITIONS

Borings at the test sites revealed similar soil stratification. The soil profile consists of an upper surface layer of compact, relatively dry, fine-to-medium nonplastic, cohesionless, windblown sand with little silt. This is underlain by a more competent marine-deposited, slightly plastic, dense-to-very-dense, lightly-to-moderately cemented calcareous silty sand known locally as gatch. The upper layer, in which all the tests were carried out, extends well below the significant zone under the footings, and is classified as poorly graded sands, SP, or well-graded sands, SW, according to the Unified Soil Classification System. The limits of the grain size distribution curves for 20 selected samples from this layer, at a depth of 0–3 m (10 ft) taken from the test sites, are shown in Figure 2. They indicate



FIGURE 2 Limits of the grading curves for the upper layer [after Ismael (7)].

that the percentage of fines passing No. 200 sieve ranges from 5 to 12 percent. The standard penetration resistance in this layer ranges between 10 and 35 blows/0.3 m and these generally increase in magnitude with depth (6). Chemical analysis on selected samples (6) indicated that quartz constitutes the principal component. The amount of carbonates varies between 10 and 20 percent, mostly in the form of calcite (calcium carbonate), thus indicating calcareous soils.

The lower layer contains between 5 and 35 percent of silt and clay and extends to a great depth over limestone bedrock. It is usually classified as silty sands, SM, or clayey sands, SC.

Soil conditions at the first site, KISR, where test series I-III were conducted, indicated a compact well-graded cohesionless sand (SW), with little silt, to a depth of 4 m (13 ft) over the lower layer. The average SPT value within the significant zone below the test footings (0.5-2 m) is 20 blows/0.3 m. The water level was encountered at a depth of 2.8 m (9.2 ft).

EFFECT OF FOOTING SIZE

Test Series I

To establish the effect of size, three sets of footings were cast in prepared excavations at a depth of 1 m (3.3 ft) below ground level. Each set consisted of four footings: 1, 0.75, 0.5, and 0.25 m (3.3, 2.5, 1.6, and 0.8 ft) square, located in three parallel lines, 10 m (32.8 ft) apart, with 10 m (32.8 ft) spacing between successive footings in each row. The tests were carried out to establish a relationship between the coefficients of subgrade reaction for footings of varying size under the same load intensity. If this is done, further work can be carried out more easily using footings of one size and the results related to larger-size footings.

The test results of this series are shown in Figure 3 in the form of pressure-settlement curves. Each curve represents the average results from three tests. From these curves the settlements corresponding to several load intensities were obtained for all footing sizes. A comparison was made between the average measured and predicted values of the settlement ratio, S_B/S_{B_1} for the different width ratios B/B_1 . Figure 4



FIGURE 4 Settlement ratio versus width ratio for footings on sand [after Ismael (7)].

shows that the data points (7) are located slightly above the curve proposed by Terzaghi and Peck (8) based on the following formula:

$$S_B = S_1 \left(\frac{2B}{B+0.3}\right)^2$$
(1)

where S_B is the settlement of a footing of width B m, and S_1 is the settlement of a 0.3-m (1-ft) square plate loaded to the same intensity. The 0.3 in the denominator becomes 1 when B is expressed in feet.

The validity of this relationship was investigated by Bjerrum and Eggestad (9). Their study of case records indicated that the increase in settlement with the dimension of the loaded area could not be represented by a single curve, and they defined an upper- and a lower-limit curve, and a curve giving an average correlation. Shown in Figure 4 are these curves and the results



FIGURE 3 Load-settlement diagrams of footings of various size [after Allison (5); depth = 1 m].

of D'Appolonia et al. (10), who have reported an appreciable scatter.

The foregoing results, showing excellent correlation with the Terzaghi and Peck relationship (8) for desert sands, are not surprising. Parry (11) has proved the dependence of the settlement ratios on soil conditions by assuming four soil conditions and calculating the settlement ratios on the basis of a direct relationship between measured SPT N values and elastic modulus E. The results indicated that for soils showing improved relative density with depth, as in the case here, the calculated settlement ratios were in close agreement with Equation 1.

Because

$$K_B = \frac{q}{S_B}, K_1 = \frac{q}{S_1}$$

therefore

$$\frac{K_B}{K_1} = \frac{S_1}{S_B} = \left(\frac{B+0.3}{2B}\right)^2$$

therefore

$$K_B = K_1 \left(\frac{B+0.3}{2B}\right)^2$$
 (2)

where K_B and K_1 are the coefficient of subgrade reaction for square footings of width B m and 0.3 m (1 ft), respectively. The 0.3 in the numerator becomes 1 when B is expressed in feet.

The implication of Equation 2, which was proposed by Terzaghi (4), is that the coefficient K_B for a footing of any size decreases, with increasing width to a minimum value of 25 percent, the coefficient K_1 for a 0.3-m (1-ft) square plate. The present tests confirmed the validity of this relationship for desert sands. The analysis by Parry (11), however, indicated that this relationship cannot be used for all soil conditions, particularly those with constant N with depth, implying a decreasing relative density with depth.

Upon completion of the above tests, it was decided to carry out further testing at the other sites using 0.5-m (1.6-ft) square footings and to employ Equation 2 to calculate K_B of larger-size footings.

COEFFICIENT OF SUBGRADE REACTION

Test Series I and IV

After the effect of size was established, a series of tests was conducted at six other sites (5), whose locations are shown in Figure 1. Three footings were cast on each site, each 0.5 m (1.6 ft) square at a depth of 1.0 m (3.3 ft). The results were compared with standard penetration tests carried out adjacent to the foundation pits. The results of the loading tests are shown in Figure 5. Results of other laboratory and in situ soil tests performed in connection with these experiments are listed in Table 1.

From the pressure-settlement diagrams of Figure 5, the coefficient of subgrade reaction was obtained. Because the curves are nonlinear, coordinates must be chosen arbitrarily. In the APPLIED PRESSURE (kPa)



FIGURE 5 Load-settlement diagrams of 0.5-m footings at seven selected sites [after Allison (5); depth = 1 m].

present study the coefficient corresponding to an applied pressure of 300 kPa (3 ton/ft²) was employed in a similar way to the analysis made by Bazaraa and Howeedy (1). The U.S. Corps of Engineers has used the deformation corresponding to 70 kPa (10 psi) for evaluation of subgrades for airfields (12). A comparison made between the two methods with respect to present test results revealed that the difference in the magnitude of K did not exceed on average 5 percent.

The relationship between the coefficient of subgrade reaction and the measured SPT N value below the footing is plotted in Figure 6. As shown, test results on the 0.5-m (1.6-ft) footings are represented by the linear relationship

$$K_{0.5m} = 2.7 N \text{ MN/m}^3$$

= 9 N ton/ft³ (3)

Extrapolating the results to those of a 0.3-m (1-ft) square plate by employing Equation 2,

therefore

$$K_1 = 4.2 N \text{ MN/m}^3$$

= 14 N ton/ft³ (4)

For a square footing of width B m, the coefficient of subgrade reaction will be given by

$$K_B = 4.2 N \left(\frac{B+0.3}{2B}\right)^2 \text{ MN/m}^3$$

or

$$K_B = 14 N \left(\frac{B+0.3}{2B}\right)^2 \text{ ton/ft}^3$$
 (5)

For rectangular foundations having dimensions of B^*L , Terzaghi (4) and Vesic (2) proposed correcting K_B for length to width effects (13), thus:

$$K_{B*L} = K_B \left(\frac{1 + 0.5 \frac{B}{L}}{1.5} \right) \tag{6}$$

TABLE 1 SOIL PROPERTIES ON EACH OF SEVEN LOAD TEST SITES [after Allison (5)]

	Location	Average N Value (blows/0.3m)	Bulk Density (kg/m ³)	Moisture Content %	Water Table Level (m)	Grading of	Sand
Α.	Sulaibiyah	30	1710	1.5	-	Medium, well	graded
в.	Sabah Hospital	25	1630	2.6	10.0	Medium, well	graded
с.	Ahmadi	15	1480	3.5	-	Coarse, well	graded
D.	Abra Kheitan	12	1520	2.4	-	Fine, poorly	graded
Ε.	Rumaithiyah	10	1630	4.9	2.6	Fine, poorly	graded
F.	Shuwaikh Indus- trial Area	10	1495	6.3	2.0	Silty, fine	
G.	KISR	20	1625	3.0	2.8	Medium, well	graded

Values for the zone 0.5-2.0 meters below ground level



FIGURE 6 Relationship between standard penetration N values and the coefficient of subgrade reaction for desert sands.

The preceding equation indicates that the value of K of a very long foundation with a width B is approximately equal to 0.67 K_B .

Because it is much easier and less expensive to perform a standard penetration test than to perform a standard plate load test, a correlation such as that given by Equations 4 and 5 can be useful to the practicing design engineers who pick up values recommended in textbooks (14) for various soil conditions. Such values are not accurate, and vary over a wide range for each soil type. Bazaraa and Howeedy (1) analyzed the results of 60 plate load tests at different sites in the United States, the United Kingdom, Australia, Czechoslovakia, Jamaica, and Hong Kong, together with the corresponding N values. The soils at these sites are different and include clayey silty sand,

fine-to-coarse sand, sandy gravel, gravel, and decomposed granite. Figure 7, reproduced from Bazaraa and Howeedy (1), gives the relationship between the standard penetration N-value and the coefficient of subgrade reaction, K. There is considerable scatter in this relationship as may be expected in view of the different soil conditions. However, about 90 percent of the data fall within the range $K = 2.25 N-15 N \text{ MN/m}^3$ (7.5 N-50 N ton/ft³). Based on a statistical analysis of the data Bazaraa and Howeedy (1) suggested, the following relationship between K and N for a 0.3-m (1-ft) square plate is shown:

$$K_1 = 6 N \text{ MN/m}^3$$
$$= 20 N \text{ ton/ft}^3$$
(7)

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FIGURE 7 Relationship between standard penetration N values and coefficient of subgrade reaction for standard plate load tests on dry cohesionless soils at various sites [after Bazaraa and Howeedy (1)].

The values for K_1 given by Equation 7 are superimposed on Figure 6 along with present test results on desert sands. The values measured herein are nearly 70 percent of those based on Equation 7. However, they are within the range of values found (1) for granular soils as shown in Figure 7. This wide range of values is attributed to the different compressibilities of the soils tested that possess different classifications.

The results of the present tests and those reported by Bazaraa and Howeedy (1) indicate much higher values than those recommended by Terzaghi (4), which were based on some limited bearing-plate data. In examining the Terzaghi values, Scott (3) observed that they can be given roughly by $K_1 = 1.2 N \text{ MN/m}^3$ (4 N ton/ft³) and recommended increasing them by 50 percent, thus

$$K_1 = 1.8 N \text{ MN/m}^3$$

= 6 N ton/ft³ (8)

In examining the above recommendations relative to present test results (Equation 4), it is evident that they are very conservative in underestimating the coefficient of subgrade reaction by 2 to 3.5 times. They are even more conservative with respect to the recently reported field data (Equation 7) from different sites around the world (1).

It should be emphasized that the standard penetration test N values employed in the analysis presented herein are the measured values with no correction applied. Recent tests and research indicated that the correction for the overburden pressure proposed by Peck et al. (15) is reasonable with respect to desert sands. Accordingly, for the shallow depth range employed herein, the corrected N value ' $\overline{N^7}$ ranges from 1.44 to 1.5 N, averaging 1.47 N for the seven test sites. If the corrected N values are used, Equations 4 and 5 will be modified to

$$K_1 = 2.85 \quad \overline{N} \quad \text{MN/m}^3$$
$$= 9.5 \quad \overline{N} \tag{9}$$

$$K_B = 2.85 \quad \overline{N} \left(\frac{B+0.3}{2B}\right)^2 \quad \text{MN/m}^3$$
$$= 9.5 \quad \overline{N} \left(\frac{B+0.3}{2B}\right)^2 \quad \text{ton/ft}^3 \tag{10}$$

It is recommended that the corrected N values (\overline{N}) be used particularly for large footings and basement slabs. The N value used in the selection of the coefficient of subgrade reaction should be the average value within the significant zone of depth B below the footing. If the water level occurs and will remain at or below a depth $D_f + B$ beneath the ground surface where D_f is the depth of footing, no correction for water table may be applied. However, if the water table rises to a depth D_w measured from ground surface, a correction for the coefficient K_B calculated from Equation 5 or Equation 10 should be applied. The reduction factor proposed by Peck et al. (15) for the allowable solid pressure is recommended herein, thus

$$C_{w} = 0.5 + 0.5 \frac{D_{w}}{D_{f} + B}$$
(11)

Another simple correlation reported (1) is the one between the coefficient of subgrade reaction and that of relative density. From an analysis of the results of 83 standard plate load tests on sand fill at Brownville, Nebraska, where the relative density, D_r , was determined at both sides of the plate, it was found that K_1 is exponentially related to the relative density D_r . Two curves are drawn to define a range that includes about 80 percent of the available field data. These curves, reproduced in Figure 8 (1), are given approximately by the equations

$$K_{1} = 300 D_{r}^{1.7} \text{ MN/m}^{3} (1,000 D_{r}^{1.7} \text{ ton/ft}^{3})$$

or
$$K_{1} = 600 D_{r}^{1.7} \text{ MN/m}^{3} (2,000 D_{r}^{1.7} \text{ ton/ft}^{3})$$
(12)



FIGURE 8 Relationship between relative density and coefficient of subgrade reaction for 0.3 m * 0.3 mstandard plate load tests at Brownville, Nebraska [after Bazaraa and Howeedy (1)].

A reasonable average relationship (1) between K_1 and D_r is given approximately by

$$K_1 = 450 D_r^{1.7} \text{ MM/m}^3$$

= 1,500 $D_r^{1.7} \text{ ton/ft}^3$ (13)

Although measurements of the relative density were not made at the sites of the present tests, the above correlation was mentioned because of its relevance and the similarity of the soil conditions. The soil conditions at the Brownville site consist of light brown, poorly graded medium-to-fine sand with a uniformity coefficient of close to 2 and a percentage passing No. 200 sieve of less than 10 percent.

INFLUENCE OF FOOTING DEPTH AND MATERIAL

Test series II was carried out to determine the influence of depth of shallow footings on the coefficient of subgrade reaction. For this purpose 24 footings were constructed at the first site, KISR; 12 of these of 1 m (3.3 ft) square, the remainder 0.5 m (1.6 ft) square. Three of each size were placed at depths of 0.5, 1, 1.5, and 2 m (1.6, 3.3, 4.9, and 6.5 ft). The footings were loaded in the same way as before and the results are shown in Figure 9.



FIGURE 9 Influence of footing depth on settlement under load [after Allison (5)].

Unlike the previous tests showing the effect of size and soil properties, these tests show little difference between the coefficients of subgrade reaction within the shallow depth range employed herein. It is likely, however, that deeper footings will have a larger coefficient of subgrade reaction and an increased factor of safety against bearing capacity failure.

EFFECT OF FOOTING MATERIAL

Test Series III

Many loading tests are performed by applying the load to a steel plate instead of a concrete slab. To find out the difference between the coefficient of subgrade reaction as determined by the two methods, the load-settlement behavior of the three 0.5m (1.6-ft) footings used in the second series of tests was compared with that of three steel plates of the same size placed on soil at the same depth. The concrete footings and the steel plates were rigid, with uniform settlement recorded during loading. The results are shown in Figure 10. Analysis of this data indicates that the coefficient of subgrade reaction for concrete footing is 20 percent larger than that for a steel plate. This may be attributed to the fact that there is clearly less friction between sand and smooth steel than there is between sand and rough concrete, and the loss of confinement may therefore be significant, resulting in more settlement under the steel plate.



FIGURE 10 Comparison of settlement of concrete and steel footings under load [after Allison (5)].

CONCLUSIONS

The results of recent field loading tests on shallow square footings at seven sites in Kuwait were analyzed to establish a simple correlation for the determination of the coefficient of subgrade reaction. Several variables were investigated, including the effect of footing size, footing material, overburden pressure, and soil properties. Based on test results, the following conclusions have been reached:

1. The footing size greatly influences the coefficient of subgrade reaction. Test results on 1-, 0.75-, 0.5-, and 0.25-m (3.3-, 2.4-, 1.6-, and 0.8-ft) concrete footings indicated close agreement with the Terzaghi formula (2).

2. The effect of footing depth in the shallow range up to 2 m (6.5 ft) on the coefficient of subgrade reaction is insignificant.

3. For the same depth and soil conditions, the coefficient of subgrade reaction for concrete footings is approximately 20 percent greater than that of a similarly rigid steel plate. This is due to a larger friction and a closer contact with soil for the concrete footings.

4. From test results, a simple correlation was established between the coefficient of subgrade reaction and the SPT (Equations 5 and 10) for desert sands. The coefficient of subgrade reaction obtained from this relationship is nearly three times larger than the recommended Terzaghi values.

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5. The simple correlation developed herein for desert sands falls within the range found by Bazaraa and Howeedy (1) from analysis of standard plate test data on granular soils in different areas around the world.

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