Nonlinear Utility in Time and Cost of Trips: Disaggregate Results from an Ordinal Methodology

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A new methodology to investigate the linearity of the systematic utility function over time-cost combinations is developed. The approach, based on stated preferences, is unique in that it requires only ordinal preferences from laboratory subjects and assumes only ordinal properties of the utility function. Requiring ordinal rather than intervally scaled preferences provides for more meaningful and cognitively simpler tasks. Assuming only ordinal properties of the utility function defines a best-case scenario for linear performance—if linearity can be rejected under ordinal conditions, it can be rejected under more restrictive conditions. The experimental design leads to geometric, statistical, and predictive tests of ordinal linearity. The methodology is applied to a sample of 12 individuals faced with time-cost combinations of representative morning commute trips, primarily to illustrate the approach. However, even in this preliminary study and using a conservative means of classification, an ordinally linear utility function is rejected in favor of a simple nonlinear specification in half of the subjects. The linear specification cannot be accepted over a nonlinear one for any subject.

The concepts of disaggregate choice and random utility maximization form the basis of many and perhaps the most appealing transportation demand models used today *(1–3)*. These models assume that an individual's preferences can be modeled by a utility function decomposable into a systematic component and a stochastic error term. The systematic utility function V is written as a function of the individual's socioeconomic characteristics and the level of service (LOS) attributes that the transportation alternative offers. For computational convenience, V is written as linear in its parameters:

$$V_j \text{ (alternative } k) = \sum_{i=1}^{N} a_{ji} f_{ji} (Z_{jik}) \qquad (1)$$

where f_{ji} is a component function of the systematic utility function corresponding to the ith of N socioeconomic or LOS variables Z_{jik}, obtained by individual j when choosing alternative k; and a_{ji} is the scaling parameter of this ith function. Two LOS variables used in most analyses of transportation alternatives are the time t and cost c of the alternatives. In practice, the component functions associated with these variables are usually linear and additive, so that Equation 1 becomes

$$V_j \text{ (alternative } k) = \sum_{i=1}^{N-2} a_{ji} f_{ji} (Z_{jik}) + a_{jt}\, t_{jk} + a_{jc}\, c_{jk} \qquad (2)$$

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However, there have been limited propositions to use nonlinear component functions in time and cost. Koppelman *(4)* refers to studies demonstrating that psychological perceptions of time and cost may not be linear in their actual values. Although discrepancies between objective and perceived values can be controlled in a laboratory setting, such studies conducted in the decision sciences have indicated nonlinear utility functions in time and cost *(5–10)*. Limited experiments conducted in the transportation field support the consideration of a nonlinear utility function *(4, 11, 12)*. There are also economically based theoretical arguments *(4, 12)* supporting nonlinear functions. However, the limited empirical results and theoretical arguments are suspect. The studies in the decision sciences deal with larger quantities of time and cost than would be encountered in most applications of transportation demand models. Also, these studies, those performed in the transportation field, and theoretical arguments deal with an intervally scaled utility function. The function used in demand models is claimed to be an ordinal one *(1)*. As argued in the next section, this distinction would invalidate both the theoretical arguments and the empirical methodologies and imply that easier cognitive tasks could be used in the laboratory.

In this paper, ordinally based arguments for considering a systematic utility function whose component functions are nonlinear in time and cost are presented. An empirical study using only ordinally stated preferences for morning commuting options is also described. The results indicate that a linear utility function cannot generally be assumed to describe preferences as functions of time and cost, even when the values of these attributes are small and even when the function is in its least restrictive, ordinal form. The results strengthen the conclusions of previous studies not only by adding more data, but by collecting the data through a more appealing methodology—one that is compatible with the ordinal nature of the utility function and that requires less difficult cognitive tasks of the subjects.

In the next section, current arguments for considering a nonlinear utility function in time and cost are shown to be incompatible with the properties of an ordinal function and ordinally based arguments for considering such a function are presented. The merits of a stated-preference, laboratory-based empirical study are then discussed. Past studies assumed stronger-than-ordinal properties of the utility function and required more difficult cognitive tasks than were necessary. In the following section, the design of the ordinally based empirical study is described. In the last section, the results, based on visual inspection of response surfaces, nonparametric tests of the assumption of a constant marginal rate of substitution, and predictive tests of linear and nonlinear ordinal specifications of the utility function, are presented. Implications and limitations of the results, along with directions for further study, are also discussed.

BACKGROUND FOR AN ORDINAL STUDY

The only LOS variables considered in this paper are time and cost. Therefore, any alternative can be specified by the associated time and cost (t_j, c_j) incurred by individual j when effecting this alternative. Individual j's systematic utility function for an alternative can similarly be specified by $V_j(t_j, c_j)$. From here on, subscript j on time-cost combinations will be dropped, both for simplicity and because the laboratory approach used can control for differences in these combinations among different individuals. With these conventions, Equation 1 becomes

$$V_j(t, c) = a_0 + a_{jt} f_{jt}(t) + a_{jc} f_{jc}(c) \tag{3}$$

where a_0 is a constant encompassing all other fixed terms. The usually encountered Equation 2 can be written

$$V_j(t, c) = a_0 + a_{jt} t + a_{jc} c \tag{4}$$

Theoretical Arguments for a Nonlinear Ordinal Function

Some economists believe that intervally scaled *(13)* utility functions exist and can be measured *(14)*. There have also been several empirical studies investigating intervally scaled utility functions *(6–9)*. But the systematic utility function used in disaggregate choice models is claimed to be an even less restrictive ordinal function *(1)*. There have also been no claims that this function possesses any of the stronger properties, such as intensity or strength of preference, implied by cardinal and intervally scaled functions *(13)*. Although the use of the function is believed to imply stronger properties, it is investigated in its least restrictive ordinal form as a conservative approach to rejecting linearity.

An ordinal function can only indicate a direction of preference. It is a function mapping its arguments into the set of real numbers such that a lower (or higher) real number indicates increased preference *(13)*. Specifically, the ordinal function implies only

$$(t_1, c_1) \,.\, \mathrm{P}_j \,.\, (t_2, c_2) \text{ if and only if } V_j(t_1, c_1) < V_j(t_2, c_2) \tag{5}$$

and

$$(t_1, c_1) \,.\, \mathrm{I}_j \,.\, (t_2, c_2) \text{ if and only if } V_j(t_1, c_1) = V_j(t_2, c_2) \tag{6}$$

where (t_1, c_1) and (t_2, c_2) are two time-cost combinations, $.\,\mathrm{P}_j\,.$ represents "is preferred to, by individual j," and $.\,\mathrm{I}_j\,.$ represents "is indifferent to, for individual j." The symbol < is used instead of > because this convention allows positive coefficients in the utility function when dealing with negatively valued attributes such as time and cost. Therefore, although V really represents a systematic disutility function, the more general term "utility function" is used except when the distinction is needed for clarity.

The implication of the ordinal nature of the utility function is that any monotonic order-preserving transformation of the function yields an equivalent function. That is, if $V_j(t, c)$ represents individual j's utility for time-cost combinations, then $V_j'(t, c)$ also represents individual j's utility for these combinations if $V_j'(t, c)$ is a monotonic transformation of $V_j(t, c)$. For example, if $V_j(t, c)$ could be described by Equation 4, it could also be described by

$$V_j'(t, c) = (a_0 + a_{jt} t + a_{jc} c)^3 \tag{7}$$

However, the linear version is normally used for computational convenience.

The importance of this implication is that it renders inappropriate the current theoretical arguments advanced for a nonlinear utility function in time and cost if the function is to be an ordinal one. These arguments *(4)* are based on the economic concept of nonconstant marginal utilities in time and cost. Because some individuals appear to have marginal utilities for time and cost that depend on the level of these variables already incurred, the marginal utilities of the systematic utility functions $[\partial V(t, c)/\partial t$ and $\partial V(t, c)/\partial c]$ should not be assumed to be constant. Because Equation 4 implies constant marginal utilities, it is not a valid representation of the systematic utility function. But, whether or not the mathematical expression of the marginal utility depends on the level of time or cost incurred depends on which of the equivalent monotonic transformations is used. To see this, the partial derivatives of V' in Equation 7 are taken with respect to time and cost. Although V' is theoretically equivalent to V in Equation 4, analysis of the marginal utilities leads to different conclusions. This difficulty arises from using the derivatives of an ordinal function to indicate something stronger than direction of preference.

However, theoretical arguments for a nonlinear ordinal function can be made by considering the marginal rates of substitution *MRS* instead of marginal utilities. Consider all time-cost combinations of equal ordinal utility. Because the utility is constant, the total derivative of the utility function among these combinations must be zero. After taking the total derivative, setting it equal to zero, and rearranging terms, the *MRS* of cost for time is

$$MRS = dc/dt = \frac{-\partial V(t, c)/\partial t}{\partial V(t, c)/\partial c} \tag{8}$$

Equation 8 implies that if an individual is to be indifferent between one alternative and a second whose time differs from the first by an amount dt, then the necessary change in the second's cost from the first's is given by the ratio of the partial derivatives of the utility function with respect to time and cost. The negative sign indicates that an increase in time requires a decrease in cost and vice versa, because the signs of both derivatives will be identical. To derive this equation, it was only assumed that the appropriate derivatives could be taken. The interpretation is based only on the assumption of the ordinal property of Relation 6.

The importance of using the *MRS* interpretation is that it is unique even for ordinal functions, and the *MRS* of Equation 4 leads to unacceptable conclusions. To see that the *MRS* is unique, let $V'(t, c)$ be related to $V(t, c)$ by a monotonic transformation g—that is, $V'(t, c) = g[V(t, c)]$. Use these definitions and the chain rule to write

$$\frac{\partial V'(t, c)/\partial t}{\partial V'(t, c)/\partial c} = \frac{\partial g[V(t, c)]/\partial t}{\partial g[V(t, c)]/\partial c}$$

$$= \frac{\partial g[V(t, c)]/\partial V(t, c) \times \partial V(t, c)/\partial t}{\partial g[V(t, c)]/\partial V(t, c) \times \partial V(t, c)/\partial c}$$

$$= \frac{\partial V(t, c)/\partial t}{\partial V(t, c)/\partial c} \quad (9)$$

The *MRS* for V' is the same as that for V, no matter what differentiable transformation is used. Although similar arguments have been made in economics *(15)*, they seem to be overlooked in transportation demand analyses.

The linear utility function used in practice, or any permissible transformation of it, leads to an *MRS* of cost for time that does not depend on the cost for time already incurred. Specifically,

$$MRS = a_t/a_c \quad (10)$$

But economic intuition and empirical studies indicate that some individuals' strengths of preference for time and cost depend on the levels of these variables already incurred. Although these dependencies cannot be used directly to invalidate a linear ordinal function, they can be used indirectly to reason that the *MRS* values depend on the levels of these variables. To see this, consider the types of strength of preference that might be exhibited.

Koppelman *(4)* describes an individual who experiences increasingly more discomfort for public transportation as the time of the trip increases. This individual will, therefore, have a stronger preference for a given decrease in travel time dt when this decrease is made from a long trip than when it is made from a short one, because the same decrease relieves more discomfort in the long trip. It follows that the individual should be willing to pay a larger sum dc to reduce the time when the trip is long than when it is short, at least if the original costs of the two trips are the same. Another individual might relax after a while so that an incremental increase will be less onerous; such an individual would pay less to eliminate it as the time incurred increases. A third individual may believe that "a minute is a minute" in the range of times considered and, therefore, would not be willing to change the amount to pay in order to eliminate an increase in time as a function of the amount of time already incurred. The magnitude of the first individual's *MRS* will be increasing in time, that of the second's decreasing, and that of the third's constant. All of these types of behavior appear plausible, but only the third is permitted by Equation 10 and the linear ordinal utility function from which it was derived.

It is likewise possible that the *MRS* depends on the level of cost already incurred. One individual may be more sensitive to a given increase in cost when the cost is high than when it is low, while another will be more sensitive when the cost is low. The first individual might be concerned with "spending more than he wants to" for the good, while the second may reason in percentage increases in cost. It follows that the first will pay more to reduce the time when the cost is low than when it is high—that is, have an *MRS* whose magnitude decreases in cost—and that the second will pay more when the cost is high than when it is low—that is, have an *MRS* whose magnitude increases in cost. Both behaviors seem plausible, but only that of a third individual, who values increased unit costs equally in the range considered and, therefore, has constant *MRS* in cost, is permitted by the linear utility function.

There are arguments *(16)* for an individual to possess both increasing and decreasing intervally scaled marginal utilities—and, therefore, both increasing and decreasing *MRSs*—depending on the amount of attribute incurred. However, these arguments are normally made when large amounts of the attributes are involved. Because this study deals with small quantities, discussion and analysis of plausible descriptions of behavior are limited to those marginal rates of substitution of cost for time that are increasing, decreasing, or constant in time and in cost.

Stated Preference Approach

Most transportation demand studies designed for quantitative, policy assessment contexts use revealed preference data *(1, 17)*. This type of data is usually expensive to obtain, and the analyst has little, if any, control over the LOS attributes acting as independent variables. In general, only one observation can be obtained for a given individual. Also, perceived levels of attributes must be represented by objectively measured levels, leading to the potential difficulties that Koppelman *(4)* discusses.

Laboratory experiments using stated preferences can reduce these difficulties. The analyst poses hypothetical alternatives to a subject, who is asked to state relative preferences among them. Because the alternatives are defined by the analyst, the analyst has complete control over their independent variables. Several observations can be obtained from the same subject. These observations can be responses to the same set of alternatives presented several times or to different sets of alternatives, whose attributes can be varied systematically. Functional forms can, therefore, be determined for each individual. Because the analyst presents the attributes of the alternatives to the subject directly, there is no discrepancy between the analyst's and the subject's perception of the attributes. The general drawback of the stated preference approach is that there is no guarantee that an individual's preferences stated in the laboratory will correspond to the individual's actions implemented in the outside environment. Still, this approach has been used successfully to predict nonlaboratory behavior *(18)*.

Like past theoretical arguments for a nonlinear utility function, however, the transportation studies that have used stated preferences *(11, 16–22)* imply that the systematic utility function has stronger than ordinal properties and, therefore, requires more difficult cognitive tasks than necessary. These studies require individuals to rate the relative value of transportation alternatives by assigning numbers on a scale that is anchored by lower and upper bounds. However, the allowable transformations of the utility functions make assigning such numbers meaningless. Given a number of ratings, many plausible specifications of the utility function could be fit through them by taking some monotonic transformation of the function.

Even if the utility function sought were intervally scaled so that the rating scheme made theoretical sense, it would not be clear that individuals could supply valid estimates of intervally scaled utilities directly. To require only ordinal information of the subjects is cognitively simpler. An analogy in which an individual must decide upon the relative temperatures of two liquids without the aid of a thermometer illustrates this. It would be easier for the individual to determine which of the two liquids is hotter than to assign temperatures to each, even though the individual might be quite familiar with the well-defined concept of temperature. The difference in cognitive difficulty between stating ordinal and intervally scaled preferences for a pair of goods would be even greater, because the individual will have little, if any, operational idea of how to assign a quantitative measure compatible with the interval scale. Researchers in other fields have also expressed concern over the ability to determine valid ratings of preferences directly *(9, 23)*.

The nonlinearity of the systematic utility function in time and cost was investigated empirically using a stated preference approach in a laboratory setting. The approach was chosen because of its appeal for investigating preferences toward systematically varied transportation alternatives. However, unlike previous studies in the transportation field, this study was ordinally based. Such a study required fewer restrictive assumptions of the systematic utility function than one based on intervally scaled data and, therefore, represented a type of best-case test. Moreover, it was compatible with the claim in the literature that the systematic utility function is ordinal. Most important, it required less difficult cognitive tasks of the subjects and, therefore, theoretically led to more valid data. The study was based on the use of preference and indifference statements, which were compatible with an ordinal function through Relations 5 and 6.

DESIGN OF EMPIRICAL STUDY

Protocol

Each subject was asked to consider his daily morning trip to work or school in an abstract mode. The mode was described only generally as not being uncomfortable, and not allowing reading or socializing. The idea was to get the individual to think only about the trade-offs between unproductive travel time and cost. The interviewer then posed the question: "Would you prefer such a trip taking t_k min and costing c_k cents or paying R cents to eliminate the time of this trip?" The values of t_k and c_k were set exogeneously as assessment parameters by the interviewer. The value of the response R was set by the interviewer, but adjusted according to the bracketing method *(9)* until the subject expressed indifference or no preference between $(0, R)$—paying R cents to eliminate the time—and (t_k, c_k). That is, a level of cost R_k was sought with this method such that

$$(0, R_k) \cdot I_j \cdot (t_k, c_k) \tag{11}$$

Using personal interviews and the bracketing method appears to represent deviations from other stated-preference-based transportation studies and should lead to more representative responses. Time was set to zero in the response time-cost pair so that the subject could think of buying out the morning commute time without having to think of an extra time parameter.

Responses to 42 (t, c) combinations were elicited for each individual. The 42 combinations were obtained by taking all combinations of 10, 20, 30, 40, 50, and 60 min with 0, 25, 50, 75, 100, 125, and 150 cents. The upper bounds on these attributes were chosen to coincide with those that the Central Ohio Transit Authority considers for transit trips in the Columbus area. The order of presentation of these 42 combinations was varied among individuals, but the presentation was ordered in such a way that the responses to combinations presented near the end of the session (which lasted about 1 hr on average) were constrained by monotonicity considerations in cost and time by responses presented near the beginning of the session *(24)*. Surprisingly few inconsistencies appeared considering the small differences used in the assessment time-cost combinations. When they did occur, the interviewer would point them out to the subject, who was then allowed to change any responses.

Use of a Response Surface

This type of data leads to an appealing geometric interpretation based on the concept of the response surface *(25)*, which was developed for analysis of choice patterns under uncertainty. The (t, c) pairs can be thought of as points in the time-cost plane. The response R can be thought of as a height above this plane. Because less cost is preferred to more, less R is preferred to more, and the relative heights rank the (t, c) combinations. The three-dimensional response surface above the time-cost plane in time-cost-response space, therefore, represents an ordinal utility function. With the 42 responses, there are 42 points on the surface equally distributed throughout the domain of the time-cost plane considered. The response along the $t = 0$ axis is also known, by construction. The rest of the surface can be estimated by interpolation.

It is convenient to represent the response surface by its isoquants—the projections of constant response in the time-cost plane. If transitivity is assumed, specifically, that $(t_k, c_k) \cdot I_j \cdot (0, R_k)$ and $(t_m, c_m) \cdot I_j \cdot (0, R_k)$ imply $(t_k, c_k) \cdot I_j \cdot (t_m, c_m)$, the isoquants represent indifference curves among (t, c) combinations. Because indifference curves represent loci of equal ordinal utility, the slopes dc/dt of these curves represent the *MRS* developed in Equation 8.

Note that this representation is model free. It assumes no behavioral properties of preferences other than continuity and transitivity, and offers a visual, completely ordinal test of the linearity of the utility function. If the function is linear, the constant *MRS* developed in Equation 10 implies that the isoquants of the response surface should be parallel straight lines. However, given the difficulty associated with the task of psychological introspection necessary for even ordinal statements, perfectly parallel indifference lines are not expected. Rather, large and systematic deviations from linearity as a function of the independent variables time and cost would occur.

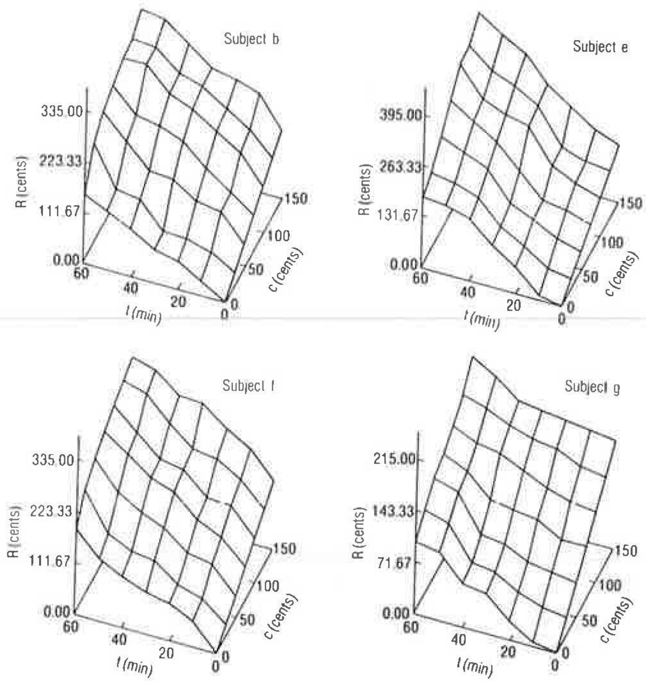

FIGURE 1 Representative response surfaces: (a) Subject b, (b) Subject e, (c) Subject f, and (d) Subject g.

RESULTS

Twelve subjects participated in the empirical study. Nine of these were graduate students in transportation at Ohio State University. The other three were Ohio State graduates with degrees in business administration. Because these 12 participants were chosen on the basis of availability and had a homogeneous educational background, the sample could not be considered representative of the general population. Tests of the general population could be a subject for further study.

Four response surfaces are presented in Figure 1 and their corresponding isoquants in Figure 2. The isoquants of Subjects b and e appear to systematically violate the requirements of linearity; those of Subject f seem to satisfy the requirements; those of Subject g violate the requirements, but not systematically. The isoquants for all 12 subjects can be found elsewhere *(24)*.

Although it is tempting to specify alternative functional forms of the systematic utility function and perform econometric fits of the parameters, it was not known how to do so without requiring stronger than ordinal properties of the utility function. A least squares fit using the 42 indifference statements would require taking the differences between the utility functions, implying that the differences are unique, at least to a positive linear transformation. Using maximum likelihood estimation based upon indifference statements and some specified binary choice model also makes stronger than ordinal assumptions, because these models assign a unique probability to choosing an alternative based upon the difference in the systematic utilities *(1, 2)*. Note that this argument implies that systematic utility functions used in current models based on random utility maximization are not ordinal.

Other quantitative means of investigating the degree to which these stated preferences satisfied the ordinal requirements of a linear utility function needed to be developed through nonparametric tests of calculated marginal rates of substitution and predictive tests of preference using ordinally calibrated parameters.

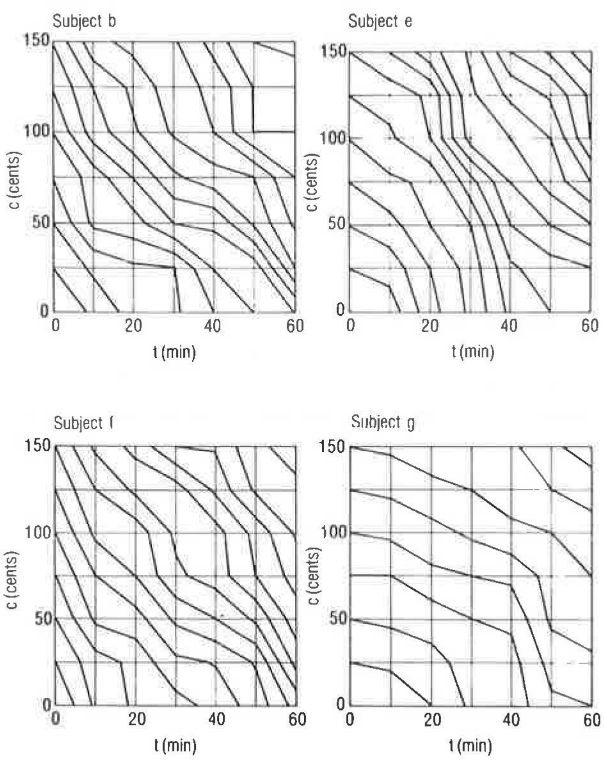

FIGURE 2 Representative isoquants: (a) Subject b, (b) Subject e, (c) Subject f, and (d) Subject g.

Nonparametric Test of Constant *MRS*

To estimate the marginal rate of substitution as a function of time and cost, the gridlike structure formed by the sample points in the time-cost plane was exploited. The grid can be seen in Figure 2, where each line represents an axis of either time or cost corresponding to a level that was used as an assessment parameter. The lines form unit boxes, the corner points of which were exogeneously set time-cost combinations for which a response was elicited. The response to a time-cost combination in the interior of one of the unit boxes can be uniquely estimated using linear interpolation among the box's four corner points. Specifically, the response is a function of the responses corresponding to the southwest, southeast, northwest, and northeast corners of the unit box—R_{sw}, R_{se}, R_{nw}, and R_{ne}, respectively; the times corresponding to the west and east edges of the unit box—t_w and t_e, respectively; and the costs corresponding to the south and north corners of the unit box—c_s and c_n, respectively. This interpolation scheme leads to a linearly generated, but perhaps nonplanar, surface above each box (see Figure 1).

Because the response forms an ordinal utility function, Equation 8 can be applied to the response function to determine the *MRS* of any point within the unit box. The *MRS* at the center $[(t_w + t_e)/2, (c_s + c_n)/2]$ of each box was taken, so that the estimate of the magnitude of the *MRS* for a given unit box was

$$MRS = \frac{R_{ne} - R_{nw} + R_{se} - R_{sw}}{R_{ne} - R_{se} + R_{nw} - R_{sw}} \cdot \frac{c_n - c_s}{t_e - t_w} \qquad (12)$$

In this way, an estimate was obtained of the *MRS* at all combinations of time levels of 5, 15, 25, 35, 45, and 55 min and cost levels of 12.5, 37.5, 62.5, 87.5, 112.5, and 137.5 cents. These estimates can be found in the literature *(24)*. These estimates would be quite approximate, given the assumption of a linearly generated surface above each box and the arbitrary point within the box at which the estimate was taken. However, these approximations tend to increase the noise level in the estimates. This, coupled with the use of tests on the ranks rather than on the magnitudes of the *MRS* values, leads to a conservative approach in rejecting ordinal linearity of the utility function.

Page's statistic *(26)* on the ranks of the calculated *MRS* values as a function of time at a given level of cost and as a function of cost at a given level of time was used. By calculating the statistic at a fixed level of either cost or time, the influence of these variables on the estimated value of the *MRS* was controlled. The results summarized in Table 1 supported the visual analyses. In Table 1, the null hypotheses are *MRS* values that are constant in time or cost. For those subjects who appeared to have constant *MRS* values in the visual inspection of the indifference curves, midvalue statistics were found. For some subjects (c, d, and g in Table 1) statistics strongly indicating an increasing *MRS* in time were found. For other subjects (a and b) the statistics indicated a decreasing *MRS* in time. Although not as strong, there was some indication of a systematic change of *MRS* as a function of cost—decreasing for Subjects g, j, and k, and increasing for Subject c. Due to the noise involved with generating the *MRS* values, these tendencies were explored further. Predictive tests were developed, consistent with the ordinal nature of the utility function, of the linear and various nonlinear specifications.

Predictive Tests

To describe the predictive test of the linear model, refer again to the unit boxes of the grid formed by the time and cost levels used in assessment. The northeast (ne), northwest (nw), southwest (sw), and southeast (se) corners of a box have coordinates (t_w + 10, c_s + 25), (t_w, c_s + 25), (t_w, c_s), and (t_w + 10, c_s), respectively, where as before, t_w and c_s are the level of time

TABLE 1 PAGE'S STATISTICS ON RANKS OF *MRS* MAGNITUDE

Subject	Alternative Hypothesis on MRS Magnitude			
	(a) Decreasing in Time	(b) Increasing in Time	(c) Decreasing in Cost	(d) Increasing in Cost
a	472.5	409.5	446.0	436.0
b	471.0	411.0	430.0	452.0
c	378.0	504.0	414.5	467.5
d	401.0	481.0	455.5	426.5
e	422.0	460.0	443.0	439.0
f	441.5	440.5	444.5	437.5
g	369.0	513.0	464.0	418.0
h	421.0	461.0	439.5	442.5
i	449.0	433.0	448.9	435.5
j	420.0	462.0	460.0	421.0
k	423.5	458.5	460.0	422.0
l	419.0	463.0	440.0	442.0

forming the west boundary and the level of cost forming the south boundary of the box. Note that by monotonicity the northeast corner of the box must be the least preferred, and the southwest corner the most preferred. For the northwest and southeast corners no preference is normatively apparent. An individual's stated preference between these two corners can be determined through the value of R assigned—the corner with lower R is preferred.

The predictive test revolved about the ability of the utility function to identify correctly the stated preferences between these two corners for each of the 36 unit boxes of the time-cost domain (Figure 2). More comparisons could have been used in the tests (there were 336 nondominated comparisons involving corner points of the unit boxes) but the number of comparisons was limited for simplicity. Comparisons were excluded whose outcomes would be dictated a priori by transitivity and the outcome of a previous comparison. For example, when $(t_k, c_k) \cdot P_j \cdot (t_m, c_m)$ was both stated by the responses and predicted by the utility function, then $(t_k, c_k) \cdot P_j \cdot (t_m + 10, c_m)$ would also be both stated and predicted.

To determine whether the northwest or southeast corner was predicted by a linearly ordinal utility function, Equation 4 and Relation 5 can be used to write that $(t_w, c_s + 25)$ is predicted to be preferred to $(t_w + 10, c_s)$ by Individual j only if

$$g[a_0 + a_{jt}(t_w + 10) + a_{jc}c_s] < g[a_0 + a_{jt}t_w + a_{jc}(c_s + 25)] \quad (13)$$

where g is any monotonic transformation of the linearly specified functions. By taking the inverse of this function, and noting that a_{jt} and a_{jc} must be positive for a disutility function monotonic in time and cost, the predictive conclusion can be written as

$$A < 25/10 \quad (14)$$

where A is a positive parameter equal to the ratio of a_{jt} to a_{jc}. If the value of A is known, whether the northwest or southeast corner of a given box is predicted to be preferred for that value of A, that is, for the calibrated linear model, can be determined.

For each individual, the 42 indifference statements were used to determine 42 values of A. Using the indifference between $(0, R_k)$ and (t_k, c_k) for an individual, Equation 4, Relation 6, and the previous reasoning,

$$A_k = (R_k - c_k)/t_k \quad (15)$$

Note that Equation 15 holds for any monotonic transformation invoked.

Not only how well the linear utility function could predict preferences was of interest, but also whether any poor predictive ability could be associated with behaviorally feasible deviations from the ordinal implications of a constant *MRS*. As previously stated, only *MRS* values increasing or decreasing in time or cost were considered as possible alternatives. Although there are many possible functional forms that could lead to these alternatives *(4, 18)*, the predictive ability of power functions of cost and time were used because of their simplicity and use in past studies. Specifically, alternatives were considered to the linear function of the form

$$V_j(t, c) = a_0 + a_{jt}t^{b_{jt}} + a_{jc}c^{b_{jc}} \quad (16)$$

The magnitude of *MRS* of this form is given by

$$MRS = \left(a_{jt}b_{jt}t^{b_{jt}-1}\right) / \left(a_{jc}b_{jc}c^{b_{jc}-1}\right) \quad (17)$$

Because the a and b parameters are positive for a monotonic disutility function in time and cost, the magnitude of the *MRS* is increasing, decreasing, or constant in time if b_{jt} is greater than, less than, or equal to 1, respectively, and in cost if b_{jc} is less than, greater than, or equal to 1, respectively.

Values of the exponents b could not be fit because an econometric fitting would imply stronger than ordinal assumptions. A number of indifference statements could theoretically be used to determine values of the independent parameters of Equation 16. Values of b_{jt} and b_{jc} were assigned arbitrarily, however, both for convenience and so that the functions would have the same number of unknown parameters as the linear model, thereby allowing a more direct comparison among the results of the predictive tests. To b_{jt} (b_{jc}) a value of 2 (1/2) was assigned for an *MRS* whose magnitude was increasing and a value of 1/2 (2) for an *MRS* whose magnitude was decreasing in time (cost). Along with a value of 1 for constant *MRS*, this convention led to the nine specifications, one of them being the linear one, summarized in Table 2. Once a specification has

TABLE 2 EXPONENT VALUES b_t, b_c FOR NINE SPECIFICATIONS OF $V(t, c)$

Time Effect on *MRS* Magnitude	Cost Effect on *MRS* Magnitude: Decreasing	Constant	Increasing
Decreasing	(1/2, 2)	(1/2, 1)	(1/2, 1/2)
Constant	(1, 2)	(1,1)	(1, 1/2)
Increasing	(2, 2)	(2, 1)	(2, 1/2)

NOTE: $V[t, c; b_t, b_c] = a_0 + a_t t^{b_t} + a_c c^{b_c}$

been chosen, the same arguments can be invoked to show that it is sufficient to know the value of A, the ratio of a_{jt} to a_{jc}, when predicting preference with a utility function given by Equation 16. An individual's stated indifference between $(0, R_k)$ and (t_k, c_k) can again be used to determine for the individual:

$$A_k = \left(R_k^{b_c} - c_k^{b_c}\right)/t^{b_t} \quad (18)$$

A small computer program was written to determine, for each of the 42 values of A, the number of times the specified utility function predicted the same direction of preference as was stated through the responses for each of the 36 northwest-southeast corner pairs. The number of correct predictions was then summed across the 42 A values and divided by the 1,512 (42 × 36) total comparisons to determine an "average percent correct" number of predictions for each of the individual's specified utility functions.

The results presented in Table 3 show that the linear model predicted the direction of preference more than 90 percent of the time for only one individual, between 80 and 90 percent of the time for two other individuals, and less than 70 percent of the time for the remaining nine individuals. For three individuals (a, d, and e) the correct number of predictions was below

TABLE 3 PERCENTAGE CORRECT PREDICTIONS OF NINE SPECIFICATIONS OF $V(t, c)$

	Utility Function Specification, (b_t, b_c)								
Subject	(1,1)	(2,1)	(1/2,1)	(1,2)	(2,2)	(1/2,2)	(1,1/2)	(2,1/2)	(1/2, 1/2)
a	58.3	48.0	61.8	58.6	52.9	60.8	55.9	49.2	65.0
b	61.7	49.8	64.2	59.8	52.4	60.3	59.1	49.5	61.8
c	66.1	80.8	57.1	68.4	75.5	62.3	64.6	78.5	42.9
d	56.0	63.2	50.8	50.7	58.2	48.4	59.7	65.3	52.8
e	55.6	72.0	41.8	56.9	68.2	51.9	50.1	60.6	37.7
f	69.4	59.5	55.4	65.9	62.0	63.0	54.9	54.6	54.1
g	83.3	74.2	78.6	77.5	75.6	74.3	75.7	69.9	77.2
h	86.1	65.8	79.0	64.5	58.5	69.0	80.6	71.8	78.6
i	62.3	61.9	65.6	55.5	57.8	56.9	66.8	67.0	68.1
j	66.9	58.3	67.4	51.4	47.4	57.6	63.3	58.9	67.9
k	60.1	60.5	49.5	60.9	62.3	61.1	57.4	55.5	51.3
l	94.4	73.8	91.3	71.5	61.8	76.7	85.7	72.6	82.2

60 percent, only marginally better than what would be expected by chance. The linear specification was best for only four individuals (f, g, h, and l). No other specification was best for as many individuals, but given that the exponents were arbitrarily chosen and that the arbitrary specification of $b_t = b_c = 1/2$ was best for three individuals (a, i, and j) the indication is that some specification using power functions could have outperformed the linear one. In addition to its generally poor performance in predicting ordinal preferences, compared to its nearest competitor, the linear specification predicted greater than 5 percent more of the preferences correctly only once, for Individual h. Of the eight individuals for whom one of the arbitrary alternative specifications predicted better than the linear one, five (a, c, d, e, and i) exhibited a decrease in the number of correctly predicted preferences of more than 5 percent when the linear specification was used.

Classification of Individuals

Table 3 and visual and statistical analyses can be combined to classify the individuals according to tendency for their *MRS* values to vary systematically with time or cost.

From Table 3 for Individuals a and b, the specifications involving $b_t = 1/2$, that is, those indicating a decreasing magnitude of the *MRS* in time, perform better than the other specifications for any of the three values of b_c. This effect is supported both by the visual inspection of these individuals' isoquants (Figure 2, Subject b) and by the relatively high rank statistic in Column *a* of Table 1. Similarly, the specification using $b_t = 2$ performs better for Individuals c, d, and e than either of the other two alternatives for any value of b_c. The visual analyses (Figure 2, Subject e) and the relatively high statistics in Column *b* of Table 1 support the conclusion that the general trend for these individuals is an *MRS* whose magnitude increases in time. The indication for Individual f is an *MRS* that is constant in time. The specifications with $b_t = 1$ performs best for any value of b_c; the rank statistic is moderate in Columns *a* and *b* of Table 1; and the slopes of the isoquants (Figure 2) show no systematic pattern as a function of time. The rank statistic for Subject g indicates a strong dependence on time. This dependence is supported by the isoquants (Figure 2) except at high values of time, where the pattern of steeper slopes with time changes drastically. Perhaps it is this change in pattern that makes $b_t = 2$ a poor predictor in Table 3. Similar inconclusive results are obtained for Individuals h, i, j, k, and l, which are conservatively classified as having mixed results as a function of time.

Only Individual k exhibits a systematic change in *MRS* as a function of cost. For any of the three possible b_t values, the specification using $b_c = 2$ predicts the greatest number of correct preferences. The corresponding rank statistic in Table 2 (Column *c*) is relatively high. And the isoquants are shallower as the cost is increased. This individual is therefore classified as having an *MRS* whose magnitude decreases in cost. Similarly, only for Individual l is there strong support for a constant *MRS* as a function of the costs considered. The specification with $b_c = 1$ performs best for any value of b_t; the rank statistics are moderate in Columns *c* and *d* of Table 1; visual inspection of the isoquants shows no systematic pattern as a function of cost. For the other 10 individuals, the results from the three analyses are either conflicting or inconclusive. This conflict could result from a preference structure similar to that of Individual g (Figure 2), which is compatible with increasing *MRS* values in some domains and decreasing *MRS* values in others. Because this type of behavior is not investigated here, these individuals can only be classified as having mixed results.

The classification results are summarized in Table 4. Although the procedure for classification was subjective in that

TABLE 4 CLASSIFICATION OF SUBJECTS

	Effect on *MRS* of	
Subject	Time	Cost
a	Decreasing	Mixed
b	Decreasing	Mixed
c	Increasing	Mixed
d	Increasing	Mixed
e	Increasing	Mixed
f	Constant	Mixed
g	Mixed	Mixed
h	Mixed	Mixed
i	Mixed	Mixed
j	Mixed	Mixed
k	Mixed	Decreasing
l	Mixed	Constant

the isoquants were visually interpreted and "relatively" high rank statistics were qualitatively determined, those individuals with weak or conflicting results in the mixed results category were classified conservatively. Strong conclusions on the impact of cost on the *MRS* for 10 individuals could not be made. Of the two remaining individuals, one exhibited a constant *MRS*, whereas the other exhibited an *MRS* whose magnitude was decreasing in cost. Strong results were obtained for more individuals when the dependence of the *MRS* on time was examined, and these results tended to discredit the assumption of a linear utility function. Ony one individual strongly showed no dependence in *MRS* on time; two showed *MRS* values whose magnitudes decreased in time; and three showed *MRS* values whose magnitudes increased in time.

None of the 12 individuals could be classified as exhibiting an ordinally linear utility function, whereas 6 could be classified as not having such a function due to a dependence on time or cost. Even if the two individuals showing no dependency on one of the variables and mixed results on the other, and the four individuals showing mixed results on both variables were classified as linear, the evidence is that other than linear specifications can be expected even at relatively low levels of time and cost.

DISCUSSION

The discussion of this study can be divided between its methodological and empirical components.

At the methodological level, a new approach has been developed and demonstrated for investigating the linearity of the systematic utility function for the time and cost of trips. The methodology would be easy to generalize to any two continuous LOS variables. It uses a laboratory, stated-preference-based approach and, therefore, allows economical collection of data that lead to systematic investigations of individuals' utility functions. Unlike approaches based on revealed preferences, an investigation of the utility function over ranges of the independent variables is easily obtained.

However, the methodology is different from others using stated preferences for transportation demand analyses in that it is ordinally based. It assumes only ordinal properties of the utility function and requires only ordinal preferences from the laboratory subjects. Only ordinal properties of the utility function are assumed because the function is claimed to be ordinal in the literature. Although current discrete choice models imply stronger than ordinal properties, a methodology that would be applicable if the function could eventually be used in an ordinal manner was desired. Also, because the methodology was used to investigate the rejection of properties, the less restrictive ordinal properties represent a conservative, best-case benchmark. Data that cannot support ordinal properties cannot support stronger ones. Finally, by requiring ordinal rather than intervally scaled preferences from the subjects, the cognitive difficulty of their tasks is reduced and the tasks are made more meaningful. These procedures should produce more valid data.

Past studies have used goodness-of-fit measures and tests of statistical significance when analyzing results. Because these types of analyses imply stronger than ordinal properties of the utility function, they were not used. Even so, three different tests of linearity could be developed at the ordinal level. Supporting results with different tests increases the confidence placed in conclusions drawn from them. One potential area of research could be devoted to understanding the situations in which the tests can give conflicting results and refining the tests so as to reduce the possibility of such situations.

At the empirical level, the pool of scarce but significant data indicating that utility functions are generally not linear in time and cost, even for the small levels encountered in urban travel, has been increased. None of the subjects could be confidently classified as exhibiting a linear function, whereas six could be confidently classified as exhibiting systematic deviations from linearity. Although the sample was not chosen to represent any general population, the absence of an across-subject consistency is somewhat disturbing. The practical implication is that even though nonlinear utility functions should be considered, a general specification does not appear possible.

Even if the results were representative of the general population, the deviations from linearity in opposite directions for different individuals and the better predictive ability of the linear utility function in the aggregate would not justify use of a linear function. The motivation for disaggregate choice theory is a behavioral one, and the utility function must be capable of describing preferences at the individual level if the models are to be marketed as being behaviorally based. Disaggregate choice theory acknowledges the possibility of different parameter values of the utility function for different segments of the population. It would be reasonable to allow different specifications, as well. Further research would be necessary to determine the distribution of functional specifications across the population and of those parameters that can be used to stratify the specifications. The methodology used in this study could prove useful in this task.

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