Forecasting Intermodal Competition in a Multimodal Environment

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In this paper, the problem of accurately describing patterns of intermodal competition in a situation in which there are a large number of alternative modes available is discussed. This research was motivated by efforts to increase the capacity and usage of the existing Hudson River crossings connecting Manhattan and northern New Jersey. This corridor is characterized by the presence of an unusually large number of distinct transportation options and a high level of transit use. In such a setting, it is important to know not just how many commuters might use a new service, but also from which existing services they would be drawn. The mathematical structure of an innovative model developed for NJ Transit and the Port Authority of New York and New Jersey to allocate demand across seven primary modes is presented. The representation of intermodal competition that this model provides is considered, and its properties are contrasted with those of some commonly used variants of the familiar logit model. Empirical estimates of the own- and cross-elasticities of demand implied by the model coefficients are broken down by mode, service attribute, and geographic area.

In recent years the situation in the corridor connecting northern New Jersey and Manhattan has been described as a crisis. In the early 1980s after a decade of relative stability, the demand for travel across the Hudson River into Manhattan began to grow. As a result of changes in the structure and growth rate of the Manhattan economy, as well as shifts in the pattern of development in northern New Jersey, peak travel demand in the trans-Hudson corridor increased substantially throughout the early years of the decade. Because of the geography of the region and the structure of its transportation network, all of these trips had to funnel through one of a limited number of river crossings. Congestion at these bottlenecks increased dramatically, generating serious needs for extra capacity and improvements in service.

NJ Transit, the agency charged with responsibility for provision of public transportation services in the state of New Jersey, responded to this need by initiating a major program of improvements to the trans-Hudson system. A wide range of proposals was developed to increase the capacity and use of the existing Hudson River crossings. In order to assess the cost-effectiveness of these proposals and design the package of improvements that would relieve the crisis in the most efficient way, NJ Transit needed a planning tool that would permit the agency to predict the responses of trans-Hudson commuters to the proposed improvements. In cooperation with the Port Authority of New York and New Jersey, NJ Transit asked Charles River Associates to develop a modal split model for the northern New Jersey-Manhattan travel market.

It was immediately apparent that in order to address the key policy questions raised by the trans-Hudson crisis and NJ Transit’s efforts to resolve it, it was essential that the model provide an accurate representation of intermodal competition in the corridor. In this complex multimodal environment characterized already by extremely heavy transit usage, policy makers and planners had to know not just how many commuters might be attracted to a new service, but also from where they would be drawn. To contribute to the solution of the trans-Hudson crisis, a transportation improvement had to draw commuters out of automobiles and other low-occupancy vehicles, and not simply cannibalize existing transit ridership.

How the model was developed and the problem of intermodal competition are described in this paper. The next section describes in general terms the form and specification of the model. The mathematical properties of this specification are then analyzed, and formulas are derived for own- and cross-elasticities of demand and contrasted with the formulas of the more common multinomial logit (MNL) model. A fourth section discusses empirical results. In the conclusion, the implications of this work for travel demand forecasting are considered.

MODEL SPECIFICATION

The model allocates travel demand across seven distinct travel modes. These include automobile, three combinations of conventional transit (bus, commuter rail with a PATH trans-Hudson link, and commuter rail to Manhattan), two fringe park-and-ride modes (using either bus or PATH for the trans-Hudson segment), and local PATH (which as a mode in itself is defined to be available only within an inner core area along the Hudson River).

The explanatory variables used to define the level of service along each trip segment are those traditionally found in mode choice models. These include variables describing ease of access and egress, wait time, transfer time, cost, and line-haul time. To take into account the multimodal trans-Hudson environment, the model incorporates separate coefficients for the different types of line-haul time to capture the distinctly different characteristics of the different line-haul technologies.

The model was formulated as a set of logistic regression equations estimated across origin-destination (O-D) pairs (I). The dependent variable in each equation consisted of the log of the ratio of the transit share for the mode in question for that O-D pair, divided by the corresponding automobile share. Six
equations were estimated—one for each transit mode. The automobile mode was thus used as the reference mode, and the automobile share was computed from the log-odds ratio predictions using the constraint that the estimated shares had to sum to one. The mathematical form of the resulting model is expressed in Equation 1.

\[
\log \left( \frac{S_i}{S_a} \right) = a_0 + a_1X_1 + \ldots + a_nX_n
\]  

(1)

where

\( S_i = \) share for Transit Mode \( i \);
\( S_a = \) share for automobile mode,
\( X_i = \) explanatory Variable \( i \) and
\( a_i = \) estimated Coefficient \( i \).

Each demand equation contains three sets of independent variables: measures describing the service offered by the subject mode; measures describing the service offered by competing alternatives (which include the reference automobile mode); and measures describing characteristics of the O-D pair itself. The latter category includes selected socioeconomic variables, as well as dummy variables specifying whether or not specific modes are available for trips between that origin and destination.

The definitions of the variables included in the model, as well as the data sources and procedures used for model estimation, are described in more detail in another paper by Neels and Mather in this Record.

**PROPERTIES OF THE MODEL**

The principal advantage of this formulation is its explicit representation of the attributes of the competing modes. The presence of these variables permits a pair of modes to be either close or distant substitutes. The degree of competition between them can vary continuously between these two extremes, and be estimated empirically. Thus, both the independence of irrelevant alternatives problem that characterizes the MNL model and the sometimes arbitrary groupings that are often found in nested logit models can be avoided.

In this respect, the Hudson model represents a considerable advance in the analysis of travel behavior in multimodal environments.

The ability of the model to capture complex patterns of intermodal competition can best be illustrated by an examination of the formulas it implies for the own- and cross-elasticities of demand with respect to level-of-service variables. The formulas for these elasticities of demand are derived in this section. The next section presents values for selected elasticities derived from the model coefficients.

Elasticity of demand for travel with respect to some level-of-service variable \( I \) is defined as

\[
n_{im} = \frac{dT_m}{T_m} + \frac{dI}{I}
\]

(2)

where

\( n_{im} = \) elasticity of demand for Mode \( m \) with respect to level-of-service variable \( I \);
\( T_m = \) volume of trips made by Mode \( m \), and
\( I = \) a level-of-service variable such as automobile travel time or bus cost.

In the multimodal framework of the model, \( I \) can describe an aspect of the level of service offered by Mode \( m \), or a measure of the level of service offered by any competing mode.

The modal split model assumes implicitly that the total number of trips remains constant, and that any change in the level of demand for a particular mode is the result of modal shifts. Equation 2 can thus be rewritten as

\[
n_{im} = \frac{dS_m}{S_m} + \frac{dI}{I}
\]

(3)

where \( S_m \) is the share of trips made by Mode \( m \).

In calculating elasticities with respect to the level-of-service variable \( I \), all other level-of-service variables are held constant. This assumption implies that

\[
dS_m = \frac{\partial S_m}{\partial I} dI
\]

(4)

Substituting Equation 4 into Equation 3 yields

\[
n_{im} = \frac{\partial I}{\partial S_m} \cdot \frac{S_m}{I}
\]

(5)

To complete the derivation of the formula for the demand elasticity, the particular functional form of the line haul mode share model must be considered and the partial derivative \( \partial S_m/\partial I \) must be evaluated.

The line-haul mode share model takes the general form

\[
\ln \left( \frac{S_i}{S_A} \right) = a_0 + b_1I_1 + \ldots + b_nI_n
\]

(6)

where

\( S_i = \) share of trips for Transit Mode \( i \),
\( S_A = \) share of trips for automobile mode, and
\( I_1, \ldots, I_n = \) explanatory variables.

The overall mode will include one such equation for each of the six line-haul transit modes. The explanatory variables can refer to the level of service offered by the subject mode, or by any competing mode. For the purposes of this derivation, all six demand equations are assumed to contain the same set of explanatory variables but different variable coefficients. Some coefficients, of course, can be equal to zero.

In computing the partial derivative, all explanatory variables except the one of interest are held constant. Thus, the explanatory variables can be folded into the constant term, and without loss of generality the system of equations can be expressed as

\[
\ln \left( \frac{S_i}{S_A} \right) = c_i + b_iI \quad (i = 1, \ldots, 6)
\]

(7)

referring to the six transit modes; and \( c_i, b_i \) are the constant terms and coefficients of variable \( I \) in Equation 7.
The share for automobiles can be computed as a residual. Solving the set of equations for $S_A$ yields

$$S_A = \frac{1}{1 + \sum_{j=1}^{6} \exp(c_j + b_j I)}$$

(8)

From Equations 7 and 8 it can be shown that

$$S_i = \frac{\exp(c_i + b_i I)}{1 + \sum_{j=1}^{6} \exp(c_j + b_j I)}$$

(9)

Therefore,

$$\frac{\partial S_i}{\partial I} = \frac{b_i \exp(c_i + b_i I)}{1 + \sum_{j=1}^{6} \exp(c_j + b_j I)}$$

$$- \left[ \frac{\sum_{j=1}^{6} b_j \exp(c_j + b_j I)}{1 + \sum_{j=1}^{6} \exp(c_j + b_j I)} \right] \frac{\exp(c_i + b_i I)}{\left[1 + \sum_{j=1}^{6} \exp(c_j + b_j I)\right]^2}$$

(10)

With the help of Equations 8 and 9, this expression can be simplified to

$$\frac{\partial S_i}{\partial I} = S_i \left( b_i - \sum_{j\neq i}^{6} b_j S_j \right)$$

(11)

Substitution of Equation 11 into Equation 5 then yields

$$n_{ij} = I \left( b_i - \sum_{j\neq i}^{6} b_j S_j \right)$$

(12)

where $n_{ij}$ is the elasticity of demand for Travel Mode $i$ with respect to level-of-service variable $I$.

The specification used for the trans-Hudson mode choice model includes the MNL model as a special case. In the standard MNL context, a demand equation of the form shown in Equations 1 and 6 for a transit mode would include only level-of-service variables associated with that mode and the reference mode of automobiles. As a result, in the formula shown in Equation 12,

$$b_j = 0 \quad \text{for all } j \neq i.$$  

(13)

If $I$ represents a level-of-service variable associated with Mode $i$, Equation 12 reduces to

$$n_{ii} = I b_i (1 - S_i)$$

(14)

which is the formula for the own-elasticity of demand implied by the multinomial logit model.

If $I$ represents a level-of-service variable associated with some Mode $i \neq j$, Equation 12 reduces to

$$n_{ij} = -I b_j S_j$$

(15)

which is, of course, the formula implied for the cross-elasticity of demand implied by the MNL model.

Note that the formula shown in Equation 15 is independent of $i$. Thus, the cross-elasticity of demand with respect to level-of-service Variable $I$ will, in the MNL model, be the same for all other modes. If one mode is improved, the MNL model predicts that it will draw share from all other modes in proportion to their current shares. In the trans-Hudson context, where there are seven distinct modes, some of which are closely related, this is a restrictive and unrealistic assumption.

**RESULTS**

Because the value for the elasticity depends on both modal shares and the value taken by the level-of-service variable, it was necessary to select a reference point in order to calculate what values the model implies for own- and cross-elasticities of demand. The point chosen represents average conditions in the Newark Division—the portion of New Jersey served by NJ Transit commuter trains running through Newark and on to Penn Station, New York. Values calculated for selected own-elasticities of demand using the formula shown in Equation 12 and the estimated mode coefficients are presented in Table 1.

The different modes presented in Table 1 differ dramatically in their sensitivity to changing levels of service. With their low modal shares, the two park-and-ride modes show the greatest sensitivity to changes in level of service. This sensitivity is especially pronounced in connection with access time, which constitutes a large fraction of the total trip time for these modes. In contrast, the two commuter rail modes show much less sensitivity to changes in the level of service. Automobiles and buses fall between these two extremes.

The elasticity values reflect the geometry of the transportation system. Although demand for direct rail is less sensitive than demand for rail with transfer to PATH to changes in travel time or travel cost, it is much more sensitive to changes in ease of access. Their differing responses to changes in access time reflect the fact that whereas rail with transfer to PATH is relatively ubiquitous, direct rail service to Penn Station, New York, is available only in the Newark Division. Demand for direct rail service from an area is thus strongly influenced by that area's proximity to the lines offering that service.

In general, the elasticity values presented in Table 1 are considerably higher than those normally found in travel demand research. This higher level of sensitivity is attributable to the large number of alternatives that are available in this region and represented in the model.

Table 2 presents own-elasticities of demand for the different modes with respect to cost, broken down by geographic area. The Newark Division, which was described briefly earlier, constitutes the southern portion of the study region. The Hoboken Division, which is served by NJ Transit commuter rail services terminating in Hoboken, constitutes the northern portion of the study region. The local PATH area, which comprises the remainder, consists of the portions of Hudson and Essex counties served directly by the PATH system.
**TABLE 1** SELECTED OWN-ELASTICITIES OF DEMAND, BY MODE, NEWARK DIVISION

<table>
<thead>
<tr>
<th>Mode</th>
<th>Elasticity of Demand with Respect to:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Line Haul Time</td>
</tr>
<tr>
<td>Auto</td>
<td>-2.69</td>
</tr>
<tr>
<td>Bus</td>
<td>-1.10</td>
</tr>
<tr>
<td>Auto-to-Bus</td>
<td>-0.95</td>
</tr>
<tr>
<td>Rail-to-PATH</td>
<td>-0.93</td>
</tr>
<tr>
<td>Auto-to-PATH</td>
<td>-1.02</td>
</tr>
<tr>
<td>Direct Rail</td>
<td>-0.37</td>
</tr>
</tbody>
</table>

**SOURCE:** Calculations from mode split model coefficients and level of service data.

**TABLE 2** OWN-ELASTICITIES OF DEMAND WITH RESPECT TO COST, BY MODE AND GEOGRAPHIC AREA

<table>
<thead>
<tr>
<th>Mode</th>
<th>Hoboken Division</th>
<th>Newark Division</th>
<th>PATH Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Auto</td>
<td>-1.52</td>
<td>-2.21</td>
<td>-1.57</td>
</tr>
<tr>
<td>Bus</td>
<td>-0.38</td>
<td>-0.64</td>
<td>-0.30</td>
</tr>
<tr>
<td>Auto/Bus</td>
<td>-1.55</td>
<td>-2.04</td>
<td>-1.07</td>
</tr>
<tr>
<td>PATH</td>
<td>N.A.</td>
<td>N.A.</td>
<td>-0.19</td>
</tr>
<tr>
<td>Rail/PATH</td>
<td>-0.49</td>
<td>-0.58</td>
<td>N.A.</td>
</tr>
<tr>
<td>Auto/PATH</td>
<td>-1.14</td>
<td>-1.25</td>
<td>N.A.</td>
</tr>
<tr>
<td>Direct Rail</td>
<td>N.A.</td>
<td>-0.24</td>
<td>N.A.</td>
</tr>
</tbody>
</table>

**SOURCE:** Calculations from mode split model coefficients and level of service data.

**TABLE 3** OWN- AND CROSS-ELASTICITIES OF DEMAND WITH RESPECT TO LINE-HAUL TIME: NEWARK DIVISION

<table>
<thead>
<tr>
<th>Demand For:</th>
<th>With Respect to Line Haul Time of:</th>
<th>Auto</th>
<th>Bus</th>
<th>Auto/Bus</th>
<th>Rail/PATH</th>
<th>Auto/PATH</th>
<th>Rail</th>
</tr>
</thead>
<tbody>
<tr>
<td>Auto</td>
<td>-2.69</td>
<td>0.04</td>
<td>0.01</td>
<td>0.30</td>
<td>0.15</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>Bus</td>
<td>0.20</td>
<td>-1.10</td>
<td>0.01</td>
<td>0.36</td>
<td>0.15</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>Auto-to-Bus</td>
<td>1.65</td>
<td>0.04</td>
<td>-0.95</td>
<td>0.30</td>
<td>0.15</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>Rail-to-Path</td>
<td>0.22</td>
<td>1.13</td>
<td>0.01</td>
<td>-0.93</td>
<td>0.15</td>
<td>0.09</td>
<td></td>
</tr>
<tr>
<td>Auto-to-Path</td>
<td>1.58</td>
<td>0.04</td>
<td>0.01</td>
<td>0.30</td>
<td>-1.02</td>
<td>0.09</td>
<td></td>
</tr>
<tr>
<td>Direct Rail</td>
<td>0.21</td>
<td>0.04</td>
<td>0.01</td>
<td>0.36</td>
<td>0.15</td>
<td>-0.37</td>
<td></td>
</tr>
</tbody>
</table>

**SOURCE:** Calculations from mode split model coefficients and level of service data.
The elasticity values in the Newark Division, where more alternatives are available, are without exception higher in absolute value than the corresponding values for the Hoboken Division. This fact emphasizes once again the effect that the presence of a large number of alternatives has on individual elasticity values. Conversely, elasticities are lower in the local PATH area because of the smaller number of trans-Hudson modes available there. In addition, the price elasticity of demand for PATH is low because of the huge share of the market that PATH commands in that area. In effect, there are few trans-Hudson commuters left to be diverted to PATH.

Table 3 presents the own- and cross-elasticities of demand with respect to line-haul time that the model implies for the Newark Division. Here the ability of this specification to provide a flexible treatment of a large number of travel alternatives is apparent. The first column of the table shows that an improvement in automobile travel time will have a major effect on demand for the two park-and-ride modes, and much less effect on demand for the more traditional transit alternatives. The second column shows that improvements in regular bus service are likely to have a much bigger effect on use of the rail with transfer to PATH option than on other transit modes. This result confirms impressions formed by NJ Transit staff based on recent shifts in patterns of demand. The third column, however, indicates that a change in automobile-to-bus travel time would be likely to have a uniform effect on the demands for other modes. The competing mode terms in Equation 1 were insignificant for automobile-to-bus mode, providing direct statistical support for the appropriateness in this case of the IIA assumption. A similar result was found in the case of the automobile-to-PATH mode. Changes in travel time for either rail with transfer to PATH or direct rail would have differential effects on demands for the other modes, although in these cases the differences are not pronounced.

CONCLUSION

The results presented are intuitively plausible, and generally conform closely to the expectations of knowledgeable observers of recent developments in the trans-Hudson travel corridor. They demonstrate the ability of this model form to provide a sensitive, accurate treatment of the complex multimodal environment of the northern New Jersey to Manhattan market. With seven primary modes and an empirically estimated pattern of intermodal competition, this model represents a considerable advance in the ability to deal with markets of this type. It has proven to be a useful, flexible tool for evaluating potential solutions to the trans-Hudson crisis.

REFERENCE