Quick Estimation of Queueing Delay for Passengers Exiting a Rapid Transit Station

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A method for estimating queueing delay for passengers exiting a rapid transit station is described. The method requires estimates of the average number of people exiting each train, average headway, and average service rate as inputs. The average delay is easily calculated with a hand calculator.

The purpose of this paper is to describe a method for estimating queueing delay for passengers exiting a busy rapid transit station. The model presented here is described in greater detail in “Passenger Delay in a Rapid Transit Station” (1).

APPLICATION

A typical rapid transit station serves two tracks, which carry trains in opposite directions. The station may have a central platform, which serves both tracks, or two side platforms, each of which serves a separate track. Passengers exiting trains may be delayed when queues form at any one of several places: (a) entrance to stairs or escalators; (b) fare gates or turnstiles; or (c) train doors. These will be referred to as "servers."

In most rapid transit systems, queues are larger exiting the station (after exiting a train) than entering the station (before boarding a train). This is not because less capacity is provided for exiting passengers. Instead, it results from the passenger arrival pattern. The arrival pattern for boarding passengers tends to be fairly steady: passengers arrive independently of each other at a rate that does not change abruptly. The arrival pattern for exiting passengers is "lumpy." If 100 passengers exit a train, then 100 passengers arrive simultaneously. A lumpy arrival pattern is more difficult to accommodate than a steady pattern. For this reason, this paper concentrates on the delays encountered by exiting passengers.

The bottleneck is the server with the smallest capacity and is usually easy to identify because it is the last place where passengers encounter a queue. Passengers encounter little or no delay after passing through the bottleneck. A well-known characteristic of queues is that when servers operate at full capacity, total delay can only be reduced by increasing the capacity of the bottleneck (2). Increasing the capacity elsewhere can only move the delay from one place to another. For example, increasing the capacity at the train door only moves a portion of the delay from the train door to the next server (perhaps an escalator). This is why servers in series are typically designed to have similar capacities.

The “time-in-station” is described in the sections that follow. The first section contains a description of how to estimate time-in-station when each track is served by its own platform; the second section contains a description of how to estimate time-in-station when both tracks are served by the same platform. Time-in-station is the time from when the train door opens until the passenger exits the station. It includes both the walking time from the train door to the exit and the time spent in queue.

PASSENGER DELAY: SEPARATE PLATFORMS

When a station has separate platforms and separate exits for each track, total passenger delay can be calculated as follows.

The bottleneck capacity, $c$, is the minimum capacity of all of the servers encountered by the passenger (stairs, fare gates, etc.). From “Passenger Delay in a Rapid Transit Station” (1):

$$D = \frac{A^2 + V(A)}{2ch} + \frac{T}{h}$$

(1)

where

$D$ = average time-in-station (hr per hr of operation),

$A$ = average number of people exiting each train,

$V(A)$ = variance of the number of people exiting each train,

$h$ = average train headway (hr),

$c$ = capacity of the bottleneck server (passengers per hr), and

$T$ = average walking time from the train door to the exit if no queue is encountered (hr).

For a given volume (passengers per hour), time-in-station tends to increase as load size increases. In essence, it is better to have 12 trains per hour, each with 50 passengers, than to have 6 trains per hour, each with 100 passengers. Time-in-station declines as the capacity of the bottleneck increases.

Sample calculations for time-in-station are given in Table 1, and the results are summarized in Table 2. Equation 1 is used to assess the relationship between time-in-station and the capacity of the fare gates. With six or fewer fare gates, the capacity is less than any other server. However, with eight or more fare gates, the capacity is greater than the stairs and the stairs are the bottleneck. Therefore, time-in-station does not decrease when more than eight fare gates are provided.
TABLE 1 STATION DATA

<table>
<thead>
<tr>
<th>Load Size Data (28 trains)</th>
</tr>
</thead>
<tbody>
<tr>
<td>111</td>
</tr>
<tr>
<td>308</td>
</tr>
<tr>
<td>271</td>
</tr>
<tr>
<td>163</td>
</tr>
<tr>
<td>366</td>
</tr>
<tr>
<td>278</td>
</tr>
<tr>
<td>179</td>
</tr>
</tbody>
</table>

Server Capacities

Train Door: 50,000 passengers/hour
Stairs: 20,000 passengers/hour
Fare Gate: 3,000 passengers/hour per gate

Walking Time to Exit: 45 seconds = .0125 hours
Headway: 4 minutes = .0667 hours

Calculations

\[
\begin{align*}
A &= \frac{\sum A_i}{n} = \frac{8397}{28} = 300 \\
V(A) &= \frac{\sum (A_i^2/n - A)^2}{n} = \frac{2738689/28 - 300^2}{28} = 7810 \\
\text{Passenger Arrival Rate} &= \frac{A}{h} = 4500 \text{ passengers per hour} \\
D &= \frac{A^2 + V(A)}{2ch} + \frac{T}{h} \frac{A}{h} = \frac{300^2 + 7810 \frac{1}{c}}{2h} + 0.0125 \cdot 4500 \\
&= \frac{733575 \frac{1}{c}}{2h} + 56.25
\end{align*}
\]

TABLE 2 TIME-IN-STATION PER PLATFORM: SEPARATE PLATFORMS

<table>
<thead>
<tr>
<th>Gates</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>C (1,000 passengers per hour)</td>
<td>6</td>
<td>12</td>
<td>18</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>D (hours per hour)</td>
<td>178</td>
<td>117</td>
<td>97</td>
<td>93</td>
<td>93</td>
</tr>
<tr>
<td>Delay/pasenger (minutes per passenger)</td>
<td>2.35</td>
<td>1.55</td>
<td>1.29</td>
<td>1.24</td>
<td>1.24</td>
</tr>
</tbody>
</table>

\*Delay/pasenger = D/(A/h).

SHARED PLATFORM

The calculation for passenger delay with a single platform serving both tracks is based on the following:

\[
\begin{align*}
A_1 &= \text{average number of passengers exiting each train on Track 1,} \\
A_2 &= \text{average number of passengers exiting each train on Track 2,} \\
V(A_1) &= \text{variance of number of passengers exiting each train on Track 1,} \\
V(A_2) &= \text{variance of number of passengers exiting each train on Track 2,}
\end{align*}
\]

The third term accounts for the “interference delay” when two trains on opposite tracks arrive at nearly the same time.

The data in Table 3 show time-in-station for a shared platform (based on load sizes given in Table 1, assuming that the arrival pattern is the same on both tracks). Notice that the delay per passenger is only slightly larger for the shared platform than for separate platforms, even though one-half as many fare gates are required.

DISCUSSION OF RESULTS

The equations are most useful in answering “what-if” questions, such as, what will be the change in passenger delay if an
Table 3: Time-in-Station: Shared Platform

<table>
<thead>
<tr>
<th>Gates</th>
<th>c (1,000 passengers per hour)</th>
<th>D (hours per hour)</th>
<th>Delay/passenger (minutes per passenger)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6</td>
<td>539</td>
<td>3.59</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td>280</td>
<td>1.87</td>
</tr>
<tr>
<td>6</td>
<td>18</td>
<td>214</td>
<td>1.43</td>
</tr>
<tr>
<td>8</td>
<td>20</td>
<td>202</td>
<td>1.35</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
<td>202</td>
<td>1.35</td>
</tr>
</tbody>
</table>

Note: $A_1 = A_2 = 300$; $V(A_1) = V(A_2) = 7,810$; $h_1 = h_2 = 0.0667$ hr; $T_1 = T_2 = 0.0125$ hr. Equal loads on platforms.

The equations are most accurate for moderately busy stations. If passenger queues are so large that they do not dissipate between the arrival of one train and the arrival of the next train on the same track, then Equations 1 and 2 will underestimate time-in-station. If load sizes are so small that queues rarely materialize, then Equations 1 and 2 will overestimate time-in-station. For this reason, the equations should generally not be used to estimate delay during off-peak hours.

Acknowledgment

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References