An Idealized Model for Understanding Impacts of Key Network Parameters on Airline Routing

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The objective of this study is to understand the impacts of the key network parameters—demand level, network size, and number of cities—on airline routing of city pairs, specifically on whether they should be served with nonstop or transfer flights. This objective is accomplished by using an approximate model under the single-hub network, at minimal cost. The proposed routing strategy uses angles to divide city pairs into two groups, based on their relative locations: those served by nonstop flights and those served by transfer flights. Costs that would be minimized include airline operating costs and the passenger costs of schedule delays, en route time, and transfer delays. The relationships between the optimal angle and network parameters are explored with a circular network configuration. The passenger demand is assumed to be homogeneous. A numerical example of applying the model to the U.S. airline network is presented. According to the model, demand has a positive and significant impact on the use of nonstop flights. However, number of cities and network size have negative and insignificant impacts. According to the model, as the time value of schedule delays alone increases, the time value of en route time alone decreases, or the time value of all delays decreases, more city pairs should be served with transfer flights. The total cost is generally not sensitive to the angle size. The model also reflects the hubbing phenomenon and the impact of demand on shaping the routing patterns in the U.S. airline network.

Routing is one of the key components of any transportation system. Carriers (e.g., trucking companies, freight airlines, or passenger airlines) cannot determine their operational plans, such as fleet and crew assignments, without knowing the routing configuration. Users (e.g., freight forwarders, shippers, or passengers) plan their shipping or travel routes based largely on routing configurations. A sound routing strategy will increase operational efficiency and be convenient for the users.

Hubs are cities—usually large ones—that carriers use as operational centers. Hubbing is a routing strategy in which passengers or goods are transported to their destination via spokes to the hubs rather than directly on nonstop flights. Because of deregulation, there has been a recent trend of using hubbing in industries, such as trucking, air freight, and air passenger transportation.

Among these industries, passenger airlines are unique in the following ways:

- Compared with goods shipped by trucks and freight airline passengers have high time values. They are generally concerned more with service frequency, en route travel time, and connection delays. For example, backtracking routes, which are common in the freight industry, are unlikely to be implemented in passenger airlines. Also, storing and consolidating methods used at the freight terminals can only be used for passenger airlines if they can be accomplished in a timely fashion by schedule coordination at the hub.
- Because of high taking-off and landing costs for the aircraft, passenger airline networks are structured with many routes and few stops along each route. The multistop routes used in the trucking industry are not appropriate for passenger airlines. Although they have many of the same features as passenger airlines, freight airlines do not have to deal with airport congestion because most of their flights are scheduled during off-peak hours. Thus, schedule coordination at the hub can be more flexible.

U.S. passenger airlines have gradually increased their hubbing activities since the passage of the 1978 Airline Deregulation Act (*1*). As a result, the number of cities with more than 1 percent of total enplaned passengers (of the U.S. network) increased from 25 in 1977 to 36 in 1981 (*2*). Under such hub-and-spoke operation, an airline's principal hub may serve as many as 100 cities with 500 flights departing daily.

Why does hubbing benefit airlines? Hubbing enables airlines to make the best use of assets such as aircraft, crews, and gate spaces. Aircraft load factor, defined as the percentage of occupied seats, can be increased by consolidating passengers destined to several cities into one spoke. Crew assignments and scheduling can be centrally supervised at the hub. Gate spaces can be used more fully by serving more flights in and out of the hub. Consequently, cost per passenger can be reduced.

Why does hubbing also benefit passengers? Although hubbing increases passengers' travel time, it can provide them more frequent flights, choice of destinations, and convenient connections without interline operation (i.e., using more than one airline in a single trip). For example, a United Airlines passenger taking any of the flights from Grand Rapids to Chicago can transfer within an hour to a flight to San Francisco. Previously, that passenger would either have had to take the single daily nonstop flight to San Francisco or arrive in Chicago to wait several hours and possibly change carriers. The same is true for other destinations west of Chicago. Fare reduction as a result of lower airline costs is another benefit to passengers.

Since hubbing provides economical operation for airlines and convenient service for passengers, hubbing should be emphasized in designing routing strategies. Both airline and

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passenger costs should be considered in determining the most efficient routing system. However, most studies of airline network routing either overlook the hubbing phenomenon or focus only on individual airline or passenger costs. They either analyze the problems empirically or apply traditional operation research techniques to obtain detailed solutions. They illustrate neither trade-offs nor cause-effect relationships among key network elements.

The objective of this study is to understand the effects of key network parameters, under the single-hub network structure and minimum-cost goal, on point-to-point (i.e., nonstop) and hub-and-spoke (i.e., transfer) operations of each origin and destination (OD) pair.

The proposed routing strategy is based primarily on relative locations of origins and destinations. The costs to be minimized include airline operating costs and passenger time costs. The key network parameters include demand level, network size, and number of cities or nodes. The demand is considered to be homogeneous and the airline network is structured in a circular configuration.

PREVIOUS STUDIES

The following literature review includes published works on airline network routing with emphasis on hubbing, first, and research methods on network routing, second.

De Vany and Garges (*3*) studied the relationship between fleet assignment and two service patterns: direct and hub-and-spoke. First, they found that the hub-and-spoke pattern offers high frequencies, which, in most cases can offset the lengthened flight times. Second, they found that this pattern establishes a feeder structure that permits greater filling of the wide-body jets and more efficient route assignments. Gordon (*4*) explored mathematically and empirically the relationships between scale economies and network shapes (including modes other than air). He concluded that

- Fully connected transportation networks are rare because of the existence of scale economies for most transportation modes,
- The greater the scale economy, the less connected is the network shape and the more concentrated the traffic pattern,
- Congestion at nodes should result in a more connected network, and
- The network shape, given a fixed-cost function, should depend on supply-demand or cost-service equilibrium.

Gordon and De Neufville (*5*) presented a method of choosing various configurations for air networks. One of their conclusions was that hub-and-spoke networks minimize overall schedule delays but point-to-point networks provide a more even service quality. Ghobrial (*6*) developed an equilibrium model of air network considering airline competition and passenger routing preference. The results suggested that network hubbing is efficient and airlines will probably find it advantageous to hub, despite the pricing penalties resulting from airport congestion imposed on airlines using major hubs. Kanafani and Ghobrial (*7*) showed hubbing to be inelastic to hub pricing and potential benefits to be gained to airports. Kanafani and Hansen (*8*) investigated empirically the effect of air network hubbing on airline productivity. Their findings indicated no direct connection between the degree of hubbing and airline cost over the 1976 to 1984 period.

O'Kelly (*9*) determined the optimal locations of a single hub and two hubs by minimizing flow-weighted distance. He found that the least-cost site in the United States generated by a single-hub model is northeast of Cincinnati, Ohio, and that, for a two-hub system, as the effects of scale economies increase, the locations of the two hubs move apart. Chan and Ponder (*10*) discussed the factors that contributed to the success of hub-and-spoke networks for the air freight industry. They also pointed out the need to study the thresholds of transition for different routing structures (e.g., single-hub, minihub, multihub, and point-to-point systems).

While these studies addressed various aspects of hubbing (e.g., aircraft technology, airport economics, equilibrium, productivity, air freight, and hub location and operation), they do not provide understanding of the trade-offs and cause-effect relationships among key network components.

Many mode-specific network routings have been examined using the traditional techniques of operation research. Most of the previously mentioned references on airline network routing also adopt this research method, which generally treats nodes and links as discrete entities. Then, either mathematical programming (e.g., integer linear programming, dynamic programming, tree-search techniques) or heuristic algorithms are applied to find an answer. Typically, an attempt is made to find an "exact" answer using the calculating power of computers. However, with these methods, the number of variables and constraints sometimes increases so fast as the network becomes larger that computing time and memory needs become prohibitive. Because detailed location-specific and demand data are required for all OD pairs, the collection and coding of these data are a massive job and a large source of errors. Clearly, these methods are ill-suited for cause-effect and trade-off analyses, in terms of human understanding, time, and cost.

By contrast, this paper adopts an "idealized model" approach, similar to the continuum approximations that were developed to study commuting, congestion, and minimum-cost-path problems in urban areas (*11, 12*). The idealized (continuous) model approach has been applied to scheduling, location, and zoning problems (*13–15*). More recently it has been used to examine many-to-one and many-to-many logistics problems. An idealized model of airline routing would approximate the service area with a continuum and would result in analytical expressions for distance traveled and other performance measures (*16*). The equations would be based on a few easily measured aggregate parameters, such as spatial densities, average demand rates, area sizes, and so on.

Although less precise, the idealized model approach is preferable to the programming and algorithm approach because it

- Requires less effort on input data preparation, solution computation, and model application;
- Is less sensitive to the scale and complexity of the network;
- Is more likely to lead to qualitative insights; and
- Is more convenient for sensitivity analysis.

The idealized model approach has been applied to both one-to-many and many-to-many networks. Few studies have been conducted on many-to-many networks. Among them, passenger networks—the subject of this paper—appear not to have been studied at all (*17*).

DESIGN OF STUDY

The airline passenger network is modeled as a single-hub network. Each OD pair can be served by one of two types of operation: point-to-point (i.e., nonstop flights) and hub-and-spoke (i.e., transfer flights through the hub). In other words, the maximum number of stops allowed for each OD pair is one. Although, in reality, most airlines have multihub systems and feature some multistop flights, this study does not address them.

Another important assumption of this study is that demand is inelastic with respect to time and cost. Because demand is fixed over time, the average schedule delay can be simply calculated as half of the average flight headway. Although understanding that demand-supply equilibrium, impact of competition, and temporal distribution are desirable goals, models including such phenomena are too complex. Also, the changes in demand as a result of airlines' marketing strategies are usually not significant within a short time period.

Other basic assumptions of the model are listed below:

- Because aircraft technology is important in shaping the routing structure, aircraft capacity is treated as an endogenous variable;
- Aircraft load factor is assumed to be constant; and
- The hub has enough gate capacity to handle all the flights.

As a common practice, airlines often maximize profits (i.e., revenues minus operating costs) subject to passenger service constraints (e.g., schedule delay, transfer delay, and line-haul time). This optimization problem (including objective function of airline profit and constraints of passenger delays) can be reduced to a Lagrangian function (a new objective function formed by summing the original objective function and the products of multipliers and delay constraints) using the Lagrange multiplier technique. These multipliers represent time values of various delays. Thus, these products become passenger time costs of various delays.

Since the passenger demands are assumed to be inelastic, maximizing profit is the same as minimizing the cost of servicing a fixed demand. Ignoring revenue portion, the above Lagrangian function can be rewritten as the sum of airline operating costs and passenger time costs. This final Lagrangian function, which can be regarded as the total cost of the airline network system, is the objective function to be minimized. It includes costs on the supply and demand sides. The supply cost is the airline operating cost; the demand cost is the passenger time cost due to schedule delays, transfer delays, and line-haul time. From these discussions it can be concluded that the minimizing cost approach adopted here is consistent with the airline's common profit-maximizing practice.

A circular network configuration assumes that all nonhub nodes are uniformly located along the circumference of a circle with a hub at the center. Real-world networks are approximated as circular networks needing only the radius of a circle and the number of nodes. The radius of the circle can be calculated by averaging all the distances between nonhub nodes and hub. Some distances can be easily expressed in terms of a few parameters *(17)*. Although this configuration is not totally consistent with reality, it represents node locations in a simple and symmetric pattern, allowing the primary issues of this study to be clearly and thoroughly explored.

A homogeneous demand pattern is used to approximate real-world demand. A constant p is assigned to each $n(n + 1)$ cell of the OD matrix to represent homogeneous demand for a network with n nonhub nodes. The greatest advantage of such a pattern is that the demand of the network can be described using only p and n. Although it is homogeneous in OD demands, traffic links that depend on routing strategy are not always homogeneous. This demand pattern is simple but does not easily accommodate the real-world situation.

MODEL DEVELOPMENT

The following symbols are used in this paper:

p = average demand per OD pair (passengers per day),
c_i = aircraft capacity of the ith link (passengers per aircraft),
l_i = stage length (i.e., the distance covered per aircraft hop from take-off to landing) of the ith link (mi),
δ = average income of air traveler (dollars per hr per passenger),
α = a fraction such that $\alpha\delta$ represents average time value of passenger schedule delay,
β = a fraction such that $\beta\delta$ represents average time value of passenger line-haul time (i.e., in-vehicle time),
γ = a fraction such that $\gamma\delta$ represents average time value of passenger transfer delay,
v_i = aircraft travel time of the ith link (hr),
n = number of nonhub nodes in the network,
d = radius of circular network (mi),
k_i = average aircraft operating cost of the ith link (dollars per aircraft-mi), and
t = average operating hours of airline (hr per day).

Aircraft operating cost per aircraft-mi (k_i) is the key element in determining total airline operating cost. Based on 1981 data of six different aircraft with capacities ranging from 115 to 500 passengers and stage lengths ranging from 200 to 2,500 mi *(1)*, k_i is a function of stage length (l_i) and capacity (c_i). k_i increases with c_i because larger aircraft consume more fuel and require larger crews for a given stage length. On the other hand, k_i decreases with l_i because a fixed portion of the operating cost for taking off and landing is independent of the stage length. In plotting these data versus c_i/l_i (i.e., passengers per aircraft-mi), a linear trend can be observed (Figure 1). Using these data, the following function for k_i is calibrated by regression:

$$k_i = a_0 + a_1\left(\frac{c_i}{l_i}\right) \tag{1}$$

where a_0 = \$4.1/aircraft-mi and a_1 = \$15.6/passenger.

Actually, a_1 can be regarded as the fixed aircraft operating cost per seat for taking off and landing. And $a_1 c_i$ is the fixed portion of airline operating cost for flying l_i. On the other hand, a_0 can be regarded as the variable unit for aircraft operating cost under cruising speed, and $a_0 l_i$ is the variable portion of the

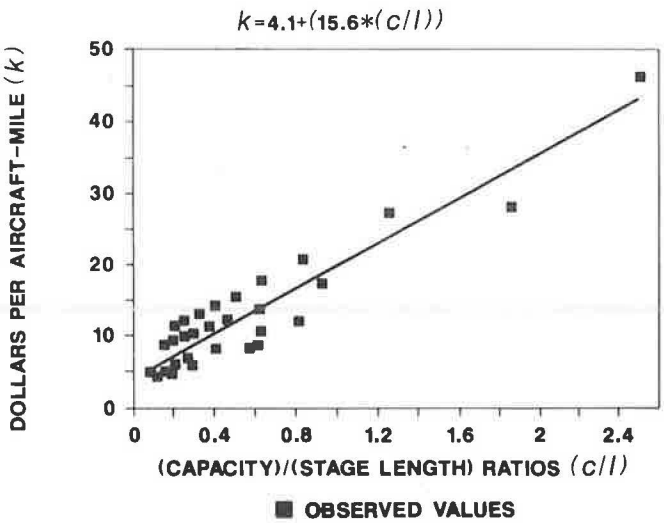

FIGURE 1 Aircraft operating cost function.

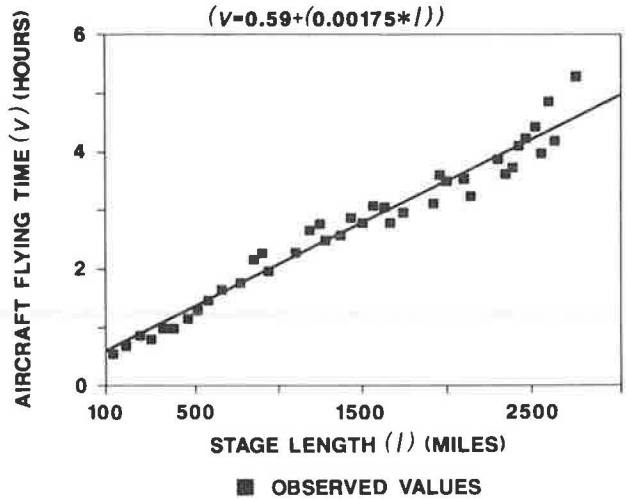

FIGURE 2 Aircraft travel time function.

airline operating cost of flying l_i. Although the linear assumption for c_i and l_i may not necessarily be in the best form for both physical meaning and data fitting, it simplifies the derivation for the rest of the model compared with other forms that could be used, such as square or product. The other forms show only marginal increase on R^2 for data fitting, however.

Aircraft travel time (v_i) is crucial in computing passenger line-haul time cost. It should include times for aircraft to take off and land and fly under cruising speed. Thus, the average overall travel speed should be smaller for shorter stage lengths because of the fixed portion of time spent on taking off and landing. Using random samples of nonstop flight data with stage lengths ranging from 100 to 2,500 mi according to system timetables published by Delta Airlines, aircraft travel time measured in hours is calculated by the difference between scheduled times at origin and destination. Travel times plotted against stage length (Figure 2) have a fairly linear functional relationship. Using these data, the following function for v_i is calibrated (also see Figure 2 about the fit) by regression:

$$v_i = a_2 + a_3 l_i \tag{2}$$

where $a_2 = 0.59$ hr and $a_3 = 0.00175$ hr/mi.

The value of a_2, which is approximately 35 min, can be regarded as the fixed taking-off and landing time regardless of the stage length. The inverse of a_3, which is approximately 570 mph, can be regarded as the cruising speed of the aircraft. This value matches reasonably well with most conventional jet planes (*18*). Based on Equation 2, the average aircraft overall travel speed drops from 490 to 340 mph when stage length decreases from 2,000 to 500 mi.

The time-value parameters (i.e, δ, α, β, and γ) are important in determining time costs for passengers. The value of δ in terms of 1981 dollars (the same monetary value as k_i) is calculated as follows. The average hourly family income of air travelers in 1979 is $14.70 (*19*). When the purchasing power of the consumer dollar (1979's is 1.25 times that of 1981) and family size counting only adults (1.65 adults per family in 1981) are considered, $12/hr per passenger results in δ.

According to the empirical studies (*20, 21*), the values of α, β, and γ ranged from 0.15 to 1.49 depending on trip purposes, transportation modes, trip length, passenger productivity, and so on. However, no clear distinction among α, β, and γ has been made. Based on how efficiently various time periods can be used by passengers, schedule delay, line-haul time, and transfer delay should have different values. Transfer delay has the highest time value ($\delta\gamma$) because that period cannot be efficiently used. Schedule delay has the lowest time value ($\delta\alpha$) because, by knowing the schedule in advance, passengers can coordinate their activities to use much of the delay period. Line-haul time has an intermediate time value ($\delta\beta$). Thus,

$$\alpha < \beta < \gamma \tag{3}$$

Reasonable values are assumed for these variables based on the above relationship. Conceptually, it is logical to assume the value of 1.0 for γ because this is the highest income a passenger can earn. Hensher (*21*) recommended 0.685 for travel time, which closely resembles the line-haul time in this study. Therefore, $^2/_3$ is assumed for β. By assuming $\gamma - \beta = \beta - \alpha$, which means that the time value difference between transfer delay and line-haul time is equal to the difference between line-haul time and schedule delay, α is assumed to be $^1/_3$.

SPLIT ROUTING

In the real world, airlines seldom use either point-to-point or hub-and-spoke operation exclusively. Two types of routings mixed with point-to-point and hub-and-spoke operations are proposed:

- Each OD pair is served, by splitting its demand, with both point-to-point and hub-and-spoke operations; and
- Destinations are served, by splitting them depending on their relative locations to the origin, with either point-to-point or hub-and-spoke operation.

The first method of routing is proved by this author (*17*) to be inferior to either point-to-point or hub-and-spoke routing (i.e., all-or-nothing in terms of demand) because total cost is a concave function of demand. Thus, only the latter method (called "split routing") is pursued further.

The idea behind split routing is to reduce the circuity (defined as the extra distance needed to serve an OD pair by hub-and-spoke rather than point-to-point operation) at certain locations for both passengers and airlines. For example, the circuity to serve OD pair AB in Figure 3 by hub-and-spoke operation is much greater than the circuity to serve OD pair AC by hub-and-spoke operation (i.e., the circuity from A to the hub to B is greater than from A to the hub to C). Thus, it is reasonable to serve the nodes closer to origin with point-to-point and the others with hub-and-spoke operation.

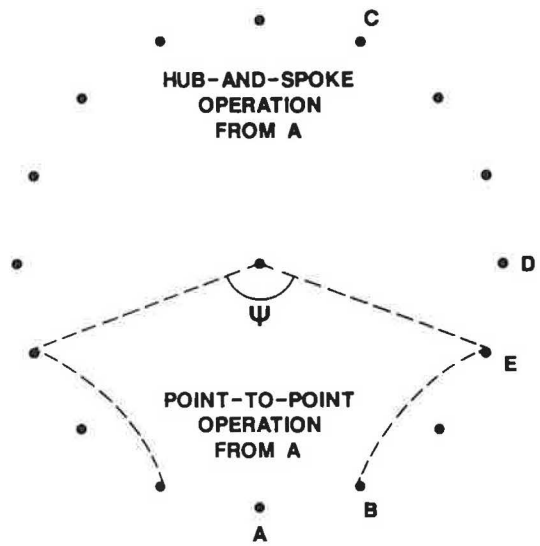

FIGURE 3 Illustration of split routing.

The problem here is to find the number of neighboring nodes on each side of nonhub node (q) served by point-to-point operation such that the total cost of split routing is minimized. Since all nonhub nodes are equally distant to the hub, it will never be optimal to serve node D and E from A in Figure 3 with nonstop and transfer flights, respectively, compared with serving them in the opposite way. Since q may vary with n, it is not a compatible measure for different network configurations. Instead, the split angle φ measured in radians (see Figure 3), corresponding to the length of arc occupied by neighboring nodes, is defined as

$$\varphi = 2\pi\left(\frac{2q + 1}{n}\right) \tag{4}$$

This split angle is used as the system measure to reflect various degrees of point-to-point operation.

Since all links have the same traffic and stage length for the hub-and-spoke operation, only one aircraft size (c) is needed. In other words, the flight frequency of all links will also be the same. Thus, the minimal transfer delays for the hub-and-spoke operation can be achieved by banking all individual flights from various origins into the first half of a common time slot (2 hours is used in this study) at the hub. Thus, passengers can transfer to their destination flight within the second half of the same time slot by spending approximately a 1-hour delay at the hub. In randomly sampling real-world connecting times at the hub from system timetables of major airlines, values range from 0.5 to 1.5 hr, which is consistent with the above assumption.

Figure 4 shows how to obtain the optimal split angle (φ^*) in terms of n, d, and p. [The derivation of each component is given in detail in the author's dissertation (*17*).] The following function of φ^* in the unit of radians is the result:

$$\varphi^* = 0.5\pi \frac{p^{0.6}}{n^{0.2} d^{0.3}} \tag{5}$$

where the constant 0.5 has the dimension of (radian) (number of nodes)$^{0.2}$ (miles)$^{0.3}$ (day)$^{0.6}$ (number of passengers)$^{-0.6}$ and the dimensions of n, d, and p can be referred to the previous definition.

Although φ^* can be numerically solved, the relationship between T_m (total cost for split routing) relative to T_m^* (optimal T_m) and φ can add more understanding. Figure 5 shows the relationship between the ratio $(T_m - T_m^*)/T_m^*$ and φ. The U-shaped curves in Figure 5 can be explained by the following trade-offs between φ and various cost components when φ increases.

- Schedule delay cost increases as a result of less frequent flights,
- Line-haul time cost decreases as a result of less circuity,
- Airline operating cost, which depends on the combining effects of smaller aircraft (increasing cost) and less circuity (decreasing cost), and
- Transfer delay cost decreases as a result of fewer transfers.

To find out the overall fit of estimated angles from Equation 5, the ratios of the difference (between estimated and theoretical angles) to the theoretical angles are calculated. It is found that 90 percent of the observations are within 20 percent difference. However, the system costs are shown to be rather flat near φ^* from Figure 5. It is found that the cost difference never exceeds 5 percent when the angle difference is within 20 percent.

The effects of changing n, d, or p to φ^* are presented in Figure 6. It shows that p has a positively stronger effect on φ^* compared with the negative effect of n and d.

By similar analyses, the effects of parameter (other than n, d, and p) variations on φ^* can be explored. Three examples are presented:

- Change time-value fraction β—By increasing β from $^2/_3$ to 1, the coefficient of Equation 5 increases to 0.58. Hence, the airline should use point-to-point operation for more OD pairs because the line-haul time savings from the circuity is more significant.
- Change time-value fraction α—By increasing α from $^1/_3$ to $^2/_3$, the coefficient of Equation 5 decreases to 0.31. Thus, the airline should use hub-and-spoke operation for more OD pairs since the schedule delay saving from scale economies is more significant.
- Change time value and time-value fractions—The following set of time-value parameters, which represents an example of low-value goods, is analyzed: $\delta = 1$ and $\alpha = \beta = \gamma = 1$ (i.e., no distinction on various time values, which is generally true for freight). The coefficient of Equation 5 decreases to 0.45. Thus, the low-value goods tend toward hubbing because they are less sensitive to both scale economies and circuity effects.

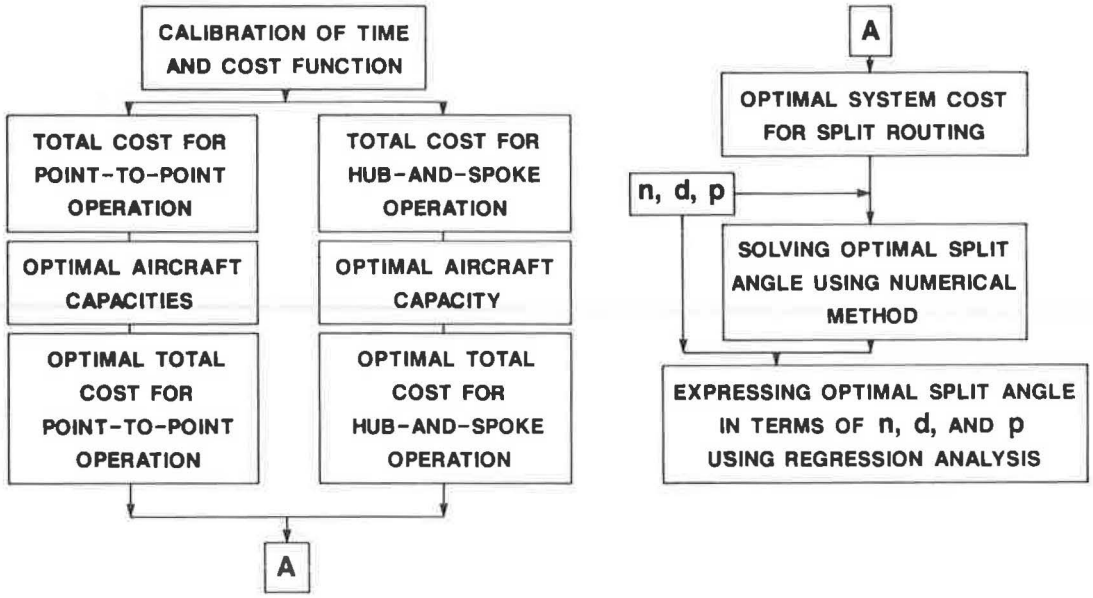

FIGURE 4 Flowchart of model derivation.

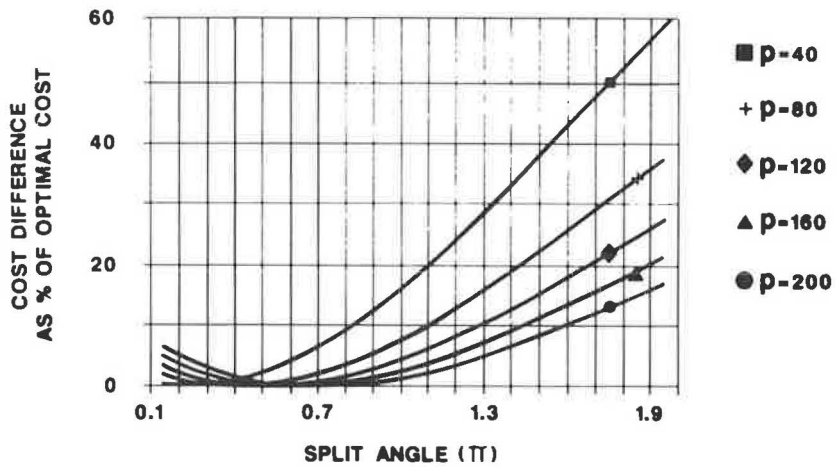

FIGURE 5 Total system cost difference versus split angle.

MODEL APPLICATION

The U.S. airline network is used as an example for application. No attempts are made here, because of the incomplete data from the airlines and the idealized model structure of this study, to verify the model with the real-world individual airline network. Rather, some qualitative implications from applications are assessed with real-world aggregate statistics.

The U.S. airline network is studied chronologically for the years 1977, 1981, and 1985. The nodes are cities and Standard Metropolitan Statistical Areas requiring aviation services. According to the criteria from FAA, nodes with more than 0.05 percent of total enplaned passengers in the network are selected for this case study. There are 150, 129, and 116 nodes for the years 1977, 1981, and 1985, respectively. Although these nodes represent only about 30 percent of the certified points in the 50 states, their passenger enplanements account for 96.8, 96.4, and 97.6 percent for the years 1977, 1981, and 1985, respectively (2).

Since only a single-hub network is considered in this study, Kansas City, Missouri, near the gravity center of the United States mainland territory, is selected as the hub. To calibrate the demand parameters of the previously developed model, only daily demand generated from each node is needed. The total daily costs of different years are compared on a common value of the 1981 dollar because the developed model is calibrated based on 1981 data.

For homogeneous demand, total daily demand generated from all nodes (TD) and the total number of nonhub nodes (n) are needed to determine the value of p:

$$\frac{TD}{n(n+1)} \tag{6}$$

The total number of passengers (10 percent samples) for the 3rd quarter in 1977, 1981, and 1985 (22, Table 11) is adjusted accordingly to obtain TD. Table 1 gives the values of the

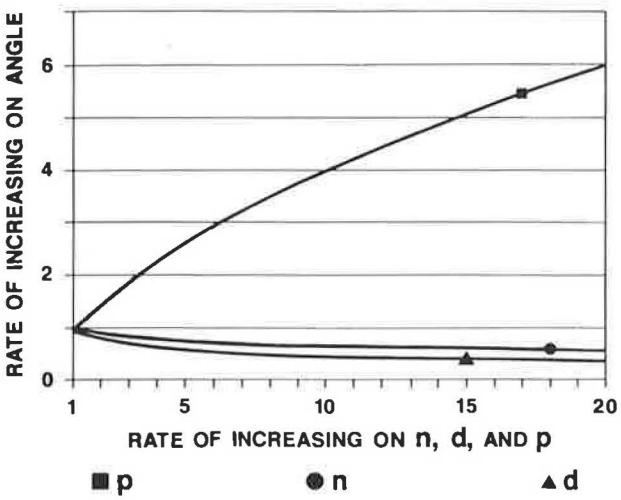

FIGURE 6 Effects of network parameters on split angle.

TABLE 1 PARAMETERS AND SYSTEMS MEASURES OF MODEL APPLICATION

	Years		
	1977	1981	1985
Network parameters			
n (no. of nodes)	149	128	115
p (passengers per day)	19	28	50
d (radius in mi)	875	903	917
System measures			
Split angle (φ, radian)	0.14π	0.18π	0.25π
No. of neighboring nodes (2*q*)	9	10	13

parameters needed and the system measure found in this section.

One additional parameter needs to be calculated for the circular network: the radius d, which is equal to the mean of all the air distances between nonhub nodes and the hub. The great circle distance in nautical miles can be calculated by the following expression (*23*):

$$\cos^{-1} [\sin (LA_O) \sin (LA_D) + \cos (LA_O) \cos (LA_D) \cos (LO_O - LO_D)] \quad (60)$$

where LA_O and LO_O are the latitude and longitude of the origin and LA_D and LO_D are the corresponding measures of the destination. By multiplying the expression by 1.15, the nautical miles are converted into statute miles. Knowing the values of latitudes and longitudes of nodes, the resulting d's for 1977, 1981, and 1985 are shown in Table 1.

The split angle (φ) covering the range of nodes served by point-to-point operation from each node was derived according to Equation 5. The values of n, d, and p for 1977, 1981, and 1985 and the values of φ* and their corresponding number of nodes (see Equation 4) served by point-to-point operation from each node are listed in Table 1.

The values of the network parameters used so far are either averaged over an entire year (e.g., the demand) or retrieved at the end of the year (e.g., the network size). However, the data may not always be available (e.g., there may be only partial OD demands in terms of time and locations). Moreover, the data are also dynamically altered (e.g., changing seasonal demands or changing network to cope with competition). In the manipulating process, errors may also be introduced into these data. Thus, the input data of the model used in the real world may not represent the "true" values. The effects of these data variations on the system measure need to be investigated.

The COV (coefficient of variation, which is equal to the ratio between standard deviation and mean) of the system measure and input parameters are used to measure the data variations. The COV of a system measure, when a particular network parameter is treated as a variable, is equal to the absolute value of the exponent of that network variable in the derived equation for that system measure, multiplied by the COV of that network variable (*17*). However, this finding is true only for the equations with exponents on their components of input parameters, such as Equation 5. For example, the estimating error of φ* resulting from the demand variation is only 60 percent of the data error from the demand itself. Since all the exponents of network parameters in Equation 5 are less than or equal to 1, the estimating errors of the system measure are never worse than the data errors from the network parameters. The robustness of the developed model has been demonstrated here.

IMPLICATIONS FROM MODEL APPLICATION

Two qualitative implications from model applications—hubbing phenomena and impact of demand on routing—are compared to real-world aggregate measures. The purpose is to show that the abstract model from this study, despite its simplifications and approximations, can still provide some insightful information by appropriate interpretation.

Hubbing Phenomenon

The findings from this study strongly indicate that it is economical to incorporate the hub-and-spoke operation into the routing strategy. In the real world, the same phenomena are observed:

- The number of enplanements is a measure of hubbing because each trip through the hub is counted as two enplanements. Thus, the larger the number, the higher the degree of hubbing. In order to be compatible for various demand levels, the percentage of total enplanements for each city is a more appropriate indicator. From the increasing percentage of total enplanements and decreasing number of cities with more than 0.05 percent total enplanements shown in Table 2 (*2*), the degree of hubbing indicated by the average percentage of total enplanements per city increased over the years. Since average percentages of both medium and small cities are stable, the large cities play a key role in shaping the hubbing network. It appears that the increasing number (but decreasing average percentage) in 1977 to 1981 and increasing average percentage (but decreasing number) in 1981 to 1985 of large cities are the driving forces behind the increased hubbing in these two periods after deregulation in 1978.
- Figure 7 shows the percentage of total enplanements for the top 10 airports (*2*). The cumulative percentages (44.9, 47.6, and 44.2 for 1977, 1981, and 1985, respectively) show the increasing then decreasing degree of hubbing. Moreover, the

TABLE 2 PERCENTAGE OF TOTAL ENPLANEMENTS BY CITY SIZE

	Years		
	1977	1981	1985
All cities (> 0.05%)			
No. of cities	158	153	124
Total %	96.8	96.4	97.6
Average % per city	0.61	0.63	0.79
Large cities (> 1%)			
No. of cities	25	36	26
Total %	68.1	70.0	72.8
Average % per city	2.7	1.9	2.8
Medium cities (0.25% to 0.99%)			
No. of cities	39	43	37
Total %	18.3	18.9	18.1
Average % per city	0.46	0.44	0.49
Small cities (0.05% to 0.24%)			
No. of cities	94	74	61
Total %	10.4	7.5	6.7
Average % per city	0.11	0.1	0.11

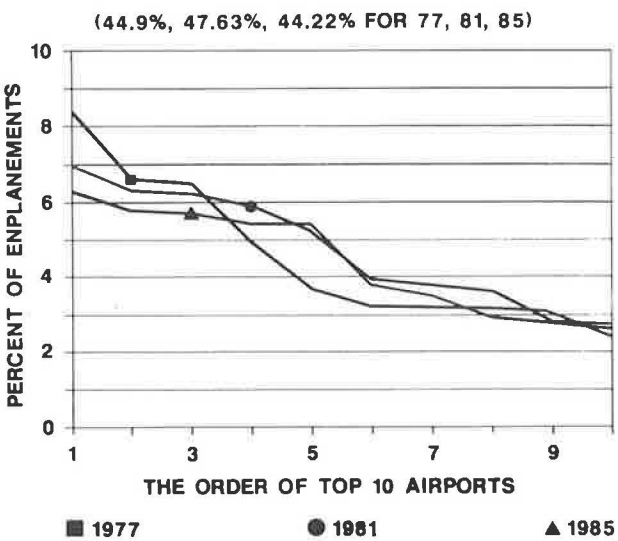

FIGURE 7 Percentage of enplanements for top 10 airports.

percentage of enplanements for the top three airports declined from 1977 to 1985. Both observations suggest that the large hubs are less concentrated (i.e., enplanements are spread more evenly) although the overall degree of hubbing of the entire network increased after the deregulation in 1978.

Impact of Demand on Routing

One of the important findings of this study is that demand has the most significant impact on shaping airline network routing in terms of providing more nonstop service. With approximately the same number of nodes and size of networks over the years (Table 1), the changing routing patterns in terms of system measure can be attributed primarily to the impact of demand. This can be verified if application results can qualitatively match real-world phenomena. According to Table 1, the optimal system measure indicates that increasing percentages of OD pairs (or passengers) are served with nonstop flights from 1977 to 1985. Some real-world aggregate statistics correspond to this observation:

- The number of OD pairs receiving nonstop flight service in the United States increased by 4 percent from 1978 to 1983 (*24*).
- The percentage of 145 cities connected with large, medium, and small cities (by FAA's definition) by nonstop flights was 31 percent, 14, percent, and 5 percent, respectively, for 1977; they increased to 34 percent, 17 percent, and 6 percent, respectively, for 1984 (*25*).
- According to the dissertation of Ghafouri-Varzand (*25*), the connectivity (measured by the connectivity index, defined as the ratio between the sum of the reciprocal harmonic mean of the actual trip times and the sum of the reciprocal harmonic mean of the ideal trip times) is significantly better in 1984 than in 1977. This indirectly implies that the larger portion of OD pairs are served with nonstop flights in 1984.
- Table 3 shows transfer enplanements as a percentage of total enplanements at several major hubs for 1977, 1981, and 1985. Generally, the decreasing percentage over the years implies that more passengers are served with nonstop services.

TABLE 3 PERCENTAGE OF TRANSFER ENPLANEMENTS AT MAJOR HUBS

	Years		
	1977	1981	1985
Chicago, Illinois	53	48	45
Dallas, Texas	55	53	52
Atlanta, Georgia	74	76	70
Denver, Colorado	49	57	54
Miami, Florida	58	63	56

From these cross-references between theoretical findings and practical observations, it appears that hubbing and the offering of more quality service with nonstop flights has occurred after the passage of the 1978 Airline Deregulation Act. This convenient route configuration can thus induce higher passenger demands, as shown from the real data. Because of the significant impact demand has on routing structure, as the model suggests, more nonstop services have been provided, as is shown from the real data.

Although higher demands were achieved by incorporating hub-and-spoke operations into routing structure over the past years, to overemphasize hubbing may not be desirable. The failure of hubbing-oriented carriers such as People's Express (although there may have been other factors contributing to the failures, such as competition and management, which were not considered in this study) supports this argument. In other words, for a new airline, the hub-and-spoke operations should be regarded as an interim tool to raise demand rather than an ultimate routing strategy. The point-to-point operation should be increasingly emphasized as the demand grows, as shown in this application.

CONCLUSIONS

The most important finding of this study is how network parameters affect the network routing pattern. Demand has positive and very significant impacts on the use of point-to-point operation. However, the number of nodes and the area

size have negative and very insignificant impacts. Other important findings regarding the model itself include the following:

- Serving an OD pair with both point-to-point and hub-and-spoke operations is less efficient than using either operation exclusively.
- Serving all OD pairs in the network with either point-to-point or hub-and-spoke operation is less efficient than dividing them into two groups by their locations, and serving them with split routing.
- As the time value of schedule delay increases or the time value of en route time or income decreases (i.e., low-valued goods), more OD pairs should be served with hub-and-spoke operation.
- The total cost is not very sensitive to aircraft capacity and system measure in the vicinity of its optimum.

Additional findings pertaining to the model applications include the following:

- The developed model performs reasonably well with limited data needed to describe the hubbing phenomenon and assess the significant impact of demand on routing structure.
- The overemphasized hub-and-spoke operation may not be efficient based on the model applications. As demand increases, point-to-point operation should be used more.
- The developed model is shown to be robust in terms of gracefully absorbing data errors from the real world.

In the real world, routing decisions are made by considering additional factors such as competition, dynamic supply-demand interaction, and resource constraints. With idealized network configurations and simplified assumptions involved in the proposed routing strategy, the developed model can neither fully represent the real airline networks nor the "optimal" system. However, the purpose is to understand the basic impacts of network parameters on proposed routing strategies through a simple and approximate model. Although the findings from this study may not be appropriate for direct application to the real world, they should provide a basis for understanding more complicated airline network routing models and for practical planning of routing systems. For example, knowing the network parameters that have insignificant effects on routing strategy should allow more time for considering other aspects of the airline system not included in this study. The proposed approach should also have limited application for other transportation modes such as buses, trucks, and railroads.

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