Optimization Strategies for Transit Systems in Urban Corridors

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SOLON-IO is a microcomputer-based interactive design procedure for the incremental optimization of performance of transit routes and transit systems based on the concept of elasticity and the total differential. The purpose of this method is to find the set of optimum service policies, such as fare and frequency changes, that management can incrementally implement through time to achieve a certain objective, such as maximizing ridership, under given performance constraints. The method is applied in an urban transit system in Minnesota and the selection of the optimum set of policy changes is illustrated. Using this method, the transit manager can determine the forecast optimum transit system performance through time as well as the optimum scheduling and pricing policies needed to reach the desired performance level. The data requirements of SOLON-IO include conventional socioeconomic and transportation information. The method is initially developed primarily for use in metropolitan transit corridors and urban transit systems in which competition among routes does not occur or is weak. Because it assumes no previous computer knowledge, this method can substantially increase management productivity.

Recent technological advances and new microcomputer-based techniques now allow the transit industry to employ innovative tools and aggressively seek, define, and fight for market niches. Engineering innovations in transportation monitoring electronics can be effectively implemented to aid the management of transit operations and improve the attractiveness of public travel. Furthermore, new interactive techniques allow the comprehensive analysis of transit scenarios throughout the life of a transit operation, and make possible the design of high-performance service alternatives based on substantially limited data needs.

Using the new techniques, transit managers can adopt the desired alternative by selecting the best options from an extensive set of policies (e.g., policies dealing with fare, frequency, and route changes). To be sure, traditional planning methods cannot deal effectively with this complex problem, especially since it is subject to continually changing constraints and often requires different solutions at different stages of development of an operation. The new techniques, on the other hand, can accomplish the desired analysis while effectively reflecting the major trade-offs and simplifications in the policy selection process.

Initially, the new techniques concentrated on being able to quickly sort through a great number of policy alternatives and advise the decision maker of the desirability of a choice or choices given the stated performance criteria (i.e., the emphasis was on the development of effective simulation tools) (*1*). Later, however, as transit managers discovered that even quick simulation tools can be time-consuming if used repetitively, it became clear that the usefulness of the new tools would hinge on their ability to make suggestions based on optimization features. To be successful, the new tools should be able to directly determine the optimum service policy (e.g., pricing and scheduling) through time for a given transit route as well as a system of transit routes. Such are the features of SOLON-IO (SOLON-Incremental Optimization), the interactive method presented here.

Although several quick-response planning methods have been developed (*1–4*), SOLON (the simulation module at the heart of SOLON-IO) was the only interactive system that began to address the time interactions between the major elements that give rise to changes in transit route performance. The method is rather straightforward and is distinguished by three unique characteristics:

- It provides solutions at any specified time or continuously through the life of a transport service by tracing the interactions between scheduling changes, route ridership, and service cost-effectiveness.
- It explicitly treats time delays (e.g., capital procurement and ridership) that hamper route performance.
- Its modules (demand, supply, and performance) operate interactively or independently and can be modified by the user. For instance, trip purpose (work or shopping) and market segments (by auto ownership, transit availability, and so forth) can be selected, and demand specifications can be updated (*1*).

SOLON initially aimed to aid management in improving the performance of transit operations at the route level. However, prior to addressing the optimization problem, it was necessary to extend the theory and the simulation tool so that it could be implemented in a system of routes. This was essential since management realistically makes performance optimization decisions based on evaluation of the performance of a complete system of transit routes, rather than of one route. Similarly, funding agencies make major funding decisions after assessing the viability of complete transit systems.

Transit system performance evaluation and policy selection could, of course, be performed without the benefit of an interactive optimization graphics system. The dynamic simulation method initially developed by this author did not have the benefit of optimization capabilities (*1*). However, inclusion of

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the optimization capabilities creates substantial savings in the time required to identify the optimum set of policies. In addition, SOLON-IO has been implemented in a way that allows easy access by decision makers with little or no computer experience. As a result, it enables experienced policy analysts to examine the consequences of expected performance improvements in greater depth and to fine tune selected policies prior to implementation. Further, it can substantially improve personnel training productivity as suggested by Twin Cities transit specialists and the managers of transit operations in several cities in Minnesota who had hands-on experience with several versions of this method. In fact, development of the management tool is a continuing process as these users make recommendations that are incorporated in the method prior to initiating its support for a transit system in a Minnesota city.

METHOD OVERVIEW

The structure of SOLON-IO can be analyzed at several levels of detail (Figure 1). At the route level, it may be pictured as a simple demand-supply model with the transit route performance sector acting as a link between supply and demand. For instance, a transit frequency increase in the service supply sector of a route results in waiting time reduction in the performance sector of that route; as level of service improves, so does route travel demand. In turn, as demand for and use of transit grow along the route, cost-effectiveness measures (e.g., load factor, operating ratio, route deficit) improve and call for service adjustments.

However, route service and pricing adjustments are limited by the financial health of the whole transit system. In particular, at the system level, SOLON-IO can be considered a combination of simulation and optimization modules with the system performance sector acting as a link between the two. For example, when expected (simulated) improvements in the ridership of a particular route call for service adjustments, the system performance sector first updates the system deficit by summing up all route deficits. The system performance is then optimized based on the system objective (e.g., maximum ridership) and subject to budget constraints. If the budget allows, system optimization recommends service adjustments and the service supply of each route is improved as necessary. Following such adjustments, performance measures are further modified and the interactions continue full circle.

Service and pricing adjustments are, further, limited by government regulations. For example, it may be impossible to increase fares in order to clear the market at given levels of transit service supply. In addition, many of the desired changes in service (e.g., number of stops and frequency modifications), supply resources (e.g., equipment and funding acquisition), and travel patterns can be accomplished only over relatively long periods of time or at infrequent time intervals (*1, 2, 4, 5*). Such substantial physical and information delays to transit supply and demand changes, together with regulatory restrictions, imply that a realistic model of transport supply-demand interactions should be able to treat the time dynamics of response to policy changes. SOLON-IO achieves this by tracing the interactions shown in Figure 1 continuously through time.

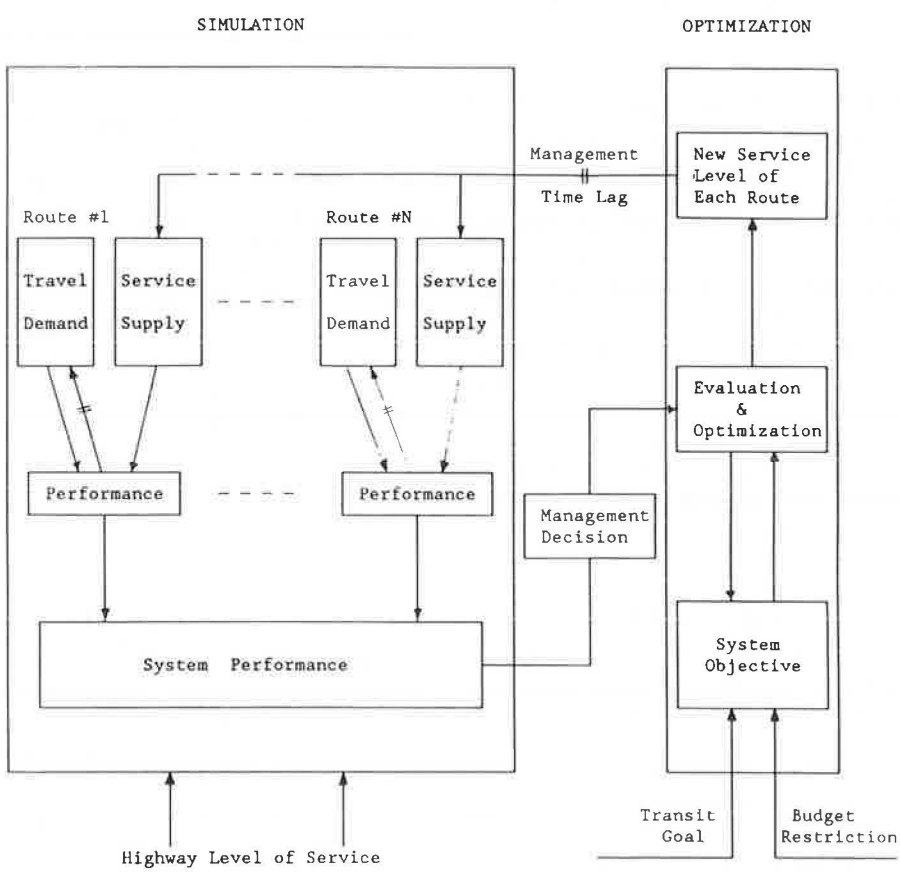

FIGURE 1 Structure of SOLON-IO.

As simulation and optimization continue to interact, each pass through the optimization module (at infrequent, user-specified intervals) determines a new value for the service level of each transit route. Each time the optimum service level is determined, the simulation module continues with simulating the supply-demand interaction until the next service change time. At these service change intervals, the user can optimize one or more indicators at a time, depending on the objective and the constraints. The optimization-simulation continues through time until the objective is reached. When this happens, optimization shuts down and simple simulation takes over until the end of the time horizon.

The tables and plots of the optimum service levels and the resulting performance indicators through the study period are the major products of SOLON-IO and are discussed in a later section. These performance records can facilitate the transit system decision making and management process by assisting managers in assessing the consequences of expected performance improvements in greater depth and by advising funding agencies in making subsidy decisions on the basis of performance potential. The rest of this paper presents a summary of the SOLON-IO system and an application example.

METHOD COMPONENTS

Tracing the simulation-optimization interactions through time is accomplished on the basis of component equations housed within each of the two major SOLON-IO modules shown in Figure 1. A summary of the contents of the modules (i.e., logic, empirical base, and data requirements) is presented in this section. This exposition focuses on the optimization module inasmuch as the simulation part is more completely documented elsewhere (*1*). Although this exposition is necessarily constrained by space, it should nevertheless expose the strengths and limitations of the methodology. The summary should further aid the reader to identify the assumptions and, therefore, the applicability of SOLON-IO to specific decision making and policy selection situations.

Simulation

Travel Demand

The demand equation incorporated in this methodology had to be capable of estimating ridership as a function of time and level-of-service measures based on initial route and system conditions only (i.e., without the need for extensive time-series data for the entire policy evaluation period) (*1*). In addition, it had to be able to consider the information delay between the time a service policy is implemented and the time residents of the service area become aware of the new service. Because of the limitations of the existing models, a dynamic demand equation was developed that fulfilled the specifications of a time-sensitive technique such as SOLON.

Assuming, for the purpose of this discussion, that the trip generation and distribution are completed, the dynamic demand equation adopted from SOLON (*1*) states that P_{t+1}, the estimated probability of selecting transit from a set of alternatives at time $t + 1$, is a function of the current probability P_t, estimated equilibrium probability P_e and a demand time constant T_P:

$$P_{t+1} = P_t + (P_e - P_t)/T_P \tag{1}$$

The demand time constant reflects the time necessary for the information on level-of-service changes to be transmitted to the population and the time for demand decisions to be modified.

The equilibrium probability, P_e, is estimated for each trip purpose (e.g., work, shopping) and market segment by a disaggregate logit specification calibrated with the data from the area of application and is generally a function of fare (W) and frequency (Q) (as well as other factors that may not be of interest here):

$$P_e = F(W, Q) \tag{2}$$

When implementing SOLON, the suggested logit parameter values, previously validated at several rural and urban areas (*1, 2, 4, 6*), can be updated with a small data sample (*7*). Alternatively, any steady-state demand model such as ULOGIT (*8*) or a simple regression model could replace the existing specification to estimate the equilibrium probability of demand. The data needs of the demand specifications include conventional demographic, socioeconomic, and trip information traditionally required by logit models (e.g., automobiles per household, travel cost, time, etc.). A summary of the data requirements for an example application of SOLON-IO in the city of Mankato, Minnesota, is presented in Table 1. Most of the updating data are entered when the method is initialized. Additional information reflecting inflation, energy costs, and other national trends could also be entered but is not necessary.

TABLE 1 DATA REQUIREMENTS FOR SOLON-IO

Code[a]	Data
Socioeconomic Data for Route *i* Corridor[b]	
--	Work population
HINC	Annual household income ($ thousands)
NCAR/NLIC	Automobiles per licensed driver
NPER	Persons per household
DISHINC	Disposable household income = HINC * 1,000 – 980 * NPER
Trip Characteristics for Route *i* Corridor	
OPTC	One-way out-of-pocket travel cost (cents): gasoline cost (automobile), fare (bus)
INVT	One-way in-vehicle travel time (minutes)
DIST	One-way trip distance (miles)
BAT	One-way bus access time (minutes)
AEGT	One-way automobile egress time (minutes)
Initial Route *i* Condition	
HEAD	Bus headway (minutes)
	Route length (miles)
	Peak-hour ridership (trips per week)
	Peak-hour operating hours per week
	Operating cost per vehicle-mile

[a]Abbreviation used in the logit model.
[b]Generally defined as the area within ½ mi from the route.

Route and System Performance

Route efficiency, effectiveness, productivity, and quality indicators describing the route performance are needed by the demand component for estimating ridership decisions during simulation. Further, the performance updates for each route are needed by the system performance component for transmitting to the system optimization module at each management decision interval (Figure 1). The library of performance indicators is extensive and reflects the knowledge gained from the earlier work by several researchers (see, e.g., *2, 9, 10*) who sought to identify the measures that can best be used to evaluate transit performance. To be sure, the user can easily create additional indicators using the output generated by SOLON. An example of a selected set appears in other work by the principal author (*1*). Similarly, the user can work with a smaller set that is generated automatically at the output and never use the library; such a small set could, for instance, be adequate for performing a preliminary analysis of a small-city transit system.

Performance indicators are determined on the basis of identities, assumptions, and simple algebraic equations calibrated in the area of application. For instance, equilibrium ridership for route i is the product of demand D_i for that route and the probability of demand P_{ei} estimated from Equation 2:

$$\text{Route } i \text{ ridership} = D_i\, P_{ei} \tag{3}$$

Similarly, total system ridership is the summation of the ridership on all N system routes:

$$\text{System ridership} = \sum_{i}^{N} D_i\, P_{ei} \tag{4}$$

As another example, the system operating ratio is defined as system fare revenue, determined from system ridership estimates (Equation 4), divided by system operating cost. Previous experience with SOLON users indicates that most systems break the operating cost down to a variable and a fixed unit cost component; further, if operating vehicle hours are not constant, the variable component is broken down to a mile-based and an hour-based component:

$$\begin{aligned}\text{System operating cost} = \sum_{i}^{N} a_i\ (\text{vehicle miles}_i)\\ + b_i\ (\text{vehicle hours}_i) + c_i\end{aligned} \tag{5}$$

where a is in dollars per vehicle mile, b is in dollars per vehicle hour, and c is in dollars. Route values for the unit costs a, b, and c are usually available from transit operations or can be determined from routinely recorded information on gas, oil, insurance, wages, maintenance, and other cost components. For small-city operations, the unit costs tend to be the same across all system routes thus simplifying the above equation. A more complete discussion of the equations involved in calculating the system performance equations is found in the Appendix.

Optimization

At specific time intervals, transit management conducts "conferences" to make decisions on needed modifications to current service on transit routes. These decisions may seek, for example, to increase system ridership by making appropriate service improvements, as necessary, on system routes. However, such improvements are subject to several constraints, the most important of which is the budgetary constraint imposed by the transit system. Alternatively, management may seek to minimize the system deficit while, at the same time, carrying a minimum ridership.

At the user-specified conference intervals, the optimization module makes suggestions to management regarding the optimum policy decisions. These suggestions are based on the information transmitted by the system performance component at each of those intervals. More specifically, using information on key system performance indicators such as total system subsidy, system load factor, operating ratio, and ridership, the optimization module determines the optimum policy that will maximize or minimize a user-selected system performance indicator. For instance, SOLON-IO can solve the following nonlinear optimization problem (see Appendix for mathematical formulation):

Select	: Optimum fare (W_i) and frequency (Q_i) for each route
To maximize	: System ridership
Subject to	: System operating ratio is not less than the target operating ratio
	Fleet size does not exceed the number of available transit vehicles

To select the optimum fare and frequency for each route that maximize system ridership subject to the above constraints, the optimization module evaluates the estimated impact of fare and frequency changes on future ridership. It accomplishes this by expressing the new probability P_{e2} (where P_e is the estimated probability following contemplated fare and frequency changes) as a function of the current probability P_{e1}; fare before and after the change (W_1 and W_2, respectively), frequency (Q_1 and Q_2); and the associated direct elasticities of P_e with respect to fare (E_W) and with respect to frequency (E_Q):

$$\begin{aligned}P_{e2} = P_{e1}\ [1 + E_W(W_2 - W_1)/W_1\\ + E_Q(Q_2 - Q_1)/Q_1]\end{aligned} \tag{6}$$

where a more complete description of the method for arriving at Equation 6 based on the definition of elasticity and the total differential is presented in the Appendix.

Based on the above relationship and the estimated elasticities (*11*), the desired system-level optimization problems, maximizing or minimizing the performance indicator selected by the user, can be formulated using fare (W_i) and service frequency (Q_i) of each route as decision variables. A summary of the application procedure of SOLON-IO and its logic to complete the simulation-optimization process through the planning period is presented in Figure 2. To date, two optimization formulations have been developed and are available in SOLON-IO:

- Maximize system ridership subject to system operating ratio or deficit constraint,
- Minimize system deficit subject to system ridership constraint.

Summary of Assumptions and Data Requirements

The previous section focused on the design and capabilities of the SOLON optimization module. Prior work by the principal author offers complete documentation of the simulation component, the assumptions on which the theory of SOLON is

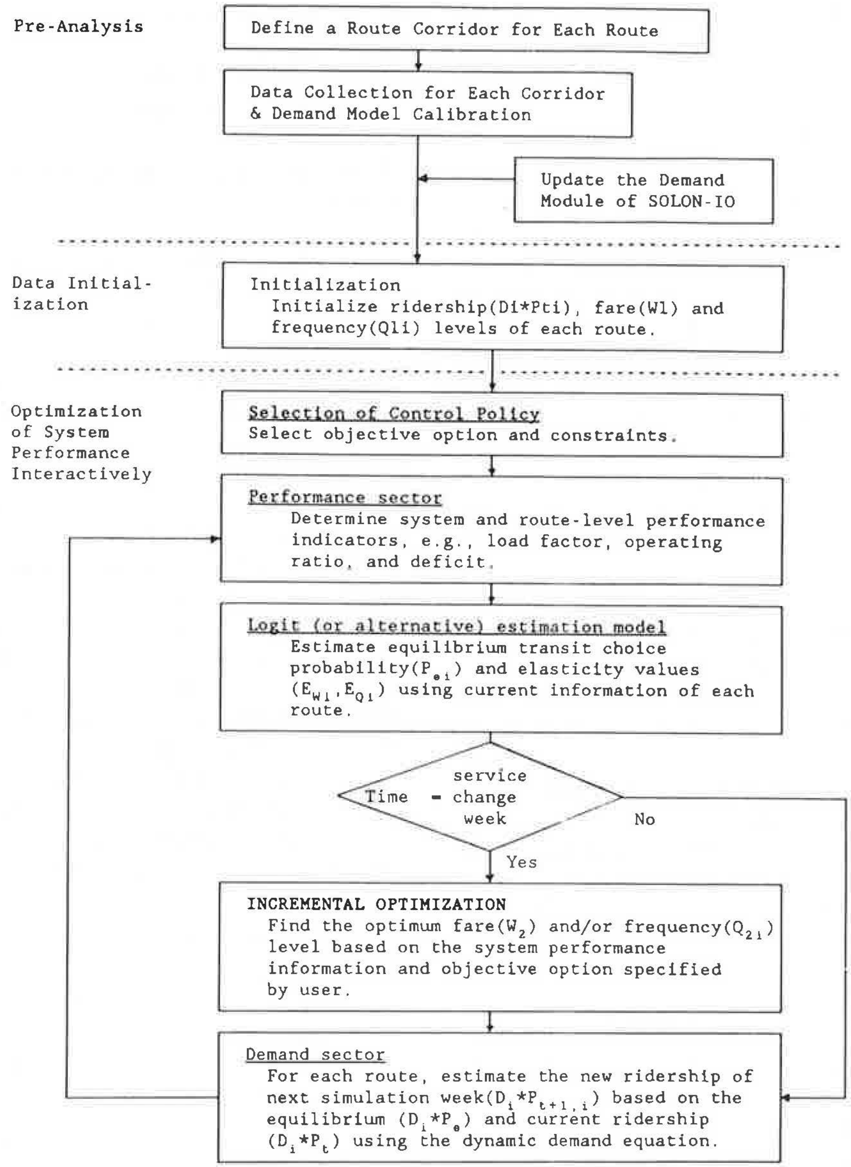

FIGURE 2 Application procedure and logic of SOLON-IO.

based, and the data required for implementation (*1*). Nevertheless, the major assumptions and data requirements are summarized here to further aid the reader to appreciate the applicability of SOLON-IO to specific decision-making situations as well as the limitations of the method.

The central assumption of SOLON is that the transit operation is almost always in a dynamic state, never quite able to reach equilibrium. This is the result of the dynamics acting between the two major driving forces (demand and supply) of the transit operation. Trip makers always seek a "bargain," that is, they would be happiest if they could travel when service is most plentiful, least used, and least costly; conversely, if given a choice, they would be quick to abandon a service that falls far short of their expectations. Similarly, under deregulation, transportation managers would not hesitate to cut service if passenger revenue fell short of projections; further, they stand ready to take advantage of opportunities offered by high, price-inelastic demand (as in peak time service) to increase fares and improve system economics.

More specifically, in the demand sector, the trip makers are continually trying to obtain the most recent information on service improvements that will enable them to take advantage of any chance to cut travel time and cost. In urban and metropolitan areas, increased consumer pressure, the large number of trip makers that can transmit information, and media reporting all contribute to quick information transfer from the transit operation to the trip maker. In particular, it has been found that the effective time constant involved in this process is approximately 3 to 9 weeks long (*2, 6*) and is a function, mostly, of current ridership level and the nature of service change. For instance, fare changes are felt quickly (3 to 5 weeks) while schedule improvements filter through more slowly (6 to 9 weeks).

In rural areas and small cities, where word of mouth, tradition, and community acceptance are often the most important factors in the transmittal and assimilation of information, the time constant is within a range of 6 to 26 weeks. From our experience, the longest time corresponds to remote areas or small towns where the mean age of the population is higher than average, such as towns in the Appalachians or northern New England.

To be sure, the value of the time constant cited here is the average of a distribution, which has been found to be near normal. Empirical methods to determine the statistics of the distribution from small samples have been developed and successfuly implemented in the course of previous work by the principal author (*2, 6*). The methods are based on controlled experiments in which new and old transit users as well as nonusers are surveyed following a fare or service change along a transit route. Survey participants indicate their awareness of the change and are asked to comment on the extent of the influence of that change on their mode choice. For instance, in a well-designed controlled experiment associated with service improvement on a transit route, the volume of new transit users and the rate with which such are generated following the improvement can be sufficient for estimating the demand time constant.

External economic and environmental factors are also assumed to influence the performance of the transit system. Since these factors, such as inflation and employment, vary with time, it is expected that they, too, play a role in the dynamics. Information on these factors can be entered by the user in the course of simulation-optimization as necessary.

A further assumption of the SOLON formulation is that transportation demand, unless otherwise driven, behaves as a first-order system (see Equation 1). While this assumption is based on data from several systems (*1, 4*), transport operations may exist for which higher-order effects cannot be easily neglected.

As mentioned earlier, the demand parameters of the simulation module of SOLON have been estimated and validated for several small and large urban and rural areas (*1, 2, 4*) based on a logit formulation and a disaggregate data set for each area. The needed data were dictated by the variables incorporated in the disaggregate logit formulation. As Table 1 suggests, these data are routinely collected by metropolitan planning agencies such as the Metropolitan Council of the Twin Cities. It is indicated in the literature (*7, 12*) and confirmed in this paper that such formulations are highly transferable under certain conditions; however, it is suggested that the policy maker update the parameters where necessary and at least every 5 years.

Updating of the demand parameters can be accomplished in any of several ways established in the literature (*7*). Updating methods range from a simple adjustment of the regression or logit coefficients so that they conform to the newly collected data to more rigorous techniques such as using Bayes' formula in conjunction with a small-sample survey of the trip or socioeconomic characteristics that are suspect of change. Survey designs that can be used to aid the potential SOLON user in surveying a small sample are found in several planning agencies [e.g., agencies that use ULOGIT for demand forecasting (*8*)] and are certainly available by current SOLON users in Minnesota such as the Minnesota Department of Transportation and the Mankato Urban System of Transportation (MUST). As mentioned earlier, the data needs of the demand specifications include conventional demographic, socioeconomic, and trip information traditionally required by logit models (e.g., autos per household, travel cost, time, etc.). (Refer to Table 1 for a summary of the data requirements for an example application of SOLON-IO in the city of Mankato, Minnesota.)

APPLICATION IN PERFORMANCE OPTIMIZATION

To illustrate the performance optimization process using the policy design features of the interactive method, SOLON-IO is applied to the transit system in Mankato, a city with a population of 30,000, 120 mi south of the Twin Cities in Minnesota [the interested reader can see other published material (*1, 4*) for applications in the Twin Cities Metropolitan Area]. Most of the data used in this application were gathered with the assistance of MUST, the area's governing body that oversees transit operation. Further, logit specifications were developed to estimate travel modal split in Mankato. For instance, the work-trip specification is based on disaggregate data from 330 households collected during a survey of bus and auto trips in Mankato in April 1986. The work-trip mode choice model and model performance statistics are presented in Tables 2 and 3, respectively. The results were also used to estimate the demand elasticities of Mankato trip makers. The route structure of the Mankato City transit system is illustrated in Figure 3.

Data initialization is the first step in applying the performance optimization method. The data requirements include conventional demographic, socioeconomic, and trip information, readily available to transport planners. While certain data needs of SOLON-IO are tied to the needs of the logit specification developed by this research team, those needs could be simplified by the use of alternative, simpler demand estimation models. A summary of data initialization, with current (May

TABLE 2 WORK-TRIP MODE CHOICE MODEL

Variable	Coefficient	*t*-Statistic
Generic		
OPTC/HINC	–0.144	–2.63
INVT	–0.061	–2.91
Automobile specific		
CONS[a]	–3.198	–4.32
DISHINC	0.907E – 4	6.56
NCAR/NLIC	3.401	5.05
AEGT	–0.380	–4.33
Bus specific		
HEAD * BAT/DIST	–0.242E – 2	–1.29

[a]CONS = 1 for auto; 0 otherwise.

TABLE 3 MODEL PERFORMANCE STATISTICS

Calculation	Result
Sum of chosen probabilities	258.66
Sum of all probabilities	330.00
Sum probability ratio (%)	78.38
$L^*(Q)^a$	–114.81
$L^*(O)^b$	–228.74
$\rho^2 = 1 - [L^*(Q)/L^*(O)]$	0.50

[a]$L^*(Q)$ = log likelihood at convergence.
[b]$L^*(O)$ = log likelihood at zero.

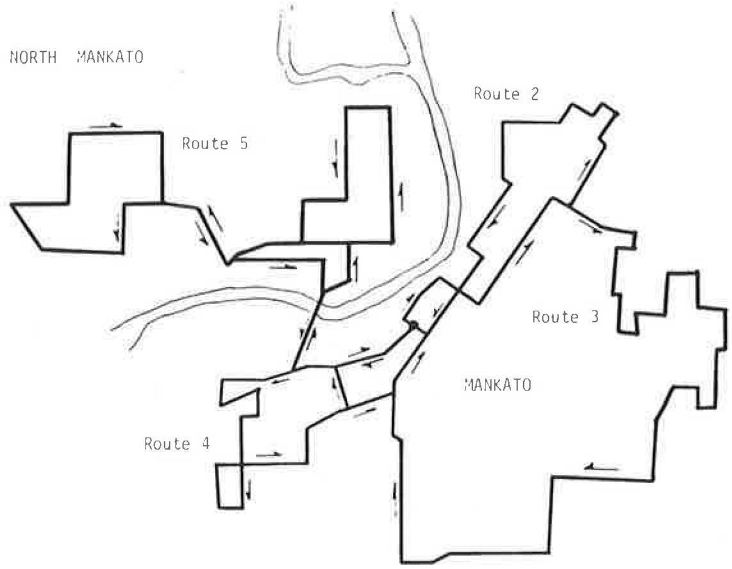

FIGURE 3 Route structure of Mankato Transit System.

1987) values from the Mankato application, is presented in Tables 4 and 5.

Selection of the desired objective option, constraints, and control variables is next performed by the user. In this application, for instance, MUST employs route frequency of service as a control policy and seeks to maximize system ridership subject to a system budget constraint (i.e., within a system deficit limit). In particular, MUST is willing to modify the frequency of one or more system routes but does not want to change the fare at this stage. The system deficit constraint is $1,000/week, a value that the SOLON-IO user soon realizes is unrealistic, at least initially. To be sure, following the initial session with the interactive tool, the user may repeat the analysis based on different choices for the above options.

Following initialization, SOLON-IO begins to operate by computing the initial value of each route and system performance indicator. Ridership, service elasticities, load factor, operating ratio, and deficit are key indicators in this application, since they provide the basis for making decisions about the service changes.

System optimization, the next stage in the process, is activated only at regular time intervals, corresponding to the points in time that management selects for making decisions regarding service improvements. In this example, the selected conference interval is 20 weeks. If optimization is not activated, the simulation module of SOLON-IO takes over and travel demand is estimated for all routes, for the next time interval (e.g., next week or month, depending on the unit selected by the user). This information is required to update the performance indicators.

TABLE 5 SOCIOECONOMIC DATA INITIALIZATION

Route Number	Work Population	Annual Household Income ($)	Persons per Household	Cars per Licensed Driver
2	2,447	28,500	2.76	0.76
3	7,043	22,400	3.10	0.72
4	2,148	28,300	2.76	0.76
5	4,330	28,700	3.36	0.75

If, on the other hand, optimization is activated, the incremental optimization module begins providing the necessary background information, so that the user can select realistic values for the problem constraints. In particular, owing to the limitations in the use of elasticities (*13*), certain restrictions are placed on the allowable range of policy (i.e., fare and frequency) changes. Specifically, at any time transit management wishes to make a policy change, fare and frequency changes are limited to within 20 percent of their value prior to the change, thus resulting in a restricted range of values for the constraints. For example, the operating ratio constraint cannot be set at a value beyond the range implied by a 20 percent fare increase and a 20 percent frequency reduction. The boundary values of the constraints resulting from this restriction are calculated and offered to the user as a guideline at each step during the simulation. Based on this guideline, the user can

TABLE 4 TRANSIT DATA INITIALIZATION

Route Number	Route Length (mi)	Frequency (buses/hr)	Peak-Hour Ridership (trips/wk)	Operating Hours/Wk	Deficit ($/wk)	Operating Ratio
2	4.6	3.0	211	18.4	477	0.21
3	9.0	3.0	1,616	28.4	832	0.54
4	4.4	3.0	280	22.4	531	0.24
5	11.1	1.5	532	20.0	498	0.39
Total			2,639		2,338	0.40

approach the desired performance level on an incremental basis. To be sure, the user may disregard the guideline and elect to set the constraints at values that imply fare and frequency changes that are substantially higher than the recommended 20 percent range. On the other hand, the user could also decide to stay well within that range should the conditions of the transit environment in the particular application allow.

The restrictions on the suggested range for the change of the control variables are particularly important when elasticity is expected to vary substantially with travel-related and other characteristics. However, it has been reported that the elasticity values remain relatively constant within city groups of similar size, urban structure, and transportation technology type (*12*). When the elasticity values are found to be stable, some of the above restrictions can be relaxed.

The background performance information of the example system during the initial week (t = 0) as well as the suggested constraint value are summarized in Figure 4. This information is presented to the user following the initialization stage. Using this information as a guideline, the user can select the desired value for each target constraint. Decision on which of the available variables to use as controls is also made at this stage.

In this application example, a desired final deficit level of $1,000/week and a wish to use only frequency change as a decision variable are the selected management policy options. However, as Figure 4 indicates, at the current condition the lowest suggested value for the system deficit, realized by changing the frequency of all routes by 20 percent, is $2,000/week. Therefore, under the elasticity restrictions, the system cannot reach the desired performance level directly at this stage and, therefore, suggests an approach to the desired deficit level incrementally. Further, the next service change will be decided upon at a management conference 20 weeks from now.

Assuming that management adopts the suggested value for the system deficit constraint and based on the selected service change option, the optimization module computes the optimum frequency level of each route, which can maximize the system ridership while bringing the total deficit within the required $2,000/week from the current value of $2,338/week. The optimum service levels determined at this step are implemented only after a period of time; this time lag reflects the management delays resulting from the needed adjustments in the rolling stock and the number of available drivers (*1*). In the meantime, the current service levels remain unchanged, and the demand and performance components of the simulation module estimate the updated ridership and other performance indicators.

At the specified service change interval—20 weeks in this example—the optimization module is again activated and provides the information illustrated in Figure 5. This information includes the system condition after the service change at t = 20 ("current condition") as well as the condition before the service change, (i.e., at t = 0) ("previous condition"). In this case, the system optimization would result in a frequency increase on one route (Route 5), but a decrease in the frequency on the remaining three routes. This implies that Route 5 users are more elastic to frequency than are the users of the remaining routes, so that the estimated revenue from the ridership increase can offset the operating cost increase following the service improvement. However, as indicated in Figure 5, the system still cannot reach the desired deficit level of $1,000/week directly. Having set the new constraint value at $1,700/week and the service-change conference interval at 20 weeks (i.e., at t = 40), the user can repeat the optimization-simulation until the desired performance level is reached. Figure 6 illustrates the optimum service levels and resulting performance variations obtained by incrementally optimizing system performance every 20 weeks. As indicated in the figure, at the end of 80 weeks the deficit decreases by 50 percent while the ridership decrease is less than 10 percent.

CONCLUDING REMARKS

SOLON-IO is a microcomputer-based method that can be used to aid transit practitioners in optimizing transit system service interactively. The new approach is based on the time-dependent interactions between transit supply and demand, and the concepts of elasticity and the total differential. The method was applied to the system-level service design, and the interactive policy selection procedure was illustrated. Using the micro-

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Control Policy : Ridership Maximization under Deficit Constraint
                 (No Fare Change)

Current System Condition      Week 0

Route  Freq. Ridership  Deficit   O.R.
  2     3.0     211       477     0.21
  3     3.0    1616       832     0.54
  4     3.0     280       531     0.24
  5     1.5     532       498     0.39
--------------------------------------
Total          2639      2338     0.40

Minimum Target Deficit at this stage by frequency change : 2000 $/week

Do you wish to change service level ?  Y/N, Retry, Quit : Y

Input Target Deficit ($/week) : 2000

Input conference interval for next service change (weeks) :  20
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FIGURE 4 Interactive policy design at Week 0.

Control Policy : Ridership Maximization under Deficit Constraint
(No Fare Change)

Previous System Condition - Week 0 Current Condition - Week 20

Route	Freq.	Rider.	Deficit	O.R.		Freq.	Rider.	Deficit	O.R.
2	3.0	211	477	0.21	———	2.4	201	366	0.24
3	3.0	1616	832	0.54	———	2.4	1580	502	0.65
4	3.0	280	531	0.24	———	2.6	255	455	0.25
5	1.5	532	498	0.39	———	1.8	562	633	0.35
Total		2639	2338	0.40			2598	1946	0.44

Minimum Target Deficit at this stage by frequency change : 1700 $/week

Do you wish to change service level ? Y/N, Retry, Quit : Y

Input Target Deficit ($/week) : 1700

Input conference interval for next service change (weeks) : 20

FIGURE 5 Interactive policy design at Week 20.

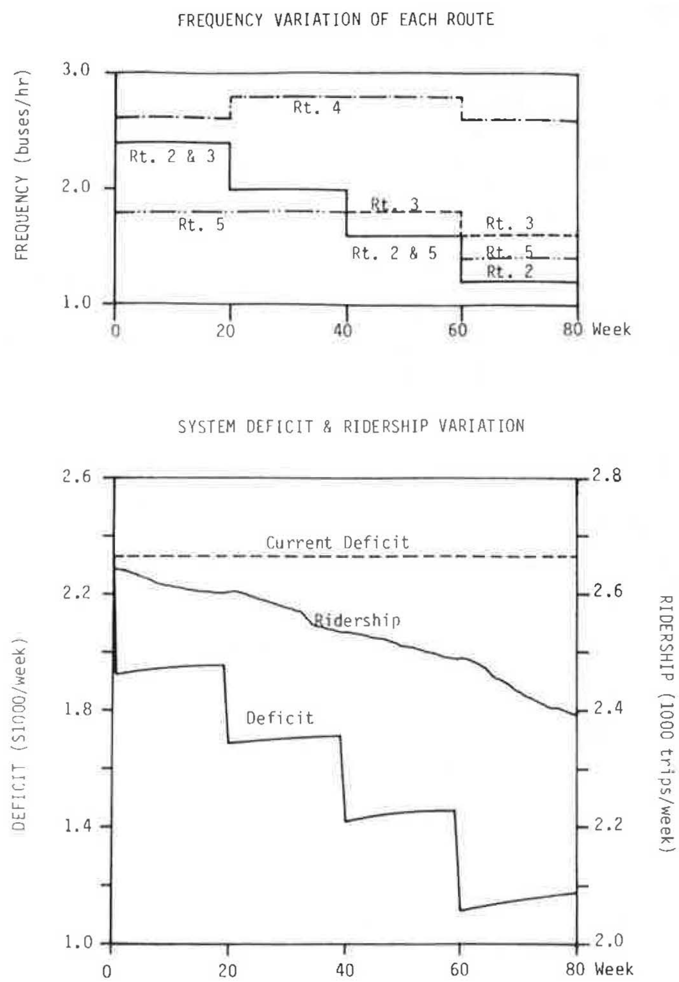

FIGURE 6 Optimum transit system performance.

computer graphics of this method, the manager can determine the optimum service policies, such as fare and frequency, that can be implemented in each route of a transit system at different time intervals to accomplish a system objective.

The main feature of the new method is the combination of system-level dynamic simulation and incremental optimization, which enables the user to find the optimum policy that can achieve a desired objective subject to constraints and the expected performance variations directly. Thus, the method can reduce substantially the amount of time and effort that the traditional simulation techniques need to reach the same conclusion.

SOLON-IO has been applied to the transit systems at different cities in Minnesota as well as several transit corridors in the Twin Cities [a validation example with data from a Minneapolis suburb can be found in other published work by the principal author (*1*)] and has confirmed the expectations regarding the benefits to be derived from inclusion of optimization features in interactive planning tools. Further, users of the preliminary versions of the software have made several recommendations; incorporation of these in the current system has made it more responsive to the complex problems of the transportation practitioner.

Several potential applications of the interactive optimization method have been suggested. For the transit system manager, being able to determine the potential best performance of the system is a most valuable asset. In addition, being able to know the suggested best course of action, or set of policies, that can be implemented to achieve that potential is a highly time-saving feature. This was especially appreciated in cases involving managers who are overloaded with work and are short on experienced personnel. A recent effort by the state to substantially improve the performance of transit systems while keeping service closely matched with actual need has presented a new area of application, in which system optimization is of high priority for both the transit manager and the funding decision maker. In this case, SOLON-IO can assist both parties in determining the ways in which service can be modified, either by increasing funding where this can be productive or by controlling costs, to allow the transit systems to aggressively seek and conquer market niches.

The restrictions of the method, which arise from the limitations in the use of elasticity, could be relaxed if the elasticities in a particular application are found to be stable. Further, dynamic optimization could be employed to find the best solutions for achieving multiple objectives under multiple constraints through time. In addition, whereas only peak travel has been addressed in this paper, the effects of off-peak travel policies on the overall transit route and system performance could also be demonstrated using this method. The method is initially developed primarily for use in metropolitan transit corridors and urban transit systems where competition among routes does not occur or is weak. Because it assumes no previous computer knowledge, it can substantially increase management productivity. SOLON-IO is designed to be implemented on a microcomputer (IBM-PC), thus minimizing the investment in computer resources needed by the user.

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APPENDIX

System Performance Equations

Using the demand probability, P_{e2}, obtained from Equation 2, with some simplifying assumptions, the major performance indicators of the transit system after the service modifications can be expressed as follows (assuming no competition between routes):

$$\text{Ridership} = \sum_i^N D_i P_{ei}$$

Total subsidy (operating cost − farebox revenue) =

$$\sum_i^N (B_{pi} Q_{2i} - D_i P_{ei} W_i)$$

$$\text{Subsidy per passenger} = \sum_i^N (B_{pi} Q_i - D_i P_{ei} W_i)/(D_i P_{ei})$$

Operating ratio (farebox revenue/operating costs) =

$$\sum_i^N (D_i P_{ei} W_i)/(B_{pi} Q_i)$$

Route load factor (ridership/capacity) = $D_i P_{ei}/(C_{pi} Q_i)$

where

N = total number of routes in the system,
D_i = total trip demand of route i,
C_{pi} = capacity coefficient of route i, and capacity
= operating hours * frequency * seats/bus
= $C_{pi} Q_i$ assuming constant operating hours, and
B_{pi} = operating cost coefficient of route i, and operating cost
= cost/vehicle-hour * total vehicle-hours + cost/vehicle-mile * total vehicle-miles
= $B_{pi} Q_i$ assuming constant operating hours.

System Ridership Maximization Problem Formulation

Select : Optimum final fare W_{2i} and frequency Q_{2i} for each route i

$$\text{To maximize}: \sum_i D_i P_{e1i} \, [1 + E_{Wi} (W_2 - W_1)/W_1 + E_{Qi} (Q_{2i} - Q_{1i})/Q_{1i}]$$

$$\text{Subject to}: \sum_i (B_{pi} Q_{2i}/\sum_i \{D_i P_{e1i} [1 + E_{Wi} (W_2 - W_1) + E_{Qi} (Q_{2i} - Q_{1i})/Q_{1i}] W_2\}$$

$$\geq \text{Target O.R.} \sum_i Q_{2i} t_i \leq \text{fleet size}$$

where t_i is round trip time of route i.

Formulation of Demand Probability

Applying the definition of elasticity and the total differential to Equation 2, the new equilibrium demand probability P_{e2} (where P_{e2} is the estimated probability following contemplated fare and frequency changes) can be be expressed as a function of the current probability P_{e1}, fare (W), frequency (Q), and the associated direct elasticities as follows:

$$dP_e = \partial P_e/\partial W * dW + \partial P_e/\partial Q * dQ$$

$\partial P_e/\partial W = E_W * P_{e1}/W_1$

$\partial P_e/\partial Q = E_Q * P_{e1}/Q_1$

Let $dP_e = P_{e2} - P_{e1}$, $dW = W_2 - W_1$, $dQ = Q_2 - Q_1$.

Then,

$$P_{e2} = P_{e1} [1 + E_W(W_2 - W_1)/W_1 + E_Q(Q_2 - Q_1)/Q_1] \quad (6)$$

where

E_W = the elasticity of P_e with respect to fare,
E_Q = the elasticity of P_e with respect to frequency, and
W_1, W_2 = fare before and after the change.

It should be noted that, in the special case that the demand function is of product form, implying constant elasticities, Equation 6 becomes

$$P_{e2} = P_{e1} [1 + E_W(W_2 - W_1)/W_1] [1 + E_Q(Q_2 - Q_1)/Q_1] \quad (6)$$

where E_W and E_Q are the estimated arc elasticities (*14, 15*).

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