# A Heuristic Shortest-Path Method for Emergency Vehicle Assignment— A Study on the Mexico City Network

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**The immediate needs of a city struck by a disaster are quick and safe evacuation of its inhabitants and prompt search and rescue operations conducted by emergency personnel. If the city's road network has suffered capacity losses because of floods, building rubble, or structural failures, a shortest-path algorithm with the aid of a computer serves as a useful tool in assigning the emergency vehicles to routes that remain open in the aftermath of the disaster. If the city network is large, the shortest-path algorithms consume large memory and execution time on a microcomputer. If the algorithm needs to operate in real-time conditions, the savings in these factors become very important. Heuristic methods have been developed to reduce the computer storage and execution time. One such heuristic method is being examined for its accuracy compared with conventional shortest-path algorithms, which build the entire shortest-path tree before selecting the path between an origin-destination pair. The suggested heuristic method alleviates the need to build the entire tree, yet proved to yield the same results as the total-path enumeration method in 99 percent of the cases when applied to the Mexico City network.**

A disaster, either natural or man-made, leaves a trail of destruction wherever it strikes. When it strikes an urban area, the consequences are collapsed buildings, damaged roads, and, above all, loss of lives and injuries to the inhabitants. In such a situation, the rescue personnel are called upon and play an important role in evacuating the injured in a quick and safe manner, so as to minimize the suffering and loss of lives. The rescue operations involve dispatching the rescue vehicles along the shortest paths to the points of incidence or to shelters.

Even though the actual transportation infrastructure may suffer only little damage, the streets are susceptible to blockage by rubble from damaged buildings and by people gathered to learn of their relatives' welfare. In such a situation, rescue vehicle personnel have no way of knowing which path would be open or which would be the quickest one to take. If computer data on the streets of the network are already available, these can be updated based on the current status of the streets and used with a shortest-path algorithm to determine the shortest paths between any two points in the network. The label-correcting method is one such algorithm.

If the network is large, the shortest-path algorithm consumes more memory and execution time on a microcomputer. In order to save memory and execute the algorithm in a quicker and more efficient way, a system of partitioning (heuristic method) has been designed. Here, when looking for the shortest-path, only those nodes that are within a restricted circular area are considered for determining the shortest path, instead of the whole network. The shortest path thus obtained is assumed to be the true shortest path. With a view to determining the accuracy of this partitioning technique, a sensitivity analysis was performed using a computer program.

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## EMERGENCY VEHICLE ASSIGNMENT

The most important objective of any Emergency Management System is to save human lives when a disaster strikes an inhabited area. The disaster could be an earthquake, a tornado, or a nuclear disaster, among others. Evacuation planning and operations are the major activities in the preparedness and response phase of emergency management. The main objective is to shift the inhabitants of danger-prone areas to disaster shelters quickly and safely. Automobiles are generally the major means of conveyance in such a situation. In order to achieve quick evacuation, the emergency vehicles must be assigned to the shortest paths between their origin points (rescue control center and evacuation areas) and destinations (evacuation areas and shelters). Some disasters, such as the September 1985 Mexico City earthquake (*1*), may cause considerable damage to the street network in a city, thereby disabling the use or reducing the capacity of some streets in the network. In such cases, because the shortest paths depend on the status of the streets, there is need for a flexible computer-aided software package, which will facilitate the addition or deletion of links of a network to reflect the real situation and then determine the shortest paths between the desired points. This would help the decision makers in assigning the emergency vehicles to the most suitable paths.

An interactive microcomputer package has been developed, known as Transportation Emergency Decision Support System (TEDSS) (*2–4*), incorporating these facilities, among others. It is a user-interactive package offering considerable flexibility in graphically updating the network conditions whenever required. A heuristic shortest-path finding method, as described later in this paper, has also been incorporated to aid decision makers in quickly assigning the emergency vehicles to desirable routes.

## HEURISTIC SHORTEST-PATH FINDING METHOD

The label-correcting method of determining the shortest path, with suitable modifications incorporating the heuristic method, has been adopted in developing the computer package.

## Label-Correcting Method

The label-correcting method is an efficient way to determine the shortest paths from a given origin (root) node to all other nodes in the network. It must be used for each origin in turn to obtain the shortest paths from that origin to all other nodes of the network. The algorithm employs a process whereby the network nodes are scanned iteratively to find a better (shorter) path than the current one, from the root to the node being scanned. This is done by comparison of labels. Every node has a label, which represents the distance from the root to that node, along the (current) shortest path. Apart from the label, each node also has a predecessor variable that keeps track of the node that precedes it in the (current) shortest path. These variables are updated when a new shorter path is found. When a link is examined, the distance to get to the beginning node of the link plus the length of the link is compared with the current label of the end node. If the current label represents a longer path, it is changed and set equal to the label of the beginning node plus the length of the link. If the current label represents a shorter path, it is kept the same. When no better path can be found from the root to any of the other nodes in the network, the algorithm terminates. The cost of travel could also be in terms of time instead of distance (*5*).

A sequence list keeps track of the nodes and maintains the most efficient order for the nodes to be examined. All the nodes that have yet to be examined as well as those requiring further examination are included in this list. Initially, the origin node, which is the only member of the sequence list, is examined by testing all the nodes that can be reached by traversing only a single link. If the label test is met, the label and predecessor lists are updated and the node is placed in the sequence list. After complete examination of a node, it is deleted from the list. When there is no node left in the sequence list, the algorithm terminates. Now, the predecessor list is traced back from a node under consideration to the root node to get the shortest path between the two (*5*).

In the computer implementation of the label-correcting method, the links are identified by their end nodes. Without elaborating, it can be said that all the nodes of the network are to be examined at least once by the algorithm (*5*). This implies the use of more computer memory and a higher execution time. The heuristic method developed is capable of bringing about savings on these.

## The Heuristic Method

The heuristic method of determining the shortest path adopted in TEDSS uses a system of partitioning. In this technique, when the label-correcting algorithm looks for the shortest path, it is made to scan only those nodes that are within a reasonable proximity to the origin and destination nodes. This area is defined by a circle with center at the midpoint of line joining the origin and destination nodes and with initial radius as mentioned in the two cases described later. If a path from an origin to a destination could not be found by examining all the nodes within this circle, the radius of search is incremented by 25 percent, and all the nodes within the new circle are examined for a possible path. Incrementation of radius continues until a path (or the shortest among the different paths formed by the nodes within the circular area) between the origin and the destination is found. This path is assumed to be the true shortest path.

There is potential for savings in microcomputer memory by using the partitioning technique. This can be realized through reduced array size allocation for the various lists such as label list, sequence list, and so on that are maintained for each node by the label-correcting algorithm. The amount of memory savings depends on the number of nodes in the network. Also, if the origin and destination nodes are nearer, the partitioning technique is capable of saving more memory and vice versa. The execution of partitioning technique is also faster in a microcomputer.

Two cases of partitioning technique have been considered:

- *Case 1.* Partitioning is done using half the straight-line distance between origin and destination nodes as the initial radius of search, with the center at midpoint of the line joining them. The method can be illustrated better by considering the following network of 8 nodes and 12 links (Figure 1). If the partitioning technique were applied to this network in order to determine the shortest paths between Nodes 3 (origin) and 4 (destination), the computer program would initially consider only Nodes 3, 4, and 5, which fall within the restricted region represented by the solid-line circle. Then, the label-correcting method searches for the shortest path. Thus, either Path 3–4, or 3–5–4 would be chosen as the shortest path, depending on their length. If no path could be found in the first attempt between Nodes 3 and 4, the radius of search is incremented by 25 percent, and so on, until a shortest path is found. The method can be appreciated better in large networks, when the origin and destination nodes are farther apart, with many connecting links.

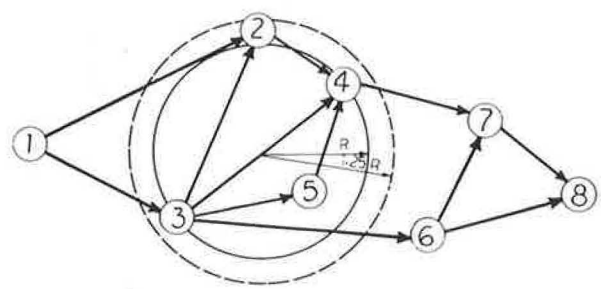


**FIGURE 1 Illustration of partitioning technique.**

- *Case 2.* This case is very similar to the first, except that the initial radius of search is 1.25 times half the straight-line distance between the origin and destination nodes. Considering the same network as in Figure 1, the restricted region of search is represented by the broken-line circle. Thus, in the initial search, Nodes 2, 3, 4, and 5 would be considered for placement in the shortest path and the shortest path would be one among 3–2–4, 3–4, or 3–5–4, depending on their lengths.

Although the partitioning method described has been designed to be computer efficient, it has a limitation. The method gives the shortest path between the origin and the chosen destination only, whereas the standard label-correcting method gives the shortest paths from the origin to all other nodes in the network. Thus, if there were a choice of destinations, the standard label-correcting method, in one run, would enable the

user to know the nearest destination, whereas the partitioning technique would have to be run a number of times (equal to the number of destinations) to determine the nearest destination. However, when the destination is fixed, the partitioning technique can be used advantageously and efficiently.

## MEXICO CITY NETWORK—THE STUDY AREA

Mexico City was shaken by a powerful earthquake on September 19, 1985, which left in its wake an emergency condition. Apart from causing heavy loss of human lives and damage to buildings, the extensive destruction left a highly restrictive transportation network and the status of the various street links in the city center was unknown to the emergency crew. This rendered the dispatch of emergency vehicles increasingly difficult, because emergency units had to make numerous attempts and diversions to reach the incident points. A comprehensive data bank on the latest street conditions and a computer program incorporating a shortest-path algorithm would have enabled dispatchers to quickly direct vehicles to incident points and shelters through the most accessible routes. It is believed that in such situations, the heuristic shortest-path method would further reduce the decision time on route selection.

This sets the background for considering the Mexico City network, on which a considerable amount of data is available to judge the usefulness and accuracy of the heuristic method. The Mexico City network, with 288 nodes and 763 links, is shown in Figure 2, where the heavy lines indicate arterial streets. It is seen from the figure that the network is of grid

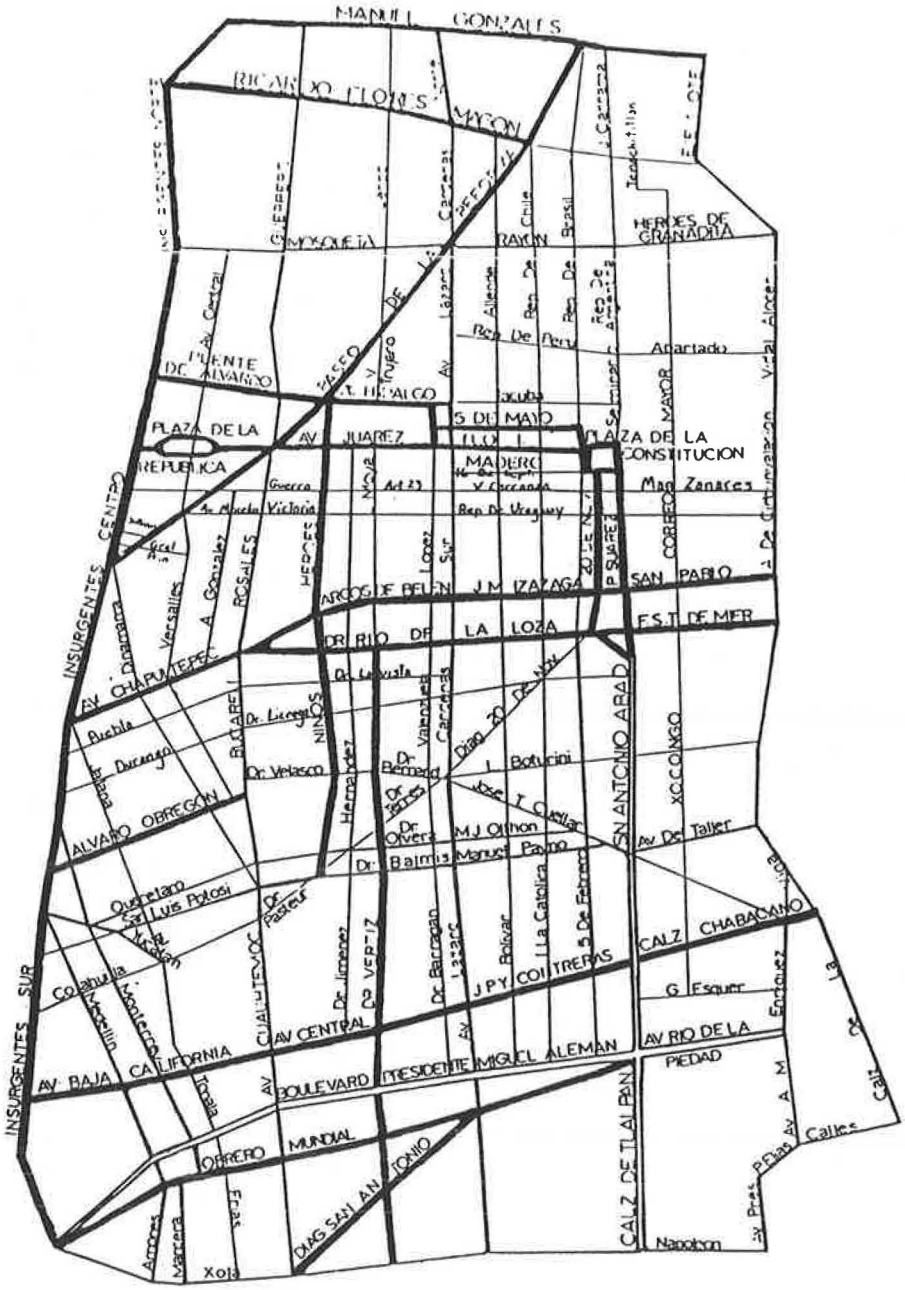


FIGURE 2 Mexico City network.

pattern predominantly. Shortest paths were established between 100 random pairs of origin-destination in the network shown. These paths were then used to assess the accuracy of the heuristic technique described.

## TWO CASES OF SHORTEST PATH

The shortest path could be either distance- or time-based.

### Distance-Based Shortest Path

The distance-based shortest path is based on the simple rule that the shortest path between any two nodes is the one for which the sum of the link lengths is the least among the various possible routes.

### Time-Based Shortest Path

Also included in the computer program is an option to use time-based shortest path. This option is useful when the emergency vehicles are to be routed along with normal flow of traffic (i.e., when the normal traffic flow is not disrupted for the sake of movement of emergency vehicles). Because the travel time in this case depends not only on the distance, but also on the traffic flow and capacity, a link performance function must be used for converting distance to travel time. A Davidson-like link performance function, given by Equation 1, has been adopted in this study. Although the original Davidson's function, developed based on queueing considerations, is asymptotic to a capacity flow (Figure 3), in the equation the travel time has been restricted to a maximum of six times the free-flow travel time, based on actual sample observations in Mexico City.

$$t_a = \min \left\{ t_a^0 \left[ 1 + J \frac{\gamma x_{a,\,peak}}{C_a - \gamma x_{a,\,peak}} \right], 6\, t_a^0 \right\} \qquad (1)$$

where

$t_a$ = travel time on link $a$,
$t_a^0$ = free-flow travel time on link $a$,
$J$ = a model parameter,
$\gamma$ = time-of-day parameter,
$x_{a,\,peak}$ = peak-hour volume on link $a$, and
$C_a$ = capacity of link $a$.

Using the actual traffic flow data of Mexico City, the values for parameters $J$ and $\gamma$ were calibrated and were found to take the following values:

$J$ = 0.5
$\gamma$ = 1.0 for day-time peak
= 0.7 for day-time off-peak
= 0.5 for nighttime peak
= 0.0 for nighttime off-peak

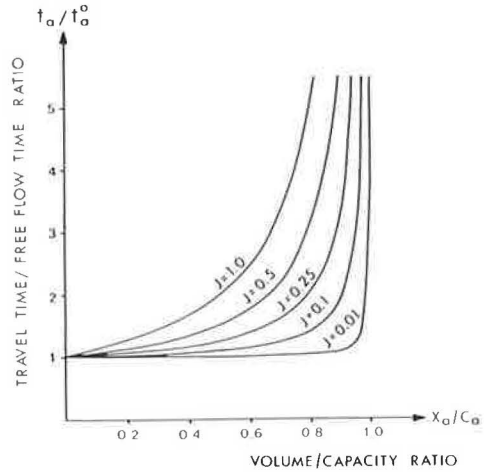


**FIGURE 3 Davidson's link performance function (5).**

## ACCURACY OF HEURISTIC METHOD (PARTITIONING TECHNIQUE) AS APPLIED TO MEXICO CITY NETWORK

With a view to determine the accuracy of shortest paths obtained by the two cases of partitioning technique discussed earlier, 100 pairs of random numbers, in the range of minimum and maximum node numbers of the network, were generated using a computer program and were considered as origin-destination nodes, thus eliminating any bias in sampling. The computer program developed accesses the input data for Mexico City network and is used with slight modifications to represent the following three methods:

- *Method 1*: Partitioning technique with initial radius of search equal to half the straight-line distance between origin and destination nodes (this was already incorporated in TEDSS),
- *Method 2*: Partitioning technique with initial radius of search equal to 1.25 times the radius as in Case 1, and
- *Method 3*: Where the whole network is considered.

For each pair of 100 random origin-destination combinations, the shortest path (both distance- and time-based) were recorded using each of the three methods just listed, and the results were compared.

An example of computer display of shortest paths obtained by the three methods using the computer package TEDSS is shown in Figure 4. The program interacts with the user and on inputting the origin and destination node numbers (or the cross-street names forming the nodes), the computer displays the shortest path, separately and as a blinking path in the network, and the shortest distance. The listing of the path can also be obtained immediately. These features would greatly assist the rescue crew in dispatching the emergency vehicles along the shortest paths. From the figure, it is seen that, to travel from Node 117 to 168, Method 1 gives a longer path as the shortest, whereas Methods 2 and 3 give the same result, which is the true shortest path.

The findings of the previous analyses appear to favor the heuristic method as shown in the comparison of computer

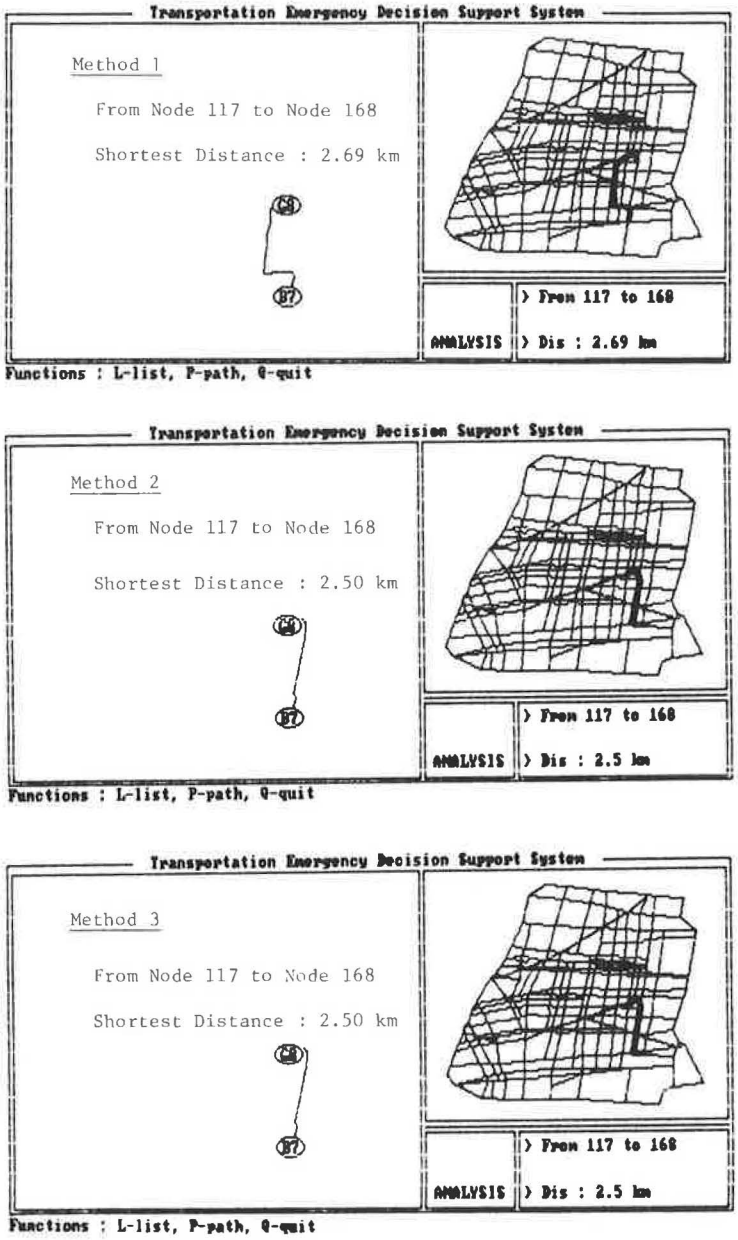


**FIGURE 4 Example of comparison of shortest paths by different methods.**

execution times in Table 1 and in the comparison of differences in shortest paths.

| | *Differences in Shortest Paths Between* | |
|---|---|---|
| *Shortest Path* | *Methods 1 and 3 (%)* | *Methods 2 and 3 (%)* |
| Distance-based | 10 | 1 |
| Time-based | 23 | 4 |

Based on these findings, the following conclusions supported by statistical evidence can be drawn:

- Ten of the 100 sample distance-based shortest paths obtained using the first method (partitioning technique with initial radius of search equal to half the straight-line distance between the origin and destination nodes) were different from and longer than the true shortest paths obtained by considering the whole network. However, based on statistical hypothesis testing, it can be concluded that only 5 percent of the shortest paths

TABLE 1 COMPARISON OF COMPUTER EXECUTION TIMES (distance-based shortest paths)

| Execution Time (sec) | Percentage of Cases | | |
|---|---|---|---|
| | Method 1 | Method 2 | Method 3 |
| ≤1 | 41 | 33 | |
| 2 | 19 | 20 | |
| 3 | 20 | 10 | |
| 4 | 11 | 16 | |
| 5 | 5 | 12 | |
| 6 | 3 | 6 | |
| 7 | 1 | 3 | 100 |

obtained by the partitioning technique are different from the true shortest paths, at 1 percent significance level. The test is shown below.

Null hypothesis: $H_0 : f = 0.05$
Alternate hypothesis: $H_1 : f \neq 0.05$

where $f$ is the true population proportion of differences in the shortest paths by the two methods. Let $\hat{f}$ represent the above proportion for the sample population. Then, based on the Central Limit Theorem (CLT), the test statistic

$$Z = \frac{(\hat{f} - f)}{S}$$

is assumed to be normally distributed with a mean of 0 and standard deviation of 1, where

$$S = \left[ \frac{f(1 - f)}{n} \right]^{1/2}$$

is the standard deviation of $\hat{f}$ and $n$ is the sample size.

For a significance level of $\alpha = 0.01$, the decision rule for a two-tailed test is

Reject $H_0$ if $Z > 2.57$ or $Z < -2.57$

Accept $H_0$ if $-2.57 \leq Z \leq 2.57$

For the data of this problem,

$$S = \left[ \frac{(0.05)(0.95)}{100} \right]^{1/2} = 0.0218$$

and

$$Z = \frac{(0.10)(0.05)}{0.0218} = 2.294$$

Hence, $H_0$ cannot be rejected and therefore it is concluded that the hypothesis is statistically valid.

- Only 1 percent of the distance-based shortest paths obtained using Method 2 (partitioning with initial radius of search equal to 1.25 times half the straight-line distance between the origin and destination nodes) was different from and longer than the true shortest paths obtained by considering the whole network. Statistical hypothesis test at $\alpha = 0.01$ also supports this conclusion.
- In the case of time-based shortest paths, the percentages (based on the sample study) for the above two cases were 23 and 4, respectively. However, based on statistical hypothesis tests, it can be concluded that only 15 percent of shortest paths obtained by Method 1 and 2 percent of those obtained by Method 2 are different from and longer than the true shortest paths obtained by considering the whole network.
- In the case of distance-based shortest paths, it is interesting to note that about 63 percent of the longer shortest paths was less than 10 percent longer than the true shortest paths.
- About 56 percent of the longer shortest time-based paths was less than 20 percent longer than the true shortest paths.

## DISCUSSION AND CONCLUSIONS

- The percentage variation in shortest paths, using the partitioning techniques, to the true shortest paths can be to some extent attributed to the configuration of the network considered. However, by using a higher initial radius of search (i.e., 1.25 times half the straight-line distance between the origin and destination nodes, and center at midpoint of the line joining them) greater accuracy appears to result in shortest-path determination without the need to consider the whole network. Thus, it may be worthwhile to adopt this heuristic method for shortest-path determination when the network is large.
- The partitioning of the network is based on the physical distance between the origin and destination. This is the case even for the shortest-path determination based on time. This, however, does not appear to be a major limitation, as depicted through the time-based shortest-path results.
- Although the execution times on a microcomputer for determining the shortest paths by the different techniques are in the range of seconds (Table 1), the savings achieved by adopting the partitioning technique may still be critical in large networks or in the process of evacuation of the seriously injured. For this study, a personal computer with a speed of 12 MHz was used. Where slower speed microcomputers are used, the savings in execution time may become even more important.
- The case study was on Mexico City network, which is of grid pattern predominantly. This may be a possible reason for greater accuracy in shortest paths obtained using partitioning techniques.

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