Analysis of Laterally Loaded Piles with Nonlinear Bending Behavior

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Analysis of the response of pile foundations to lateral loads requires accurate characterization of the behavior of the pile and the soil surrounding the pile. The common assumption of pile linearity in bending may not be valid in many cases. A model for representing nonlinear pile bending behavior in the analysis of laterally loaded piles is introduced. The fourth order differential equation governing the response of a laterally loaded pile is solved iteratively by a finite difference technique. The method converges to a solution featuring displacement-compatible soil resistance and curvature-compatible bending moments along the length of the pile. The ability of the method to represent nonlinear pile bending behavior is illustrated. The ability of the model to predict observed behavior is illustrated by application of the model to lateral load test case histories.

The response of pile foundations to lateral loads depends on the interaction between the piles and the surrounding soil. Analysis of this response requires accurate characterization of the behavior of both the pile and the soil surrounding the pile. Commonly used existing methods for analysis of the lateral load response of single piles consider the nonlinearity of the soil resistance but treat the pile as a linear, elastic beam. The assumption of pile linearity may not be valid in many cases. This paper considers the effect of this assumption on the behavior of laterally loaded piles and introduces a method for representing nonlinear pile bending behavior in the analysis of laterally loaded piles.

PREVIOUS WORK

The problem of calculating the response of a single, laterally loaded pile has been studied by many investigators using different approaches including methods of applied elasticity (*1*), finite element analysis (*2, 3*), boundary element methods (*4*), and subgrade reaction methods (*5*). While each of these methods has advantages in particular applications, the subgrade reaction methods have become the most commonly used for practical analysis of routine problems.

Subgrade reaction methods of lateral load analysis, in which the soil resistance is described by *p-y* curves, have been used for many years and have been recently summarized (*6*). Such methods for analysis of laterally loaded piles reported in the literature, however, almost uniformly treat the pile as a linear, elastic material. Nakai and Kishida (*3*) incorporated pile bending nonlinearity into an incremental finite element model, which iteratively solved for the deflected shape of the pile using a Rayleigh-Ritz procedure. Their model showed reasonable prediction of load-deflection behavior when applied to several case histories. For piles loaded into their nonlinear bending range, Reese (*6*) suggests analytical determination of the pile flexural stiffness corresponding to the maximum bending moment and use of that stiffness for the entire pile in a linear pile analysis.

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ANALYSIS OF LATERALLY LOADED PILES WITH NONLINEAR BENDING BEHAVIOR

The response of laterally loaded piles embedded in an elastic medium is usually described by the beam-on-elastic-foundation theory of Hetenyi (*7*). Equilibrium of an element of a beam subjected to axial and transverse loading was shown by Hetenyi to be described by the differential equation:

$$d^2M/dx^2 + Q(d^2y/dx^2) - p = 0 \quad (1)$$

where

M = bending moment;
x = position along axis of beam;
Q = axial load;
y = deflection normal to axis of beam; and
p = transverse loading along length of beam.

If the beam is assumed to exhibit linear bending behavior, the bending moment may be taken as the product of the constant beam flexural stiffness, EI, and the induced curvature, in which case the governing differential equation is of the familiar form:

$$EI(d^4y/dx^4) + Q(d^2y/dx^2) - p = 0 \quad (2)$$

For the analysis of laterally loaded piles in soil, this equation is commonly expressed in difference form and solved by a numerical procedure that iterates toward a solution in which the unit soil resistance, p, is compatible with the lateral pile deflection, y.

The bending moment induced in a pile exhibiting nonlinear bending behavior is not proportional to the pile curvature. Defining a secant pile flexural stiffness, $(EI)_s$, as the ratio of the bending moment to the induced curvature, the bending moment may be represented as

$$M = (EI)_s\,(d^2y/dx^2) \quad (3)$$

Substitution of Equation 3 into Equation 1 and expansion of the first differential term leads to the general governing differential equation for a laterally loaded pile with nonlinear bending behavior:

$$(EI)_s(d^4y/dx^4) + 2[d(EI)_s/dx]\,(d^3y/dx^3) + [Q + d^2(EI_s/dx^2)]\,(d^2y/dx^2) - p = 0 \qquad (4)$$

This governing equation may be solved numerically by discretizing the pile into segments of length, h, and expressing the differentials in difference form, which yields a banded system of simultaneous equations expressed, letting $R = (EI)_s$, in the form:

$$A_iy_{i+2} + B_iy_{i+1} + C_iy_i + D_iy_{i-1} + E_iy_{i-2} = 0 \qquad (5)$$

where

$$\begin{aligned} A_i &= R_i + (R_{i+1} - R_{i-1})/2 \\ B_i &= 2R_{i-1} - 6R_i + Qh^2 \\ C_i &= 10R_i - 2R_{i+1} - 2R_{i-1} - 2Qh^2 + E_sh^4 \\ D_i &= 2R_{i+1} - 6R_i + Qh^2 \\ E_i &= R_i + (R_{i-1} - R_{i+1})/2 \end{aligned}$$

SOLUTION OF PILE RESPONSE EQUATION

An iterative procedure was developed to solve the governing equation in difference form accounting separately for both the nonlinearity of the soil resistance and the bending nonlinearity of the pile. The proposed procedure retains the conventional iteration toward displacement-compatible soil resistance, used in programs like COM624 (*5*), wherein the soil resistance is taken as the negative product of the pile displacement and a secant soil modulus. The procedure iterates until the computed displacement is within some small tolerance of the displacement from the previous iteration, as shown in Figure 1a. The proposed procedure also iterates simultaneously toward a curvature-compatible bending moment. In this process, the actual bending moment at some point along the pile is taken as the product of the curvature of the pile at that point and a secant flexural stiffness. The secant flexural stiffness is varied until the curvature computed by a particular iteration is within some small tolerance of the curvature used in the previous iteration, as shown in Figure 1b. By this simultaneous iteration procedure, the solution converges to one in which the soil resistance is compatible with the pile displacement and the bending moments are compatible with the induced pile curvature.

This simultaneous iteration procedure has a number of attractive features. It is a relatively simple procedure, both conceptually and computationally. It can be incorporated into existing programs for lateral load analysis with a moderate amount of programming effort. The bending characteristics of the pile are described by one or more moment-curvature diagrams and may be varied along the length to accommodate tapered, segmented, or composite piles. While it does not provide a rigorous method for representation of nonlinear pile behavior, the method is consistent with that conventionally used to represent the soil resistance. Considering that the moment-curvature behavior of a pile is likely to be much more reliably known than the unit soil resistance–displacement behavior of the soil, the uncertainty associated with this numerical method of representation of pile nonlinear bending behavior is small.

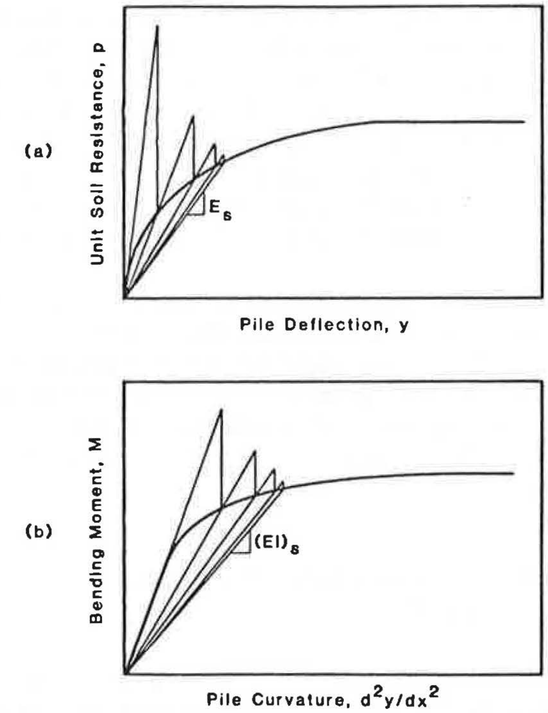

FIGURE 1 Schematic diagram of iteration procedure used to converge to (a) deflection-compatible soil stiffness and (b) curvature-compatible pile flexural stiffness.

MOMENT-CURVATURE BEHAVIOR OF PILES

Piles generally exhibit nonlinear bending behavior as a result of one or both of two mechanisms. First, yielding of the pile material(s) may lead to a nonlinear moment-curvature relationship. Nonlinearity as a result of material yielding can occur in steel, concrete, and timber piles when bending moment-induced stresses in the outer fibers of the pile section exceed the elastic range of the pile material. Second, the flexural stiffness of the pile may vary as a result of changes in geometry of the moment-resisting pile section. This effect is usually the result of cracking of the pile section and is typically observed only in concrete piles.

Steel piles are generally not subject to cracking under normal conditions. Large pile curvatures, however, may induce compressive and tensile stresses that may exceed the yield strength of the steel, particularly in portions of the pile section in which residual stresses are high. If additional loads cause further curvature, the associated bending moment will not increase proportionally. The theoretical maximum bending moment for which a steel pile will have constant flexural stiffness may be determined from simple beam theory. The moment-curvature relationship for steel piles stressed beyond the yield stress of the steel may be calculated based on the cross-sectional shape of the pile and an assumption of elastic, perfectly plastic steel behavior (*8*).

Concrete is used in pile foundations in the form of reinforced concrete, as in drilled, cast-in-place piles, or in precast, prestressed piles. Concrete piles, however, may develop tensile cracking even at moderate curvatures. Since tensile stresses cannot be transmitted across a crack, the moment-curvature relationship becomes nonlinear as soon as cracking develops. The nonlinear portion of the moment-curvature relationship for a concrete pile cannot be calculated as easily as that of steel pile since the bending behavior depends on the constitutive behavior of both the concrete and the reinforcing steel or prestressing strands. Analytical models are available (*6, 9*), however, that are capable of predicting the moment-curvature behavior of reinforced concrete and prestressed concrete sections.

The moment-curvature behavior of timber piles may have significant nonlinearity at moderate curvatures because of the material nonlinearity of wood. Ultimate bending moments and initial flexural stiffnesses appear to vary from pile to pile (*10*). The nonlinear bending model is well suited for incorporation into a probabilistic analysis of laterally loaded timber pile response.

ILLUSTRATION OF EFFECTS OF PILE NONLINEARITY

The nonlinear bending model was applied to two reported case histories of laterally loaded piles. The case histories were both drilled shafts, one constructed in stiff clay and the other in loose sand, or soft clay. In each case, the response of the pile was analyzed by assuming linear bending behavior and then assuming nonlinear bending behavior. The results are then compared with the observed response of the piles.

Johnson et al. Test

Results of a full-scale field lateral load test on an aged drilled shaft were presented by Johnson et al. (*11*). The shaft was 18 in. (0.46 m) in diameter by 34.5 ft (10.5 m) long including a 36-in. (0.9 m) diameter underream. The shaft extended through expansive clays, which had fractured the shaft at the neck of underream by the time it was tested 18 yr after construction. The shaft was loaded to failure with one intermediate unload-reload cycle. Yielding of the shaft was observed at failure.

Johnson et al. analyzed the response of the shaft with a constant computed flexural stiffness and the Reese and Welch (*12*) stiff clay *p-y* criteria and calculated load-displacement response stiffer than observed. The authors suggested that the actual flexural stiffness was less than computed.

Analysis of the load test with a smaller, constant flexural stiffness as indicated in Figure 2 gave reasonable agreement at lateral loads below about 28 kips (125 kN). At lateral loads above 28 kips (125 kN), however, linear pile analyses are unable to predict the large deflections observed in the field, as shown without the unload-reload cycle in Figure 3. Assuming that the maximum bending moment of the shaft was reached at a lateral load of 28 kips (125 kN), the bilinear moment-curvature relationship marked "Nonlinear Pile" in Figure 2 may be taken to represent the bending behavior of the shaft. Solution with this nonlinear moment-curvature relationship predicts the load-displacement response over the entire range of loading shown in Figure 4. The superior ability of the nonlinear model to predict the observed load-displacement response for this case history is apparent.

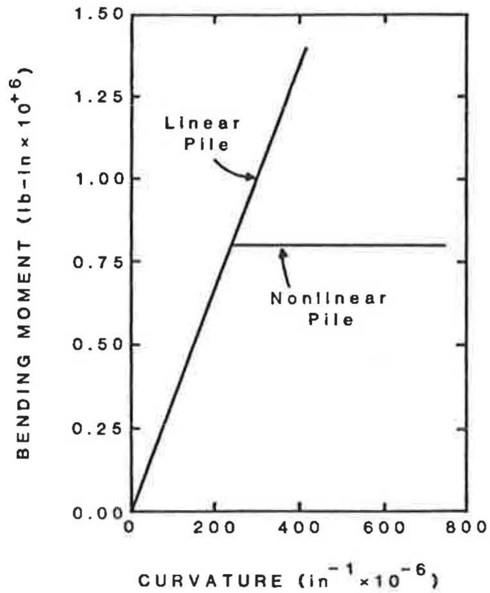

FIGURE 2 Moment-curvature diagrams for linear and nonlinear bending behavior of drilled shaft (*11*).

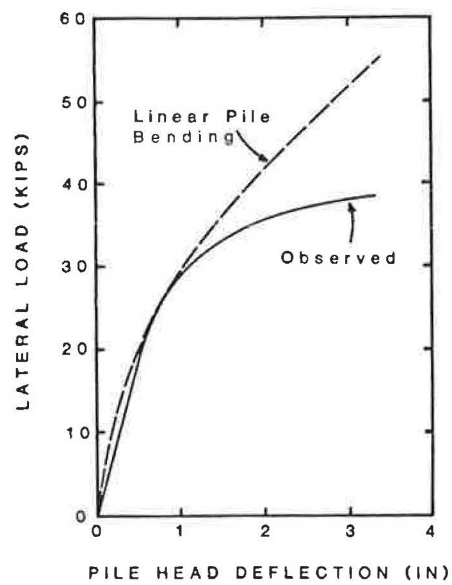

FIGURE 3 Comparison of observed response with response predicted by linear pile bending model (*11*).

Tsuji et al. Test

A lateral load test originally reported by Tsuji et al. in Japanese was described by Tominaga et al. (*13*). The test was performed on a 47.2-in. (1200 mm) diameter, cast-in-place bored pile, installed in loose sand, or soft clay, with standard penetration resistance of 4 to 5.

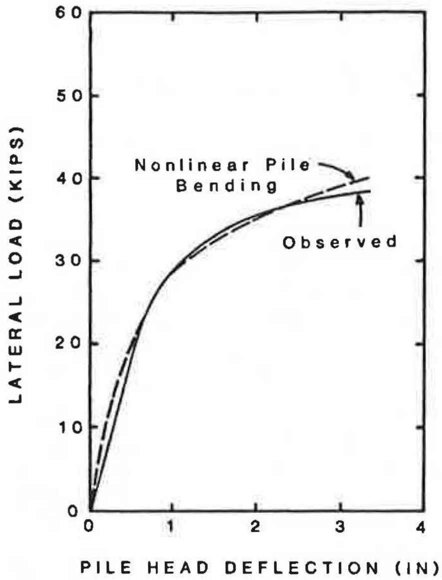

FIGURE 4 Comparison of observed response with response predicted by nonlinear bending model (*11*).

The uncracked pile was reported to have a flexural stiffness (*EI*) of 1.389 × 10^{12} lb-in.^{2}. Tominaga et al. developed an expression for the ultimate unit soil resistance as

$$P_{ult} = 0.1126z^2 + 9.21z$$

in pounds per inch where z = depth in inches. Using these values and assuming bilinear *p-y* behavior, Tominaga et al. computed the response of the pile by varying the initial soil stiffness until reasonable agreement with the observed behavior was obtained. By this procedure, good agreement was obtained at lateral loads below approximately 132 kips. Above this level, however, the linear model is unable to predict the rapidly increasing deformations attributed by Tominaga et al. to cracking of the pile.

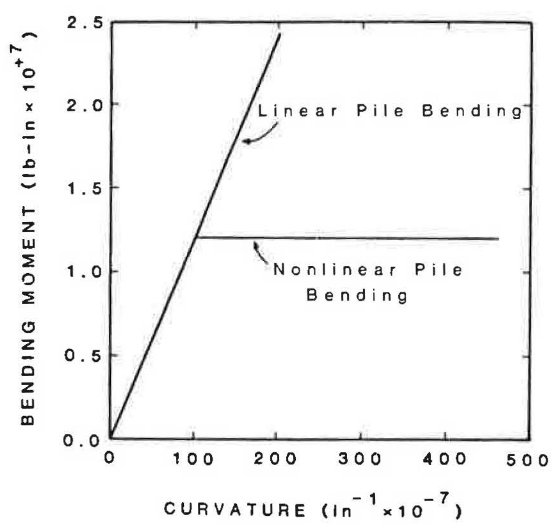

FIGURE 5 Moment-curvature diagrams for linear and nonlinear bending behavior of drilled shaft (*13*).

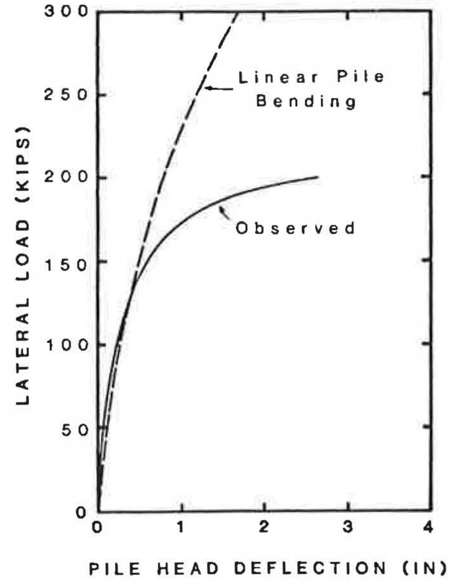

FIGURE 6 Comparison of observed response with response predicted by linear pile bending model (*13*).

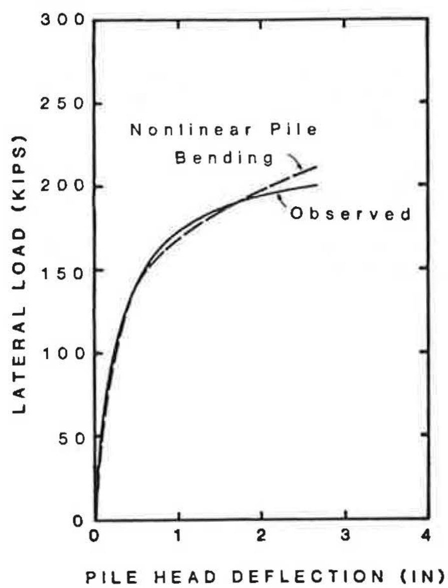

FIGURE 7 Comparison of observed response with response predicted by nonlinear pile bending model (*13*).

A similar procedure was used in the present investigation. Using *p-y* curves of the hyperbolic tangent form (*14, 15*), which is considered to be more representative of the actual nonlinear soil behavior than the simple bilinear model, the response of the pile was computed assuming linear pile bending behavior. The moment-curvature relationship was that indicated for linear pile bending in Figure 5. This analysis also indicates good agreement between computed and observed behavior at lateral loads below 132 kips as shown in Figure 6. Assuming that the ultimate bending moment was reached at a lateral load of 132 kips, the bilinear moment-curvature relationship marked "Nonlinear Pile Bending" in Figure 5 may be taken to represent the bending behavior of the pile. Solution of the pile response with this nonlinear moment-curvature relationship gives the load-displacement curve shown in Figure 7. Again, the ability of the nonlinear model to predict the large pile deflections associated with nonlinear pile bending behavior can easily be seen.

SUMMARY

A methodology for analysis of the lateral load response of piles with nonlinear bending behavior has been developed. Application to case histories of lateral load tests on piles and drilled shafts loaded into their nonlinear bending ranges indicates that the model is capable of predicting the large deflections often associated with the lateral load response of such piles. The model is expected to have particular application to reinforced concrete drilled shafts and to timber piles.

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