# Unified Design of Piles and Pile Groups

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**A unified design of piles and pile groups is proposed wherein capacity, residual compression, negative skin friction, and settlement are related. First, the location of the neutral plane is determined. Then, the adequacy of the structural strength of the pile is checked and followed by an analysis of the settlement of the pile foundation, applying the concept of an equivalent footing placed at the neutral plane. Finally, the adequacy of the pile bearing capacity is verified. For structural capacity at the neutral plane, dead load and dragload are considered together, but live load is excluded. For settlement, all stress increase in the soil is considered, not just that of the dead load acting on the pile foundation. For bearing capacity, dead and live loads are considered, but dragload is excluded. The design is iterative, inasmuch as the choice of load and pile length will have an interactive influence on all aspects: location of neutral plane, dragload, structural capacity, and settlement, as well as bearing capacity.**

Conventionally, or traditionally, when designing piles and pile groups, design for bearing capacity and design for settlement are considered separately and are not influenced by each other. In the simplest principle, design for bearing capacity consists of determining the allowable load—the service load—on the pile by dividing the capacity by a factor of safety. Settlement occurs when the service load on the piles stresses the soil, causing the soil to consolidate and compress. Usually, the methods of calculation are very simple. For instance, a common approach is to take the settlement of piles in sand to be equal to 1 percent of the diameter of the head of an individual pile plus the "elastic" compression of the pile under the load. For the case of an essentially shaft-bearing pile group in homogeneous clay soil, Terzaghi and Peck (*1*) recommended taking the settlement of the group as equal to that calculated for an equivalent footing located at the lower third point of the pile embedment length and loaded to the same stress and over the same area as the pile group plan area (Figure 1). For other approaches, see Meyerhof (*2*).

More complex methods for calculating settlement use elastic halfsphere analysis or finite element techniques. Vesic (*3*) and Poulos and Davis (*4*) presented several such analytical approaches toward calculating settlement on single piles and pile groups. Generally, it is assumed that before load is applied to the pile foundation, no stress is present in the pile or piles.

For the case of piles installed through a multilayered soil deposit, where upper layers settle because of, for instance, a surcharge on the ground surface or a general groundwater lowering, a settlement calculation of the pile group is often not performed. (The design practice seems to be to trust that the settlement will somehow be taken care of by including loads from downdrag in the bearing capacity analysis. Sometimes, on the other hand, the dragload is added to the service load and some settlement calculation is carried out for this combined load—a totally erroneous approach.)

Provided that the piles have been installed to reach well into competent soils and that no weaker soil layers exist below the pile toe elevation, this approach of including the dragloads in the bearing capacity and settlement analyses is mostly safe, albeit excessively costly. However, the problem of negative skin friction is one of settlement and not of bearing capacity (i.e., the magnitude of the dragload is of no relevance to the bearing capacity of the pile). Furthermore, the allowable load on the pile should be governed by a combined (unified) approach considering soil resistance and settlement inseparably acting together and each influencing the value of the other.

## LONG-TERM MEASUREMENTS OF LOAD AND SETTLEMENT

Observations show that for piles bearing on very competent material, negative skin friction can result in very large dragloads. Bjerrum et al. (*5*) measured dragloads amounting to about 4,000 kN on 0.5-m-diameter steel test piles installed to bedrock through 55 m of clay soil settling under the influence of a recent surcharge.

If a pile is long enough or if the ratio of its unit circumferential area to its cross-sectional area is large enough, the induced stress could exceed the material strength (i.e., the structural capacity of the pile). In the field tests reported by Bjerrum et al. (*5*), the piles were driven to rock, and the induced dragload exceeded the available toe resistance, forcing the pile to penetrate into the rock. This effect is cyclic, as discussed by Fellenius (*6*). Obviously, the toe force developed during the pile driving must have been smaller than the dragload.

Immediately after a pile is installed in the soil, the soil begins to reconsolidate from the disturbance caused by the installation of the pile, whether the pile was driven or otherwise installed. Fellenius and Broms (*7*) and Fellenius (*6*) reported load measurements in 0.3-m-diameter concrete piles driven into a 40-m-thick clay deposit and into an underlying sand layer. Immediately after the driving, the load in the pile was small, about equal to the free-standing weight of the pile before the driving. The reconsolidation of the clay after the driving took about 5 months. During this time, negative skin friction developed and the dragload induced amounted to about 350 kN corresponding to about one-third of the maximum dragload, which developed during the following several years of observations (*6, 8*). The settlement of the ground surface

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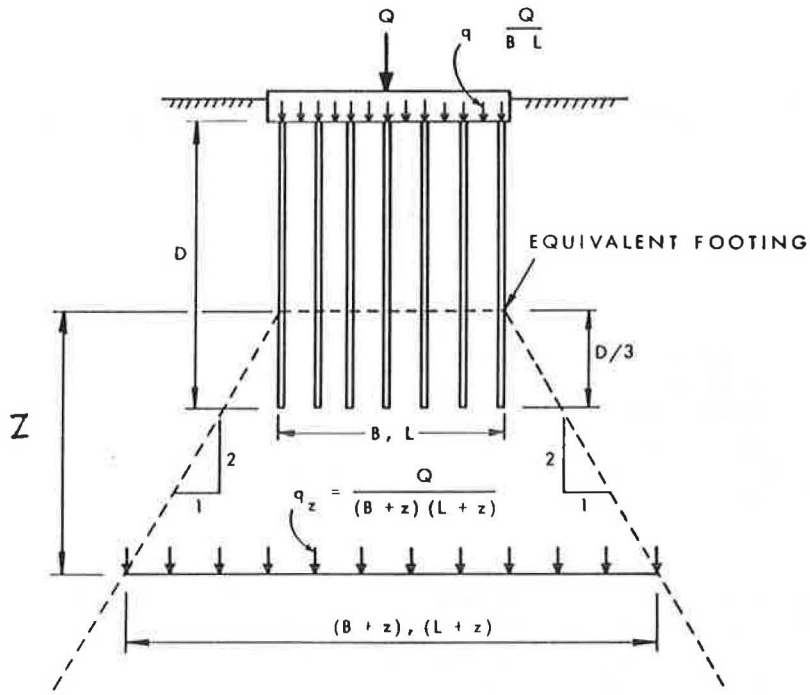


**FIGURE 1 Calculations of settlement for a pile group in homogeneous clay soil using the equivalent footing concept (*1*).**

associated with the 5-month reconsolidation was interpolated from measurements over a longer period of time and found to be smaller than about 2 mm. The distribution with depth of relative displacement between the pile and the clay was even smaller, of course.

The test is particularly interesting because it involved the effect of applying a static load to the pile head and not just observations of the development of the dragload in the pile. Applying a static load to the pile head caused the dragload in the pile to be reduced by the magnitude of the load applied. As the load became permanent, however, the negative skin friction built up again and the end effect was that the dragload in the pile was added to the load applied to the pile head. At the end of the test, the dragload was fully developed. The maximum load in the pile was 1,750 kN, consisting of a dead load of 800 kN and a dragload of 950 kN.

The negative skin friction was fully mobilized to a depth of about 25 m after a relative displacement of about 2 mm as measured at a distance smaller than about 0.5 m away from the pile. Measured at a distance of 5 m away from the test piles, the mean relative displacement approached 6 mm.

The magnitude of the movement necessary for shear resistance to develop was observed by Walker and Darvall (*9*), who reported that a mere 35-mm settlement of the ground surface due to a surcharge placed around single piles driven in clay was sufficient to develop negative skin friction down to a depth of 18 m. Settlement distribution with depth was not reported.

When loading a 49-m-long instrumented pile that had been driven through an embankment and into consolidating clay and then monitored during a 10-yr period, Bozozuk (*10*) found that a reversal of direction of shear forces down to a depth of 20 m occurred after a relative movement between the pile head and the ground surface of no more than about 4 mm. At depth, the relative movement between the pile and the soil near the pile was much smaller than 4 mm.

Bjerrum et al. (*5*) reported that negative skin friction causing large dragloads developed along about 55-m-long piles driven in clay at a site where the settlement under a recent fill amounted to 2 m. However, the same authors also reported that about an equal magnitude of dragload developed on 41-m-long piles that were driven through an adjacent 70-year-old fill of the same height and in the same type of soil in which an ongoing surface settlement of only 1 to 2 mm per year was observed.

In the referenced observations, which report results from field investigations performed on three separate continents extremely small movement was all that was needed to generate shear stress or to reverse the direction of shear along the pile-soil interface. Of course, on other occasions, larger relative movements could be necessary before the shear forces are fully developed. For instance, Vesic (*3*) suggested the rather large relative movement of 15 mm as a general requirement. However, the evidence suggests that such large movement is the exception and that the very small movement is the rule. Vesic's suggestion should be understood more to indicate when settlement around a pile foundation could start to cause problems.

There is a far-reaching consequence of a very small movement being all that is needed to fully transfer shear between the pile and the soil. The pile is immensely more rigid than the soil; with time, there will always be small movement in a soil, meaning that small relative displacements between a pile and the soil will occur and these are large enough to develop shear forces along the pile. The conclusion is that all piles experience negative skin friction and dragloads.

A consequence of the small displacement required to mobilize or to reverse the direction of shear forces along a pile is that live loads and dragloads are not additive (*6, 10*).

## CALCULATION OF THE MAGNITUDE OF UNIT SHAFT RESISTANCE

For the analysis of shaft resistance, Johannessen and Bjerrum (*11*) and Burland (*12*) established that the unit resistance is proportional to the effective overburden stress in the soil surrounding the pile. The constant of proportionality is called beta-coefficient, $\beta$, and is a function of the earth pressure coefficient in the soil, $K_s$, times the soil internal friction, $\tan \phi'$, times the quotient of the wall friction, $M = \tan \delta'/\tan \phi'$ (*13*). Thus, the unit negative skin friction, $q_n$, follows the following relations:

$$q_n = \beta \sigma_v'$$

$$\beta = M K_s \tan \phi'$$

(See the Notation section for more information on the symbols used.) Bjerrum et al. (*5*) found that the beta-coefficient in a soft silty clay ranged between 0.20 and 0.30. Kraft et al. (*14*) summarized several methods of determining shear transfer for piles in clay.

## PILE LOAD DISTRIBUTION AND LOCATION OF THE NEUTRAL PLANE

There must always be equilibrium between the sum of the dead load applied to the pile head and the dragload and the sum of the positive shaft resistance and the toe resistance. The depth where the shear stress along the pile changes over from negative skin friction into positive shaft resistance is called the neutral plane. The location of this plane is also where there is no relative displacement between the pile and the soil.

Provided the shear stress along the pile does not diminish with depth and that there is some toe resistance, the neutral plane lies below the midpoint of a pile. If the soil below the neutral plane is strong, the neutral plane lies near the pile toe. The extreme case is for a pile on rock, where the location of the neutral plane is at the bedrock elevation. For a predominantly shaft-bearing pile "floating" in a homogeneous soil with linearly increasing shear resistance, the neutral point lies at a depth that is about equal to the lower third point of the pile embedment length (assuming that the negative skin friction is equal to the positive shaft resistance, the toe resistance is small, and the load applied to the pile head is a third of the bearing capacity of the pile). It is interesting to note that this location is also the location of the equivalent footing according to the Terzaghi-Peck approach (*1*) (Figure 1).

With larger toe resistance, the elevation of the neutral plane lies deeper into the soil. If an increased dead load is applied to the pile head, the elevation of the neutral plane moves up.

Figure 2 illustrates the distribution of load in a pile subjected to a service load, $Q_d$, and installed in a relatively homogeneous soil deposit, where the shear stress along the pile, as induced by a relative displacement, is proportional to the effective overburden stress. It is assumed that any excess pore pressure in the soil has dissipated and the pore pressure is hydrostatically distributed. For reasons of simplicity, the shear stress along the pile is assumed to be independent of the direction of the displacement (i.e., the magnitude of the negative skin friction, $q_n$, is equal to the magnitude of the unit positive shaft resistance, $r_s$). It is also assumed that a toe resistance, $R_t$, has been mobilized.

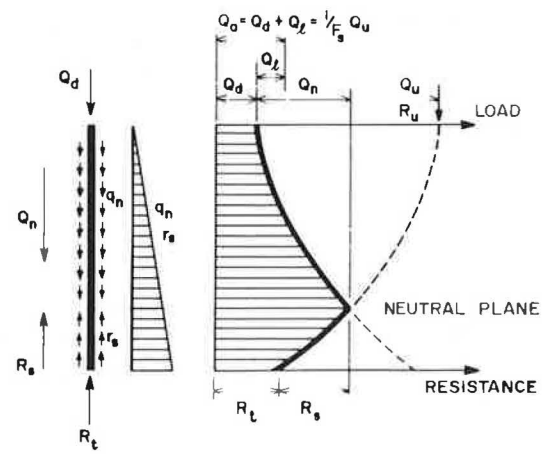


**FIGURE 2 Definition and construction of the neutral plane.**

The dragload, $Q_n$, is the sum (integral) of the unit negative skin friction along the pile. The total shaft resistance below the neutral plane, $R_s$, is the sum of the unit shaft resistance along the pile.

These conditions determine the location of the neutral plane as shown in the diagram.

## SETTLEMENT OF A PILE

The neutral plane is, as mentioned, the location where there is no relative displacement between the pile and the soil. Consequently, whatever the settlement in the soil is as to its magnitude and vertical distribution, the settlement of the pile head is equal to the settlement of the neutral plane plus the compression of the pile caused by the applied dead load and the dragload combined.

Illustrated in the "Load and Resistance" segment of Figure 3 is how the elevation of the neutral plane changes with a variation of the load, $Q_d$, applied to the pile head. Notice also that the magnitude of the dragload changes as $Q_d$ changes.

Assume that the distribution of settlement in the soil around the pile is known and follows the "Settlement" portion of the diagram in Figure 3. As illustrated in the diagram for the case of the middle service load, by drawing a horizontal line from the neutral plane to intersect the settlement curve, the settlement of the pile at the neutral plane and, thus, the settlement of the pile head can be determined. The construction in the figure is valid both for a small settlement that diminishes quickly with depth and for a large settlement that continues to be appreciable well below the pile toe.

The construction in Figure 3 has assumed that the toe resistance is fully mobilized. If the settlement is small, it is possible that the toe movement is not large enough to mobilize the full toe resistance. In such a case, the neutral plane moves

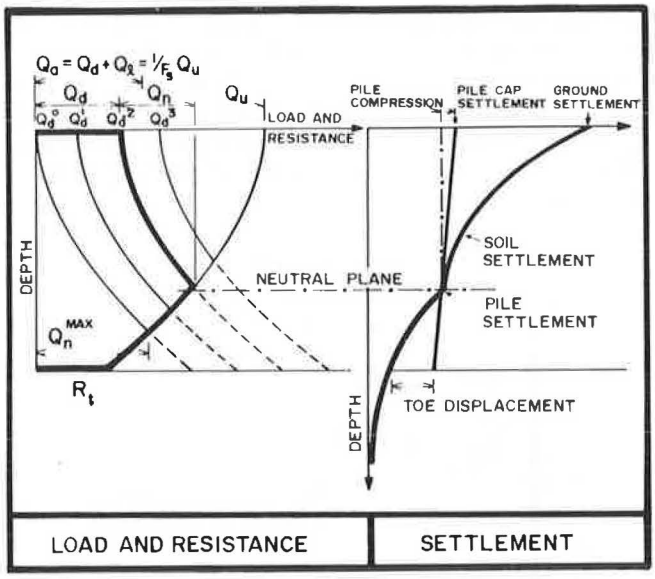


**FIGURE 3 Unified design for capacity, negative skin friction, and settlement.**

to a higher location as determined by the particular equilibrium condition.

For a driven pile, the toe movement necessary to mobilize the toe resistance is about 1 to 2 percent of the pile toe diameter. For bored piles, the movement is larger. However, in cases where the toe movement is too small for the full toe resistance to be mobilized (less toe resistance results in a raising of the location of the neutral plane), the settlement is normally not an issue.

## REDUCTION OF SKIN FRICTION BY MEANS OF BITUMEN COATING

Bjerrum et al. (*5*) demonstrated the efficiency of coating piles with bitumen to reduce the negative skin friction. Walker and Darvall (*9*) presented a comparison between bitumen coated and uncoated steel piles, and Clemente (*15*) reported measurements of dragloads on coated and uncoated concrete piles. Fellenius (*16, 17*) discussed some practical aspects of bitumen coating of piles to reduce negative skin friction.

Other papers comparing measurements in plain and coated piles were published by Endo et al. (*18*), Okabe (*19*), Mohan et al. (*20*), Velloso et al. (*21*), Fukuya et al. (*22*), Lee and Lumb (*23*), and Keenan and Bozozuk (*24*).

## DESIGN ASPECTS

### Fundamentals

In all the papers referenced in the foregoing, the emphasis was on the dragload. When the referenced authors reported observations of deformation and settlement, the main use of these was to calculate the loads in the pile. As to the consequence of the negative skin friction on the design, it was discussed in terms of reduction of the pile-bearing capacity or of the allowable load, not in terms of settlement.

In contrast, this paper suggests that the problem in designing for negative skin friction is one of settlement and not of bearing capacity (i.e., the magnitude of the dragload is of no relevance to the bearing capacity of the pile). Furthermore, the allowable load on the pile should be governed by a combined, unified approach considering soil resistance and settlement inseparably acting together and each influencing the magnitude of the other.

The published records of measurements of movements associated with negative skin friction indicate that extremely small relative movements—on the order of 1 mm—are sufficient to generate negative skin friction. Because of the considerable difference of stiffness between a pile and soil, all piles are subjected to relative movements of this magnitude. Therefore, all piles are subjected to negative skin friction, not just those in soils that settle significantly, and, in all piles, a neutral plane develops.

As mentioned, at the elevation of the neutral plane, there is no relative movement between the pile and the soil. Consequently, the settlement of the pile head is that of the soil at the neutral plane plus the "elastic" compression of the pile for the dead load and the dragload. To determine the location of the neutral plane, an analysis of the load distribution in the pile must first be performed.

Negative skin friction and the consequent dragload on piles cannot be treated separately from the settlement occurring in the soil, the pile movement relative to the soil, and the shaft and toe resistances of the pile.

The suggested design approach is essentially the same for all piles, whether single or in a group; whether installed in a soil that settles significantly under the influence of a surcharge, groundwater lowering, or other cause, or in a soil that does or does not experience appreciable settlement; and whether the piles are essentially toe bearing, shaft bearing, or both toe and shaft bearing. The design principles are equally applicable on piles in clay as on piles in sand, or in other coarse-grained soil, or in multilayered soil.

To understand the design principle, it is important to realize that the live load and the dragload do not combine and that two separate loading cases must be considered: dead load plus dragload, but no live load, and dead load plus live load, but no dragload. Furthermore, a rigid, high-capacity pile will experience a large dragload but small settlement, whereas a less rigid, smaller capacity pile will experience a smaller dragload but larger settlement. Also, while the dragload is caused by settlement or, rather, relative displacement, the dragload does not generate settlement, and no pile will settle more than the ground surface nearest the pile.

The design is carried out considering four aspects interactively: location of the neutral plane, structural capacity, settlement, and bearing capacity.

### Neutral Plane

As a first step in the design, the neutral plane must be determined. The neutral plane is located where the negative skin friction changes over to positive shaft resistance. In other words, the neutral plane is determined by the requirement that the sum of the applied dead load plus the dragload is in equilibrium with the sum of the positive shaft resistance and the toe resistance of the pile. The neutral plane is located at the intersection of two load distribution curves construed as follows. First, as was illustrated in Figure 2, a load distribution

curve ("forcing-load" curve) is drawn from the pile head and down, with the load value starting with the applied dead load and increasing with the load because of negative skin friction taken as acting along the entire length of the pile. Second, a load distribution curve ("resistance" curve) is drawn from the pile toe and up, starting with the value of the toe resistance and increasing with the positive shaft resistance.

Correct determination of the two load distribution curves is important for a correct design. Several theories exist whereby the capacity of a pile can be determined from soil data. Most of these theories have been developed by correlation to results of static loading tests. However, most static tests, even the ones that have been well instrumented, share one fallacy, namely, the measurements of the load induced in the test pile by loading the pile head fail to consider the load induced by the installation and the reconsolidation of the soil after the installation—the residual load, or residual compression, in the pile. Every pile will be subjected to a residual compression by the time it is instrumented for the static test. Assuming that the pile and the soil are unstressed before the load was placed on the pile head brings a large error into the interpretation of the test and the correlation of the data with theory.

The sometimes observed reduction of unit shaft resistance with depth can be satisfactorily explained by means of introducing a small residual compression into the observed pile compression under the applied load. So can the concept of the critical depth (*2*), which states that below some depth, called the "critical depth," both the unit shaft resistance and the unit toe resistance are constant and independent of the effective overburden stress below the critical depth. This does not mean that the critical depth concept is wrong, only that the issue is more complicated than thought. For a discussion, see also Vesic (*25*) and Hanna and Tan (*26*). To determine the load distribution curves requires reliable information on the soil strength properties. Then, for most problems encountered in practice, the theoretical analysis using the previously mentioned method of beta-coefficient on the effective overburden stress is preferred over any total stress, or "elastic" method. The analysis should be supplemented with information from penetrometer tests. Results from static test loading, preferably on piles that are instrumented with at least telltales, would be of significant assistance. However, it is beyond the scope of this paper to discuss the method for determining the load distribution or the many factors influencing the results of a static loading test.

In practice, a pile group will consist of piles of varying length installed to a capacity that varies between the piles. If each pile in the group were able to carry an equal portion of the dead load on the pile group, the location of the neutral plane would vary considerably between the piles. In reality, provided the pile cap is reasonably rigid, there is an equalization between the piles inasmuch as part of the dead load is transferred from "softer" piles to "stiffer" piles. In the process, the location of the neutral plane is equalized too. Of course, the neutral plane cannot be a horizontal or even plane, but must be shaped like a rolling surface. The location of the neutral plane determined according to the foregoing is therefore a mean location. This concept can be used to determine the load distribution between piles in a given pile cap considering known pile lengths, installation behavior, and so on.

## Structural Capacity

The structural capacity is governed by the structural strength of the pile material at the neutral plane for the combination of dead load plus dragload—live load is not to be included. (At or below the pile cap, the structural strength of the embedded pile is determined as a short column subjected to dead load plus live load, but dragload is not included).

At the neutral plane, the pile is confined and it is suggested that the limiting value of maximum combined load be determined by applying a safety factor of 1.5 on the pile material strength (steel yield or concrete 28-day strength or long-term crushing strength of wood).

It should be realized that if both the negative skin friction and the positive shaft resistance, as well as the toe resistance values, are determined assuming soil strength values "erring" on the strong side, the calculated maximum load in the pile will be on the conservative side (and the neutral plane located deep down into the soil).

As illustrated in Figure 3, a reduction of the dead load on the pile will result in a lowering of the location of the neutral plane but have a proportionally smaller effect on the magnitude of the maximum load in the pile.

## Settlement

As also demonstrated in Figure 3, the settlement of the pile head is determined by first calculating and plotting the distribution of settlement of the soil and then drawing a horizontal line from the neutral plane to intersect the settlement curve. The settlement of the pile is equal to the settlement of the soil at the elevation of the neutral plane plus the "elastic" compression of the pile from the sum of the dead load and the dragload.

To predict settlement correctly is difficult, in particular, in view of the dearth of results from field tests on pile groups. Until data become available from well-instrumented field tests having emphasized the measurement of settlement and deformation rather than the loads, the author suggests the following approach.

The settlement calculation should be carried out according to conventional methods for the effective stress increase caused by surcharge, groundwater lowering, and any other aspect influencing the stress in the soil. The calculation should include the dead load acting on the pile(s) as applied at the level of the neutral plane on an equivalent footing having the same size as the pile cap. Neither the dragload nor the live load should be included in the calculation. (Some judgment must be exercised as to whether a live load is of a duration that it should be considered at least partially a dead load or if it truly is temporary.)

A condition for the suggested approach is that the movement at the pile toe must be equal to or exceed the movement required to mobilize the ultimate toe resistance of the pile. (In most soils, this required movement is about 1 to 2 percent of the pile toe diameter of driven piles and about 5 to 10 percent of the toe diameter for bored piles). If the movement is smaller than this, the toe resistance will not be fully mobilized and the neutral plane will move to a higher elevation. In a design case where the toe resistance value is difficult to estimate or where it is variable, for instance in the case of toe-jetted piles, a

conservative estimate of the settlement is obtained by disregarding the toe resistance when construing the location of the neutral plane.

The settlement calculations emphasize the settlement of the soil layers located below the neutral plane and must include the compression of silt and sand layers in the soil profile. This makes it important to carry the investigation of the soil conditions at a site to a sufficiently large depth and to include a representative amount of sampling and laboratory testing of the soils located below the pile toe. As a minimum, an investigation should include static cone penetrometer tests and sampling of all layers encountered with undisturbed samples taken of all cohesive soils.

The settlement calculation of noncohesive soils should not be based on the use of a constant "elastic" modulus, but on the tangent modulus approach, which considers that compression of soil does not increase linearly with increase of stress. The Janbu unified settlement theory, as detailed by the *Canadian Foundation Engineering Manual* (*27*), is particularly useful for calculating settlement in deep profiles of cohesive, as well as noncohesive, soils. The manual contains reference values of moduli for use in estimating soil compression.

It should be realized that if both the negative skin friction and the positive shaft resistance, as well as the toe resistance, are determined assuming soil strength values "erring" on the weak side, the calculated location of the neutral plane will be located higher up in the settlement diagram (i.e., the settlement of the pile will be calculated on the conservative side).

As illustrated in Figure 3, a reduction of the dead load on the pile will result in a lowering of the neutral plane and, therefore, a reduction of the settlement of the pile.

## Bearing Capacity

The dragload must not be included when considering bearing capacity (i.e., the dragload is of no consequence for the analysis of soil bearing failure). Therefore, for bearing capacity consideration, it is incorrect to reduce the dead load by any portion of the dragload.

The dead load should only be reduced because of insufficient structural strength of the pile at the location of the neutral plane, where the pile is subjected to the combination of dead load and dragload, or by a necessity to lower the location of the neutral plane in order to reduce the amount of settlement.

The consideration of the bearing capacity in the design of a pile, or of a group of piles, amounts to making a check of the safety against plunging failure of the pile(s). In such a case, the pile moves down along its entire length and the negative skin friction is eliminated. Therefore, the load to apply on the pile in the bearing capacity analysis is the combination of dead and live loads. Dragload is not to be included.

Normally, when the pile capacity has been determined from results of a static loading test or analysis of data from dynamic monitoring, a factor of safety of two or larger ensures that the neutral plane is located below the midpoint of the pile. When the capacity is calculated from soil strength values, the factor of safety should not be smaller than three.

## Special Considerations

It is clear from the foregoing that all piles will be subjected to negative skin friction and experience dragload. However, piles installed where soil settlement is small will not constitute a problem, unless the structural capacity of the pile is exceeded. Furthermore, the maximum dragload induced in a straight and vertical pile is not dependent on whether the settlement of the soil is large or small. However, for piles that are inclined, large settlement will force the pile to bend. For this reason, where large settlement is expected, it is advisable to avoid inclined piles in the foundation or, at least, to limit the inclination of the piles to values that can accept the settlement without its inducing excessive bending in the piles.

Piles that are bent, doglegged, or damaged during the installation will have a reduced ability to support the service load in a downdrag condition. Therefore, the unified design approach postulates that the pile installation is subjected to stringent quality control directed toward ensuring that the installed piles are sound and that bending, cracking, and local buckling do not occur.

## Counteracting Negative Skin Friction

When the design calculations indicate that the settlement could be excessive, increasing the pile length or decreasing the pile diameter could improve the situation. When the calculations indicate that the pile structural capacity is insufficient, increasing the pile section, or increasing the strength of the pile material, could improve the situation. When such methods are not practical or economical, the negative skin friction can be reduced by the application of bituminous coating or other viscous coatings to the pile surfaces before the installation (*15–17*). For cast-in-place piles, floating sleeves have been used successfully.

## Design Case History

A pile group consisting of 10 piles is to be installed at a site where the soil profile consists of a 2-m-thick fill recently placed over 8 m of soft-to-firm clay on a 4-m layer of sand below which lies a 20-m-thick layer of slightly overconsolidated, silty clay deposited on dense mixed granular soil, an ablation glacial till. The groundwater table is located at a depth of 2 m and the pore pressure is hydrostatically distributed throughout the profile. Details of the soil properties are given in Table 1.

The 10 piles consist of 300-mm-diameter pipe piles driven closed-toe to a total embedment of 38 m. The pile cap area is 3.5 m by 5.0 m = 17.5 $m^2$.

The piles will be concrete filled after driving and the allowable structural load of the pile at the neutral plane is 2100 kN. The intended allowable service load on the piles is 1400 kN of which 1200 kN is dead load and 200 kN is live load. Thus, the stress over the equivalent footing area is 686 kPa.

The bearing capacity of the pile is 2520 kN, when calculated in accordance with the beta-method and neglecting the critical depth concept. For the applied load of 1400 kN, the factor of safety against bearing failure is 1.80. The detailed results of the calculations are summarized in Table 2 and graphically presented in Figure 4.

TABLE 1 SOIL PROFILE AND SOIL PROPERTIES

| Depth Range (m) | Type | Unit Weight (kN/m³) | $c'$ (kPa) | $\beta$ | $m$ | $m_r$ | $j$ | $\sigma_p' - \sigma_0'$ (kPa) |
|---|---|---|---|---|---|---|---|---|
| 0–2 | Fill | 18.0 | – | 0.50 | – | | – | – |
| 2–12 | Clay | 15.5 | 15 | 0.25 | 40 | 200 | 0 | 0 |
| 12–16 | Sand | 20.0 | – | 0.45 | 250 | – | 0.5 | – |
| 16–36 | Clay | 17.4 | 0 | 0.35 | 180 | 550 | 0 | 120 |
| 36– | Till | 21.0 | – | 0.60 | 400 | – | 0.5 | – |

NOTE: The bearing capacity coefficient, $N_t$, of the till is 50. The letters $m$, $m_r$, and $j$ indicate the modulus numbers and the stress exponent for the soil. The letter $c'$ indicates the effective cohesion intercept and the symbols $\beta$, $\sigma_0'$, and $\sigma_p'$ indicate the beta coefficient, the existing effective stress in the soil, and the preconsolidation pressure, respectively.

TABLE 2 RESULT OF BEARING CAPACITY AND RESISTANCE CALCULATIONS

| Factor | Determined Value |
|---|---|
| Applied loads | |
| $Q_{tot}$ (kN) | 1400 |
| $Q_d$ (kN) | 1200 |
| $Q_l$ (kN) | 200 |
| Ultimate resistances | |
| $R_{ult}$ (kN) | 2520 |
| $R_s$ (kN) | 1750 |
| $R_t$ (kN) | 770 |
| Dragload, $Q_n$ (kN) | 660 |
| Maximum load, $Q_d + Q_n$ (kN) | 1860 |
| Depth, $D_{NP}$ (m) | 24.5 |

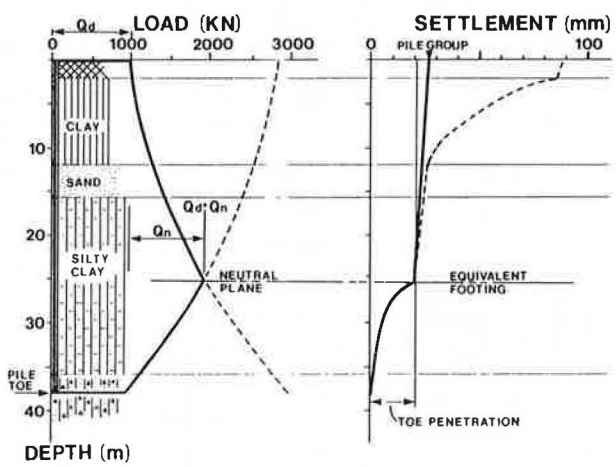


**FIGURE 4 Design case history. Graphical presentation of the results of the design calculations.**

The settlement for the pile group is calculated for the increased stress from the fill and the dead loads on the piles acting on and below the neutral plane. The stress from the equivalent footing is distributed according to the 2:1 method. The calculations indicate that the pile cap would settle slightly less than 30 mm and that the pile toe penetration into the till is about 15 mm, which movement is enough to develop the toe resistance assumed in the calculation of the neutral plane location.

## NEED FOR RESEARCH

The proposed unified design approach shares one difficulty with all other approaches to pile group design, namely, there is a lack of thorough and representative full-scale observations of load distribution in piles and of settlement of pile foundations. With regard to settlement observations, the lack is almost total with respect to observations of settlement of both the piles and the soil adjacent to pile foundations.

In a typical design case, the shaft and toe resistance for a pile can only be estimated within a margin. For instance, in a given case, a designer cannot know whether the critical depth concept should be included in the calculation of resistance even when a routine static loading test is carried out. To provide the profession with reference cases for aid in design, it is very desirable that sturdy and accurate load cells be developed and installed in piles to register the load distribution in the pile during, immediately after, and with time after the installation. Naturally, such cells should be placed in piles subjected to static loading tests, but not exclusively in these piles (*28, 29*).

The greatest perceived need lies in the area of settlement observations. (It is paradoxical that pile foundations are normally resorted to for reasons of excessive settlement. Yet, the design is almost always based on a capacity rationale with disregard of settlement.) Actual pile foundations should be instrumented to determine both the settlement of the piles and the distribution of settlement in the soil near the piles. No instrumentation for study of settlement should be contemplated without an inclusion of piezometers.

## CONCLUSIONS

A unified design of piles and pile groups is proposed wherein capacity, residual compression, negative skin friction, and settlement of both pile and soil are related. The design is carried out in four main steps: first, the location of the neutral plane is determined, then the adequacy of the structural strength of the pile is checked, followed by an analysis of the settlement of the pile foundation applying the concept of an equivalent footing placed at the neutral plane, and, finally, the adequacy of the bearing capacity of the pile is verified.

For structural capacity at the pile cap level, dead and live loads are considered together; for structural capacity at the neutral plane, dead load and dragload are considered together, but live load is excluded.

For settlement, all stress increase in the soil is considered, not just that of the dead load acting on the pile foundation. For bearing capacity, dead and live loads are considered, but dragload is excluded.

The design analysis is iterative, inasmuch as the choice of load and pile length will have an interactive influence on all

aspects: the location of the neutral plane, the structural capacity, the settlement, and the bearing capacity.

## NOTATION

$\beta$ = beta-coefficient = $M K_s \tan \phi'$
$M$ = quotient of wall friction = $\tan \delta'/\tan \phi'$
$\delta'$ = angle of effective pile-soil friction
$\phi'$ = angle of effective internal soil friction
$K_s$ = ratio of horizontal to vertical effective stress
$Q_{tot}$ = allowable or applied total load
$Q_d$ = allowable or applied dead load
$Q_l$ = allowable or applied live load
$Q_n$ = dragload
$q_n$ = unit negative skin friction
$R_{ult}$ = ultimate resistance
$R_s$ = pile shaft resistance
$r_s$ = unit pile shaft resistance
$R_t$ = pile toe resistance
$r_t$ = unit pile toe resistance
$D$ = pile embedment depth
$D_{NP}$ = depth to the neutral plane
$m$ = modulus number–virgin curve
$m_r$ = modulus number–reloading curve
$j$ = stress exponent

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greater depths, the controlling failure mechanism is assumed to be a horizontal flow failure of sand around the pier (*14*). A similar analysis principle was followed in this study to derive the ultimate resistance expression for the sloping ground surface condition (*10*). Although the presence of the ground slope will influence the mobilized passive resistance near the ground surface, the flow failure pattern, which governs at greater depths, is not significant for rigid piers having a relatively short length.

The total lateral resistance is given by the following expression (2):

$$Pu = \gamma H\ [H\,(S_{1\phi} + 3K_0\,S_{3\phi}) + bS_{2\phi} - K_a b] + c\,[H\,(S_{1c} + S_{3c}) + bS_{2c} - 2b\,K_a^{0.5}] \tag{8}$$

where

$$S_{1\phi} = \frac{\lambda 2 \tan\Omega \tan\beta}{(\tan\theta \tan\beta + 1)^2}\,[(\tan\theta \tan\beta + 1) \times (3 + 4\tan\phi \tan\beta) - (2\tan\phi \tan\beta)] \tag{9}$$

$$S_{2\phi} = \frac{2\lambda 2}{\tan\theta \tan\beta + 1}\,(1 + \tan^2\phi) \tag{10}$$

$$S_{1c} = \frac{2\tan\Omega \tan\beta}{(\tan\theta \tan\beta + 1)^2}\,[\lambda 1\,(1 + 2\tan\theta \tan^2\beta + \tan\beta) + 2\tan\beta\,(\tan\theta \tan\beta + 1) - \tan\beta] \tag{11}$$

$$S_{2c} = \lambda 1 = \frac{1 + \lambda 1 \tan\phi}{\tan\theta \tan\beta + 1} \tag{12}$$

$$S_{3\phi} = (\tan\phi - \tan\Omega) \times \left[\tan\beta - \frac{\tan^4\beta \tan^3\theta + \tan^3\beta \tan^2\theta}{(\tan\beta \tan\theta + 1)^3}\right] \tag{13}$$

$$S_{3c} = \tan\beta - \frac{\tan^3\beta \tan^2\theta + \tan^2\beta \tan\theta}{(\tan\beta \tan\theta + 1)^2} \tag{14}$$

$$\lambda 1 = K1\,K_{pc} \tag{15}$$

$$\lambda 2 = \frac{K1}{2}\left(K_{p\phi} + \frac{K2}{\cos\beta}\right) \tag{16}$$

$$K1 = \frac{\cos\beta\,(\tan\theta \sin\beta + \cos\beta)}{H} \tag{17}$$

$$K2 = \frac{\tan\beta \sin\beta}{\tan\theta \tan\beta + 1} \tag{18}$$

$$K_{pc} = \frac{1}{(\tan\theta \sin\beta + \cos\beta)\,(\sin\beta - \cos\beta \tan\phi)} \tag{19}$$

$$K_{p\phi} = \frac{\tan\beta\,(\cos\beta + \sin\beta \tan\phi)}{(\tan\theta \tan\beta + 1)\,(\sin\beta - \cos\beta \tan\phi)} \tag{20}$$

where

$b$ = pier diameter,
$H$ = depth below ground surface,
$\beta$ = $45 + \phi/2$,
$\theta$ = ground surface slope angle,
$\gamma$ = soil effective unit weight,
$\phi$ = angle of internal friction,
$c$ = cohesion intercept,
$K_a$ = coefficient of active earth pressure,
$K_0$ = at-rest earth pressure coefficient
$\Omega$ = angle that defines the wedge size in front of the pier.

## COMPUTER PROGRAM LTBASE

LTBASE [Lateral pier analysis including base and slope effects (*10*)] is a computer program for the load-deflection analysis of laterally loaded piles and drilled piers. The program uses the finite difference technique to solve the nonlinear simulation model formulated using the subgrade reaction approach. The program is coded in FORTRAN77 computer language, and the source code was compiled using the Microsoft FORTRAN77 version 3.2 compiler. The compiled code is linked to the MS-FORTRAN runtime library FORTRAN.L87, which supports an 8087 math coprocessor. The Microsoft 8086 object linker version 3.02 was used in the linking process. Double precision arithmetic is used throughout the program to enhance the accuracy of the solution.

The computer program was written for an IBM-compatible PC, and solves for deflection, bending moment, soil reaction, and soil moduli values as functions of depth under a given set of loads. The base resistance model and the ultimate resistance expressions presented in the previous section were implemented in the program. The structure of the computer program, LTBASE, consists of a main program and seven subroutines. The authors modified the subroutines LPILE1 and SOIL 2, given in the computer program LPILE (*8*), and used them in developing LTBASE.

The solution technique adopted in LTBASE requires that the soil be replaced by a set of nonlinear springs that conceptually define soil response curves. Such curves define the soil resistance, $p$ (force per unit length along the pile) as a function of pier deflection, $y$. In general, the shape of a soil response curve, usually referred to as a $p$-$y$ curve, is defined by the coefficient of lateral subgrade reaction, $K_{ho}$, and the ultimate soil resistance, $P_u$. As of this time, the sloping ground surface ultimate resistance expression has not been validated for the undrained clay soil condition (i.e., $\phi = 0$). Also, it is assumed that the $K_{ho}$ values are unaffected by the presence of a sloping ground surface.

Several methods of formulating $p$-$y$ curves have been developed (*3, 5, 6, 9, 12, 15–18*). Although each method uses many of the same soil parameters, the equations are formulated differently because each method was developed in conjunction with a particular set of field or laboratory tests. The soil response curves generated by LTBASE are according to the

procedures described by Reese et al. (*15*) and Murchison and O'Neill (*16*), based on experimental data reported by Parker and Reese (*6*) for sands and according to the unified method developed by Sullivan for clays (*7, 16*).

## COMPUTING TECHNIQUE

Although the mathematical formulation of the problem resulted in a number of simultaneous linear equations, the nature of the problem presented here is nonlinear and requires an iterative approach for solution. The reactions that are generated in the soil because of the pier deformations under a given loading condition must be such that the equations of static equilibrium and compatibility are satisfied.

LTBASE initially assumes zero base resistance. Values of lateral soil moduli and associated deflections are successively computed until convergence is achieved under the applied loads. Convergence to the correct solution is judged to have been obtained when the differences between deflections computed in Step $i+1$, $y_{i+1}$, and Step $i$, $y_i$, are less than a specified tolerance criterion.

The moment and horizontal shear resistance at the base are then determined using the evaluated base deformations. New deflection values along the pier are then calculated using the applied loads at the top and the computed moment and horizontal shear resistance at the base.

The computed new deflection values result in different soil reaction along the length of the pier, and different moment and horizontal shear resistance at the base from those used in the previous step. Therefore, using the new computed base resistance, new deflection values are computed along the pier. The procedure is repeated until convergence is achieved. Convergence to the correct solution is achieved when the following convergence criterion is reached:

$$y_{i+1} - y_i < \text{convergence tolerance}$$

where the convergence tolerance is specified by the user. A flow chart that outlines the computational sequence followed to evaluate the load-deflection response is presented in Figure 3.

## LOAD DEFLECTION AND FACTOR OF SAFETY

The nonlinear lateral load-deflection response is evaluated incrementally. The calculation algorithm proceeds by computing the response under an initial loading condition equal to one-fourth of the design load specified by the user. The initial loads are then incremented within the program and the lateral response is evaluated. This procedure is continued until the deflection at the top of the pier exceeds a user-specified deflection criterion. The evaluated response for each load increment includes lateral deflection, moment, shear, soil modulus, and soil reaction, along the pier length.

The ultimate resistance of the soil is defined as the pier resistance corresponding to the user's specified limiting deflection. The factor of safety is evaluated by dividing the ultimate resistance, based on the deflection criterion, by the design load specified by the user.

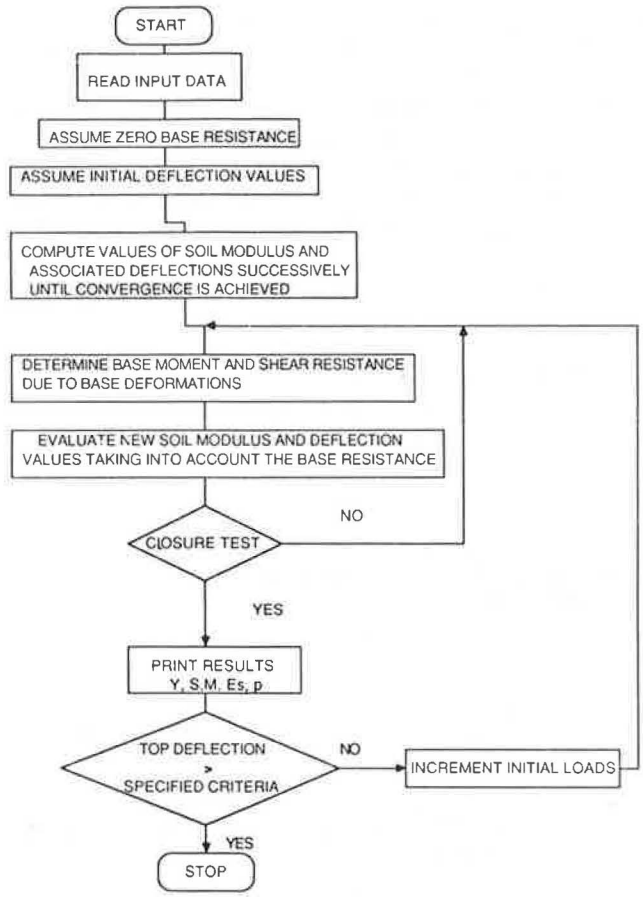


**FIGURE 3 Flowchart of the computer program LTBASE.**

The program is also capable of internally increasing the pier length. The length of each increment is equivalent to an element length, which is specified by the user in the data input file. The program proceeds by evaluating the factor of safety for the original input length. If this factor of safety is found to be less than a limiting factor of safety criterion, specified by the user, the program internally increments the length. The analysis is then performed using the new length. If the evaluated factor of safety is greater than or equal to the specified criterion, the run is terminated and the results are printed.

The maximum number of elements that can be added to the original length is three. If adding a new element will lead to placing the base of the pier in a soil with lower strength than that initially existing at the base of the pier, the pier capacity will be overpredicted. Caution should be exercised in interpreting the results for such cases.

## PROGRAM CAPACITY

The capacity of the program is determined by the size of the variables in the COMMON statements. Currently the storage required for the sum of the code, data, and constants blocks is about 300 K. Maximum number of nodes and elements corresponding to this capacity is 101 and 100, respectively. The capacity could be increased by increasing the dimensions of the variables in the COMMON statement. However, the memory limitations of the computer system will be the controlling factor. Also, it should be noted that as the number of elements