Effect of Grade on the Relationship Between Flow and Occupancy on Freeways

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In this paper, 30-sec data for the median lane of a freeway section with significant grades are used to explore how the uncongested flow-occupancy function is affected by grade, as well as to examine the implication of this effect for incident detection logic. The results accord with intuition: significant grades cause differences in speed at affected locations, resulting in differences in occupancy for a given flow. The size of the difference depends not only on the grades at each location but also on the vertical profile of the upstream sections and on the prevailing flow. Distinct flow-occupancy functions are required for each affected location, and because the effect of the grade might vary with environmental and traffic operating conditions, it appears desirable to continuously update these functions on line in traffic management systems. This task appears to be feasible and is expected to improve the efficiency of occupancy-based detection algorithms.

The relationship among speed, flow, and occupancy can be described as the kingpin of most freeway traffic management systems. For example, incident detection, which is one of the major components, is generally occupancy based, but there are implicit assumptions about the nature of that relationship. It therefore seems natural that a thorough understanding of traffic operations should be vital to efficient freeway traffic management. Accordingly, in recent years, some research effort has been focused on better understanding patterns in flow-occupancy-speed data (*1–7*). In one series of papers (*3–7*), catastrophe theory evolved as a reasonable three-dimensional representation of the flow-occupancy-speed pattern, and on this basis, an alternative incident detection logic has been suggested (*7, 8*). All of the earlier work, however, has used only data pertaining to "ideal" freeway conditions, that is, for locations with no grade or other geometric constraints, and from time periods without inclement weather. The reason for thus confining the earlier analyses is that there were hints in the data that weather and geometric conditions (in particular, grade) would confound the task of seeking a fundamental understanding of traffic operations.

Understanding the effect of grade on traffic operations is vital for the designers and operators of freeway traffic management systems. Most such systems, at least in North America, use some variety of the California-type algorithms for detecting incidents (*9*). In essence, the principle of these algorithms is that a significant change in the traffic operation pattern (usually represented by some function of occupancy) between successive detector stations is indicative of the presence of an incident between those stations. Threshold values are used to indicate how significant a change is required before an incident is declared. If, as appears intuitively obvious, relatively steep upgrades and downgrades can cause natural changes in traffic operations between successive detector stations, then it is easy to see that the presence of changes in grade presents difficulties in the detection of incidents on freeway segments.

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These difficulties are partly alleviated in some systems by varying thresholds from location to location. This works if the location-specific thresholds can be accurately calibrated, but further difficulties might be presented if the effect of grade varies over time, depending on weather and traffic conditions. It should be noted that some systems in operation today do not allow for location-specific thresholds. In such situations, to reduce false alarms due to upgrades, the systemwide threshold can be set so high that most incidents are not detected. Conversely, natural reductions in occupancy due to downgrades might require a low threshold that could result in many false alarms at other locations.

The purpose of this paper is to provide some insights into how grade might affect the flow-occupancy-speed relationship and to explore whether a single three-dimensional model, such as the catastrophe theory model, might adequately represent these varying effects. It has already been suggested that such a model might usefully complement existing algorithms by providing a logic system for detecting incidents on the basis of data from a single detector station (*7*). If this model automatically captures the effect of grade, then it would be particularly useful for detecting incidents at locations where operations are affected by relatively steep upgrades and downgrades. A related paper (*10*) examines the effect of weather on the flow-occupancy relationship.

The text of this paper is structured into four substantive sections, followed by a summary. First, details of the data are provided. Next, a brief review of the catastrophe theory representation of data that is unaffected by grade follows. Then the dependence of the flow-occupancy pattern on grade is empirically explored and reconciled with the theory. The implications for incident detection are discussed before the summary.

DATA

The study site was a segment of the Queen Elizabeth Way near Hamilton, Ontario, Canada. It contains level, upgrade, and downgrade sections (Figures 1 and 2). The Burlington Skyway Freeway Traffic Management System records 30-sec averages of speeds, flows, and occupancies in each of at least two lanes

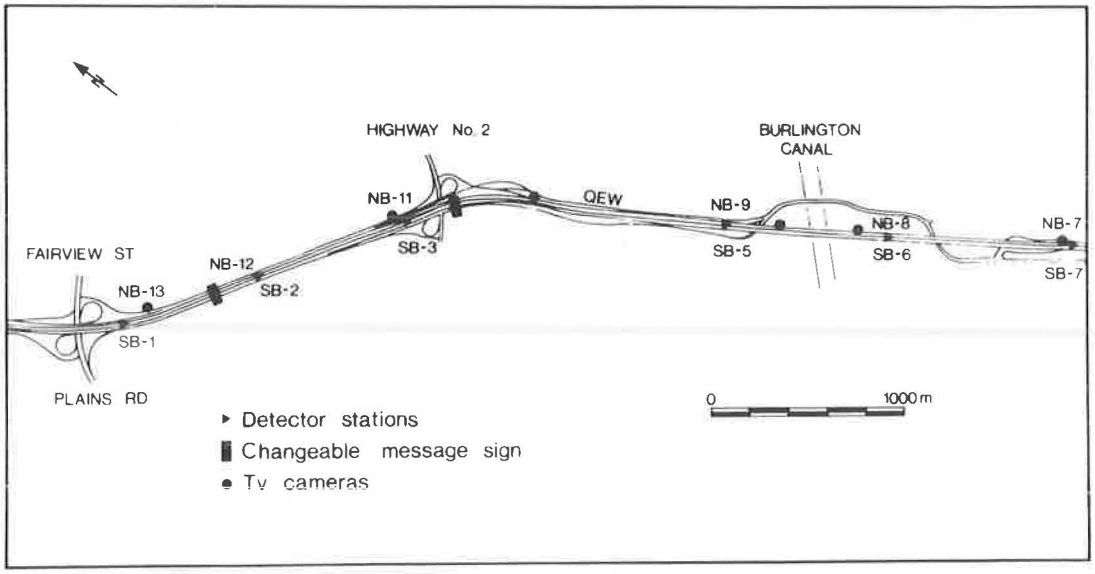

FIGURE 1 Schematic diagram of the Skyway Freeway Traffic Management System on the Queen Elizabeth Way, Ontario, Canada.

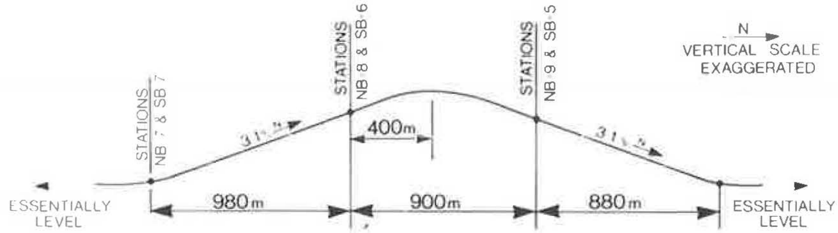

FIGURE 2 Vertical profile of the Burlington Bay Skyway on the Queen Elizabeth Way, Ontario, Canada (vertical scale is exaggerated).

at each of six southbound (SB) and six northbound (NB) stations on the segment of interest. Two related papers (*7, 10*) were based on operations at Station NB-7. The data set that is the basis of this paper consists of samples of 30-sec data for the median lane at Station NB-7, as well as those stations at which operations are thought to be affected by grade, that is, NB-8, NB-9, SB-5, SB-6, and SB-7. Details of the vertical profile at each of these stations are summarized in Table 1.

TABLE 1 DETAILS OF VERTICAL PROFILE

	Approximate Grade	
Station	At Station	Upstream
NB-7	Level	Level
NB-8	+3%	Level
NB-9	−3%	+3%
SB-5	+3%	Level
SB-6	−3%	+3%
SB-7	Level	−3%

The data that were originally extracted represented normal behavior, as well as behavior before, during, and after nine incidents that occurred during good weather near the stations of interest during the week ending August 15, 1986. For this paper, it was decided to confine the analysis to the uncongested data. This decision was partly based on information contained in scatter plots such as Figure 3, which is taken from one of the related papers (*7*). This plot demonstrates that it is only the uncongested flow-occupancy data (in the tightly defined Area A) that can sensibly be described by a functional relationship. Inclusion of the widely scattered congested data (Area B) would probably distort the relationship. In addition, given the premise of this paper, all that is of interest for the moment is to define the limits of uncongested operation (Area A) to see what effect changes in grade have on them.

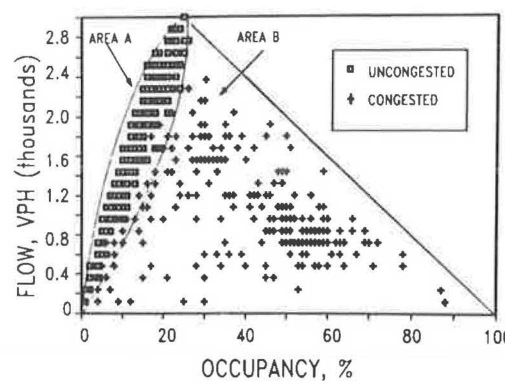

FIGURE 3 Scatter plot of 30-sec flow-occupancy data in the median lane, for Station NB-7 before, during, and after six downstream incidents.

To obtain data sets consisting of only uncongested data, congested data were deleted according to the following rules:

Rule 1. Data for stations affected by an incident were deleted for the duration of reported incidents. As discussed elsewhere (*7*), the beginning and end of an incident were marked by distinct changes in speed at the station immediately upstream.

Rule 2. Any occasional data point caused by slow-moving vehicles or shock waves, identified by a drop in speed of at least 10 km/hr from average speed for the occupancy at that point, was deleted.

Although Rule 2 appears to be somewhat arbitrary, it should be pointed out that 95 percent of the data deletions were in accordance with the more credible Rule 1. In the end, the number of uncongested data points ranged from 341 for Station NB-8 to 840 for Station SB-7. (The reason for this spread is that six of the nine incidents affected operations at Station NB-8, whereas operations at SB-7 were virtually unaffected by any of the incidents.)

The analysis was done with median lane data for reasons detailed elsewhere (*7*). In essence, because of the presence of trucks, patterns in the shoulder lane data are harder to distinguish. In particular, the distinction between congested and uncongested operation is not as clear-cut. Equally important, it appears feasible to base incident detection logic on the median lane data alone (*7*).

REVIEW OF RELATED WORK

The data in Figure 3 are from Station NB-7, where operations are unaffected by grade. As indicated earlier, uncongested data occur in a tightly defined area (designated Area A), whereas congested data caused by downstream incidents are scattered in a roughly triangular area (Area B). Changes from uncongested to congested operation and vice versa are marked by a smooth change in flow and occupancy, with sudden discrete speed changes of the order of 15–25 km/hr between consecutive 30-sec intervals (*7*).

In earlier work (*5–7*) this behavior was found to be reasonably explained by catastrophe theory, which takes its name from the sudden discrete changes in one variable during smooth changes in other related variables. The suggested representation, shown in Figure 4, accommodates discrete changes in speed during smooth change in flow and occupancy—the pattern observed in the data on which Figure 3 is based. Uncongested operations unaffected by grade occur along a tightly defined line on the upper surface of Figure 4, and on breakdown to congested conditions, operations fall directly to the lower surface where they become highly variable and remain until recovery.

In this paper, it is the uncongested operations on the upper surface that are of interest. The working hypothesis in this exploration is that if these operations are affected by grade, this factor merely affects the location of the tightly defined line on the surface; or, in effect, that a single upper surface can feasibly model the varying effects of grade on uncongested operation. In this paper, however, it is simply the flow-occupancy curve, not the flow-occupancy-speed surface, that is examined to investigate the grade effect.

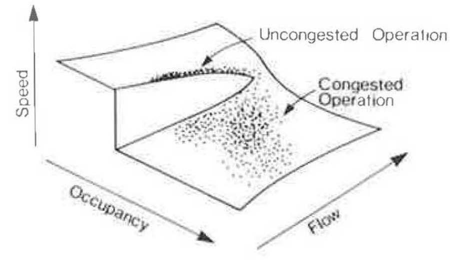

FIGURE 4 Conceptualization of traffic operations on a catastrophe theory surface.

This review has been intentionally brief because full details are given in the background papers (*5, 6*) and more detailed reviews are provided elsewhere (*7, 10*).

METHOD AND RESULTS

The first step in the analysis was to extract the uncongested data according to the rules specified earlier. Then, for each station, the mean flow for each occupancy value was plotted to obtain some initial insights on the functional form of the flow-occupancy relationship and on the possible effects of grade. Visual inspection of these plots, three of which are shown in Figure 5, indicates apparent differences among the stations and that a function of the form flow = a[occupancy^{b_1}] might be appropriate. This form is also consistent with that used in examining the effect of weather (*10*).

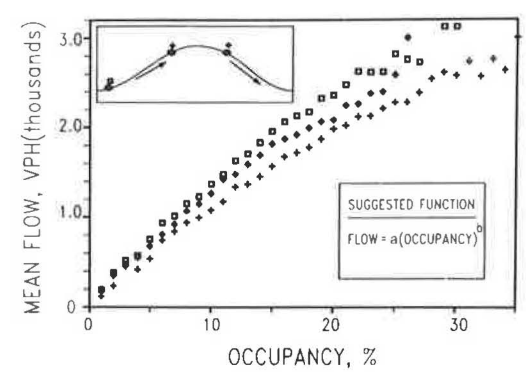

FIGURE 5 Mean value of flow versus occupancy for three stations (uncongested data only).

The next steps were to calibrate the function for each station and to statistically test whether different functions are required for each station. The calibration was carried out by first doing the usual linear transformation on this type of model. In effect, a and b_1 were estimated by doing linear regressions of ln[flow] on ln[occupancy]. Estimated values for the intercept, ln[a], and the slope, b_1, of the log-linear models are presented in Table 2, along with the statistics of the fits. These statistics, along with plots such as Figure 6 (which shows how the fitted functions relate to the flow-occupancy data), indicate that the model form and the fits are quite reasonable for the entire occupancy range.

TABLE 2 REGRESSION ANALYSIS ON UNCONGESTED DATA SETS

	ln[a]		b_1			Number of Data
Station	Coefficient	t Ratio	Coefficient	t Ratio	R^2	Points
NB-7	5.32	272.3	0.824	109.4	0.949	649
NB-8	5.10	245.9	0.822	111.7	0.948	341
NB-9	5.24	321.8	0.820	118.5	0.959	603
SB-5	5.28	244.6	0.800	96.1	0.948	503
SB-6	5.10	264.8	0.846	113.0	0.955	605
SB-7	5.41	367.3	0.819	130.1	0.953	840

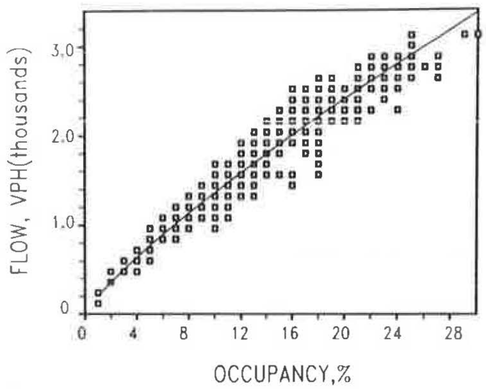

FIGURE 6 Uncongested flow-occupancy data and fitted function for Station NB-7.

The question that remains is whether there are significant differences among the fits, that is, whether a single set of coefficients can be used for groups or all of the stations. To answer this, Station NB-7 was used as a base for comparison because, it will be recalled, operations there are thought to be unaffected by grade. The Station NB-7 data were combined with that for each of the other days, and for each combination, regressions were performed to estimate coefficients for the following function:

$$\ln[\text{flow}] = \ln[a]' + b_1' (\ln[\text{occ}]) + b_2'(D) + b_3'(D)(\ln[\text{occ}]) \quad (1)$$

where D is a dummy variable assigned a value of 1 for data points for station NB-7 and 0 otherwise. If there are significant differences between the functions for Station NB-7 and the station of interest, one or both of b_2' and b_3' would be significant. The coefficients for the other station would be $\ln[a]'$ and b_1'—the same as those in Table 2. If a separate function is required, the coefficients for NB-7 in Table 2 can be reproduced by simply summing the relevant coefficients in Equation 1 (i.e., $\ln[a]' + b_2'$, and $b_1' + b_3'$, to get $\ln[a]$ and b_1, respectively).

The results of this set of regressions, presented in Table 3, indicate that each of the other stations requires a different function from NB-7, the station at which operations are unaffected by grade. It is not surprising that the two stations on upgrades, NB-8 and SB-5, both require different functions from NB-7. What is mildly surprising at first glance is that functions for the two stations on downgrades, NB-9 and SB-6, and the function for SB-7, which is on a level section, are all significantly different from that for NB-7. However, close inspection of Figure 2 reveals that NB-9 and SB-6 are both just beyond relatively long upgrades, so it appears likely that operations there might still be adversely affected by the preceding upgrade. Also, SB-7 is at the foot of a long downgrade, so it might be expected that speeds there would be faster than speeds at a station preceded by level freeway (such as NB-7). This expectation is indirectly confirmed by the negative dummy variable coefficients for the NB-7/SB-7 comparison, which suggest that for a given flow, SB-7 has a higher occupancy, and therefore a faster speed, than NB-7.

A similar set of regressions was performed to determine whether there are significant differences between stations that appear similar in location with respect to grades. NB-8 and SB-5 are both about two-thirds of the way up the 3 percent grade. For this pair of stations, the results of the dummy variable regression, also presented in Table 3, indicate that there is a significant difference between the functions for these stations. Similarly, there is a significant difference between the functions for NB-9 and SB-6, even though both appear to be on identical downgrades. This would indicate that it is difficult to specify how operations at a station might be affected by grade by merely specifying the grade and its length. What came before the grade of interest also appears to be important. This creates a problem if the aim is to be able to identify the flow-occupancy function for a station by specifying its geometrics, because to specify the various combinations of grade, length of grade, and grade "history" would require a large number of combinations. This issue will be explored further in the next section.

DISCUSSION OF RESULTS

In Figure 7, the final regression lines for the three northbound stations are plotted. In many ways, this plot is a confirmation of intuition: if an upgrade is large enough and long enough to affect traffic operations at a station, then it will result in a higher occupancy for a given flow. What is not intuitively obvious is that the difference in occupancy between two functions increases with flow. For example, the expected difference in occupancy between NB-7 and NB-6 increases from about 2 percent at a flow of 1,000 vph to about 5 percent at a flow of 2,000 vph. Examination of the actual data on which the functions are based indicates approximate upper 95th percentile values for these numbers of 10 percent and 25 percent, respectively. This finding has important implications

TABLE 3 REGRESSION ANALYSIS ON DATA FOR PAIRS OF STATIONS

Section Pair[a]	ln[a] Estimated Coefficient	ln[a] t Ratio	b_1' Estimated Coefficient	b_1' t Ratio	b_2' Estimated Coefficient	b_2' t Ratio	b_3' Estimated Coefficient	b_3' t Ratio
NB-7, NB-8	5.10	171.2	0.822	77.8	0.239[b]	6.73	−0.0070	−0.54
NB-7, NB-9	5.24	355.5	0.820	131.7	0.115[b]	4.36	−0.0091	−0.86
NB-7, SB-5	5.28	246.4	0.800	96.8	0.0665[b]	2.26	0.0149	1.33
NB-7, SB-6	5.10	264.1	0.846	112.7	0.239[b]	8.71	−0.0308[b]	−2.91
NB-7, SB-7	5.41	369.9	0.819	131.0	−0.0713[b]	−2.91	−0.0039	−0.39
NB-8, SB-5	5.28	248.0	0.800	97.4	−0.173[b]	−4.71	0.0219	1.63
NB-9, SB-6	5.10	233.3	0.846	99.5	0.124[b]	4.68	−0.0217[b]	−2.06

[a]In each case, data for the first station listed are used as the basis for comparison; $D = 1$ for that station.
[b]Dummy variable coefficient is significant at the 5-percent level of significance.

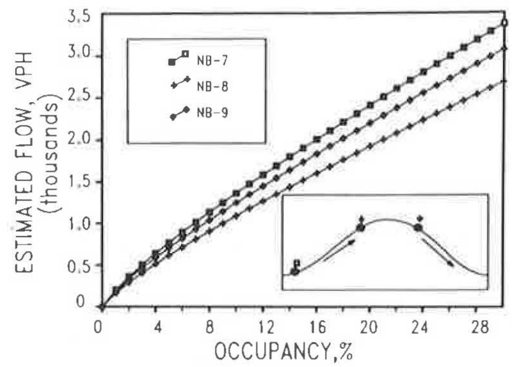

FIGURE 7 Regression lines for three northbound stations.

for incident detection logic, so it will be dealt with in the next section.

The question of whether the varying effects of grade can be explained by catastrophe theory remains to be explored. This explanation would be particularly desirable because one of the premises for developing the theory is that it might be able to capture the entire range of freeway operations within a single model. One way that catastrophe theory might model the effect that grade has on operations is suggested by the definition of occupancy. That definition implies that an upgrade causes uncongested occupancy to increase if it causes vehicles to reduce speed and occupy a detector for a longer time. Because of the curvature of the surface on which the catastrophe theory model is built, lower but uncongested speeds would occur closer to the drop-off. In other words, the grade that produces the slowest operating speeds will produce an uncongested flow-occupancy function that is close to the edge of the upper surface (higher occupancy and lower speed for a given flow), whereas faster speeds on more gentle grades will produce points higher up and to the left of this edge. This has not been modeled mathematically at present, but it appears feasible that the catastrophe theory model can accommodate this feature of traffic operations.

IMPLICATIONS FOR INCIDENT DETECTION AND FURTHER WORK

It is quite common for incident detection logic to be based on a comparison of occupancy (or on some related measure, or both) between successive detector stations. If the difference exceeds some preset threshold, then an incident is tentatively declared. It appears obvious from the present results that because grade might cause natural changes in occupancy, such comparative algorithms would require specific thresholds for each pair of stations to be compared. Although this has undoubtedly already been recognized in some systems, such recognition has been in an ad hoc fashion in those systems with which the authors are familiar and not on the basis of functions such as those in Figure 7.

An important implication of the finding that the magnitude of the downward shift in occupancy due to grade increases as flow increases is that the threshold for the spatial difference in occupancy between affected stations should depend on the flow. The preliminary indication is that this dependency might be reasonably accounted for by "normalizing" the spatial difference in occupancy by dividing it by the occupancy at the upstream station. Most versions of the California-type algorithms do compare this relative spatial occupancy difference against a threshold, but it is also necessary to compare the actual spatial occupancy difference against another threshold. Because this latter threshold should clearly depend on flow, it appears desirable for such algorithms to directly or indirectly incorporate the flow-occupancy function for each detector station for the prevailing operating conditions.

Most of the time, on most freeways, station-specific calibration of comparative algorithms is not a problem because, conceivably, detector data can be used to give a ready-made set of functions calibrated for each station, for day or night, during normal weather conditions. The problem is that it is difficult, if not impossible, to specify the entire range of freeway operating conditions and have a separate function for each possible condition. For example, there are many degrees of weather conditions, and it would be necessary to specify what a condition is in terms of its effect at a given station. A possible solution that is being explored in ongoing work is to update the uncongested flow-occupancy functions on line. Experience with 30-sec data suggests that this approach is likely to bear fruit.

It should also be pointed out that thresholds are required even for incident detection logic systems that are not based on comparing stations, such as that of point detection (smoothing) and that of the flow-occupancy-speed–based model suggested previously (7). In such cases, the importance of being able to explain the effect of grade on operations is that it would be necessary to identify the uncongested flow-occupancy function for each detector station at all times. In principle, it is desirable to know, for each flow value, the prevailing occupancy limit for uncongested operation.

SUMMARY

The results presented in this paper are in accord with intuition and reality: operations on a freeway can be affected by its vertical profile, and in effect, upgrades can cause an increase in occupancy for a given flow. Because this effect might vary over time with environmental and other traffic operating conditions, it has been suggested that incident detection logic should be capable of continuously updating the flow-occupancy function at each detector station.

Three useful directions for further work are indicated. First, it appears feasible and fruitful to explore how uncongested flow-occupancy functions can be continuously updated for each detector station. This process should automatically capture the confounding effects of weather, but it might still be desirable to explore how a combination of inclement weather and grade affects operations. The second direction is a more detailed exploration of how occupancy-based thresholds in comparative incident detection logic might be allowed to depend on flow at stations where operations differ because of grade. Finally, the results of trying to capture the variety of uncongested flow-occupancy functions with a single catastrophe theory model should be interesting. Conceptually, this appears feasible, but the issue will remain unresolved until the flow-occupancy-speed relationship can be mathematically transformed into a catastrophe theory surface. Also, it is necessary to examine whether grade affects the widely scattered congested operation and, if so, whether such an effect can be modeled. For the present purposes, however, this issue is not of primary interest, because the main purpose of this paper was to seek improvements to incident detection logic by being better able identify the limits of uncongested operation.

ACKNOWLEDGMENTS

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