Bridge Performance Prediction Model Using the Markov Chain

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As part of a study to develop a comprehensive bridge management system for the Indiana Department of Highways (IDOH), a bridge performance prediction model using the Markov chain was developed. The model can be used to predict the percentages of bridges with different condition ratings as well as to develop performance curves of bridges. The Markov chain, a probability-based method, was used in the model to reflect the stochastic nature of bridge conditions. The study exhibited the power of the Markov chain approach in prediction or estimation of future bridge conditions. The procedure, although simple, was found to provide a high level of accuracy in predicting bridge conditions.

A major objective of a bridge management system is to assist bridge managers in making consistent and cost-effective decisions related to maintenance and rehabilitation of bridges. The decision making, either at the network level or at the project level, is based on current and future bridge conditions. Therefore, it is essential for a bridge management system to be capable of accurately predicting future bridge conditions.

A bridge performance model was developed for a bridge management system for the Indiana Department of Highways (IDOH). The model was developed using the Markov chain for predicting future bridge conditions. Predictions can be made for the condition rating of bridges and for the percentage of bridges at different condition ratings, both of which are important for a bridge management system.

Knowing the percentage of bridges at different condition ratings at present, a bridge manager may wish to know the percentage distribution in the future. Also, knowing the present condition rating of a bridge, he may want to predict the condition rating of the bridge in a given year. This model provides a tool for these predictions. To use the model, one simply has to input the present percentage distribution of bridge conditions or the present condition rating of a bridge.

This paper presents a brief introduction to the concept and use of the Markov chain approach. The development of transition matrices is also discussed. Their applications to the development of a bridge performance model are explained through examples.

DATA BASE

The complete data base included about 5,700 state-owned bridges in Indiana. A sample data set was selected from bridges on State Roads 1, 2, 3, 4, 14, 16, 46, and 57. To evaluate the effects of bridge type and climate on bridge performance, structures were divided into steel and concrete bridges, and bridges in northern and southern regions were studied separately. The sample data set consisted of 170 concrete bridges and 106 steel bridges.

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MARKOV CHAIN APPROACH

The Markov chain as applied to bridge performance prediction is based on the concept of defining states in terms of bridge condition ratings and obtaining the probabilities of bridge condition transition from one state to another. These probabilities are represented in a matrix form that is called the transition probability matrix, or simply transition matrix, of the Markov chain. If the present or the initial state of the bridges is known, the future condition can be predicted through multiplication of the initial-state vector and the transition probability matrix.

Using the FHWA bridge-rating system, bridge inspectors employ a range from 0 to 9, with 9 being the maximum rating number for near-perfect condition (*1*). Ten bridge condition ratings are defined as 10 states, with each condition rating corresponding to one of these 10 states. For example, condition rating 9 is defined as State 1, rating 8 as State 2, and so on. Without repair or rehabilitation, the bridge condition rating decreases as the bridge age increases. Therefore, there is a probability that the condition will make a transition from one state, say i, to another state, j, during a given period of time, which is denoted $P_{i,j}$. Table 1 shows the correspondence of condition ratings, states, and transition probabilities.

Let the transition probability matrix of the Markov chain be P, given by

$$P = \begin{vmatrix} P_{1,1} & P_{1,2} & \cdots & P_{1,10} \\ P_{2,1} & P_{2,2} & \cdots & P_{2,10} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ P_{10,1} & P_{10,2} & \cdots & P_{10,10} \end{vmatrix} \quad (1)$$

Then the state vector for any time T, $P^{(T)}$, can be obtained

TABLE 1 CORRESPONDENCE OF CONDITION RATINGS, STATES, AND TRANSITION PROBABILITIES

		R=9	R=8	R=7	R=6	R=5	R=4	R=3	R=2	R=1	R=0
		S=1	S=2	S=3	S=4	S=5	S=6	S=7	S=8	S=9	S=10
R=9	S=1	$P_{1,1}$	$P_{1,2}$	$P_{1,3}$	$P_{1,4}$	$P_{1,5}$	$P_{1,6}$	$P_{1,7}$	$P_{1,8}$	$P_{1,9}$	$P_{1,10}$
R=8	S=2	$P_{2,1}$	$P_{2,2}$	$P_{2,3}$	$P_{2,4}$	$P_{2,5}$	$P_{2,6}$	$P_{2,7}$	$P_{2,8}$	$P_{2,9}$	$P_{2,10}$
R=7	S=3	$P_{3,1}$	$P_{3,2}$	$P_{3,3}$	$P_{3,4}$	$P_{3,5}$	$P_{3,6}$	$P_{3,7}$	$P_{3,8}$	$P_{3,9}$	$P_{3,10}$
R=6	S=4	$P_{4,1}$	$P_{4,2}$	$P_{4,3}$	$P_{4,4}$	$P_{4,5}$	$P_{4,6}$	$P_{4,7}$	$P_{4,8}$	$P_{4,9}$	$P_{4,10}$
R=5	S=5	$P_{5,1}$	$P_{5,2}$	$P_{5,3}$	$P_{5,4}$	$P_{5,5}$	$P_{5,6}$	$P_{5,7}$	$P_{5,8}$	$P_{5,9}$	$P_{5,10}$
R=4	S=6	$P_{6,1}$	$P_{6,2}$	$P_{6,3}$	$P_{6,4}$	$P_{6,5}$	$P_{6,6}$	$P_{6,7}$	$P_{6,8}$	$P_{6,9}$	$P_{6,10}$
R=3	S=7	$P_{7,1}$	$P_{7,2}$	$P_{7,3}$	$P_{7,4}$	$P_{7,5}$	$P_{7,6}$	$P_{7,7}$	$P_{7,8}$	$P_{7,9}$	$P_{7,10}$
R=2	S=8	$P_{8,1}$	$P_{8,2}$	$P_{8,3}$	$P_{8,4}$	$P_{8,5}$	$P_{8,6}$	$P_{8,7}$	$P_{8,8}$	$P_{8,9}$	$P_{8,10}$
R=1	S=9	$P_{9,1}$	$P_{9,2}$	$P_{9,3}$	$P_{9,4}$	$P_{9,5}$	$P_{9,6}$	$P_{9,7}$	$P_{9,8}$	$P_{9,9}$	$P_{9,10}$
R=0	S=10	$P_{10,1}$	$P_{10,2}$	$P_{10,3}$	$P_{10,4}$	$P_{10,5}$	$P_{10,6}$	$P_{10,7}$	$P_{10,8}$	$P_{10,9}$	$P_{10,10}$

Note: R = Condition Rating

S = State

$P_{i,j}$ = Transition Probability from State i to State j

by the multiplication of the initial-state vector $P^{(0)}$ and the transition probability matrix P:

$$P^{(T)} = P^{(0)} * P * P \ldots * P = P^{(0)} * P^T \qquad (2)$$

Thus, a Markov chain is completely specified when its transition matrix P and the initial-state vector $P^{(0)}$ are known. Because the initial-state vector $P^{(0)}$ is usually known for a bridge management system, the main problem of the Markov chain approach in this study is to determine the transition probability matrix.

TRANSITION PROBABILITY MATRIX

Since 1978 all federally supported bridges have been inspected every 2 years. The inspection includes ratings of individual components such as deck, superstructure, and substructure as well as of the overall bridge condition. Unless rehabilitation or repair is applied, bridge structures deteriorate gradually so that the bridge condition ratings either are unchanged or change to a lower number in the 2-year rating period. That is, a bridge condition rating should monotonically decrease as the bridge age increases. Therefore, the probability $P_{i,j}$ is null for $i > j$, where i and j represent the states in the Markov chain.

For the purpose of management, two kinds of predictions need to be performed: (a) the percentage of bridges with a particular condition rating at any given time and (b) the bridge conditions at different bridge ages.

Percentage of Bridges with a Particular Condition Rating

From the data base, the number of bridge-condition transitions from one state to another state was obtained. Let $n_{i,j}$ denote the number of transitions from State i to State j within the time period; then the number of bridges in State i before the transition can be defined as

$$n_i = \sum_j n_{i,j} \qquad (3)$$

It can be proved (2) that the estimated transition probability is

$$\hat{P}_{i,j} = \frac{n_{i,j}}{n_i} \qquad (4)$$

Consequently, the transition matrix is determined and the prediction can be performed by using Equation 2.

As an example, the transition matrix for deck conditions of concrete bridges in northern Indiana was obtained by this method. Because bridges are inspected every 2 years, a 2-year transition period was used; that is, $p_{i,j}$ was the probability of transition from State i to State j in a 2-year period. The numerical results of n_i, $n_{i,j}$, and the corresponding transition matrix are given in Tables 2 and 3.

Bridge Conditions at Different Bridge Ages

To develop a bridge performance curve, it is necessary to predict bridge conditions at different ages. The transition

TABLE 2 NUMBER OF STATE TRANSITIONS OF DECK CONDITION FOR CONCRETE BRIDGES

state	$n_{i,1}$	$n_{i,2}$	$n_{i,3}$	$n_{i,4}$	$n_{i,5}$	$n_{i,6}$	$n_{i,7}$	$n_{i,8}$	$n_{i,9}$	$n_{i,10}$	Σ
i=1	1	6	5	2	0	0	0	0	0	0	n_1=14
i=2	0	20	51	12	2	6	0	0	0	0	n_2=91
i=3	0	0	114	36	9	4	0	0	0	0	n_3=163
i=4	0	0	0	45	12	0	3	0	0	0	n_4=60
i=5	0	0	0	0	18	7	1	0	0	0	n_5=26
i=6	0	0	0	0	0	18	2	1	0	0	n_6=21
i=7	0	0	0	0	0	0	5	0	0	0	n_7=5
i=8	0	0	0	0	0	0	0	1	0	0	n_8=1
i=9	0	0	0	0	0	0	0	0	0	0	n_9=0
i=10	0	0	0	0	0	0	0	0	0	0	n_{10}=0

Note: The sample data were randomly chosen from the concrete bridge condition data from nothern Indiana.

TABLE 3 TRANSITION PROBABILITY MATRIX OF CONCRETE DECK CONDITION IN NORTHERN INDIANA

state	$p_{i,1}$	$p_{i,2}$	$p_{i,3}$	$p_{i,4}$	$p_{i,5}$	$p_{i,6}$	$p_{i,7}$	$p_{i,8}$	$p_{i,9}$	$p_{i,10}$
i=1	0.071	0.429	0.357	0.143	0.000	0.000	0.000	0.000	0.000	0.000
i=2	0.000	0.220	0.560	0.132	0.022	0.066	0.000	0.000	0.000	0.000
i=3	0.000	0.000	0.699	0.211	0.055	0.025	0.000	0.000	0.000	0.000
i=4	0.000	0.000	0.000	0.750	0.200	0.000	0.050	0.000	0.000	0.000
i=5	0.000	0.000	0.000	0.000	0.692	0.269	0.038	0.000	0.000	0.000
i=6	0.000	0.000	0.000	0.000	0.000	0.857	0.095	0.048	0.000	0.000
i=7	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
i=8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000
i=9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
i=10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

matrix used in this case is different from that in percentage prediction, which gives the proportion of bridges with a particular condition rating in a given year. Bridges with different ages may have the same condition ratings. However, a performance curve provides a direct relationship between bridge condition and bridge age.

The transition matrix presented in Table 3 cannot be used to develop bridge performance curves, because this matrix is independent of bridge age; in other words, it is not homogeneous with respect to bridge age. However, a Markov chain requires the homogeneity of a transition matrix (2). Therefore, to avoid overestimating or underestimating the bridge condition, a different approach, called a zoning technique, was used to obtain the transition matrix. This approach had been used by Butt et al. for the development of pavement performance curves in a previous study (*3*).

Unlike in percentage prediction, a 1-year transition period was used in developing the performance curve. That is, p_{ij} was the transition probability from State i to State j in a 1-year period. Bridge age was divided into groups, and within each age group the Markov chain was assumed to be homogeneous. Groups consisting of 6 years were used, and each group had its own transition matrix, which was different from those of the remaining groups.

To make the initial computations simple, an assumption was made that the bridge condition rating would not drop by more than one state in a single year. Thus, the bridge condition would either stay in its current state or make the transition to the next lower state in 1 year. The transition matrix therefore has the following form:

$$P = \begin{vmatrix} P(1) & q(1) & 0 & 0 & 0 & 0 & 0 \\ 0 & p(2) & q(2) & 0 & 0 & 0 & 0 \\ 0 & 0 & p(3) & q(3) & 0 & 0 & 0 \\ 0 & 0 & 0 & p(4) & q(4) & 0 & 0 \\ 0 & 0 & 0 & 0 & p(5) & q(5) & 0 \\ 0 & 0 & 0 & 0 & 0 & p(6) & q^{(6)} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix} \quad (5)$$

where $q(i) = 1 - p(i)$.

It should be noted that the lowest recorded rating number in the data base was 3, indicating that bridges are usually repaired or replaced at a rating not less than 3. Consequently, the corresponding State 7 has the transition probability $p(7) = 1$.

To estimate the transition matrix probabilities, for each age group the following nonlinear programming objective function was formulated:

$$\min \sum_{t=1}^{N} |S(t) - E(t,P)| \quad (6)$$

subject to $0 \leq p(i) \leq 1, \quad i = 1, 2, \ldots, 6$

where

N = 6, the number of years in one age group;
P = $[p(1), p(2), \ldots, p(6)]$, a vector of length 6;
$S(t)$ = average of condition ratings at time t; and
$E(t,P)$ = estimated value of condition rating by Markov chain at time t.

The objective function was to minimize the absolute distance between the actual bridge condition rating at a certain age and the predicted bridge condition for the corresponding age generated by the Markov chain with the probabilities obtained by the nonlinear programming. The solution to this function was obtained by the gradient projection method (*4*).

To find the trend of a performance curve, a polynomial regression procedure (*5*) was performed first. The results of the regression were taken as the average condition ratings to solve the nonlinear programming. This method is explained in the following section through an example.

The maximum rating of bridge condition is 9, and it represents a near-perfect condition of a bridge component. It is almost always true that a new bridge has condition rating 9 for its deck, superstructure, and substructure. In other words, a bridge at age 0 has condition rating 9 for its components with unit probability. Thus, the initial-state vector $p^{(0)}$ for the deck, superstructure, or substructure of a new bridge is always (1, 0, 0, ..., 0), where the numbers are the probabilities of having condition ratings 9, 8, 7, ..., and 0, respectively, at age 0. That is, the initial vector of the first group for developing the bridge performance curve is known. Group 2 takes the last-state vector of Group 1 as its starting-state vector. In general, Group n takes the last-state vector of Group $n - 1$ as its starting-state vector. The rest of the work to obtain the overall bridge performance curve or performance curve for bridge components is nothing but to conduct the following matrix multiplications:

$$p^{(1)} = p^{(0)} * P$$

$$p^{(2)} = p^{(0)} * P2$$

$$\vdots$$

$$p^{(i)} = p^{(0)} * p^{i} \quad (7)$$

where $p^{(i)}$ represents the condition-state vector at age i.

APPLICATIONS

Once the transition matrix has been obtained, the prediction of the future condition by the Markov chain becomes a matter of simple multiplication of matrices. As stated earlier, this study used the Markov chain technique for two kinds of predictions: the percentage of bridges with a particular condition rating at any given time and the performance curve of bridges. The two applications are discussed in the following paragraphs.

Example of Percentage Prediction

The percentage of bridges with particular condition ratings in the base period can be readily obtained from the record of bridge condition ratings. For example, the fraction of concrete bridges with different deck condition ratings in northern Indiana in 1978 can be used as the initial-state vector:

$$P^{(0)} = (0.096, 0.559, 0.272, 0.059, 0.015, 0.000, 0.000, 0.000, 0.000, 0.000)$$

That is, in 1978 there were 9.6, 55.9, ..., and 0.0 percent of concrete bridges with deck condition ratings of 9, 8, ..., and 0, respectively.

Suppose that the expected percentages in 1984 are required, that is, the percentages after three rating periods, because bridge conditions are evaluated every 2 years. Using the transition probability matrix given in Table 3, the problem is only to get the three-step probability vector $P^{(3)}$ with the initial-state vector $P^{(0)}$ and the transition probability matrix P. From Equation 2,

$$P^{(3)} = p^{(0)} * P^{3} = (0.000, 0.009, 0.350, 0.323, 0.157, 0.110, 0.045, 0.006, 0.000, 0.000)$$

Figures 1 and 2 show the comparison of the actual and the predicted values for deck condition of concrete bridges in northern Indiana and steel bridges in southern Indiana, respectively, recorded in 1984.

The chi-squared goodness-of-fit test (*6*) was used to measure the closeness of the predicted and the recorded values. The computed chi-square is given by

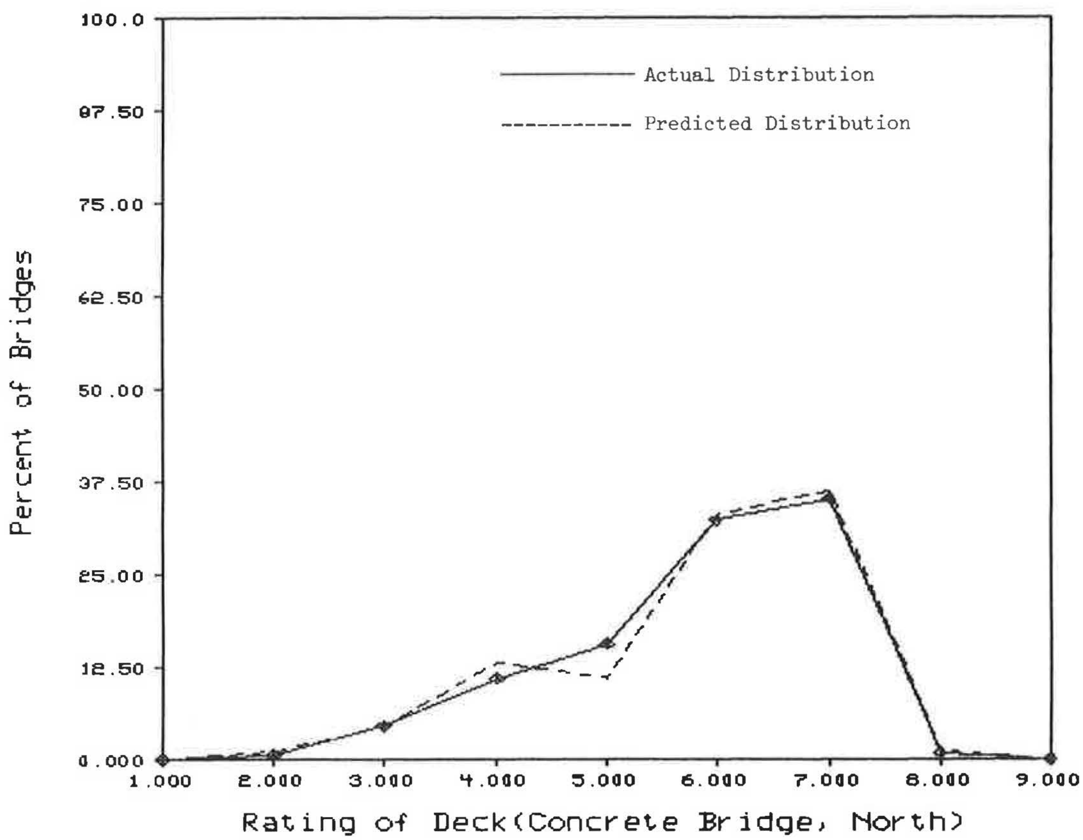

FIGURE 1 Actual versus predicted percentage of concrete bridges in northern Indiana in 1984 by deck condition rating.

TABLE 4 CHI-SQUARED GOODNESS-OF-FIT TEST OF PERCENTAGE PREDICTION OF DECK CONDITION RATING: CONCRETE BRIDGES, NORTHERN INDIANA

Rating	R_i	E_i	$(R_i-E_i)^2$	$\frac{(R_i-E_i)^2}{E_i}$
9	0.0%	0.0%	0.0	--
8	1.1%	0.9%	4.0×10^{-6}	4.4×10^{-4}
7	36.3%	35.1%	1.4×10^{-4}	4.1×10^{-4}
6	33.0%	32.3%	4.9×10^{-5}	1.5×10^{-5}
5	11.0%	15.6%	2.2×10^{-3}	1.4×10^{-2}
4	13.1%	11.0%	4.8×10^{-4}	4.4×10^{-3}
3	4.4%	4.5%	1.0×10^{-6}	2.2×10^{-5}
2	1.1%	0.6%	2.5×10^{-5}	4.2×10^{-3}
1	0.0%	0.0%	0.0	--
Σ	100.0%	100.0%		$x^2=0.024$

$CHI^2_6[x^2 > 0.024] > 0.995$

$$\chi^2 = \sum_{i=1}^{k} \frac{(R_i - E_i)^2}{E_i} \tag{8}$$

where

k = number of observations,
R_i = recorded value of the ith observation,
E_i = expected value of the ith observations, and

χ^2 has a chi-squared distribution with $k - 1$ degrees of freedom.

The results are shown in Tables 4 and 5. As can be seen from Figures 1 and 2 and the statistical test results, the predicted values are very close to the recorded values.

Example of Performance Curve for Bridge Component

The deck performance curve of a concrete bridge in northern Indiana is used as another example. As mentioned in the last section, the initial-state vector of the first group for the deck, superstructure, or substructure of a new bridge is always (1, 0, 0, ..., 0). Therefore, the major problem is to obtain the transition matrix for bridge decks.

A polynomial nonlinear regression model was assumed as follows:

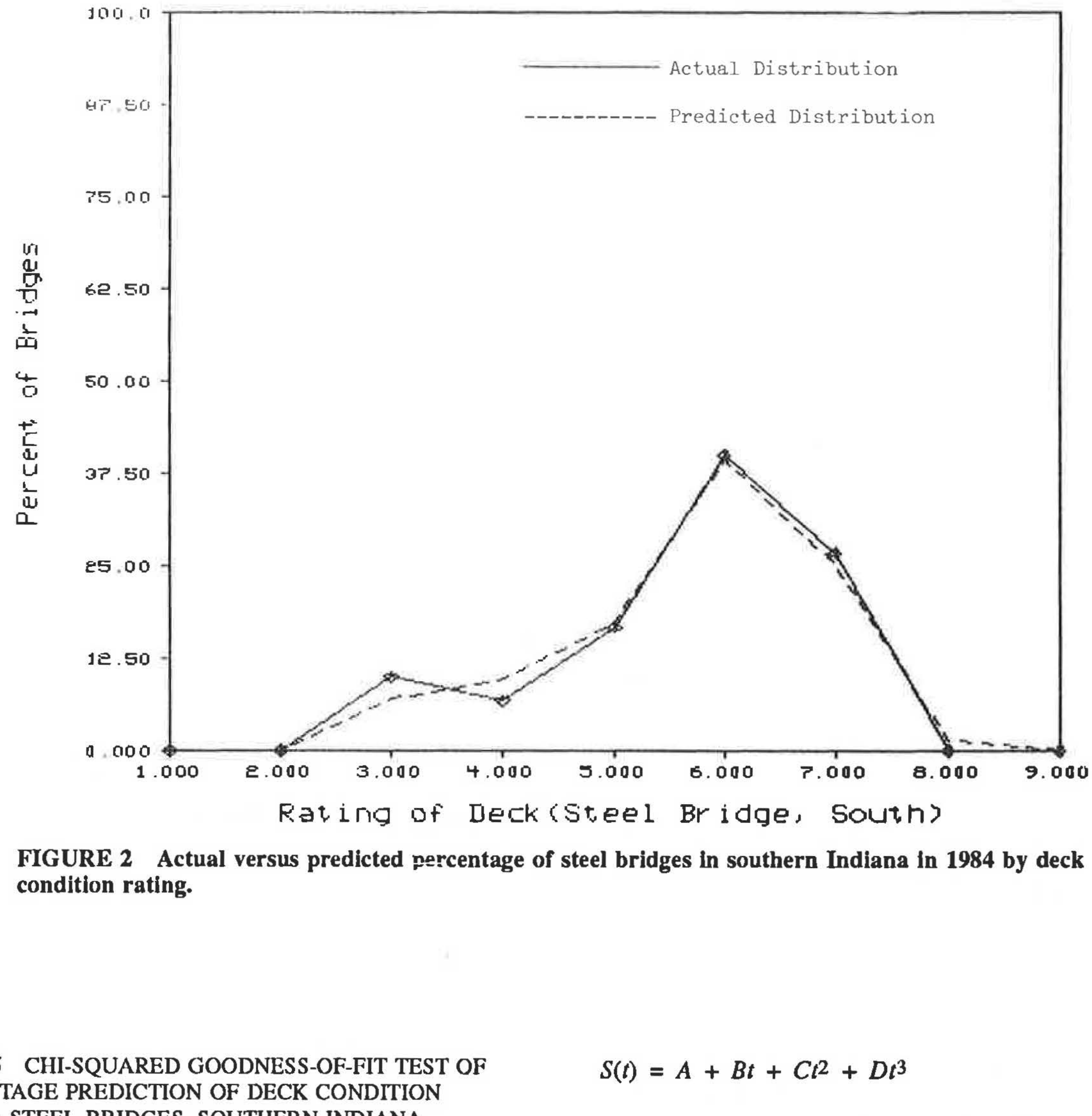

FIGURE 2 Actual versus predicted percentage of steel bridges in southern Indiana in 1984 by deck condition rating.

TABLE 5 CHI-SQUARED GOODNESS-OF-FIT TEST OF PERCENTAGE PREDICTION OF DECK CONDITION RATING: STEEL BRIDGES, SOUTHERN INDIANA

Rating	R_i	E_i	$(R_i-E_i)^2$	$\frac{(R_i-E_i)^2}{E_i}$
9	0.0%	0.1%	1.0×10^{-6}	1.0×10^{-3}
8	0.0%	1.4%	2.0×10^{-4}	1.4×10^{-2}
7	26.7%	25.2%	2.2×10^{-4}	8.9×10^{-4}
6	40.0%	39.3%	4.9×10^{-5}	1.2×10^{-4}
5	16.6%	17.3%	3.6×10^{-5}	2.1×10^{-4}
4	6.7%	9.7%	9.0×10^{-4}	9.3×10^{-3}
3	10.0%	7.0%	9.0×10^{-4}	1.2×10^{-2}
2	0.0%	0.0%	0.0	--
1	0.0%	0.0%	0.0	--
Σ	100.0%	100.0%		$x^2=0.038$

$CHI_6^2[x^2 > 0.038] > 0.995$

$$S(t) = A + Bt + Ct^2 + Dt^3 \quad (9)$$

where

t = bridge age or number of years since last major reconstruction,
$S(t)$ = bridge deck condition rating, and
A–D = coefficients to be determined.

Using the SAS (7) statistical package, the coefficients were obtained on the basis of the sample data, as shown below:

$$S(t) = 9.0 - 0.30266t + 0.00895t^2 - 0.00009t^3 \quad (10)$$

The values of $S(t)$ obtained from Equation 10 were used to solve the nonlinear programming function in Equation 6. This solution provided transition probabilities corresponding to Equation 5 for different bridge age groups. Table 6 shows the transition probabilities for the nine age groups. For example, $p(1) = 0.69$ in Group 1, which means that 69 percent of the bridges in Group 1 (aged 6 years or less) that are in State 1 (condition rating 9) at present would remain in State 1 and that the remaining 31 percent of the bridges would deteriorate to State 2 (condition rating 8) in a 1-year period.

An example set of computations is given in the following. Using Equation 5 and information from Table 6, the transition matrix for Group 1 was obtained:

$$P = \begin{vmatrix} 0.69 & 0.31 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.77 & 0.23 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.92 & 0.08 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.91 & 0.09 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.90 & 0.10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.79 & 0.21 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix} \quad (11)$$

The initial-state vector of Group 1 was $p^{(0)} = (1, 0, 0, \ldots, 0)$. Therefore, the state vector of Group 1 for year t can be obtained by Equation 7. For example, the state vectors for year 0 through year 6 are given below:

$$p^{(0)} = (1, 0, 0, 0, 0, 0, 0, 0, 0, 0)$$

$$p^{(1)} = p^{(0)} * P = (0.69, 0.31, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00)$$

$$p^{(2)} = p^{(0)} * P2 = (0.48, 0.45, 0.07, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00)$$

$$p^{(3)} = p^{(0)} * P3 = (0.33, 0.49, 0.17, 0.01, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00)$$

$$p^{(4)} = p^{(0)} * P4 = (0.23, 0.48, 0.27, 0.02, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00)$$

$$p^{(5)} = p^{(0)} * P5 = (0.16, 0.44, 0.36, 0.04, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00)$$

$$p^{(6)} = p^{(0)} * P6 = (0.11, 0.39, 0.43, 0.06, 0.01, 0.00, 0.00, 0.00, 0.00, 0.00)$$

TABLE 6 TRANSITION PROBABILITIES FOR DIFFERENT BRIDGE AGE GROUPS

	p(1)	p(2)	p(3)	p(4)	p(5)	p(6)
Group 1	0.69	0.77	0.92	0.91	0.90	0.79
Group 2	0.42	0.86	0.86	0.69	0.72	0.56
Group 3	0.65	0.91	0.91	0.92	0.97	0.76
Group 4	0.10	0.07	0.96	0.95	0.98	0.93
Group 5	0.10	0.10	0.94	0.98	0.99	0.11
Group 6	0.10	0.10	0.99	0.98	0.95	0.99
Group 7	0.10	0.10	0.83	0.99	0.97	0.99
Group 8	0.10	0.10	0.57	0.92	0.95	0.99
Group 9	0.10	0.10	0.10	0.55	0.97	0.48

Then $p^{(6)}$ obtained above for Group 1 was taken as the initial-state vector of Group 2, and the corresponding transition matrix of Group 2 was used to continue the procedure.

By this procedure, the bridge condition at any time t can be predicted in terms of initial-state vector $p^{(0)}$ and transition matrix P. Figure 3 shows the deck performance curve of concrete bridges in northern Indiana obtained by this method. Performance curves can be developed similarly for other bridge components.

The trend of the predicted performance curve matched the actual bridge condition data well. The results indicated that bridge deck ratings dropped quickly at the beginning of a bridge's life, then became more stable as the bridge age increased, and dropped quickly again after the deck condition rating reached 5 or less. It should be noted that bridge condition ratings are subjective judgments of bridge inspectors and thus the trend may reflect inherent human bias. For example, bridge inspectors are generally reluctant to rate a condition "perfect" after the initial year and they also tend to consider the condition as rapidly deteriorating after the rating has reached 5.

A chi-squared goodness-of-fit test(*6*) was performed, and the results indicated that the difference between the predicted performance and the least-squares polynomial performance was not significant at $\alpha = 0.05$, as shown below:

$$\chi^2 = \sum_{i=0}^{60} \frac{(R_i - E_i)^2}{E_i} = 0.072$$

$$\mathrm{Chi}^2_{60}\ (x^2 \geq 0.072) > 0.995 > \alpha = 0.05$$

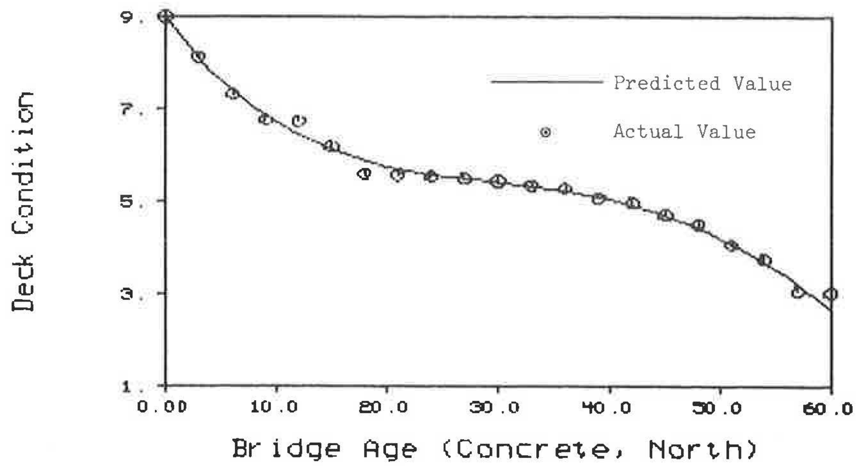

FIGURE 3 Performance curve of concrete-bridge deck condition in northern Indiana.

CONCLUSIONS

An accurate estimate of the future condition of bridges is essential for an effective bridge management system. The Markov chain is a powerful and convenient tool for estimating future bridge performance. The results obtained by a Markov chain model are particularly useful if dynamic programming is used for optimization in a bridge management system, because the transition probabilities are the basic parameters to determine before one can solve a dynamic program (*8*). Furthermore, performance curves give bridge managers a quantitative view of bridge conditions that is useful in selecting rehabilitation strategies.

A Markov chain is completely specified when its transition matrix P and the initial-state vector $p^{(0)}$ are known. Usually, the initial condition is known in a bridge management system. So the main task in using the Markov chain is to develop the transition probability matrix. In the present study, the effects of bridge age on bridge condition were emphasized. The effects of other factors on bridge performance, such as truck traffic and climate, are currently under study. The more factors a model considers, the closer it is to the reality of the problem.

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