Elastic Buckling Strength of Buried Flexible Culverts

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Buckling of buried flexible culverts is defined as the loss of resistance to flexural deformations. Alternative theories are described. Then the procedures used by the most common current design codes are summarized and their limitations explained. The continuum theory is proposed as the best available approach to evaluate the buckling strength of buried flexible culverts because it most realistically models the soil properties and geometry. The suggested means of applying the continuum theory is presented. Example calculations show how the continuum theory results compare with those of existing codes. A major conclusion is that the substantial reduction in stability as structure size increases, as indicated by most approaches, is not correct according to the continuum theory. Commonly used theories are shown to be very conservative in most cases compared with the continuum theory. However, in some cases, for example shallow burial, the reverse may be true.

A characteristic feature of corrugated metal culverts is their bending flexibility. Early in the history of long-span metal culverts, workers recognized that these flexible structures could potentially fail by buckling (*1*). In fact, various buckling collapses have been observed in the field. Unfortunately most of these are not documented in the literature, and there is also ongoing debate as to which cases involving distorted structures constitute buckling. A structure buckles elastically or inelastically when compressive membrane forces act to reduce the flexural stiffness so that there is no resistance to lateral movement.

Currently, codes of practice and design handbooks use a variety of procedures for estimating buckling strengths of flexible structures [e.g., the American Association of State Highway and Transportation Officials (AASHTO) (*2*), the American Iron and Steel Institute (AISI) (*3*), the American Water Works Association (AWWA) (*4*), and the Ontario Ministry of Transportation and Communications (OMTC) (*5*)], although in some cases there is no requirement for the largest of these, known as long span structures, to be designed for buckling (*2, 3*). The design procedures are generally based on the Winkler (i.e., elastic spring) soil model, and are largely empirical in nature.

This paper begins with a brief review of the theoretical buckling analyses and code procedures, and comparisons are made with available test data. An approach to the problem based on the elastic continuum model is then described that permits rational predictions of culvert buckling strength. Finally, a number of example problems are considered, to demonstrate how the elastic continuum theory differs from existing design rules.

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DEFINITION OF BUCKLING

Buckling is directly associated with the effect of changes in geometry on structural stiffness (i.e., geometrical nonlinearity). The simplest type of buckling analysis deals with the structure in its initial position. This is illustrated in Figure 1 for the Euler buckling problems (i.e., a straight column). If large in-plane forces (hoop thrusts N) are present and the structure deforms slightly (displacement W), then moments (M) are generated as a result of the in-plane forces acting at some eccentricity. The hoop thrusts therefore induce further bending and so decrease the effective flexural stiffness. Linear buckling theories involve the calculation of the hoop thrusts, which lead to zero flexural stiffness in the initial position.

For a soil-supported structure, the combined flexure stiffness of the complete soil-structure system must be considered, and in general soil support increases the buckling strength of the metal culvert significantly (Figure 2). Although not generally appreciated, the soil provides resistance to incremental deformations inward as well as outward.

A structure may or may not become unstable at critical load levels predicted by the linear buckling analysis. Deformations at lower load levels lead to changes in geometry. The loss of flexural stiffness then may never occur, or alternatively it may develop at load levels less than those predicted by linear analysis. A nonlinear analysis involving the study of incremental equilibrium in the deformed state is necessary to determine whether the critical load calculated using linear theory is a useful measure of buckling strength.

In this paper, buckling will be used to refer to the theoretical loss of resistance to flexural deformations. In practice this may be manifest by the development of wavelike deformations or flattening on the circumference, perhaps followed by catastrophic collapse (flattening, however, does not necessarily mean buckling). Elastic buckling means that buckling is initiated before the metal structure yields, whereas inelastic buckling means that the buckling response occurs after yield. Yield may occur after elastic buckling is initiated, but the response will nevertheless be called elastic buckling.

Only elastic buckling is addressed in this paper. More theoretical work is needed to determine how structural yield can influence buckling strength.

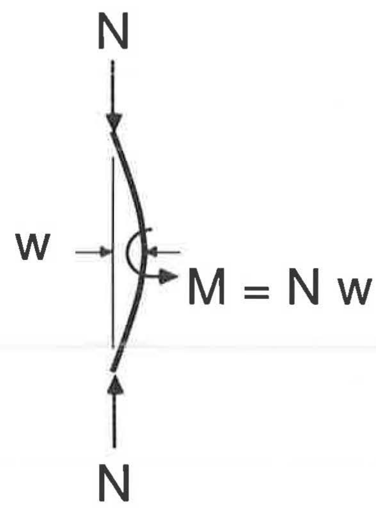


FIGURE 1 Euler buckling.

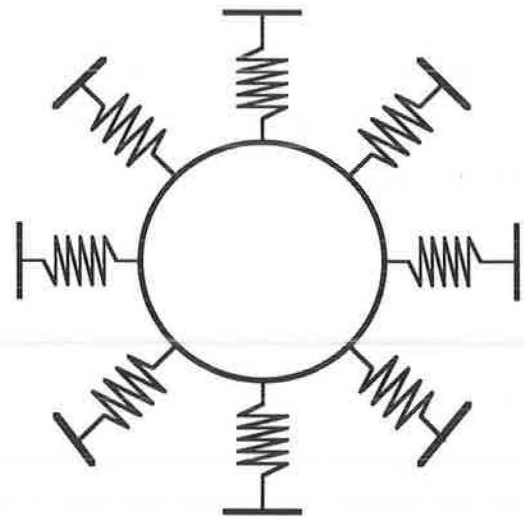


FIGURE 3 Winkler model.

DISCUSSION OF BUCKLING THEORIES

Both linear and nonlinear buckling theories have been developed for buried flexible cylinders.

Linear Theories

Linear theories have generally focused on the linear elastic buckling strength as a number of buckles form around the circumference of a uniformly stressed circular structure. The ground support restrains structural movement and therefore increases stability.

The ground support at the interface can be modeled using a series of elastic springs, as in the Winkler theory (Figure 3), where the spring stiffness is called the coefficient of soil reaction (*1*, *6*, *7*). Unfortunately, ground resistance to structural movements is a complex function of structural geometry, burial depths, and soil properties. The difficulty in using the Winkler model lies in estimating the spring stiffness. Three approaches are possible for doing this:

1. Experimental measurements of buckling strength can be used to back calculate spring stiffness. An extensive testing program to cover the full range of culvert types, soil conditions, and burial depths is needed for this empirical approach to be reliable.
2. Spring stiffness can be expressed as a simple function of some measurable soil properties such as soil modulus (*1*, *7*). This approach can only be an uncertain approximation because the influences of structure size and shape, burial depth, the geometry of the backfill zone, and the embankment soil condition are not defined.
3. A rigorous theoretical analysis can be used in which the material properties and geometry of the soil system are modeled [e.g., Duns and Butterfield (*8*)]. One such approach is introduced in this paper.

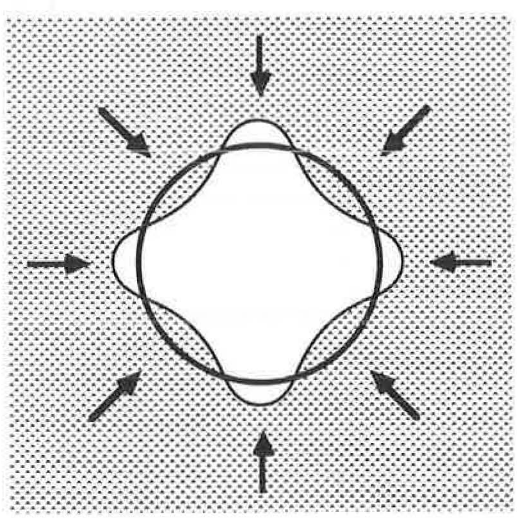


FIGURE 2 Flexible pipe buckling.

The elastic continuum model is a useful tool for assessing the extent of ground restraint at the soil-structure interface [e.g., Forrestal and Herrmann (*9*)]. Because this model represents the whole soil region, it has the potential to reveal how soil quality and quantity, hoop thrust distribution, and other factors influence buckling strength (Figure 4). Rational designs for burial depth (*10*) and the zone of select backfill (*11*) are therefore possible.

Nonlinear Theories

It is certainly important to consider the possibility that buried structures may be imperfection sensitive (i.e., deformations before buckling may reduce buckling strength below that predicted from linear theory). A number of workers have developed nonlinear buckling theories for buried structures (*12*–*14*), and it has been established that structures are not imperfection sensitive when earth loads induce the ring thrusts. However, there may be substantial decreases in buckling

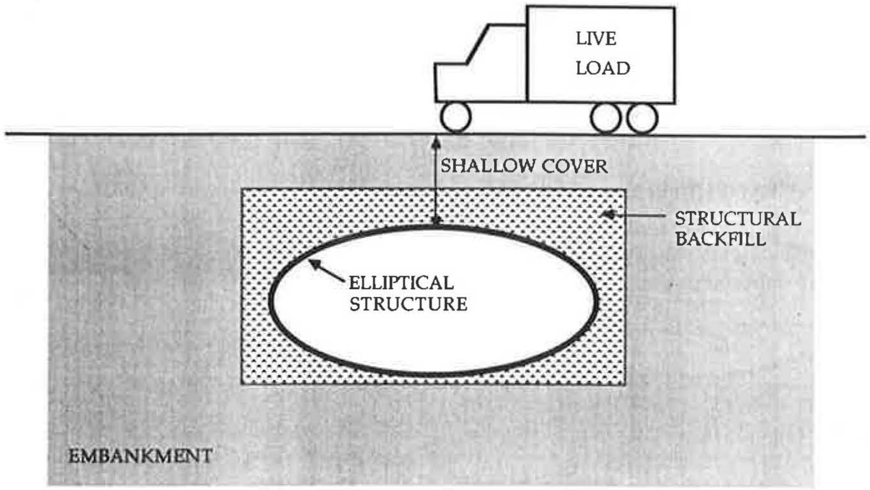


FIGURE 4 Factors considered in the continuum model.

strength when external fluid or internal vacuum loadings predominate.

Current Design Procedures

The design procedures outlined by AASHTO (*2*), AISI (*3*), AWWA (*4*), and OMTC (*5*) all feature calculations of elastic buckling strength based on the Winkler soil model. The first two are only loosely based on the Winkler theory, because they employ a single soil stiffness value that was selected on the basis of one set of experiments (*15*). The other two models feature variable spring stiffness, with empirical corrections for the effect of finite burial depth. AWWA has spring stiffness given as a linear function of constrained soil modulus, whereas the Ontario code provides a list of spring stiffness values for various soil types and densities.

COMPARISON WITH TEST DATA

Field test results indicate that hoop thrust in flexible metal culverts is nonuniformly distributed [e.g. Selig, Lockhart, and Lautensleger (*16*), Selig and Musser (*17*), and Beal (*18*)]. Theoretical analysis indicates that maximum hoop thrust controls the elastic stability (*19*). Therefore the test data used in this paper are limited to those cases in which the static structural response could be analyzed to evaluate soil modulus E_s, and where maximum hoop thrust around the structure at buckling could be estimated. The data, plotted in Figure 5, are from Allgood and Ciani (*20*), Howard (*21*), Gumbel (*22*), and Crabb and Carder (*23*). Also shown in Figure 5 are lines corresponding to (a) elastic continuum theory for a smooth, uniformly stressed, deeply buried cylinder in homogeneous ground (*24*), (b) Winkler theory in which spring stiffness k_s is given in terms of soil, Young's modulus E_s and Poisson's ratio υ_s by

$$k_s = E_s/(1 + \upsilon_s)$$

(c) AASHTO (*2*), and (d) AISI (*3*). Critical hoop thrust N_c is normalized using structure flexural rigidity EI and tube radius R. Results are expressed as a function of stiffness ratio

$$8E_s^* R^3/EI$$

where

$$E_s^* = E_s/(1 - \upsilon_s^2).$$

In these calculations the soil Poisson's ratio is assumed to be 0.3, but Young's modulus E_s is back calculated from static deformation response.

Clearly the approaches outlined in AASHTO (*2*) and AISI (*3*) are simplistic and can yield both excessively conservative and excessively unconservative solutions. The fact that these solutions do not account for shallow burial and other factors exacerbates the problems.

The Winkler solution, using the relationship between spring stiffness and soil modulus, is better but also does not follow the experimental trends satisfactorily, particularly for very flexible structures. The best fit line for the test data is almost parallel to the continuum theory line, which is effectively an upper bound to the experimental results. The difference between theory and experiment probably results from the nonlinear nature of soil behavior. Secant modulus, as calculated from static soil-structure response, may be consistently different from the soil modulus that controls buckling.

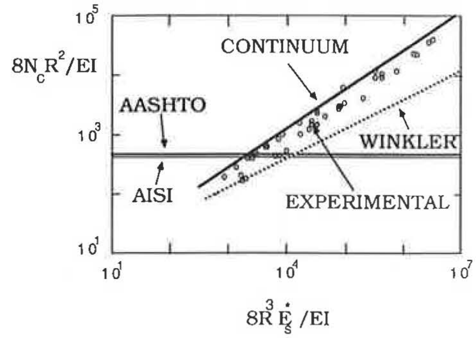


FIGURE 5 Comparison of buckling theories with experimental results.

The remainder of the paper deals with the use of elastic continuum theory predictions of buckling strength after calibration to give lower bound rather than upper bound buckling strength predictions.

APPLICATION OF CONTINUUM THEORY

The critical hoop thrust N_c, which elastically destabilizes the metal culvert, is conveniently expressed as

$$N_c = \phi N_{ch} R_h R_s \quad (1)$$

where ϕ is a calibration factor to account for experimental variation and soil nonlinearity; N_{ch} is the buckling strength of a uniformly stressed, deeply buried circular culvert in homogeneous ground; R_h represents the correction factors for shallow burial and the geometry of the backfill zone; and R_s is a correction factor for culvert shape.

Calibration Factor

Statistical analysis of test data presently available suggests that a value of 0.55 gives a reasonable lower bound for granular soil (*24*). The calibration factor for clay material should probably be less.

Deeply Buried Culvert

The critical thrust N_{ch} for a smooth, circular, deeply buried culvert of radius R and flexural stiffness EI is given by

$$N_{ch} = \frac{(n^2 - 1)EI}{R^2} + \frac{E_s^* R}{2n + (1 - 2\upsilon_s)/(1 - \upsilon_s)}, \quad (2)$$

which is minimized with respect to harmonic number n, an integer greater than or equal to 2 (*25*). For typical flexible metal culverts, that is, $EI/E_s^*R^3 \leq 10^{-2}$, Equation 2, reduces to

$$N_{ch} \approx 1.2(EI)^{1/3}(E_s^*)^{2/3}. \quad (3)$$

For these same deeply buried flexible structures, buckling wavelength is given approximately by [Moore (*10*)]:

$$\lambda = 2\pi(4EI/E_s^*)^{1/3}, \quad (4)$$

which increases as soil stiffness E_s^* is reduced.

Backfill Geometry

Backfill geometry effects can be examined using various solutions for the linear buckling problem. To date, two idealized configurations have been considered, as shown in Figure 6:

1. A circular culvert buried close to the ground surface in homogeneous soil. This solution was obtained using the finite element method (*10*). Correction factors R_{hs} are shown in Figure 7 relative to the stiffness ratio $4EI/E_s^*R^3$ for various ratios of crown cover depth h to culvert radius R. The soil-culvert interface is smooth (frictionless).

2. A deeply buried circular culvert in a circular zone of backfill. A closed-form analytical solution (*11*) was obtained

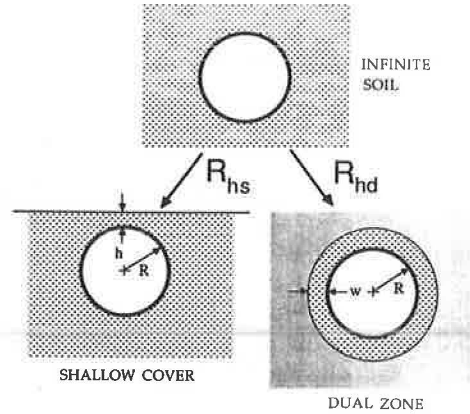


FIGURE 6 Backfill geometry correction factors.

for this case for various ratios of backfill zone width w to culvert radius R. Shown in Figure 8 are R_{hd} values for $w/R = 0.1$ and a range of modulus ratios E_o^*/E_s^*, where $E_o^* = E_o/(1 - \upsilon_o^2)$ characterizes the stiffness of the material surrounding the select backfill.

To obtain Figures 7 and 8, the soil Poisson's ratio was taken as 0.48. However, the effect of changing Poisson's ratio on the value of correction factors is small.

The wavelength of the buckling deformation can lengthen significantly as the parameter w, representing backfill quantity, or h, representing burial depth, is reduced.

Noncircular Culverts

The linear finite element buckling analysis can also be used to examine the buckling strength of noncircular structures (Figure 9). Elliptical culverts have been examined (*26*). The results show that the buckling strength of a deeply buried ellipse is approximately equal to that of a circular tube of

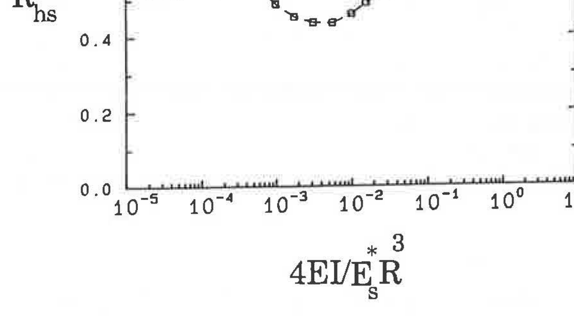


FIGURE 7 Correction factor for shallow cover.

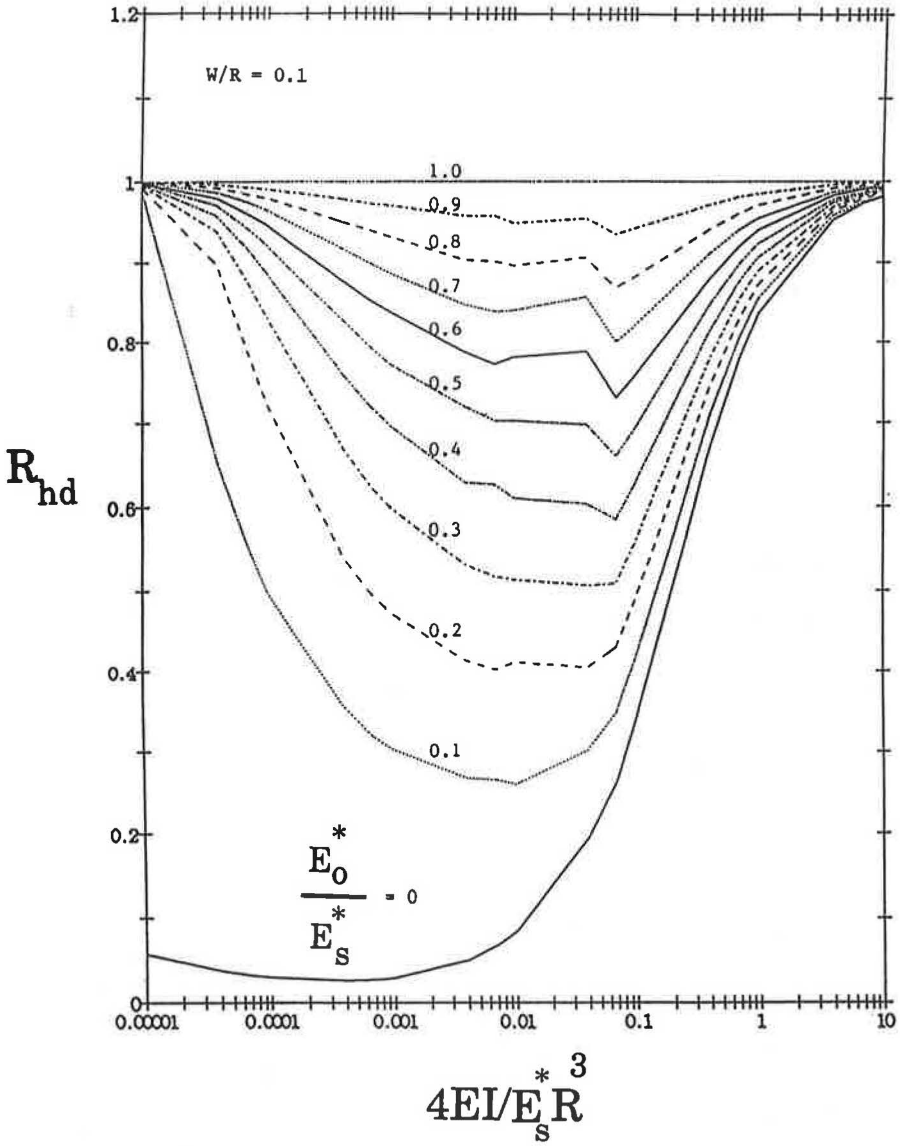


FIGURE 8 Correction factor for dual zone of soil.

equal circumference (i.e., $R_{se} = 1$). Thus the stability of shallow buried ellipses can be evaluated from Figure 7, where R in the stiffness ratio $4EI/E_s^*R^3$ is equated to the ellipse circumference divided by 2π. The burial depth ratio h/R in Figure 7 is equated to burial depth over the half span for the ellipse.

Nonlinear buckling analysis of shallow buried elliptical structures may be needed to confirm the validity of these findings obtained using linear buckling theory.

Factor of Safety

The safety factor F is defined as

$$F = \frac{N_c}{N_m}, \tag{5}$$

for critical thrust N_c from Equation 1 and maximum thrust N_m. Maximum thrust is best calculated using static finite element analysis.

Soil Modulus

Naturally, an important step in using the continuum theory lies in estimating E_s^*, because buckling strength primarily arises from the soil restraint, as shown by Equation 3. The comparison between measured and predicted buckling thrust shown in Figure 5 was based on secant soil modulus backfigured from experimental data, and the theory has been calibrated on that basis. Reasonable lower bound values for secant soil modulus are therefore needed for design.

EXAMPLE CALCULATIONS

A series of example problems will be given to demonstrate the implications of the continuum theory solution. Shown in Figures 10 to 13 for various cases are the ratios of buckling thrust to the thrust that induces wall crushing by material yielding. Four different methods are used to estimate this ratio, namely the continuum model and the procedures out-

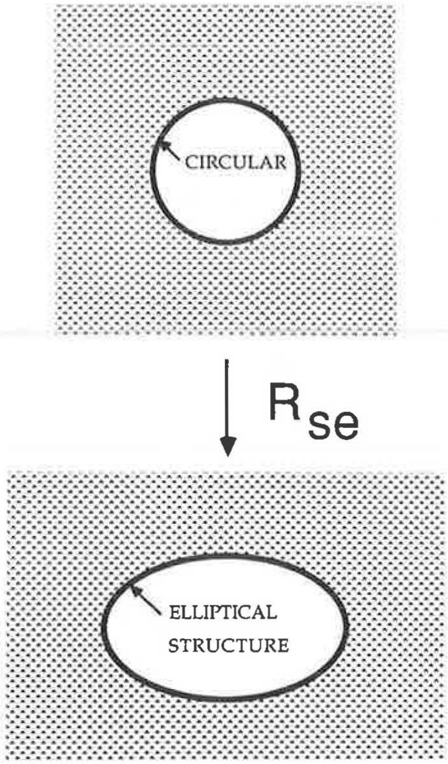


FIGURE 9 Correction factor for structural shape.

lined by OMTC (*5*), AASHTO (*2*) and AISI (*3*). Table 1 contains parameter values used in these calculations.

First, the effect of backfill quality and the size of the backfill zone are examined (Figure 10). The buckling strength of deeply buried 25-ft-diameter circular culverts is considered in turn for low-stiffness backfill, a thin ring of good-quality soil, a more extensive soil envelope, and finally good-quality soil alone. The continuum theory suggests that for low-stiffness soil, buckling precedes wall crushing. As the quantity of good-quality soil increases, buckling strength steadily improves until material strength controls stability. The continuum theory can be used to make a rational assessment of these various types of ground support.

The Ontario code permits an assessment of soil stiffness and its influence on buckling strength but is not able to rationally predict the effect of backfill quantity. Neither AASHTO nor AISI suggest how backfill quality influences buckling strength. For this problem, they appear unconservative for low-stiffness soil but overconservative for stiff ground.

CASE	CONTINUUM	OMTC	AASHTO	AISI
A	0.7	0.3	1.2	1.1
B	0.9	1.0	1.2	1.1
C	1.9	1.0	1.2	1.1
D	2.7	1.0	1.2	1.1

FIGURE 10 Effect of backfill conditions on ratio of buckling to thrust yield stress.

CASE	CONTINUUM	OMTC	AASHTO	AISI
D	2.7	1.0	1.2	1.1
E	2.7	0.9	0.7	1.1
F	2.7	0.85	0.5	0.4
G	2.7	0.8	0.2	0.4

FIGURE 11 Effect of size and shape of culvert on ratio of buckling to thrust yield stress.

A significant difference between Winkler soil models and the continuum theory lies in the perceived effect of structural size and shape on elastic buckling. Compared in Figure 11 are predictions of the buckling-to-yield ratio for two deeply buried circular culverts (spans 25 and 40 ft) and two deeply buried elliptical structures (span 25 ft, height 15 ft; and span 40 ft, height 29 ft).

The continuum model suggests that buckling thrust N_c is independent of culvert size and shape. In each case the buckling thrust is 2.7 times the thrust that induces wall crushing.

AASHTO and AISI indicate that there is a substantial reduction in stability as span increases for the same shape. The radius of curvature at the crown of the elliptical culverts is larger than the radius of circles of equal span. Thus OMTC and AASHTO both suggest that buckling is more likely for elliptical culverts than for circular culverts of equal span. Size and shape effects in OMTC are small. It is well known that the AASHTO and AISI buckling equations contradict field experience in that long-span culverts currently in service are performing satisfactorily, whereas these methods indicate that the culverts are overloaded. This partially explains the fact that long-span structures are currently exempted by AASHTO and AISI from satisfying the buckling criteria. There is no reason to believe that long spans are less susceptible to buckling failure than the smaller span structures. Thus the assessment of long-span structures for the possibility of buckling failure is desirable for reasons of safety and economy. The proposed continuum method should make this possible.

CASE	CONTINUUM	OMTC	AASHTO	AISI
D	2.7	1.0	1.2	1.1
H	0.5	0.2	1.2	1.1

FIGURE 12 Effect of cover depth on ratio of buckling to thrust yield stress.

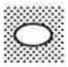
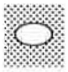
CASE		CONTINUUM	OMTC	AASHTO	AISI
E		2.7	0.9	0.7	1.1
I		3.5	1.1	0.7	1.1

FIGURE 13 Effect of culvert wall thickness on ratio of buckling to thrust yield stress.

Ground support contributes significantly to the buckling strength of metal culverts. As cover over the culvert crown is reduced, ground restraint decreases over the structure and load capacity is reduced. Predictions of buckling thrust relative to yield thrust for a deeply buried circular culvert and a shallow buried structure are compared in Figure 12. Soil and culvert properties remain unchanged. Once again four predictions are shown for each culvert case.

Neither AASHTO (*2*) nor AISI (*3*) include an assessment of the influence of shallow burial on buckling strength. Both the OMTC (*5*) design approach and the continuum theory indicate that significant reductions in buckling strength occur as cover is decreased. The former makes use of a number of empirical corrections for shallow burial. The latter is a theoretical procedure for estimating burial depth effects given directly by the continuum model. The failure mode has converted from ring crushing to elastic buckling with the decrease in crown cover. The empirical buckling equations given by AASHTO and AISI may be quite unconservative for this problem. Rational predictions of minimum cover can be made based on the continuum buckling theory. These complement empirical guidelines for minimum cover such as presently in use by OMTC and those based on analyses of stability using soil failure and limiting bending moment.

To complete the examples, the buckling strengths of deeply buried elliptical culverts for different sets of wall moment of inertia I and wall area A (resulting from a change in plate thickness) have been compared in Figure 13. Both empirical solutions suggest that the ratio of critical thrust to crushing force is unchanged. The other models indicate that wall crushing becomes even more dominant as plate thickness is reduced. It appears that elastic buckling is relatively more significant for the thicker steel plates used commonly on long-span structures.

In general, then, the empirical models currently incorporated in AASHTO (*2*) and AISI (*3*) include corrections for culvert span that are questionable, and cannot account for the effects of ground modulus or burial depth on buckling strength. For the structures considered, continuum theory predicts considerably higher buckling strengths for good-quality backfill. It does suggest, however, that buckling strength may control shallow cover situations in which stability is reduced significantly. The Ontario code (*5*) is, in general, the most conservative of the four theories.

SUMMARY AND CONCLUSIONS

The elastic buckling of flexible metal culverts has been considered. An examination of experimental results indicated that linear buckling theory based on the elastic continuum ground model provides a better estimate of buckling strength than other methods [Winkler models, AASHTO (*2*) and AISI (*3*)]. The continuum model is based on well-defined soil parameters and can consider the effect of shallow cover, the quality and quantity of backfill used to support the corrugated metal structure, and the culvert shape.

A procedure has been described for predicting metal culvert elastic buckling strength. With this procedure, the stability of both circular and elliptical structures can be evaluated for deep and shallow burial in homogeneous ground. The stability of deeply buried circular structures surrounded by a finite envelope of backfill can also be assessed. Rational design of structure backfill and minimum cover height is now possible. Linear buckling solutions of this type are suitable when hoop thrust is generated from earth loads rather than from fluid pressure or internal vacuum.

A number of example problems were considered in order to examine the implications of the new procedure. Estimates

TABLE 1 GEOMETRY AND MATERIAL PROPERTIES FOR EXAMPLE CALCULATIONS

Problem	Span (ft)	Rise (ft)	E_s, E' (lb/in.^{2})	I (in.^{4}/in.)	A (in.^{2}/in.)	R_h	R_t (ft-in.)
A	25	25	500	0.166	0.343	1	12-6
B	25	25	4,000	0.166	0.343	0.33 (w/R = 0.1)	12-6
C	25	25	4,000	0.166	0.343	0.70 (w/R = 0.3)	12-6
D	25	25	4,000	0.166	0.343	1	12-6
E	25	15	4,000	0.166	0.343	1	16-8
F	40	40	4,000	0.166	0.343	1	20
G	40	29	4,000	0.166	0.343	1	28-4
H	25	25	4,000	0.166	0.343	0.2 (h/R = 0.12)	12-6
I	25	15	4,000	0.108	0.228	1	16-8

NOTE: For steel $E = 30 \times 10^6$ lb/in.^{2} and yield stress $= 33 \times 10^3$ lb/in.^{2}. For soil $\upsilon_s = 0.33$.

of buckling strength were compared with calculations based on three other design procedures [OMTC (*5*), AISI (*3*), and AASHTO (*2*)].

The empirical elastic buckling equations specified in AASHTO and AISI suggest that buckling strength is independent of soil stiffness and burial depth. They also indicate that substantial reductions in stability occur as structural size is increased. These trends are believed to be incorrect.

The Ontario code used a linear buckling solution based on the Winkler model, in which modulus of subgrade reaction is treated as a material constant and empirical corrections are included for considering burial close to the ground surface. It is believed to be very conservative.

The continuum theory solution indicated that for typical deeply buried culverts, buckling strength is a function of the flexural stiffness of the structure and ground modulus rather than is span or perhaps even shape. However, the continuum theory also demonstrates that shallow burial or poor backfill can reduce stability dramatically. The assessment of buckling strength for long-span culverts is currently not required by AASHTO or AISI, probably because of the excessively conservative nature of the empirical buckling equations when used for those structures. However, it is important to assess the buckling strength of all flexible metal culverts, and continuum solutions are believed to yield rational and reliable estimates of stability that enable all structures to be considered.

Further developments of the continuum solutions are envisaged, using a linear finite element buckling solution. Studies of various field installations should provide valuable data for comparisons with the model.

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