Optimum Geometric Shapes of Precast Concrete Arch Structures of 24-, 30-, and 40-Ft Spans

PAUL A. ROWEKAMP, JAMES J. HILL, AND THEODOR KRAUTHAMMER

The results of a structural analysis of elliptical-shaped precast concrete arch structures and circular-shaped arches being considered by the Minnesota Department of Transportation are summarized in this paper. These arch structures were analyzed to compare the effect of geometry on structural performance. They were analyzed using the finite element method by placing identical load conditions on each arch. Half of each arch was modeled, based on symmetry, with no rotation allowed at the arch crown. The effects of cracking, critical stresses, and displacements were tabulated. Temperature stresses and shrinkage of the concrete were also introduced into the shape comparisons. Conclusions are included which indicate the optimum geometric shape and considerations for further analysis of different loading combinations.

The application of arch structures in transportation systems is not new. Nevertheless, many aspects of the structures' behavior are not well understood, and therefore studies are being conducted by several researchers to enhance knowledge in this area. In April 1987, an analytical study was initiated to review the differences between an elliptical-shaped arch and several proposed arches being developed by the Minnesota Department of Transportation (Mn/DOT). The circular shapes chosen have rise-span ratios of approximately 1:3 to 1:4 and radii less than 25 ft. The structures were compared by subjecting finite element models of each arch to identical loads and reviewing the resulting stresses and deflections. This study and its conclusions were based on a computer analysis and did not include field testing. However, the results of this study will be used as a basis for the development of future field testing.

The arches vary in span from 24 to 44 ft and have vertical openings from 8 to 14 ft high. They have a constant thickness of 10 in. and support an HS20 loading. The circular shapes were designed for use over small rivers or streams and were not intended for traffic passage through the arch opening. The arches are generally manufactured in 6-ft-wide panels that are placed side by side to form the required roadway width. The circular shapes are presently being designed, but have not yet been built.

P. A. Rowekamp and J. J. Hill, Minnesota Department of Transportation, Transportation Building, St. Paul, Minn. 55155. T. Krauthammer, Department of Civil and Mineral Engineering, University of Minnesota, 500 Pillsbury Drive S.E., Minneapolis, Minn. 55455.

ARCH SHAPES

Included in this paper are the analyses of two different types of 24- and 30-ft elliptical and circular arches (see Figures 1 and 2). The first type is labeled as an arch "without legs." For the 24-ft span the vertical opening for the arch without legs is 8 ft high and for the 30-ft span the opening is 11 ft high. The arches labeled "with legs" have exactly the same geometric shape as the arches without legs except that a portion has been added at the base to obtain a higher vertical clearance.

In the case of the circular arches the added leg is actually an extension of the curve that defines the arch shape. For the elliptical arches the added leg is a vertical strut added at the base. The added leg on the 24-ft elliptical arch increases the vertical opening by 25 in. for a total opening of 10 ft. For the 30-ft arches the leg increases the height by 28 in. for the elliptical arch and 32 in. for the circular arch. This results in a vertical height of 13 ft 8 in. for both structures.

The shape of each arch and geometric comparisons of the structures are shown in Figures 1 and 2.

PROCEDURE

Ten different structures were analyzed using the finite element method. The structures were modeled using a series of beam elements connected end to end to form the geometric shape of the arch. The actual element chosen to model the structures was a two-node beam element that is one of the simplest elements available for use with this method. Nodes or joints are used to define the beginning and ending point of each element (see Figure 3). After defining the material and section properties of each element, a computer program combines this information to form the stiffness matrix. Given the stiffness matrix, the applied loads and the boundary conditions of the structures, the deflections and stresses at each node are calculated (*1*).

Two different computer programs were used: ADINA (*2*), on the IBM 4341 mainframe computer at the Civil Engineering Department of the University of Minnesota, and STAAD3 (*3*), a commercially available program that was run in house at MnDOT. Sample problems run by each program resulted in nearly identical data output.

The thickness and material properties were identical for all the arches. They were all analyzed using 4,000 lb/in.^{2} concrete, which has an elastic modulus of approximately 3605 kips/in.^{2}

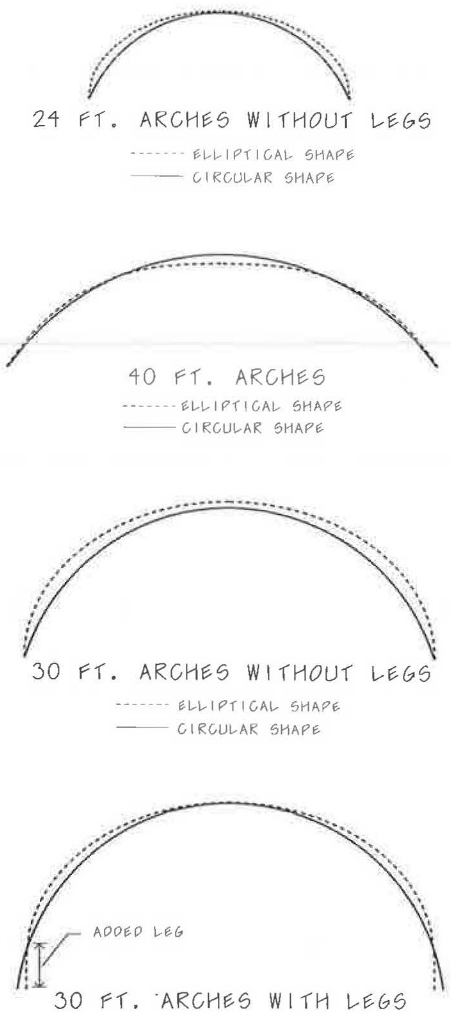

FIGURE 1 Arch profiles.

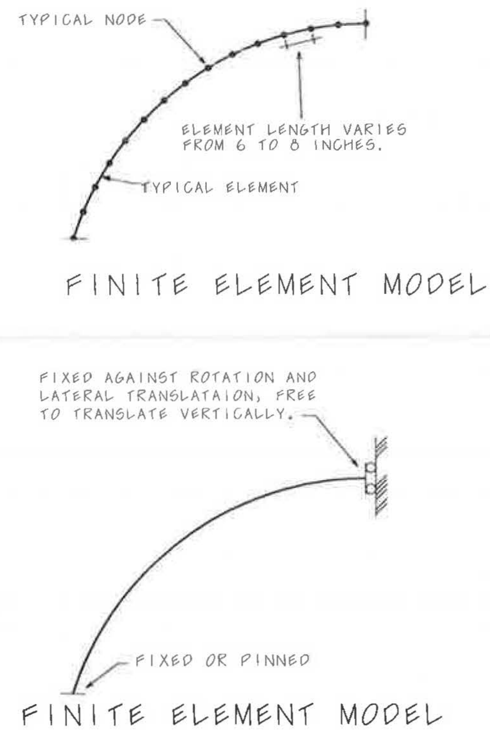

FIGURE 2 Finite element models.

and a Poisson's ratio of 0.2. All arches had a constant thickness of 10 in. and a concrete density of 150 lb/ft^3. A 12-in.-wide section was used to compute the cross-sectional area and moment of inertia for each beam element. The 12-in. width was also used for computing the dead weight of the arch and the loads induced by the soil supported by the structure.

LOAD CONDITIONS

A total of eight different load cases were applied to the 24- and 30-ft arches (see Figure 4). They included:

1. Two ft of soil over the entire structure (assumed soil weight = 120 lb/ft^3) plus a live load surcharge equivalent to 240 lb/ft^2.
2. A layer of soil equal in height to half the radius of the arch plus a live load surcharge equivalent to 240 lb/ft^2.
3. Sixteen in. of soil topped by an 8-in. concrete slab plus a live load surcharge equivalent to 240 lb/ft^2.
4. Two ft of soil plus a concentrated live load of 3,200 pounds (truck axle load of 32,000 pounds spread laterally over 10 ft) placed at the midspan of the arch. A second set of four

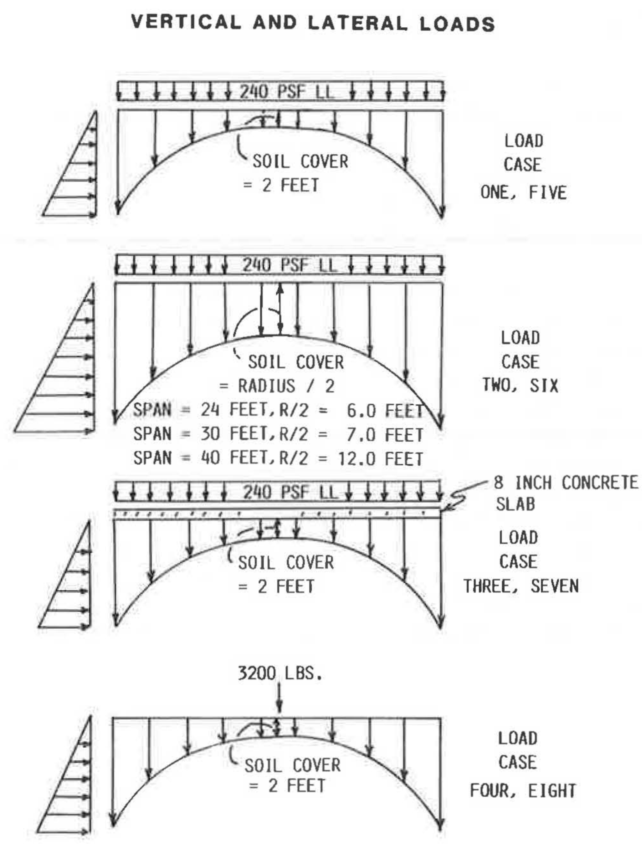

FIGURE 3 Loading conditions.

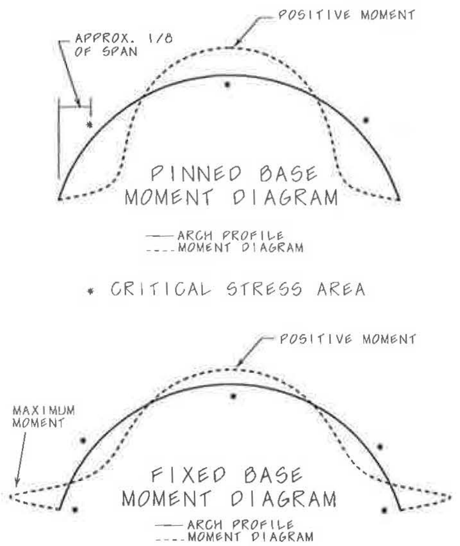

FIGURE 4 Moment diagrams.

load cases included the same vertical loads as previously described plus a hydrostatic lateral load using an equivalent soil weight of 30 lb/ft^3 (see Figure 4). These lateral loads were applied to the 24- and 30-ft arches but not to the 40-ft arches. The arches were analyzed using both a fixed and a pinned boundary condition at the base because the true base fixity of each arch is unknown. It should be noted that the load cases applied in this study are an attempt to model conditions that will be encountered in the field. As with any analysis of this type, there is a degree of uncertainty in approximating actual field conditions. However, the load conditions applied are the same for each type of arch and should give valid results when used for comparison purposes.

MODELING TECHNIQUES

Because each arch is symmetric and all loads were applied in a symmetrical fashion, only half the arch needed to be modeled. However, an important boundary condition must be defined before using this shortcut. Specifically, no rotation can be allowed at the crown of the arch and this node must be free to translate vertically and fixed laterally (see Figure 3). The base of the arch is assumed to be fixed or pinned, depending on the actual condition under consideration. For the analysis of the arches in this study, each load case included both the fixed and the pinned base condition.

EFFECT OF CRACKING

The stress at which concrete is assumed to crack in tension is $7.5\sqrt{f_c}$ (*4*), or 474 $lb/in.^2$ for 4,000 $lb/in.^2$ concrete. Because the exterior face of arches will be in contact with soil and the interior face may be subjected to moisture from stream flow or condensation, a primary design concern is to keep the structure relatively free from cracking. Although all concrete structures are subject to temperature and shrinkage cracking, the main concern is to limit the tensile cracks caused by dead and live loads and temperature effects. By limiting the cracking the chance for moisture to penetrate the concrete will be reduced, which will limit reinforcement deterioration. Cracking will also affect the flow of forces in the structure. Once a portion of a concrete beam is considered a cracked section it is not uncommon for the moment of inertia to decrease by as much as 50 percent. Because the magnitude of the deflection is inversely proportional to the moment of inertia, the deflections may increase substantially if the structure cracks in areas where maximum deflections are likely to occur. The shape of the moment diagram will also shift as a result of cracking.

The analysis carried out in this study neglected the effect of the steel reinforcement and considered the concrete to be a linearly elastic, isotropic, and homogeneous material. This assumption may be valid for an uncracked section but does not hold true once the concrete cracks in tension. In reviewing the results, there are many cases in which the tensile stresses are in excess of the cracking limit and in some cases they are over 2,000 $lb/in.^2$. Concrete tensile stresses cannot reach this level but are included here as a means of comparison. If the tabulated stress at a critical point in one arch is 2,000 $lb/in.^2$ and in another arch it is 800 $lb/in.^2$, the concrete will very likely have cracked in both cases and the load will have been transferred to the tension steel. Because the stresses shown are often above cracking they are not likely to be the actual stresses in the structure, but they do give an indication of the relative stress levels for comparison purposes.

CRITICAL STRESSES

The final stresses included both axial and bending effects and were computed using the equation $P/A + Mc/I$ = final stress. Generally there are three critical areas of each arch that should be checked for maximum tension stresses. These include:

1. The inside face of the arch at the crown;
2. The outside face of the arch, about 45 percent up from the bottom (approximately the eighth point of the arch span); and
3. The stress at the base of the arch (for the arches with a fixed base condition). Figure 4 shows the critical areas where tension stresses are usually at a maximum.

RESULTS

The shape of the moment diagram is fairly similar for all of the load cases (depending on whether the base is fixed or pinned) and for all the arches analyzed. The typical moment diagram for the 30-ft circular arch is shown in Figure 4. As expected, the areas of maximum moment coincide with the locations of maximum tensile stresses. For the 40-ft arches, the maximum moment generally occurs at the crown when the ends of the arch are pinned or fixed. For the 24- and 30-ft arches, the maximum moment occurs near the eighth point of the span for the pinned case, and at the base for the fixed case. A summary, listing the maximum tensile stresses for the exterior face, the interior face, and the base of each arch is provided in Tables 1–4. Load Case 3 produced applied loads and resulting stresses similar to Load Case 2, hence the results for these cases are not included in this paper.

TABLE 1 MAXIMUM TENSION STRESSES, 40-FT ARCHES, VERTICAL LOADS ONLY

	STRESSES IN PSI		
	INTERIOR FACE (CROWN) Maximum Stress	EXTERIOR FACE (1/8 POINT) Maximum Stress	BASE OF ARCH Maximum Stress
ELLIPTICAL			
Load Case No. 1			
Fixed Base Condition	327	213	289
Pinned Base	437	309	-0-
Load Case No. 2			
Fixed	1240	951	1518
Pinned	1696	1464	-0-
Load Case No. 3			
Fixed	340	225	308
Pinned	457	340	-0-
Load Case No. 4			
Fixed	655	285	367
Pinned	777	390	-0-
CIRCULAR			
Load Case No. 1			
Fixed Base Condition	-0-	-0-	-0-
Pinned Base	-0-	-0-	-0-
Load Case No. 2			
Fixed	91	26	434
Pinned	351	404	-0-
Load Case No. 3			
Fixed	-0-	-0-	-0-
Pinned	-0-	-0-	-0-
Load Case No. 4			
Fixed	243	-0-	-0-
Pinned	291	2	-0-

(-0- Indicates no tension)

A summary of the crown deflections and the effect of a 100 degree Farenheit temperature change are provided in Tables 5 and 6. The crown was chosen as a reference point for comparison because it is the area of maximum tensile stress for the 40-ft arches. Other structures also exhibit high stresses in this area. It is also the location of maximum vertical deflection. Deflection data for the pinned base condition is provided in Table 5. These results are approximately two times higher than the results using a fixed base condition.

TEMPERATURE EFFECTS

Stresses resulting from changes in temperature have also been analyzed. The resulting stresses and deflections for a temperature change of 100 degrees Farenheit are included in Table 6. A 100 degree range was used to allow for ease of interpolation of actual temperature changes. An increase in temperature will cause an upward deflection at the crown and this in turn will cause tension stresses on the exterior (top) side at the crown and compression on the interior (bottom) side. A temperature decrease will cause opposite behavior.

A coefficient of thermal expansion of concrete equal to 0.000006 (5) was used for this analysis. The coefficient for shrinkage is 0.0002, which is equal to a temperature drop of 33 degrees Fahrenheit. The effect of shrinkage and a temperature drop of 30 degrees would be similar to a temperature drop of 60 degrees. For a 60 degree temperature drop the stresses and deflections induced would be 60 percent of those listed in Table 6, and are quite substantial for all the arches.

The moments induced by temperature effects for the pinned end case were zero at the base and reached a maximum at the crown. For the fixed end case the maximum moment occurred at the base, then the moment diagram changed sign and reached a second critical point at the crown where the magnitude was approximately 50 percent of the moment at the base. The axial loads induced from temperature changes were very small and were neglected when computing the tensile stresses.

ANALYSIS OF 40-FT ARCH

As seen from Table 1, which summarizes the maximum stresses and their locations, the maximum tension stress for the 40-ft elliptical arch occurred at the underside of the crown of the arch. For Load Cases 2 and 4, the stresses calculated in the analysis far exceeded the cracking stress of 474 lb/in.^{2}. For Load Case 2 with pinned ends, the tension stresses on the outside of the arch also exceeded cracking in an area about 45 percent up from the base of the arch (the eighth point of the arch span). With the base fixed, the stress of 1,500 lb/in.^{2}

TABLE 2 MAXIMUM TENSION STRESSES, 30-FT ARCHES, NO LEGS

	STRESSES IN PSI					
	VERTICAL LOAD ONLY			VERTICAL AND LATERAL LOAD		
	INTERIOR FACE (CROWN)	EXTERIOR FACE (1/8 PT.)	BASE OF ARCH	INTERIOR FACE (CROWN)	EXTERIOR FACE (1/8 PT.)	BASE OF ARCH
ELLIPTICAL						
Load Case 1						
Fixed Base	243	313	992	169	220	685
Pinned Base	516	767	-0-	380	571	-0-
Load Case 2						
Fixed	628	727	1970	481	560	1565
Pinned	1200	1622	-0-	948	1288	-0-
Load Case 4						
Fixed	534	323	910	460	240	673
Pinned	800	731	-0-	664	536	-0-
CIRCULAR						
Load Case 1						
Fixed Base	46	50	252	-24	-32	48
Pinned Base	157	227	-0-	28	54	-0-
Load Case 2						
Fixed	239	228	736	99	77	381
Pinned	516	629	-0-	274	331	-0-
Load Case 4						
Fixed	343	113	336	272	41	132
Pinned	472	275	-0-	343	107	-0-

(-0- Indicates no tension)

TABLE 3 MAXIMUM TENSION STRESSES, 30-FT ARCHES, WITH LEGS

	STRESSES IN PSI					
	VERTICAL LOAD ONLY			VERTICAL AND LATERAL LOAD		
	INTERIOR FACE (CROWN)	EXTERIOR FACE (1/8 PT.)	BASE OF ARCH	INTERIOR FACE (CROWN)	EXTERIOR FACE (1/8 PT.)	BASE OF ARCH
ELLIPTICAL						
Load Case 1						
Fixed Base	438	571	1300	316	415	917
Pinned Base	812	1162	-0-	590	839	-0-
Load Case 2						
Fixed	1026	1213	2653	802	948	2033
Pinned	1779	2344	-0-	1392	1824	-0-
Load Case 4						
Fixed	719	543	1223	597	390	839
Pinned	1068	1065	-0-	846	746	-0-
CIRCULAR						
Load Case 1						
Fixed Base	159	192	623	32	41	249
Pinned Base	376	535	-0-	143	220	-0-
Load Case 2						
Fixed	499	520	1437	265	263	831
Pinned	976	1210	-0-	569	701	-0-
Load Case 4						
Fixed	462	225	646	335	81	272
Pinned	681	530	-0-	448	217	-0-

(-0- Indicates no tension)

TABLE 4 MAXIMUM TENSION STRESSES, 24-FT ARCHES, NO LEGS

	STRESSES IN PSI					
	VERTICAL LOAD ONLY			VERTICAL AND LATERAL LOAD		
	INTERIOR FACE (CROWN)	EXTERIOR FACE (1/8 PT.)	BASE OF ARCH	INTERIOR FACE (CROWN)	EXTERIOR FACE (1/8 PT.)	BASE OF ARCH
ELLIPTICAL						
Load Case 1						
Fixed Base	141	184	566	110	147	470
Pinned Base	313	502	-0-	257	423	-0-
Load Case 2						
Fixed	308	372	1079	247	306	915
Pinned	630	938	-0-	526	804	-0-
Load Case 4						
Fixed	393	220	618	362	183	522
Pinned	575	513	-0-	519	434	-0-
CIRCULAR						
Load Case 1						
Fixed Base	10	-0-	76	-0-	-0-	-0-
Pinned Base	63	78	-0-	6	5	-0-
Load Case 2						
Fixed	77	51	243	15	-0-	95
Pinned	196	222	-0-	92	96	-0-
Load Case 4						
Fixed	255	69	182	223	39	95
Pinned	334	153	-0-	277	85	-0-

(-0- Indicates no tension)

was more than three times higher than the stresses in the circular arch. The 40-ft circular arch also exhibited a maximum stress in the area at the underside of the crown. However, the stresses did not exceed 500 lb/in.^{2} for any of the four Load Cases. It is also important to note that for Load Cases 1 and 3 the entire arch remained in compression. For the fixed base condition, only one of the four load cases caused tensile stress at the base; Load Case 2 produced a stress of 434 lb/in.^{2}.

The maximum deflections for both shapes occurred under Loading 2. With hinged ends, a deflection of 0.72 in. downward occurred for the elliptical shape, with 0.26 in. for the circular shape (see Table 5).

In summary, the stress and deflection data for the 40-ft arches show results that would favor use of the circular arch shape over that of the elliptical shape. After reviewing the geometric profile in Figure 1, it becomes evident why the results turned out as they did. The elliptical arch has a noticeable flat spot near the crown and rises up at a steeper slope from the base than the circular arch. The result was that at the crown the elliptical and circular arches had nearly identical axial loads. The crown moments in the elliptical arch, however, were at least two times higher than they were in the circular. These high moments were induced by the flattening out of the arch and result in high tensile stresses at the underside of the crown.

ANALYSIS OF 30-FT ARCH WITHOUT LEGS

Unlike the 40-ft arches, for Load Cases 1 to 3, the 30-ft arches did not produce a maximum tensile stress at the underside of the crown. For the fixed base condition, the maximum tensile stress occurred at the inside face of the arch at the base and under vertical load, but the magnitude was nearly twice the cracking stress. For the pinned base condition, the maximum stress occurred at the eighth point of the arch span. For the elliptical shape, the tensile stress at this point was approximately 20 percent lower than it was at the base. Under both vertical and lateral load, the tensile stress at the base dropped by 300 lb/in.^{2} but was still nearly 50 percent higher than the cracking stress.

As expected, the addition of lateral load decreased the magnitude of the tensile stresses. However, for the 30-ft elliptical arch without legs, all four load cases still produced tensile stresses above 500 lb/in.^{2} when lateral loads were included in the analysis. Thus all 8 of the load cases (see Figure 4) applied to the elliptical shape induced stresses that exceed the cracking stress. The 30-ft circular arch without legs performed quite well under seven of the eight load conditions applied in this study. The stresses were usually well below the cracking stress and the deflections were quite small. However, Load Case 2, with vertical load only, did produce tensile stresses above 500 lb/in.^{2} for the fixed and the pinned base condition, but these stresses were still two to four times less than those produced by the same load case on the elliptical arch.

The 30-ft circular arch without legs produced maximum moments at the same locations as those of the 30-ft elliptical arch without legs. For the fixed base condition the maximum moment usually occurred at the base and for the pinned base condition the maximum moment usually occurred near the eighth point of the arch span. The major difference between the two arch structure types was the magnitude of the max-

TABLE 5 SUMMARY OF DEFLECTIONS AT CROWN OF ARCHES, PINNED BASE CONDITION

	Deflections in inches					
	VERTICAL LOADS ONLY			VERTICAL AND LATERAL LOADS		
ARCH TYPES	LOAD CASE NO. 1	LOAD CASE NO. 2	LOAD CASE NO. 4	LOAD CASE NO. 1	LOAD CASE NO. 2	LOAD CASE NO. 4
24 FOOT ARCHES						
*ELLIPTICAL WITHOUT LEGS	-0.09	-0.16	-0.10	-0.07	-0.14	-0.09
*CIRCULAR WITHOUT LEGS	-0.02	-0.05	-0.04	-0.02	-0.04	-0.03
30 FOOT ARCHES						
*ELLIPTICAL WITH LEGS	-0.38	-0.79	-0.38	-0.29	-0.64	-0.29
WITHOUT LEGS	-0.21	-0.46	-0.20	-0.17	-0.38	-0.18
*CIRCULAR WITH LEGS	-0.16	-0.37	-0.18	-0.09	-0.24	-0.11
WITHOUT LEGS	-0.07	-0.17	-0.09	-0.04	-0.12	-0.06
40 FOOT ARCHES						
*ELLIPTICAL WITHOUT LEGS	-0.19	-0.72	-0.23	The 40 foot arches were not analyzed for vertical plus lateral load.		
*CIRCULAR WITHOUT LEGS	-0.03	-0.26	-0.08			

(-) Downward Deflection

imum moments. At the base the axial load in the circular arch was approximately 15 percent higher than it was for the elliptical arch, but the moments in the elliptical arch were from two to five times greater than those in the circular arch. At the eighth point the axial load in the elliptical arch was virtually identical to the circular arch, but the elliptical arch exhibited bending moments that were from two to four times greater than they were for the circular arch. At the crown, the circular arch had a slightly higher axial load when compared with the elliptical arch. However, the bending moment produced in the elliptical arch was higher than that of the circular arch.

In summary, the results show that the circular structure acts more like a true arch, with high axial load and low bending moment, when compared with the elliptical shape, which has relatively equal axial load but higher bending stresses.

ANALYSIS OF 30-FT ARCH WITH LEGS

The addition of a leg to the 30-ft arches caused the tensile stresses to increase by about 30 percent for the elliptical shape and nearly a 100 percent increase for the circular shape when compared to the arches without legs (see Table 3).

When only vertical loads were applied, all four of the load cases analyzed caused tensile stresses to exceed the cracking level in both the circular and elliptical arch. This occurred for both the fixed and pinned base condition. However, it should be noted that the tensile stresses in the elliptical shape were approximately twice as high as those of the circular shape.

When vertical and lateral loads were applied, only one of the load cases produced stresses greater than 500 lb/in.^{2} in the circular shape. All four of the load cases resulted in stresses higher than the cracking stress for the elliptical shape, where the tensile stresses were anywhere from 1.5 to 4 times higher than those of the circular arch.

The location on the arch where the maximum tensile stresses occurred changed very little for the arch with legs as compared with the arch without legs. For the fixed base condition the maximum stresses generally occurred at the base of the arch. For the pinned condition the maximum stresses occurred near the eighth point of the span. More precisely, for the arch without legs and a pinned base the maximum moment and maximum tensile stress occurred at a point approximately 57 in. up vertically from the base. For the fixed base condition the maximum tensile stress on the exterior face occurred at a point approximately 72 in. up vertically from the base. This maximum stress occurs at a higher point for the fixed base condition because the moment curve changes sign in moving up from the base to this point of high stress. In the case of a pinned base the moment is zero at the base and the curve does not change sign before reaching this critical stress point (see Figure 4).

The addition of the vertical leg to the arches caused the crown deflection to nearly double for both shapes. The maximum deflection recorded for the circular arch was 0.37 in. downward compared to 0.79 in. for the elliptical.

If a preferred shape must be chosen the circular arch is favored over the elliptical shape for the case of a 30-ft arch with legs. The circular shape produced tensile stresses and deflections that were consistently lower than the elliptical shape. However, it is important to point out that although the circular shape did produce lower stresses, these stresses still exceeded 500 lb/in.^{2} for Load Cases 1 to 4 without lateral load. When the previously defined lateral load was applied to the circular arch the stresses fell to less than the cracking

TABLE 6 SUMMARY OF TEMPERATURE EFFECTS

100 DEGREE TEMPERATURE CHANGE
TENSILE STRESSES AND DEFLECTIONS
Stresses in PSI, Deflections in inches

ARCH TYPES		CROWN DEFLECTION	TENSILE STRESS AT BASE	TENSILE STRESS AT CROWN
24 FOOT ARCHES				
*ELLIPTICAL				
WITH LEGS	HINGED	+/- 0.14	No Tension	+/- 110
	FIXED	+/- 0.18	+/- 435	+/- 215
WITHOUT LEGS	HINGED	+/- 0.15	No Tension	+/- 148
	FIXED	+/- 0.19	+/- 612	+/- 294
*CIRCULAR				
WITHOUT LEGS	HINGED	+/- 0.15	No Tension	+/- 166
	FIXED	+/- 0.18	+/- 603	+/- 324
30 FOOT ARCHES				
*ELLIPTICAL				
WITH LEGS	HINGED	+/- 0.18	No Tension	+/- 79
	FIXED	+/- 0.21	+/- 292	+/- 152
WITHOUT LEGS	HINGED	+/- 0.18	No Tension	+/- 103
	FIXED	+/- 0.22	+/- 403	+/- 203
*CIRCULAR				
WITH LEGS	HINGED	+/- 0.19	No Tension	+/- 88
	FIXED	+/- 0.22	+/- 303	+/- 169
WITHOUT LEGS	HINGED	+/- 0.19	No Tension	+/- 117
	FIXED	+/- 0.22	+/- 420	+/- 229
40 FOOT ARCHES				
*ELLIPTICAL				
WITHOUT LEGS	HINGED	+/- 0.29	No Tension	+/- 137
	FIXED	+/- 0.35	+/- 591	+/- 261
*CIRCULAR				
WITHOUT LEGS	HINGED	+/- 0.28	No Tension	+/- 139
	FIXED	+/- 0.34	+/- 516	+/- 273

(-) Downward Deflection

level for all but one load case (*R*/2 soil cover, Load Case 2). Hence, unless the designer is assured the field conditions will provide a substantial amount of lateral load, the 30-ft arches with legs should be considered very carefully before selection.

Perhaps a more suitable method of acquiring the extra headroom would be to build a short abutment wall that would support the arch. A properly designed wall could provide a base that would not affect the structural integrity of the arch but still allow the required additional vertical clearance. Extra height required for clearance may also be achieved by changing the height of the abutment. If the taller arch section is used, the amount of fill placed above the arch should be limited.

ANALYSIS OF 24-FT ARCH WITHOUT LEGS

The areas of critical tension stress for the 24-ft elliptical arch without legs were the same as those of the 30-ft elliptical arch for the eight loading cases analyzed here. One major difference between the 24- and 30-ft elliptical arches was the magnitude of the stresses. The 24-ft elliptical arches had a maximum stress 33 percent less than those of the 30-ft elliptical arches. However, for the fixed base condition, the tension stresses at the base were equal to or exceeded the cracking stress for all eight load cases and for Load Case 2 they were nearly two times the cracking stress. For the pinned end condition, the stresses at the eighth point of the span exceeded the cracking level for all four load cases for vertical loads only, and were less than 500 lb/in.^{2} for two of four load cases when lateral load was included. A maximum deflection of 0.16 in. was recorded for Load Case 2 with pinned ends. Deflections for the other load cases were less than or equal to 0.14 in., which is negligible for a 24-ft span.

Of all the arches analyzed in this study, the 24-ft circular arch exhibited the best all around structural performance. The highest tension stress was 40 percent less than the theoretical cracking stress, and in most instances was less than 100 lb/in.^{2} (see Table 4). The deflections for this structure were also small.

In summary, a comparison of the 24-ft elliptical arch without legs and the circular arch without legs yielded a result identical to the 40- and 30-ft arches; the circular arch had much lower tension stresses and deflections and performed better than the elliptical shape for the load cases presented here. It seems that the cause for the higher tension stresses in the elliptical shape was the result of the flatter crown geometry and sharper change in slope along the arch. At the eighth point the elliptical shape developed a moment 3.5 to 5.5 times higher than the circular arch, whereas axial loads were virtually identical. At the crown, the elliptical shape developed a moment two to four times higher than the circular shape, whereas the circular arch had an axial load only 15 percent higher.

ANALYSIS OF 24-FT ELLIPTICAL SHAPE WITH LEGS

The 24-ft elliptical arch with legs has a vertical opening of 10 ft and in this study is noted as a "24-ft elliptical arch with legs." Its shape was derived by adding 24-in. vertical legs to the standard 24-ft elliptical shape. The proposed 24-ft circular arch does not have an added leg and thus has a vertical opening 2-ft shorter than that of the elliptical arch with legs.

For the elliptical shape with legs, the stresses were above the cracking stress for all eight load cases, although they did drop approximately 25 percent when lateral load was added. The tensile stresses at the base were nearly 10 percent higher than those at the eighth point and the deflections were about twice as high as those of the elliptical arch without legs.

Like the 30-ft arches, the preferred method of achieving the extra vertical clearance may be to add a short abutment wall.

CONCLUSIONS AND RECOMMENDATIONS

After reviewing the results of the 24-, 30-, and 40-ft arch data, it is evident that circular arch geometry is preferred over elliptical shapes for the load combinations presented here. The tensile stresses and deflections developed by the circular arches were consistently lower than those of the elliptical shape.

As discussed earlier, the main reason for the difference is related to the geometric shape. A comparison of the 24- and 30-ft shapes, with the crown elevation of each arch being nearly equal, is shown in Figures 1 and 2. This allows for comparison of the curved portions of each arch, showing that the elliptical arch has a steeper slope at the base and is flatter near the crown. A review of the 40-ft arches in Figure 1 shows similar characteristics, including a pronounced flat area at the crown of the elliptical shape.

It should be pointed out, however, that the elliptical shape may prove more effective than a circular shape for other criteria or load cases that were not analyzed by the authors.

The results also show that temperature changes can cause large tensile stresses that cannot be ignored in the design. The present American Association of State Highway and Transportation Officials' code requires similar structures to be analyzed for the effects of a 35 degree temperature rise and a 45 degree temperature fall.

Another important topic that was not investigated in this report is the effect of lateral translation of the footings. In most instances, the thrust from the arch will react on the footing at an angle that may cause outward lateral movement of the footing. If the footings are allowed to translate outward, the tensile stresses at the underside of the crown will greatly increase. The lateral translation can be significantly reduced by using a pile foundation or by anchoring the footings into bedrock.

As mentioned earlier in this paper, the accuracy in idealizing actual field conditions by means of a computer model cannot be easily verified. However, in field measurements of elliptical arch structures the footing translations have been quite low. The effects of soil structure interaction, footing movement, and soil arching are all unknowns that enter the analysis. The only way to validate the effectiveness of the load distributions chosen is to experimentally test small- or full-scale models of each arch. Such testing will provide actual interface pressures, stresses, and deflections and give an indication of the effects of soil structure interaction. However, even though the load distributions and material properties used in this study may not exactly match the actual field conditions, they do provide a good basis for comparing arch geometry and the effects of vertical loads.

Mn/DOT is presently working to fine tune its final selection of arch shapes. This work has included an analysis to optimize the rise-span ratio for each circular shape and computing the effects of moving HS20 live loads over arches with shallow fills. Future work will include the effects of construction loading and the monitoring of full-scale arches to compare the analytical results with actual field data.

REFERENCES

1. K. J. Bathe. *Finite Element Procedures in Engineering Analysis.* Prentice-Hall, Inc., Englewood Cliffs, N.J., 1982.
2. *ADINA Users Manual.* ADINA Engineering, Report AE 81-1, Sept. 1981.
3. Research Engineers. STAAD-III Program User Manual. Marlton, N.J., 1986.
4. *Standard Specification for Highway Bridges*, 13th ed. Section 8.5, American Association of State Highway and Transportation Officials, Washington, D.C., 1983.
5. C. K. Wang and C. G. Salmon. *Reinforced Concrete Design.* Intext Press Inc., New York, 1973.

DISCUSSION

NEAL FITZSIMONS

10408 Montgomery Avenue, Kensington, Md. 20895.

For the practicing engineer, governmental or private, this paper can be too easily misinterpreted. The authors seem to say that field observations of cracking in the soffit of a few arch elements of one or two bridges created great concern for their durability and that this study was undertaken to understand why the cracks occurred and to provide the basis for new geometries that do not have this problem. Several pages of detailed computer printout of maximum tension stresses are provided that show the reader that in a circular geometry the maximum face stresses are less than those in the elliptical geometry for a series of eight static load cases, all of which have an overfill of only 2 ft. From this highly theoretical set of results, the authors seem to imply that a circular arch would be more durable than an elliptical arch.

It is implicitly assumed in this paper that the soffit cracks (in an arch element of an elliptical bridge) that appear to be the original cause for concern [see Bathe (*1*) in the paper] are caused by tension stresses induced by static loading, not by other causes such as craning during construction or improper backfilling procedures. This is despite the fact that dozens of bridges (involving hundreds of arch elements) built to identical specifications in Australia, Europe, and the United States, are visually crack free. Some of these structures are more than 20 years old. In only one case (other than in Minnesota), soffit cracks in a single arch element very probably caused by craning were observed by inspectors and judged to warrant additional monitoring. Although still under observation, there appears to be little likelihood that the durability of the structure has been compromised.

There also seems to be an implicit assumption in the paper that cracks are to be avoided as a "primary design concern," even those that are less than 0.01 in. in width, which is a widely accepted standard for permissible widths without compromising durability. The ideal of visually crack-free concrete is desirable, but practitioners generally accept that crack control is a reasonable strategy for producing durable structures.

Because the parabola is widely recognized as the ideal geometry for a uniformly arch structure in terms of tension-free stresses, it is strange that this was not studied rather than circular segments. There is no rationale presented for the selection of the circular section, nor is the parabola even mentioned. In his 1937 book on continuous structures and arches, Charles Spofford writes "Segmental arches are seldom used for bridges, but inasmuch as they are susceptible, if of uniform cross section, to precise analysis, they are treated fully in Chapter VI." Of course, the reason for the elliptical section is that it has hydraulic characteristics more desirable than the circular or the parabolic arch. Because the primary design concern is the passage of water under the arch (otherwise there would be no need for the structure), the elliptical geometry has been used for centuries for this purpose.

There are some theoretical questions about this paper. Why was an approximate method such as finite element method (FEM) compared with the "precise" elastic analysis? What were the "errors of closure" in the authors' FEM calculations? I have made more than a few FEM analyses of arches and found that they are sensitive to the number of nodes and that for the spans studied, 50 or more elements were needed to keep the errors of closure within acceptable limits. Also, in one case, the authors used the same number of nodes, 42, for the elliptical geometry as they used for the circular geometry. Because the length of the elliptical arch is greater, this calculation would have a greater error of closure than would the circular.

Neglected in this study is the effect of steel reinforcement and it therefore does not use an interaction diagram to determine stresses at the interior and exterior surfaces. The effect this has on the results is not discussed by the authors. Of course, it is the reinforcement that "liberated" concrete arch bridge design from being a mere copy of the stone arches. Being able to accept some moment-induced tensile stress without significant cracking is the reason that reinforced concrete arch bridges can be designed with geometries that enable the structure to perform its primary function more efficiently.

Instead of using a "tire print" and distributing the wheel load longitudinally, the study uses a load wedge of 3,200 lb. This is unnecessarily unrealistic. Further, the study indicated that the arch elements were 6 ft wide, therefore only one wheel on an arch element would give its maximum load. However, a given wheel load is 16,000 lb, which divided by 6 would give a load of 2,667 lb rather than the 3,200 lb that assumes a 10-ft lane. Also, a moving wheel load would produce only transitory crack openings, giving water little chance to penetrate upward into the soffit.

In the conclusion, the authors write "This allows for comparison of the curved portions of each arch showing that the elliptical arch has a steeper slope at the base and is flatter near the crown." Of course it is! This is the basic difference that makes the elliptical arch preferable to the circular arch for stream crossings.

In summary, although this paper provides some interesting results from applying a highly theoretical set of conditions to a highly theoretical set of arches using FEM, it does not provide a practical basis for selecting arch geometries in real-world situations. Despite technical caveats that are scattered through the paper, readers who are not familiar with short-span arches might receive the erroneous impression that elliptical sections should be avoided in favor of circular segments solely because they are theoretically less durable.

AUTHORS' CLOSURE

The authors would like to thank FitzSimons for his discussion comments. His design work for the manufacturer of elliptical arches has no doubt given him a good background in the design and analysis of such structures.

However the authors would like to clear up several apparent misunderstandings brought forth in the discussion. In the first paragraph, FitzSimons states that all eight load cases had overfill depths of only 2 ft. As shown in Figure 4 of the paper and described in the text, Load Cases 2 and 6 had overfill depths in excess of 12 ft.

The authors were also surprised that the discussion included comments concerning durability. The purpose of the study was to compare the effects of arch geometry on the anticipated state of stress. The issue of durability has not been investigated.

Several other comments in the discussion address crack control and the authors' primary design concern to limit cracking. One particular sentence in the discussion noted that the authors limited cracks, "even those that are less than 0.01 in. in width." In the report the authors actually write "a primary design concern is to keep the structure relatively free from cracking." They go on to say that "Although all concrete structures are subject to temperature and shrinkage cracking, the main concern is to limit the tensile cracks caused by dead and live loads and temperature effects." There is no reference made to not allowing cracks of width less than 0.01 in. or of cracking causing the durability to be compromised.

FitzSimons also poses the question of why a circular shape was used instead of a parabolic shape, which produces a tension-free structure under uniform load. This question is answered by examining an arch with 2 ft of fill at the crown. This results in fill heights of from 8 to 13 ft at the base, depending on the span of the arch. Because of this difference in soil depth, the loads at the base may be from 4 to 6 times higher than the load at the crown, producing a load diagram that is far from uniform and diminishing any advantage of using a parabolic shape.

Arches that have higher curvatures will tend to be in more of a compressive mode than will arches with lower curvatures. A circular shape seems to be an optimum choice between the two groups and was therefore considered in this study. Nevertheless, designers may wish to consider other shapes for particular projects after a careful investigation of their performance.

The discussion also recommends using at least 50 beam elements to model the entire structure and questions the authors' decision in one particular case to use 42 elements to model both the elliptical and circular shape. However, further investigation reveals that the sum of the element lengths for each arch differs by less than ½ of 1 percent for that particular set of arches. Because symmetry was used in the modeling, the shapes were analyzed using 42 elements for half of the arch, which is equivalent to using 84 elements to model the full arch. This is far in excess of the recommended 50 and the very small difference in structure length will produce negligible closure error.

Concerning an elastic analysis, there is no real justification to use an elastic analysis for studying the behavior of reinforced concrete structures beyond the inception of cracking. From that point on this structural behavior enters the nonlinear domain and an approximate analytical method is required.

Steel reinforcement was not included explicitly in the analysis because the major advantage of arch structures is in resisting load through compressive action. The contribution of steel, although important, is a secondary parameter under these conditions. If flexure is taken into consideration, the overall depth of the cross section becomes significantly more important. Increasing the amount of steel has a small effect on the moment of inertia compared with increasing the depth of the cross section. The selection of reinforcement was carried out using conventional techniques, and this has been done in a later phase of the present study.

With regard to the choice of loads, as discussed in the paper, the same loads were consistently applied to each shape and the relative behavior was compared accordingly. As far as transient loads are concerned, dynamic analysis of these systems has not been performed during this phase of the study. It is known that dynamic loads may affect such structures well beyond "transitory crack openings" and it is recommended that such considerations be addressed in the future.

The later part of the discussion highlights the fact that the elliptical shape provides greater area for the flow of water through the opening. Of the structures analyzed in this study, the elliptical shape allowed from 2 to 9 percent more flow. If the required flow area becomes a critical design requirement, the elliptical shape would prove more effective than the circular shape. However, the authors believe that the results of this study show that if a slight reduction in flow area can be permitted, a circular arch shape could be used which, for the load cases analyzed herein, should produce smaller tensile stresses within the structure.

Additional research is needed to address questions related to the ultimate behavior of the structures, soil structure interaction, and dynamic effects. It is also worth noting that in 1987 the California Department of Transportation was granted $600,000 to study arch structures and soil structure interaction. They are currently in the process of finalizing the design of these structures, using circular shapes with thicknesses of less than 10 in.

Again the authors would like to thank FitzSimons for his comments. They hope that the discussion and response clarify the issues with respect to the study. The authors would be happy to provide any further information upon request.

Publication of this paper sponsored by Committee on Culverts and Hydraulic Structures.