New Algorithm for Solving the Maximum Progression Bandwidth

HUEL-SHENG TSAY AND LIANG-TAY LIN

Two popular computer programs, MAXBAND and PASSER II, are widely used in obtaining the maximal bandwidth. However, these bandwidths may not be realized or only be partly realized if the resultant signal timings are actually applied on the arterial. This phenomenon can be observed from field tests or from a time-space diagram. In this paper two examples demonstrate the problem. A new algorithm is proposed for solving the bandwidth problem and provides the user with a more realistic maximum progression bandwidth. The algorithm uses a general mixed-integer programming formulation, and a program BANDTOP based on this formulation has been developed to obtain the real progression bandwidth. It has been tested on street networks in Taiwan, where it has proved very effective. The major variation from traditional methods is that the bandwidth has a saw-toothed pattern in both directions instead of parallel and uniform. Any vehicle in the segment is allowed to travel through the entire section of an arterial with at most one stop.

The coordination of traffic signals on the arterial is an effective way of reducing stops, delay, and excessive fuel consumption. Previously, signal settings were determined from the time-space relation of signal timing and traffic flow by manual methods. As researchers begin to use computers to increase analysis flexibility and reduce computational effort, it becomes possible to develop new approaches that take into account more variables and complex equations involved in reflecting the real situation. The United States and many other nations use Maximal Bandwidth (MAXBAND) and Progression Analysis and Signal System Evaluation Routine (PASSER) II, which are based on maximizing two-way traffic through bandwidth. MAXBAND and PASSER II can automatically take traffic demands to determine two-way progression bandwidths and to provide users with other information, such as cycle length, phase sequence, offsets, phase lengths, and a time-space diagram for practical use. Here, the bandwidth is defined as the vehicles within a time interval, present at a given traffic signal or point, that can travel through downstream signals of an arterial without stopping.

The progression bandwidth of MAXBAND was mainly calculated from Little's general mixed-integer linear programming formulation (*1–5*). It obtains a global optimum of bandwidth, cycle length, offsets, and phase sequence with no starting solution. MAXBAND also has the capability to allow small deviations from the arterialwide progression speed on individual links (*6*). PASSER II is a macroscopic, deterministic program that obtains the optimal timing settings from maximizing progression bandwidth in both directions. It was developed by Messer through Little's half-integer synchronization and expansion of Brook's algorithm by selecting the offsets that minimize the total interference to the progression band (*2, 7–10*). The newest version of PASSER II-84 can analyze the phase sequence of any NEMA style from two to eight phases and find minimum delay through fine-tuning of the offsets while allowing the maximization of bandwidth to dominate (*11*). The heuristic optimization technique used in PASSER II does not produce the widest possible green bands, unlike MAXBAND, which guarantees a global optimum (*12*).

Since both approaches have impressive bandwidths through time-space diagrams, a substantial number of practicing traffic engineers may use the output as the arterial signal timing settings. Unfortunately, these bandwidths actually will not be realized or only be realized in fraction. The phenomena can be observed from the field or time-space diagrams. One could argue that many fairly restrictive hypotheses related to these bandwidth approaches exist. The assumptions include a uniform platoon, no platoon dispersion, low volumes, and no or few vehicles entering the arterial from side streets; but situations corresponding to these assumptions are rare and unreasonable.

Many traffic engineers prefer a maximization of synchronized green phases using time-space diagrams to satisfy the public's demand. Since the assumptions made in the two programs are unrealistic, the use of MAXBAND and PASSER II output on an arterial may result in unexpected stops, delay, and even more fuel consumption to the entire system. It is necessary therefore to develop a new algorithm for solving the maximum progression bandwidth that allows the driver to travel at the design speed without any stop. In this paper, as the first step, two examples define the existing problem of bandwidths obtained from MAXBAND and PASSER II. Then, a new algorithm is developed to provide users a real maximal bandwidth without interference. A complete mixed-integer programming formulation of the new algorithm is proposed and discussed in detail. Finally, the new algorithm is tested on street networks and proved effective.

THE EXISTING PROBLEM

When the lights are red, queues build up as a result of turning movements into the arterial at the previous intersection before the appearance of green. The queue includes not only turning vehicles from the previous intersection but also the vehicles that do not pass through the arterial at the end of the last

Graduate School of Transportation and Communication Management Science, National Cheng Kung University, Taiwan, Republic of China.

green time. The phenomena are quite obvious and should not be neglected at any intersection during the entire day.

Although the assumptions of no or very few flow left on the arterial and entering the arterial from side streets were made by MAXBAND and PASSER II, the two programs still allow the user to specify a queue clearance time at any intersection in either direction. A queue clearance time can either be a fraction of the cycle length or actual time units to deal with queues (*1, 10*). The program then adjusts the through vehicles to arrive at the intersections after the queue has cleared and leave the intersections with the queue as a part of the band. This puts a jog into the through band, advancing it upon leaving the intersection by an amount equal to the queue clearance time (*1*). In other words, MAXBAND and PASSER programs admit the existence of queue at each intersection and try to use the queue clearance time to handle this unavoidable situation. If a queue clearance time is being considered at each intersection of the arterial, however, its maximal bandwidth will be severely affected and sometimes reduced to a very small value. In addition, since queue clearance time is an arbitrary number specified by the user, it is difficult to provide the user with guidelines for setting a reasonable value at a particular intersection.

Figures 1 and 2 show the time-space diagrams of PASSER II-84 and MAXBAND for Zin-Wha Arterial with four intersections in Tainan City, Taiwan. Both programs were run on the same information of arterial configuration and traffic flows as input. The major difference between the two figures lies in the dot points in PASSER II that represent signal green time but indicate red time in MAXBAND and vice versa. From Figures 1 and 2, it can be seen that, in the outbound direction, as the light of intersection 2 (MING-CHEN) turns green, the queueing vehicles at this intersection will move toward the adjacent downstream intersection [i.e., intersection 3 (MING-SEN)] and have to wait at the red light at this intersection. The newly arriving vehicles then join the existing queue to form a new composite queue at intersection 3.

Based on the first-come first-served principle, the vehicles involved in the composite queue have to use the front portion of the next green time. It equals the time needed to depart the total queue at saturation flow rates as the signal turns green. Hence, the incoming through-band vehicles cannot cross intersection 3 unless all queues have cleared. Under such a circumstance, most of the vehicles in the through band are hindered and have to stop. This phenomenon can also be observed from the trajectories of several leading vehicles at intersection 3 in Figures 1 and 2. Here, any intersection that has the bandwidth located in the very front of green time but with a different band location of green time at an adjacent upstream intersection is the critical intersection. The bandwidth of MAXBAND and PASSER II will be affected or decreased at each critical intersection.

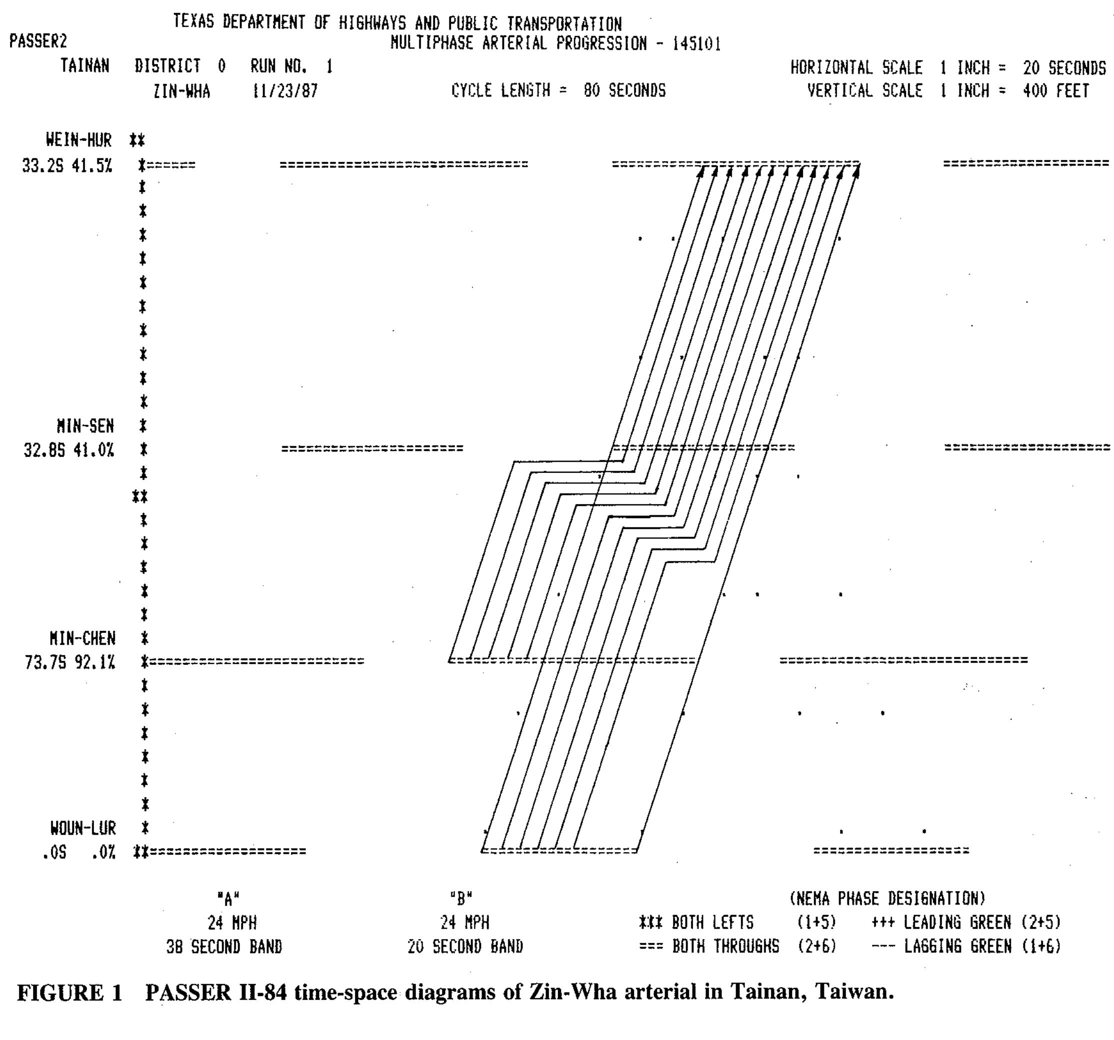

FIGURE 1 PASSER II-84 time-space diagrams of Zin-Wha arterial in Tainan, Taiwan.

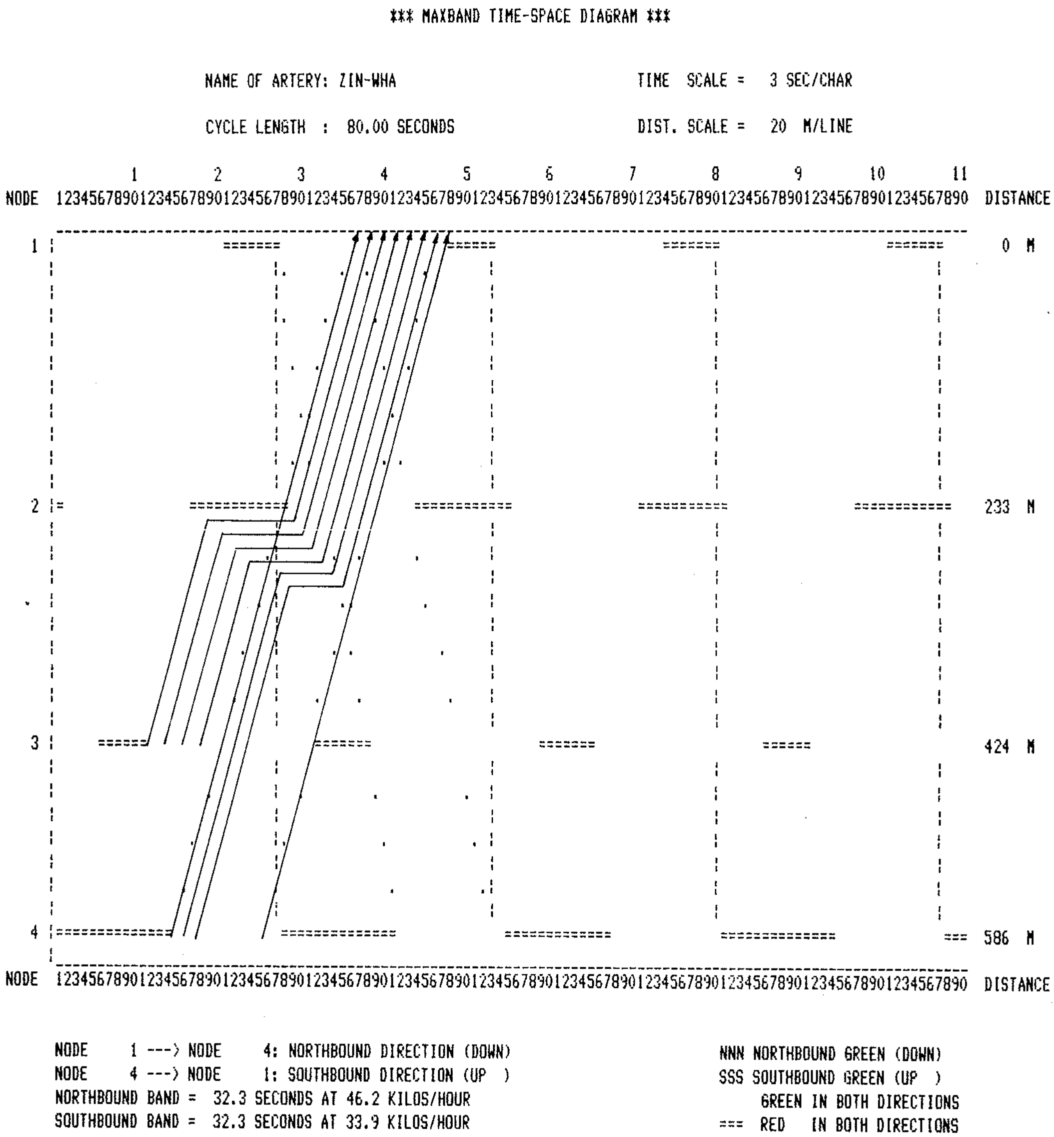

FIGURE 2 MAXBAND time-space diagrams of Zin-Wha arterial in Tainan, Taiwan.

One may consider using queue clearance time to avoid the problem encountered at the critical intersection. However, because queue clearance time set in MAXBAND and PASSER II attempts to clear the queue due to turning movements, it cannot handle the composite queue involving the existing queue and incoming upstream vehicles. In addition, the user has to specify queue clearance time at any intersection. It is impossible to know which intersection needs queue clearance time and what value should be used. Even if the value is assumed, the critical intersection will soon be changed to another intersection based on the output of MAXBAND and PASSER II. The existing problem still remains unsolved.

At this point, several questions arise concerning the critical intersection, for example, intersection 3 in Figures 1 and 2. How much time is needed to clear the composite queue? Can we prespecify the queue clearance time to prevent this phenomenon? How many seconds of the maximum bandwidth from the two programs will not be used due to the composite queue? Will this value just be equal to the time required to relieve the queue? As a result, how many seconds of bandwidths that vehicles can travel from the critical intersection till the last intersection of the designated arterial without stopping will be realized?

Three outputs of MAXBAND and PASSER II have been tested on three arterials in two cities in Taiwan, Tainan and Taipei, under various flow conditions. The results showed that these bandwidths could not be realized and their signal timing plans gave more stops and delay than the original one. Traffic engineers in both cities could not explain the reason. One claimed it was probably due to different driver behavior and cultural background as programs developed in the United States may not be suitable in other countries. In fact, the problems mainly come from inaccurate progression theory because of unrealistic assumptions involved in the two programs.

According to several tests in the field which utilized the bandwidths obtained from PASSER II and MAXBAND, at least double the time needed to clear the composite queue will not be available at the critical intersection. That is to say, if the width of green band minus double the composite queue clearance time is less than zero, the progression band will not be available under given conditions. For example, in Figure 1, the outbound bandwidth (38 sec) will not be realized if the composite queue requires more than 19 sec to clear at intersection 3. This is the serious drawback of current MAXBAND and PASSER in practical applications; therefore, it becomes necessary to develop an algorithm that can obtain the real maximum bandwidth. One way to perform this study is to discuss the progression theory used in MAXBAND as the first step. The following section serves this purpose.

MAXBAND FORMULATION

The time-space diagram of MAXBAND showing green bands was presented by Little et al. (*1*) and is shown in Figure 3. Inbound and outbound green bands pass through signals S_h and S_i. Quantities with bars refer to inbound reds, are drawn solid, and above inbound reds need not coincide (*1*). The definitions of variables shown in Figure 3 are as follows:

- b = outbound bandwidth,
- S_i = ith signal ($i = 1, \ldots, n$),
- r = outbound red time at S_i,
- W_i = time from right side of red at S_i to left edge of outbound green band,
- $t(h, j)$ = outbound travel time from S_h to S_i,
- $\phi(h, i)$ = time from center of an inbound red at S_h to the center of a particular outbound red at S_i,
- Δ_i = time from center of r_i to nearest center of r_i, and
- τ_i = queue clearance time.

A general mathematical programming formulation of MAXBAND given by Little et al. (*1*) is presented in the following. All variables and symbols are based on Figure 3 except that signal h is replaced by symbol i and signal i is substituted by $i + 1$. It is defined as $x = x(i, i + 1)$, for $x = t, \bar{t}, m, \phi, \bar{\phi}$.

$$\text{Max } b + k\bar{b}$$

$$ST\ (1\text{-}k)\bar{b} \geq (1\text{-}k)kb$$

$$1/T_2 \leq Z \leq 1/T_1$$

$$W_i + b \leq 1 - r_i \qquad i = 1, \ldots, n$$

$$W_i + b \leq 1 - r_i \qquad i = 1, \ldots, n$$

$$(W_i + \overline{W}_i) - (W_{i+1} + \overline{W}_{i+1}) + (t_i + \bar{t}_i) + \delta_i l_i - \bar{\delta}_i l_i - \delta_{i+1} l_{i+1} + \bar{\delta}_{i+1} \bar{l}_{i+1} - m_i = (r_{i+1} - r_i) + (\bar{\tau}_i + \bar{\tau}_{i+1}), \qquad i = 1, \ldots, n-1$$

$$(d_i/f_i)\, Z \leq t_i \leq (d_i/e_i)Z, \qquad i = 1, \ldots, n-1$$

$$(\bar{d}_i/\bar{f}_i)\, \bar{Z} \leq \bar{t}_i \leq (\bar{d}_i/\bar{e}_i)Z, \qquad i = 1, \ldots, n-1$$

$$(d_i/h_i)Z \leq (d_i/d_{i+1})t_{i+1} - t_i \leq (d_i/g_i)Z, \qquad i = 1, \ldots, n-2$$

$$(\bar{d}_i/\bar{h}_i)Z \leq (\bar{d}_i/\bar{d}_{i+1})\, \bar{t}_{i+1} - t_i \leq (\bar{d}_i/\bar{g}_i)Z, \qquad i = 1, \ldots, n-2$$

$$b, \bar{b}, Z, W_i, \overline{W}_i, t_i, \bar{t}_i \geq 0$$

$$m_i = \text{integer}$$

$$\delta_i, \bar{\delta}_i = 0, 1$$

where

- K = target ratio of inbound to outbound bandwidth;
- T = cycle length;
- $Z = 1/T$ = signal frequency;

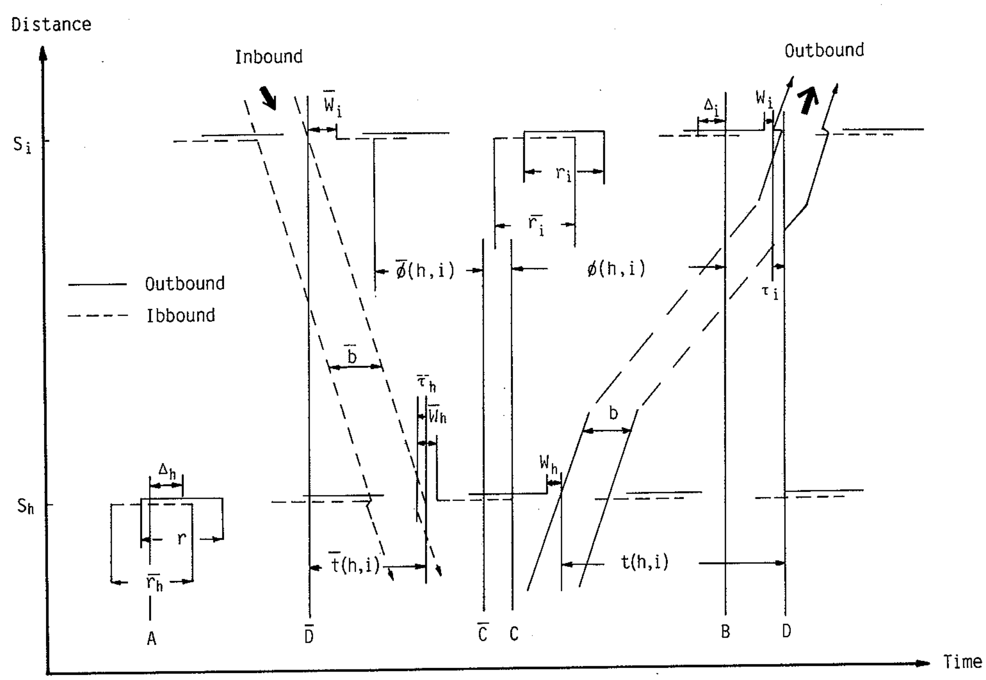

FIGURE 3 Time-space diagrams of MAXBAND showing green bands (*1*).

T_1, T_2 = lower and upper limits on cycle length (i.e., $T_1 \leq T \leq T_2$);
$d(h,i)$ = distance between S_h and S_i outbound;
d_i = $d(i,i+1)$ = distance between outbound intersections i and $i+1$;
e_i, f_i = lower and upper limits on outbound speed;
$1/h_i, 1/g_i$ = lower and upper limits on change in outbound reciprocal speed; and
t_i = travel time from outbound intersection i to intersection $i+1$ = $(d_i/V_i)Z$, V_i = travel speed.

This formulation has the following deficiencies:

1. The progression band cannot be fully realized or can only be partly realized.
2. Queue clearance time is prespecified by the user. In fact, it cannot be a fixed value and will be varied with the queue length of each intersection. This value should be determined internally through the computation of traffic flow movements.
3. The traffic flow model is oversimplified. No account is taken of secondary flows turning from side streets and platoon dispersion (*6*).
4. The time lag between the remaining portion of green time after the band and start of red time and the time difference between the beginning of green time and of bandwidth in either direction are not clearly distinguished. Only a variable W_i is used to represent this time difference and may cause confusion.
5. A symbol error exists in the inbound speed change of the above MAXBAND formulation. For consistency, the equation should be changed to the following form:

$$(\bar{d}_{i+1}/\bar{h}_{i+1})Z \leq (\bar{d}_{i+1}/\bar{d}_i)\bar{t}_i - \bar{t}_{i+1} \leq (\bar{d}_{i+1}/\bar{g}_{i+1})Z$$

MATHEMATICAL FORMULATION OF NEW ALGORITHM

Based on the above discussion, a new algorithm for obtaining the real maximal bandwidth will be developed in this section. To explain the new algorithm more easily, similar notations and definitions considered in the MAXBAND formulation are used. The following described variables refer to the above section unless otherwise specified. Major features of this new algorithm compared to the MAXBAND formulation are:

1. Divide the time between the start of green time and of the bandwidth into two parts: queue clearance time (Q_i) and incoming flow clearance time (H_i) at each intersection. Here, the queue clearance time is used to clear queues due to turning vehicles during red time and through vehicles that do not go through the arterial at the end of the last green time. Incoming flow clearance time represents the time needed by the incoming vehicles that come from the adjacent upstream intersection but excludes vehicles in the through band, to depart the upstream intersection.
2. Specify the time lag from the right side of the bandwidth to the left edge of outbound red as W_i at intersection i. It is noted that this new definition of W_i is different from the one used in the MAXBAND.
3. Add a composite queue clearance time constraint.
4. Add a constraint that guarantees the progression bandwidth to be fully used by vehicles without stopping.
5. Add the minimum green time of side streets.
6. Provide the selection of eight left-turn phase patterns.
7. Find the minimum cycle time by making minor changes of the objective function and constraints under given bandwidths in both directions.

A time-space diagram of the new algorithm concerning green bands is given in Figure 4. All variables involved and equations derived later are based on the relationships shown in this figure. Any variable with a bar represents the inbound flow. Otherwise, it indicates the outbound flow. Since Little et al. have provided detailed information to derive some equations (*1*), similar derivations of these equations are omitted and only the final equations with the newly added variables will be shown.

Objective Function

$$\text{Max } b + \bar{b} \tag{1}$$

Constraints

Geometric Relationship

$$t_i + \bar{t}_i + (1/2)(r_i + \bar{r}_i) - (1/2)(r_{i+1} + \bar{r}_{i+1}) + (Q_i - Q_{i+1}) + (H_i - H_{i+1}) + (\overline{W}_i - \overline{W}_{i+1}) + \Delta_i - \Delta_{i+1} = I_i \tag{2}$$

where I_i is an integer.

Offsets

$$\text{OFF}_i = t_i + (Q_i - Q_{i+1}) + (H_i - H_{i+1}) \tag{3}$$

$$\overline{\text{OFF}}_i = \bar{t}_i + (\overline{Q}_{i+1} - \overline{Q}_i) + (\overline{H}_{i+1} - \overline{H}_i) \tag{4}$$

Common Cycle

In the coordinated signal intersections of an arterial, every intersection within the segment has the same cycle length. Therefore,

$$Q_i + H_i + b + W_i + r_i = 1 \tag{5}$$

$$\overline{Q}_i + \overline{H}_i + \bar{b} + \overline{W}_i + \bar{r}_i = 1 \tag{6}$$

Bandwidth

To guarantee a real bandwidth, the following equations have to be added as bandwidth constraints:

$$H_{i+1} \geq H_i + Q_i \tag{7}$$

$$\overline{H}_i \geq \overline{H}_{i+1} + \overline{Q}_{i+1} \tag{8}$$

From equations (7) and (8), the final shape of the progression bandwidth will be a saw-toothed pattern.

Queue Clearance Time

Before discussing the queue length, it is necessary to explain the arrival types of incoming vehicles from the adjacent

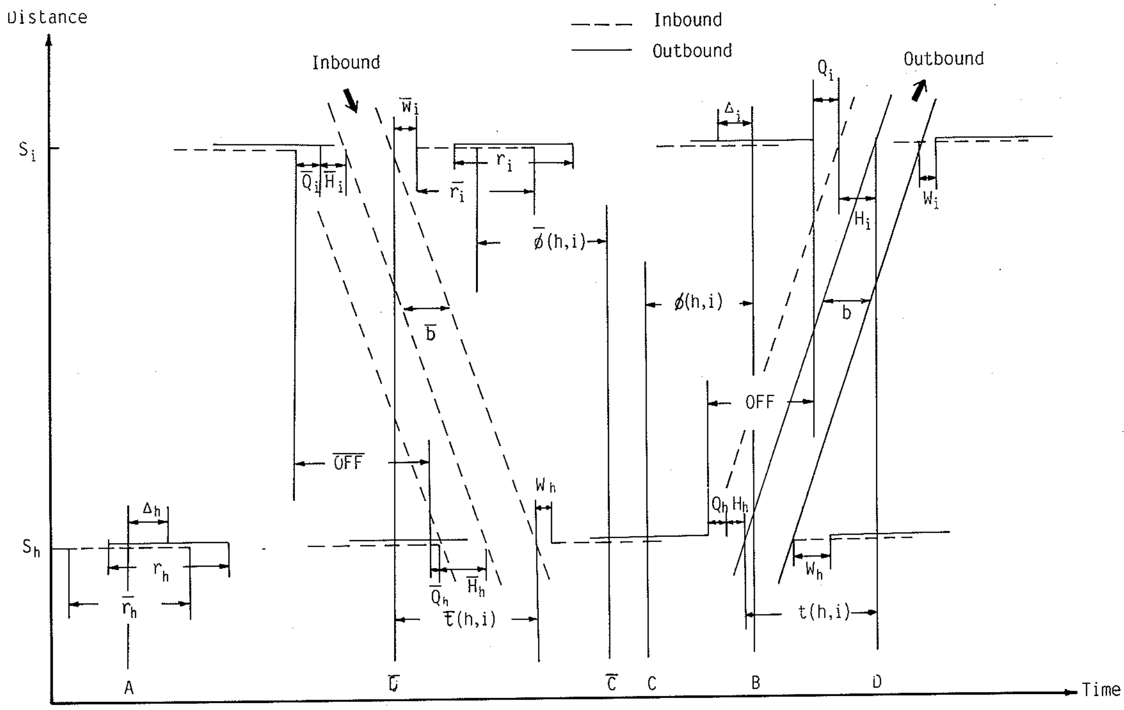

FIGURE 4 Time-space diagrams of new algorithm.

upstream intersection. The vehicle arrival type depends on through traffic volume, turning vehicles, and timing plan at the upstream intersection. Two types of arriving vehicles exist. Vehicles departing from the upstream intersection during green belong to type 1. Similarly, vehicles leaving the upstream intersection during red are referred to as type 2.

For outbound flow, the arrival rate of type 1 from upstream intersection i to intersection $i+1$ is

$$\lambda_{i+1,1} = V_{i,T}/[(g_i/C) \times 3600 \times N_{i+1}] \tag{9}$$

where

$\lambda_{i+,1}$ = arrival rate of type 1 at intersection,
$V_{i,T}$ = through traffic volume at intersection i,
g_i = green time of intersection i,
N_{i+1} = number of lanes at intersection $i+1$, and
C = cycle time.

The arrival rate of type 2 at the intersection $i+1$ is

$$\lambda_{i+1,2} = (V_{i,R} + V_{i,L}) / [(r_i/C) \times 3600 \times N_{i+1}] \tag{10}$$

where

$\lambda_{i+1,2}$ = arrival rate of type 2 at intersection $i+1$,
$V_{i,R}$ = right turn volume at intersection i,
$V_{i,L}$ = left turn volume in the opposite approach at intersection i, and
r_i = red time interval at intersection i.

Because the model has to satisfy the bandwidth constraint given in equations (7) and (8), two cases can be drawn to show the relationships of any two neighboring intersections.

Case 1: $W_{i+1} \geq W_i$. The time lag from the right side of the bandwidth to the left edge of outbound red at an intersection is greater than or equal to that of the adjacent upstream intersection. This can be displayed in Figure 5(a). The queuing vehicles $QV_{i+1,1}$ of intersection $i+1$ in the outbound direction can be obtained from the following equation:

$$QV_{i+1,1} = \lambda_{i+1,2} \times [r_i - (W_{i+1} - W_i)] \times C \quad \text{if } W_{i+1} \geq W_i \tag{11}$$

Case 2: $W_{i+1} \leq W_i$. This case is shown in Figure 5(b). Queuing vehicles at intersection $i+1$ are

$$QV_{i+1,2} = [\lambda_{i+1,1} \times (W_i - W_{i+1}) + \lambda_{i+1,2} \times r_i] \times C \quad \text{if } W_{i+1} < W_i \tag{12}$$

where $QV_{i+1,2}$ is queuing vehicles at intersection $i+1$ under case 2.

After obtaining queuing vehicles and assuming saturation flow rates, the queue clearance time of outbound flow becomes

$$Q_{i+1} \geq SH_{i+1} \times \lambda_{i+1,2} \times [r_i - (W_{i+1} - W_i)] \quad \text{if } W_{i+1} \geq W_i \tag{13}$$

or

$$Q_{i+1} \geq SH_{i+1} \times [\lambda_{i+1,1} \times (W_i - W_{i+1}) + \lambda_{i+1,2} \times r_i] \quad \text{if } W_{i+1} < W_i \tag{14}$$

where SH_{i+1} is saturation flow headway for outbound time flow at intersection $i+1$. Similarly, the queue clearance time of inbound flow is

$$\overline{Q}_i \geq \overline{SH}_i \times \bar{\lambda}_{i,2} \times [\bar{r}_{i+1} - (\overline{W}_i - \overline{W}_{i+1})] \quad \text{if } \overline{W}_{i+1} \leq \overline{W}_i \tag{15}$$

or

$$\overline{Q}_i \geq \overline{SH}_i \times [\bar{\lambda}_{i,1} \times (\overline{W}_{i+1} - \overline{W}_i) + \bar{\lambda}_{i,2} \times \bar{r}_{i+1}] \quad \text{if } \overline{W}_{i+1} > \overline{W}_i \tag{16}$$

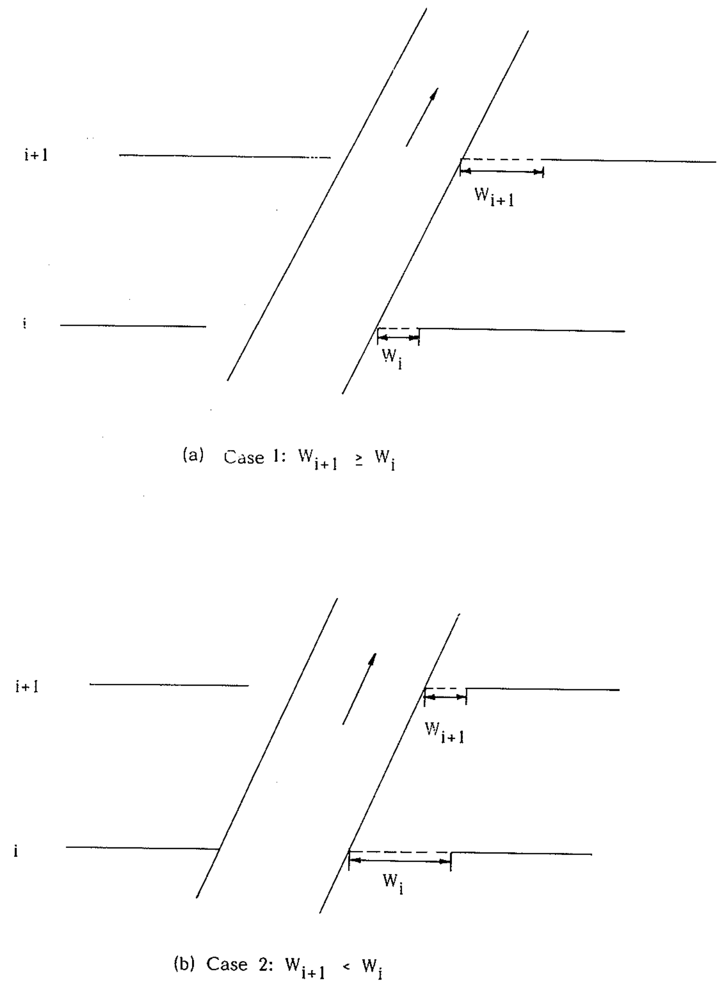

FIGURE 5 Two cases of queueing vehicles.

Because most of the green time of downstream intersection $i+1$ will be used by the queue clearance time and incoming flow clearance time, it is recommended equations (14) and (16) be used as the queue clearance time of the outbound flow and inbound flow, respectively.

Bandwidth of Each Intersection

The bandwidth constraint shown by equations (7) and (8) gives a real bandwidth for vehicles to travel through all downstream intersections of an arterial without interference. This new algorithm also provides additional progression opportunities at intersections in both directions except the through bandwidth. For example, vehicles move before the left edge of through band during green time and can arrive at the last intersection without stopping for the outbound direction. This is defined as the bandwidth of each intersection; the bandwidth is saw-toothed. The bandwidths of intersection i in either direction are represented in the following two equations:

$$b_i = b + Q_i + H_i \tag{17}$$

$$\bar{b}_i = \bar{b} + \overline{Q}_i + \overline{H}_i \tag{18}$$

From equations (17) and (18), it can be determined that vehicles outside the through band need to stop at most once to pass through the entire arterial.

Directional Bandwidth Weight

We can set up weights for different directions:

$$\bar{b} = K\,b \tag{19}$$

where K is a relative weight ratio between inbound and outbound bandwidths.

If K is greater than 1, the inbound bandwidth is wider than the outbound one.

Minimum Green Time

This constraint guarantees that each side street has a minimum green time to prevent overdelay of vehicles from the side street and give pedestrians enough time to cross the arterial safely.

$$\tfrac{1}{2}(r_i + \bar{r}_i) - \Delta_i \geq MIG_i$$

$$r_i \geq MIG_i$$

$$\bar{r}_i \geq MIG_i \tag{20}$$

where MIG_i is minimum green on side street at intersection i.

Cycle Limit

$$1/C_2 \le Z \le 1/C_1 \tag{21}$$

Speed Limit

For outbound flow:

$$(d_i / f_i)Z \le t_i \le (d_i/m_i)Z \tag{22}$$

For inbound flow:

$$(\bar{d}_i / \bar{f}_i)Z \le \bar{t}_i \le (\bar{d}_i/\bar{m}_i)Z \tag{23}$$

where m_i , f_i are lower and upper limits of outbound speed.

Speed Change Limit

For outbound flow:

$$(d_i/h_i)Z \le (d_i/d_{i+1})t_{i+1} - t_i \le (d_i/n_i)Z \tag{24}$$

For inbound flow:

$$(\bar{d}_{i+1}/\bar{h}_{i+1})Z \le (\bar{d}_{i+1}/\bar{d}_i)\bar{t}_i - \bar{t}_{i+1} \le (\bar{d}_{i+1}/\bar{n}_{i+1})Z \tag{25}$$

where $1/h_i$, $1/n_i$ are lower and upper limits on change in outbound reciprocal speed.

Left-Turn Phase

Eight possible patterns of left-turn green phases exist in this new algorithm. The eight left-turn phase patterns are shown in Figure 6. Let

g_i = outbound green time for through traffic,
l_i = time allocated for outbound left-turn green at intersection i, and
R = common red time in both directions to provide for side street movements.

This gives

$$r_i = R + \bar{l}_i$$
$$\bar{r}_i = R + l_i$$
$$r_i + g_i = 1$$
$$\bar{r}_i + \bar{g}_i = 1 \tag{26}$$

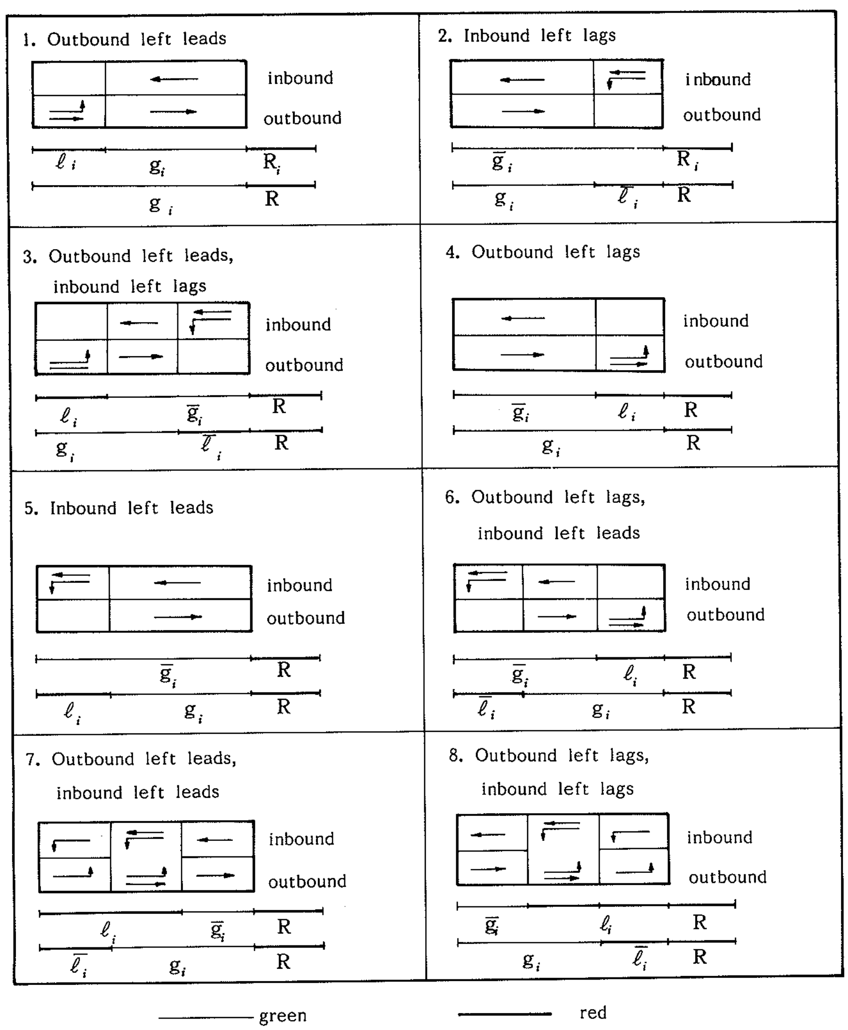

FIGURE 6 Eight possible patterns of left-turn phases.

TABLE 1 EIGHT POSSIBLE PATTERNS OF LEFT-TURN PHASE IN TERMS OF l_i, $\bar{l}_i$, AND 0-1 VARIABLES

Left-turn phase pattern	Δ_i	β_i	$\bar{\beta}_i$	α_i	$\bar{\alpha}_i$
1	$-\frac{1}{2}\ell_i$	0	1	1	0
2	$-\frac{1}{2}\bar{\ell}_i$	0	1	0	1
3	$-\frac{1}{2}(\ell_i+\bar{\ell}_i)$	0	1	1	1
4	$\frac{1}{2}\ell_i$	1	0	1	0
5	$\frac{1}{2}\bar{\ell}_i$	1	0	0	1
6	$\frac{1}{2}(\ell_i+\bar{\ell}_i)$	1	0	1	1
7	$-\frac{1}{2}(\ell_i-\bar{\ell}_i)$	0	0	1	1
8	$\frac{1}{2}(\ell_i-\bar{\ell}_i)$	1	1	1	1

These eight phases Δ_i (time from center of inbound red to nearest center of outbound red at intersection i) can be expressed by l_i and $\bar{l}_i$. The relationships of eight left-turn phases related to l_i and $\bar{l}_i$ are given in Table 1. Furthermore, each left-turn phase can be represented by the following general equation:

$$\Delta_i = (1/2)[(2\beta_i - 1)\alpha_i l_i - (2\bar{\beta}_i - 1)\bar{\alpha}_i \bar{l}_i] \tag{27}$$

where α_i, β_i, $\bar{\alpha}_i$, $\bar{\beta}_i$ are 0–1 variables.

The values of α_i, β_i, $\bar{\alpha}_i$, $\bar{\beta}_i$ corresponding to each left-turn phase are also given in Table 1.

Based on the previous discussion, a complete mathematical programming formulation of this new algorithm is given as follows:

$$\text{MAX } b + \bar{b}$$

$$ST\ \bar{b} = K b$$

$$1/C_2 \le Z \le 1/C_1$$

$$t_i + \bar{t}_i + (1/2)(r_i + \bar{r}_i) - (1/2)(r_{i+1} + \bar{r}_{i+1}) + (Q_i - Q_{i+1}) + (H_i - H_{i+1}) + (\overline{W}_i - \overline{W}_{i+1}) + \Delta_i - \Delta_{i+1} = I_i \qquad i = 1, \ldots, n-1$$

$$OFF_i = t_i + (Q_i - Q_{i+1}) + (H_i - H_{i+1}) \qquad i = 1, \ldots, n-1$$

$$\overline{OFF}_i = \bar{t}_i + (\overline{Q}_{i+1} - \overline{Q}_i) + (\overline{H}_{i+1} - \overline{H}_i) \qquad i = 1, \ldots, n-1$$

$$Q_i + H_i + b + W_i + r_i = 1 \qquad i = 1, \ldots, n$$

$$\overline{Q}_i + \overline{H}_i + \bar{b} + \overline{W}_i + \bar{r}_i = 1 \qquad i = 1, \ldots, n$$

$$H_{i+1} \ge H_i + Q_i \qquad i = 1, \ldots, n-1$$

$$\overline{H}_i \ge \overline{H}_{i+1} + \overline{Q}_{i+1} \qquad i = 1, \ldots, n-1$$

$$Q_{i+1} \ge SH_{i+1} \times [\lambda_{i+1,1} \times (W_i - W_{i+1}) + \lambda_{i+1,2} \times r_i] \qquad i = 1, \ldots, n-1$$

$$\overline{Q}_i \ge \overline{SH}_i \times (\bar{\lambda}_{i,1} \times (\overline{W}_{i+1} - \overline{W}_i) + \bar{\lambda}_{i,2} \times r_{i+1}] \qquad i = 1, \ldots, n-1$$

$$b_i = b + Q_i + H_i \qquad i = 1, \ldots, n$$

$$\bar{b}_i = \bar{b} + \overline{Q}_i + \overline{H}_i \qquad i = 1, \ldots, n$$

$$(d_i/f_i)Z \le t_i \le (d_i/m_i)Z \qquad i = 1, \ldots, n-1$$

$$(\bar{d}_i/\bar{f}_i)Z \le \bar{t}_i \le (\bar{d}_i/\bar{m}_i)Z \qquad i = 1, \ldots, n-1$$

$$(d_i/h_i)Z \le (d_i/d_{i+1})t_{i+1} - t_i \le (d_i/n_i)Z \qquad i = 1, \ldots, n-2$$

$$(\bar{d}_{i+1}/\bar{h}_{i+1})Z \le (\bar{d}_{i+1}/\bar{d}_i)\bar{t}_i - \bar{t}_{i+1} \le (\bar{d}_{i+1}/\bar{n}_{i+1})Z \qquad i = 1, \ldots, n-2$$

$$(1/2)(r_i + \bar{r}_i) - \Delta_i \ge MIG_i \qquad i = 1, \ldots, n$$

$$r_i \ge MIG_i \qquad i = 1, \ldots, n$$

$$\bar{r}_i \ge MIG_i \qquad i = 1, \ldots, n$$

$$\Delta_i = (1/2)[(2\beta_i - 1)\alpha_i l_i - (2\bar{\beta}_i - 1)\bar{\alpha}_i \bar{l}_i] \qquad i = 1, \ldots, n$$

$$b, \bar{b}, Z, W_i, \overline{W}_i, H_i, t_i, \bar{t}_i, OFF_i, \overline{OFF}_i, Q_i, \overline{Q}_i \ge 0$$

$$I_i = \text{integer}$$

$$\alpha_i, \beta_i, \bar{\alpha}_i, \bar{\beta}_i = 0 \text{ or } 1$$

It should be noted that one may consider only part of this complete mathematical formulation to obtain the band, depending upon the user's requirements. If fewer constraints are included for analysis, the user obviously will have a wider progression bandwidth. To solve this mixed-integer programming problem, a variety of solution methods can be considered. If solving the new algorithm optimally through the above formulation is needed, the major consideration lies in the effectiveness of the mixed-integer programming packages. The Linear, INteractive and Discrete Optimizer (LINDO) (*13*) is considered here to solve this formulation. Although the free input form to run LINDO is easy to prepare, this new algorithm still requires substantial effort in learning how to formulate and create an input file to run LINDO. As far as sensitivity analysis and future applications are concerned, it is better to write a computer program that obtains the real progression bandwidth automatically based on the proposed formulation. A FORTRAN-based program named the BANDwidth of Timing Optimization Program (BANDTOP) has been developed to find the maximum progression bandwidth in both directions. It is a user-friendly program that improves the computational efficiency and ease of use by traffic engineers. BANDTOP provides much flexibility and convenience in responding to the changes of formulation or arterial configuration. It is noted that this new bandwidth program gives the optimal solution. BANDTOP can be run on PCs, VAX, IBM, or CDC and has been considered to generate progression signal timing plans for real-time traffic control systems in the cities of Keelung and Taichung, Taiwan.

AN EXAMPLE

The new bandwidth algorithm has been tested on the Zin-Wha Arterial in Tainan City, south of Taiwan. Four intersections exist in this arterial. Figure 7 gives the network geometry and traffic flows on these four intersections. The inputs for this new algorithm consist of the order and distances of signals between intersections, traffic flows and capacities, range of speed, left-turn phase pattern, acceptable range of cycles, and the target ratio of bandwidths on different directions. The user can either specify the green splits at each intersection as a fraction of the overall cycle or calculate them through Webster's formula (*1*). The upper and lower limits of speed in this example are assumed to be 50 km/hr and 30 km/hr, respectively. The saturation flow headway equals 2.07 sec based on a recent study (*14*).

Through given information, BANDTOP finds the optimal signal timing plan for four intersections and its time-space diagram is shown in Figure 8. From Figure 8, this new bandwidth approach clearly produces offsets and other signal timing parameters. The maximum bandwidths in both directions are 14.2 sec. Any vehicle within this band, unlike MAXBAND and PASSER II, can travel through all downstream intersections without stopping. As far as intersection 2 (Ming-Chen) is concerned, 20.6 sec outbound and 28.7 sec inbound bandwidths exist. Similarly, at intersection 3 (Min-Sen), outbound and inbound bandwidths have 33.7 sec and 14.3 sec, respectively. The new algorithm also recognizes partial progression opportunities over the shorter sections of the arterial. The partial progression bandwidth becomes wider as a vehicle moves toward downstream intersections. This is important because through this partial progression bandwidth one can conclude that vehicles outside the through band will need to stop at most once to pass the entire section of an arterial. In other words, vehicles will not stop or stop only once to travel through the arterial if the timing plan generated from BANDTOP is to be implemented.

To make a consistent comparison, the same network information and traffic flows were used to prepare the inputs for

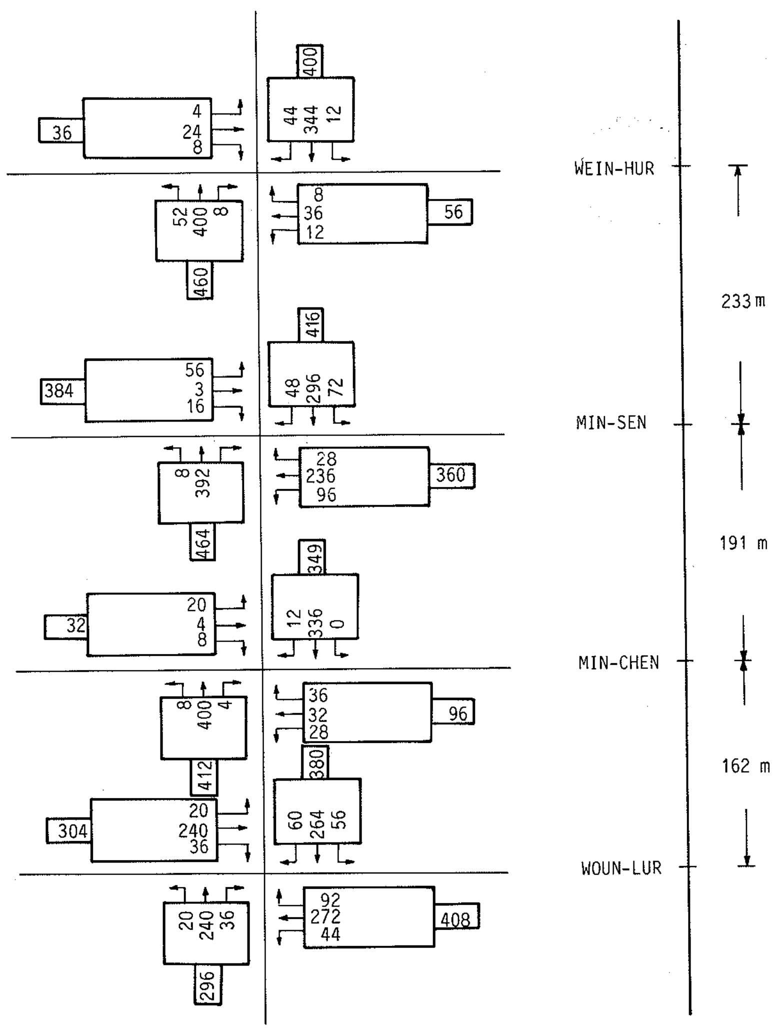

FIGURE 7 Geometrics and traffic volumes of Zin-Wha arterial in Tainan, Taiwan.

<< T I M I N G   P L A N >>

		CYCLE	G1	Y1	AR1	G2	Y2	AR2	OFFSET
INTERSECTION	4	80	55	3	2	15	3	2	0
INTERSECTION	3	80	39	3	2	31	3	2	10
INTERSECTION	2	80	55	3	2	15	3	2	9
INTERSECTION	1	80	34	3	2	36	3	2	11

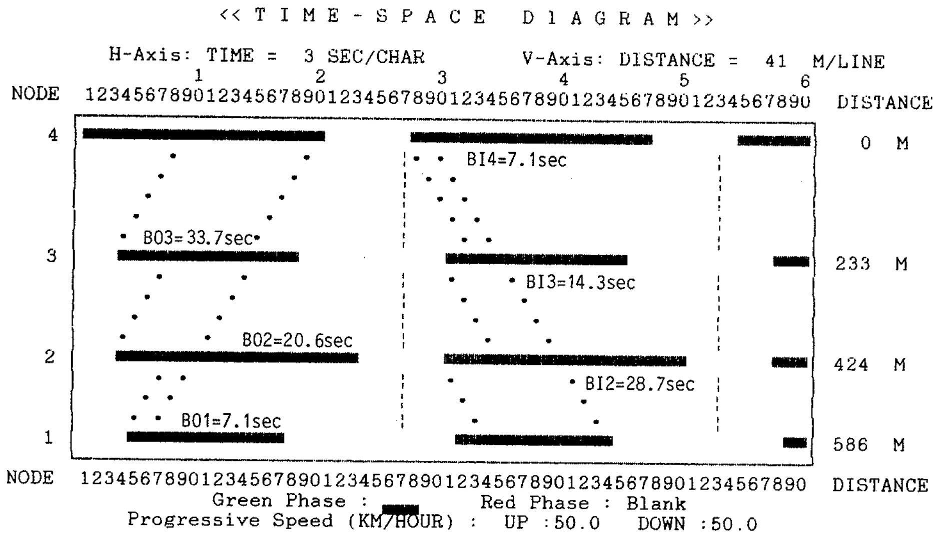

FIGURE 8 Output of BANDTOP for Zin-Wha arterial in Tainan, Taiwan.

TABLE 2 COMPARISON OF BANDTOP, MAXBAND, AND PASSER II SYSTEM PERFORMANCE THROUGH TRANSYT AND NETSIM

	System Performance	5 Intersections			4 Intersections		
		BANDTOP	MAXBAND	PASSERII	BANDTOP	MAXBAND	PASSERII
TRANSYT	PI	52.9	58.55	56.15	32.8	34.83	34.04
	Average Delay (sec/veh)	13.81	15.19	14.17	12.42	12.78	12.38
	Stop (%)	49	54	54	48	53	52
NETSIM	Average Delay (sec/veh)	28.27	30.9	29.03	25.0	23.48	24.46
	Stop (stops/veh)	0.94	1.12	1.04	0.83	0.82	0.9

MAXBAND and PASSER II. Figures 1 and 2 give time-space diagrams of two programs. By comparing Figure 8 with Figures 1 and 2, the new algorithm obviously produces a more reliable and acceptable progression bandwidth than MAXBAND and PASSER II in practical applications. Signal timing plans obtained from BANDTOP, MAXBAND, and PASSER II for four and five intersections are also analyzed by running TRANSYT-7F and NETSIM to evaluate their system performance. Results are given in Table 2. From this table, it can be seen that stops and average vehicle delay of BANDTOP are almost all less than those of MAXBAND and PASSER II. Figure 9 shows the computer output of BANDTOP for five intersections. The computer time of running BANDTOP on a PC/AT for three cases is available in Table 3. It takes only 36 sec and 72 sec to obtain the optimal solutions for four and six intersections with a math coprocessor 80287-10 on a PC/AT. BANDTOP uses far less computing time than MAXBAND.

Because the input file of BANDTOP is similar to that of MAXBAND, users only need to make a slight modification of MAXBAND input to run BANDTOP. Details of conversion described in the BANDTOP user's manual will be released in the near future. It should be emphasized that this general mixed-integer mathematical programming formulation does not always guarantee the achievement of a feasible solution under given arterial configuration and traffic flows. If no feasible solution is available, it means that no real progression bandwidth can be realized through given conditions. Under such circumstances, the user may need to change arterial information, target ratio of bandwidths, or the number of

<< T I M I N G P L A N >>

		CYCLE	G1	Y1	AR1	G2	Y2	AR2	OFFSET
INTERSECTION	5	90	60	3	2	20	3	2	2
INTERSECTION	4	90	57	3	2	23	3	2	0
INTERSECTION	3	90	62	3	2	18	3	2	11
INTERSECTION	2	90	62	3	2	18	3	2	7
INTERSECTION	1	90	43	3	2	37	3	2	6

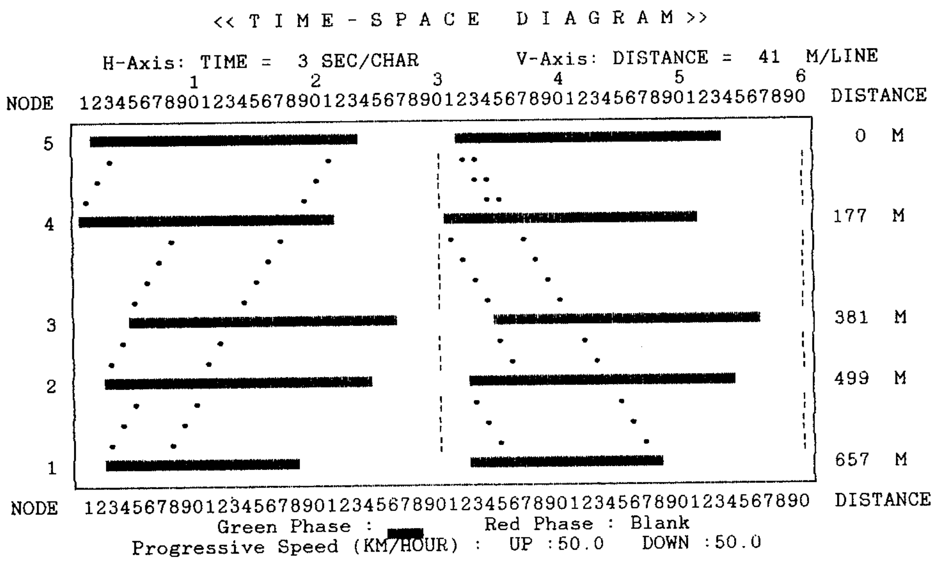

FIGURE 9 Output of BANDTOP for five intersections.

TABLE 3 COMPARISON OF BANDTOP RUNNING TIME ON PC/AT FOR THREE CASES

Number of Intersections	PC/AT Without Math Coprocessor	PC/AT With Math Coprocessor 80287-10
4	134 sec	36 sec
5	175 sec	46 sec
6	283 sec	72 sec

intersections considered in that segment to obtain the progression bandwidth in two directions.

CONCLUSIONS

From the research conducted in this work, it can be calculated that this new algorithm to find the maximal bandwidth in developing an arterial signal timing plan has many advantages over MAXBAND and PASSER II:

1. Unlike the current bandwidth approach, this approach guarantees that any vehicle in the progression band can travel through all downstream signals without stopping. Vehicles outside the bandwidth will need to stop at most once to pass the entire section of the arterial.
2. It provides several features in practical applications. The program calculates queue clearance time and incoming flow clearance time automatically, provides eight left-turn phase patterns for selection, sets the minimum green time on side streets, and gives the target ratio of direction flow.
3. BANDTOP shows a better system performance than MAXBAND and PASSER II according to the stops and average vehicle delay of two real examples tested on NETSIM and TRANSYT-7F. The real progression bandwidth should have a saw-toothed shape. In addition, the partial progression bandwidth becomes wider as a vehicle moves toward downstream intersections.

4. If no feasible solution can be obtained from the new algorithm, it means that under given conditions no real progression bandwidth is available on the arterial. The user may change the number of intersections considered in the segment or run other kinds of signal timing packages for the designated arterial.

5. The new algorithm can consider leading and lagging phase patterns at each intersection. The use of a leading or lagging phase will result in a wider bandwidth for the arterial but increase the delay of vehicles from side streets.

6. Based on the proposed formulation, BANDTOP provides the optimal solution of bandwidth and requires less computer time to obtain the arterial signal timing plan and its time-space diagram. BANDTOP has been used successfully as a part of generating timing plan software at two real-time traffic control systems in the cities of Keelung and Taichung in Taiwan.

Through field tests, the new approach can give better and more reliable progression bands than MAXBAND and PASSER II. Therefore, it is recommended that this new algorithm be used to obtain the maximum bandwidth if the resultant signal timing plan is to be implemented.

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DISCUSSION

EDMOND C.-P. CHANG

Texas Transportation Institute, Traffic Operations Program, Texas A&M University System, College Station, Tex. 77843-3135.

PASSER II and MAXBAND are two computerized signal timing programs currently available and popularly used for optimizing signal timing plans based on the maximum progression bandwidth concept. The maximum bandwidth approach can simultaneously optimize signal timing settings to provide the maximum weighted sum of arterial progression bands in both directions of an arterial street. This paper describes a modification of the constraints on the determination of the locations of the progression band with respect to the start of the arterial green times. This study examines the resultant reformulation being implemented in the original MAXBAND program and investigates the run-time efficiency after replacing the existing mixed-integer programming technique through microcomputer applications. The major difference between this enhanced algorithm and the original MAXBAND progression solution is that the enhanced algorithm provides wider progression bandwidths travelling farther toward the downstream intersections. It claims that the saw-toothed progression bandwidth generated by this formulation can allow some vehicles to travel through the arterial with at most one stop.

This study has raised several interesting points. First, the perception of the progression concept and definition of the maximum progression bandwidth may sometimes be misinterpreted. Second, it should be pointed out that this algorithm should only be considered an enhancement to the original MAXBAND algorithm, as the formulation and the computer program remain almost the same. The only new term being introduced by the authors is the modified "W_i" variable, which is used to provide the preset maximum queue clearance settings. Third, since no simulation or field control validation of the new algorithm has been performed, other than the computer run-time evaluation, serious reservations exist concerning the effectiveness and validity of the enhanced algorithm. Fourth, it should be clearly stated that the significant improvements on the run-time efficiency from the test case examination were due primarily to the commercially available LINDO code in the program. This replaces the inefficient execution of the 1973 version of the Mixed-Integer Linear Programming Code (MILP) for solving this complex optimization problem. Fifth, due to the feasible number of intersections in the solutions and the mathematical characteristic of the algorithm for not being able to find a feasible solution, sincere reservations exist about the intended use of this algorithm, as reported, for the real-time signal control applications.

The maximum progression, or the maximum bandwidth concept, is designed to provide the specific time intervals in which vehicles have the opportunity to travel through the downstream signalized intersections without having to stop. The existence of this arterial "Progression Bandwidth," or "Progression Opportunity," is an optimum time period whose existence is conditioned upon the interactions of cycle length, phase sequence, coordinated offsets, phase length, and, most important, intended target progression speeds. It should be noted that the progression bandwidth may be totally independent of the physical vehicle trajectories. "Vehicular Trajectories" represent the locations of vehicles arriving at a certain time. They can be used to examine whether vehicles following certain trajectories can travel through arterial streets. The existence of the progression band, however, does not guarantee that there will be certain vehicles lined up in the progression band. This simply means that the opportunities do exist in that time period for those vehicles that wish to follow the average target progression speed, and they may take advantage of the progression band to travel through without having to stop. However, the realization of the progression opportunities still depends on whether and how the designed progression bands can be utilized by the platooned and random vehicular arrivals in the field.

Progression may not work very well in those cases where target progression speeds were not set according to realistic operating speeds. Also, it may not function properly under those instances in which the progression phenomenon simply cannot exist because of heavy vehicular queue spillback or intersection blockage during arterial green times due to the overcongested operating conditions. However, most progression-based signal timing programs, such as PASSER II and MAXBAND, do allow users to adjust progression bands to some extent through the queue clearance features to tailor the progression time-space diagram to the potential queues observed in the field. On the other hand, multiple solutions may also exist to the same progression problems for given combinations of progression design speeds and coordinated background cycle lengths. This phenomenon is particularly noticeable in the coordinated two-phase operations. Therefore, the realization of the arterial progression bandwidth design approach depends heavily on whether the predicted progression can be achieved or fine-tuned according to actual vehicular performance during coordinated arterial traffic signal system operations.

It should be clearly stated that this algorithm can only be considered an extension to the original MAXBAND program because most of the MAXBAND formulation and all the program features remain exactly the same. Phase sequence, cycle length, green time, and offset optimization already existed in the original MAXBAND and PASSER II model. In addition, the benefits of using combinations of different traffic signal phase sequences to achieve a wider arterial progression bandwidth calculation were demonstrated in several earlier studies. Realistically, the new definition introduced by the authors only serves to modify the existing "W_i" variable to provide a crude estimation of the maximum queue clearance settings without having to add a detailed traffic flow prediction model. The basic question that still remains is whether the saw-toothed type of progression bandwidth can provide a better scheme than either the constrained MAXBAND-TRANSYT-7F progression approaches or the system delay offset fine-tuning optimization used in the PASSER II-84 approach.

Nevertheless, this paper did illustrate the significant run-time reduction that can be achieved by replacing the existing optimization code in the MAXBAND program to solve complicated optimization problems. This study examined the effectiveness obtained by replacing the relatively inefficient 1973 version of the MIL with the commercially available LINDO code. The inefficiency of the optimization algorithm and the commercial availability of Mixed-Integer Linear Programming Optimization codes have been commonly recognized. In parallel to this investigation, the maximum bandwidth program, MAXBAND 86, was enhanced by the Texas Transportation Institute in 1986 to simultaneously maximize the weighted sum of progression bandwidths on all the arteries of a signalized network. During its development, the same recommendations were made on the program run-time efficiency. It was decided to emphasize the network formulation and traffic engineering interpretation of the optimization results for developing the generalized arterial network optimization program. The results of an in-house study made by the Federal Highway Administration indicate that approximately 90 percent to 95 percent of the computer CPU time was spent on several subroutines of the MILP code of the MAXBAND 86 program during several test case runs. Therefore, the differences in run-time efficiency are contributed primarily to the replacement of the optimization code in MAXBAND, as all the other algorithms remain practically the same.

As indicated in this paper, this algorithm does suffer, as expected, from the inherent limitations of the number of intersections that can be feasibly analyzed to reach practical solutions efficiently. For the algorithm to provide feasible solutions, three important elements must exist. First, the arterial street directions must be given much larger amounts of green time than the cross street direction to provide the chance of generating a wider progression bandwidth farther toward downstream intersections. Second, to fully take advantage of the early start strategy for advancing the green times provided by the program, the saw-toothed type of progression approach will tend to favor those signal systems having short-spaced intersections, a large operating speed differential, and heavy turning traffic from side streets. This also implies that the algorithm tends to encourage the arterial vehicles travelling much faster than traffic turning from cross streets or slower vehicles. Third, full realization of this saw-toothed type of progression bandwidth relies heavily on the existence of equal amounts of green times to achieve maximum arterial progression.

As summarized from the above observations, the most successful operations of this enhanced algorithm are best suited for arterial signal systems having short spacing, small numbers of signals, large amounts of arterial green times, and almost optimum zero-offset coordination traffic operating conditions. In these cases, an alternative computerized signal operation can also be implemented through a series of two-phase signals with real-time green split adjustments without having to use the sophisticated Mixed-Integer Linear Optimization Problem for optimizing the one-line operation. At the same time, because of inherent limitations on the mathematical formulation due to the introduction of more constraints to the original optimization problem, the system does sometimes suffer from not being able to reach a feasible solution. Therefore, serious reservations exist concerning the intended use of this enhanced MAXBAND algorithm for real-time traffic signal system control. Consequently, it is highly recommended that implementation of this enhanced algorithm be reserved until realistic validation studies, through either simulation studies or field controlled experiments, can be made available for further evaluation. Simulation studies, through either the TRANSYT-7F or NETSIM program evaluations, for examining the potential effectiveness of the enhanced algorithm versus the conventional constant progression bandwidth approach will be beneficial.

AUTHORS' CLOSURE

Chang's discussion mainly concerns the concept and application of the new algorithm. From his discussion, several points are raised due to the misunderstanding of this algorithm. Each of these points will be discussed here.

Chang, in his second paragraph, states that the new algorithm introduces only a modified "W_i" variable that is used to provide the preset maximum queue clearance settings. Actually, in our paper, seven major characteristics of the new algorithm compared to the MAXBAND are clearly discussed in the beginning of the section "Mathematical Formulation of New Algorithm." The new algorithm uses three new variables: queue clearance time (Q_i), incoming flow clearance time (H_i), and time lag (W_i) for each intersection to take into consideration the clearing of queued vehicles before the arrival of platoons in the progression band. Thus, any vehicle within the band can travel through downstream intersection without stopping. All these variables need not be preset but are internally calculated based on the requirements of different arriving flow volumes. On the other hand, to assure vehicles in the through band cross the critical intersection without stopping, PASSER II and MAXBAND try to use the concept of preset queue clearance time. Nevertheless, it is impossible to know which intersection needs the queue clearance time and what value it should take. Even with an assumed or preset queue clearance time, the critical intersection will soon be shifted to another intersection according to the progression theory used in PASSER II and MAXBAND. The existing problem still remains unsolved. The new algorithm, however, can overcome this problem by introducing those three new variables at each intersection.

The discussant, in the second and fifth paragraphs, has emphasized that the new algorithm should only be considered as an enhancement to the original MAXBAND algorithm because most of the MAXBAND formulation and all the program features remain exactly the same. In our paper, we list complete formulations of the MAXBAND and the new algorithm separately in the context for comparison. Even part of the output of the MAXBAND and BANDTOP are also shown in Figures 2 and 8. The proposed algorithm uses a new progression concept and theory to handle the existing problem encountered by MAXBAND and PASSER II. A new mathematical programming formulation has been developed and enhanced LINDO is applied to obtain a saw-toothed pattern of the bandwidth instead of a parallel and uniform one. Obviously, it is different from the original MAXBAND and PASSER II. The discussant, however, strongly objects to the term of a "new" algorithm.

The discussant mentions the importance of performing simulation study through either TRANSYT-7F or NETSIM and field tests to the new algorithm and its program BANDTOP. We certainly agree; some of simulation results and comparisons among BANDTOP, PASSER II, and MAXBAND on two arterials with four and five intersections have been shown in Table 2. At present, BANDTOP is used as a part of computing software for timing plan generation on a new real-time traffic control system, named the Traffic Responsive and Uniform Surveillance Timing System (TRUSTS), located in the cities of Keelung and Taichung in Taiwan. Many results can be obtained from the field and considerations have been given to perform further evaluation and to improve the TRUSTS performance and current BANDTOP version.

The discussant tries to reinterpret the concept of bandwidth in the third and fourth paragraphs. In the paper, we have pointed out the existing progression problem if the output of PASSER II and MAXBAND is implemented directly. The vehicles in the front portion of the through band will be hindered and have to stop at the critical intersection. This problem mainly comes from inaccurate progression theory with unrealistic assumptions made in the PASSER II and MAXBAND. The phenomenon can be easily observed from the field and time-space diagrams. Although the discussant, in his fourth paragraph, tries to use a preset queue clearance time to tailor the progression time-space diagram to the potential queues observed in the field, the existing problem still cannot be solved. This is simply because queues are varying from time to time and the estimated or observed queues are only suited for a particular time and day. It is uneconomical for the users to check the potential queue of each intersection in the field every time. In other words, if the current output of the MAXBAND and PASSER II with an impressive bandwidth operates through time-space diagrams without additional manual adjustments, it will provide practicing traffic engineers with false information in determining whether to choose to implement the signal timing plan.

The discussant mentions in the seventh paragraph that for the new algorithm to provide a feasible solution, three important elements must exist. In fact, none of these three points are accurate. First, he states that ". . . the arterial street direction *must* be given *much* larger amounts of green time than the cross street direction. . . ." From Figure 8 of the paper, it can be seen that the arterial has 34 sec green versus 36 sec green of the cross street at intersection 1 and 39 sec green of the arterial versus 31 sec of the minor street at intersection 3. Second, he states that ". . . the approach will tend to favor those signal systems that have short-spaced intersections, a large operating speed differential, and heavy turning traffic from side streets." The new algorithm, in fact, can handle long-space intersections even over one cycle travel time of one block distance based on the integer value of I_i in Equation 2. That is the reason why we claim the new algorithm has the general mixed-integer formulation. Since the new algorithm considers the general case, it will be able to deal with various kinds of street types and flow patterns. Furthermore, the optimal travel speed in Figures 8 and 9 remains 50 km/hr (31 mph) for both directions. We do not understand why the discussant concludes that the new algorithm tends to favor a large operating speed differential, heavy turning traffic, and the arterial vehicles travelling much faster than traffic turning from cross streets or slow vehicles. Third, he mentions that ". . . bandwidth relies heavily on the existence of equal amounts of green time to achieve maximum arterial progression." Here we are not sure whether the green time refers to the bandwidth or the green time interval of that intersection. In either case, it is not true, as Figures 8 and 9 demonstrate.

In the last paragraph, the discussant summarizes his observations and states, ". . . the most successful operations of this enhanced algorithm are best suited for arterial signal systems having short spacings, small numbers of signals, large amounts of arterial green times, and almost optimum zero-offset coordination traffic operating conditions." Some of the points have been explained above. Similarly, the offsets shown in Figures 8 and 9 ranging from 2 to 11 sec reveal that the new algorithm is not only suited for almost zero-offset coordination. Besides, the new algorithm can handle various types of intersections and traffic flow. In our paper, we mention that if no feasible solution can be obtained from the new algorithm, it means that no real progression bandwidth is available on the arterial under given conditions. This is due to the proposed optimization model that assures any vehicle in the band can travel through all downstream intersections without stopping and outside the band with at most one stop. The bandwidth should be the saw-toothed shape. Therefore, we agree to the point that the new algorithm is suited for small numbers of signals, but not exclusively, because the optimal solution relies mainly upon the traffic flow movement and block distances of intersections under consideration in the segment.

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