Analysis of Platoon Dispersion with Respect to Traffic Volume

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Platoon dispersion depends on friction. The authors hypothesize that this friction can be external and internal and that the internal friction depends on traffic density and volume. Platoon dispersion should consequently be influenced by traffic density and volume. This hypothetical relationship was confirmed by limited field observations and simulations. Platoon dispersion increases with increasing volume up to a maximum value, which is observed at lower volumes than capacity. Dispersion decreases with further increasing volume and becomes nearly zero at volumes near capacity. If further studies confirm the observed relationship, platoon dispersion could be calculated directly from densities or volumes and travel times, applied to Robertson's model on a link by link basis, thus improving the results of traffic signal coordination models.

Traffic signals have the effect of grouping vehicles into platoons that leave the intersection under saturated flow conditions. As the platoon moves down the arterial it tends to spread out over time and space. This phenomenon of platoon dispersion is an important factor in traffic signal coordination. Robertson's model allows one to calculate the histogram of a platoon at different points downstream from an intersection. The basic equation that determines the degree of dispersion is

$$F = \frac{1}{1 + \alpha\beta t} = \frac{1}{1 + Kt}$$

The value of dispersion K (a capital letter is used here to avoid confusion with the density k) in this equation depends on the type of road under study and on the degree of friction. McCoy et al. (*1*) described platoon dispersion in lower friction environments, and the TRANSYT 7F manual allows one to choose a K value depending on several degrees of friction ranging from low to high as in figure 1.

Figure 1 shows that platoon dispersion increases with friction. It would be useful to have a more quantitative measure of friction that could take explicitly into account an independent variable like service volume or volume divided by saturation flow. Service volumes or saturation flows would be established considering the many independent variables such as corridor width, percentage of heavy vehicles, and so on, and would be particular to a given arterial type. If friction could then be explained by service volumes, the basic dispersion equation would become a function of service volumes and travel times. Platoon dispersion could then be integrated on a link basis into the traffic signal coordination model.

The aim of this paper is to describe the concepts of such a

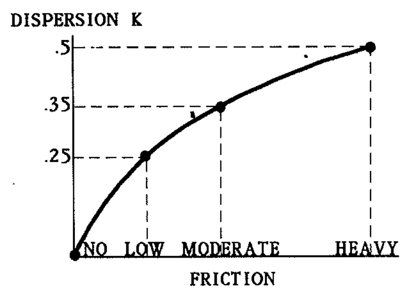

FIGURE 1 Platoon dispersion as a function of friction (TRANSYT 7F).

relationship based on field observations, microscopic simulations, and theoretical considerations.

CONCEPT AND THEORETICAL CONSIDERATIONS

One can easily observe that platoon dispersion is a function of the physical features of the highway facility and of the interaction between vehicles in the traffic stream. The friction concept introduced by May (*2*) in 1959 is very useful in the analysis and explanation of platoon dispersion. Friction results from two phenomena: internal friction between moving vehicles and external friction related to roadway features. External and internal friction are implicitly considered in the concept of level of service, which takes into account such variables as number of lanes and their widths, vertical and horizontal alignment, and the location of the arterial in the city.

The internal friction reflects the individual driver's comfort, safety, and freedom to maneuver; the greater the traffic volumes, the more adverse the effects of friction. Conceptually, when volume and density are zero, friction also is zero and increases with increasing volume and density up to a certain maximal value, which occurs at less than the maximal volume (level of service E). As volume increases, friction diminishes and approaches zero at capacity, when every vehicle is moving at nearly the same speed. Thereafter, friction again increases with increasing density and should be maximal (at least theoretically) at jam density. Figure 2 illustrates this conceptual relationship.

The purpose of the following paragraphs is to show whether there is a rational explanation of this interrelationship to be found in the different phases of driver behavior as volume and density increase. May (*3*) and Leutzbach (*4*) distinguished three zones of different driver behavior. May identified the three zones on the volume-density curve: constant speed, constant volumes, and constant rate of change of volumes with

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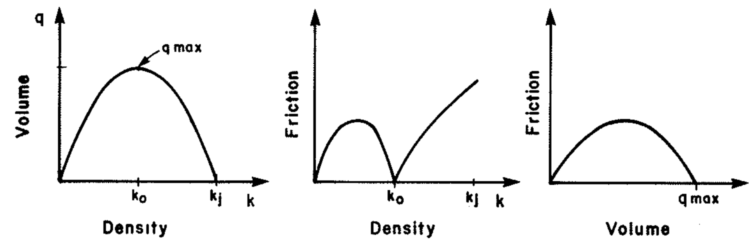


FIGURE 2 Conceptual relationship between friction, density, and volume.

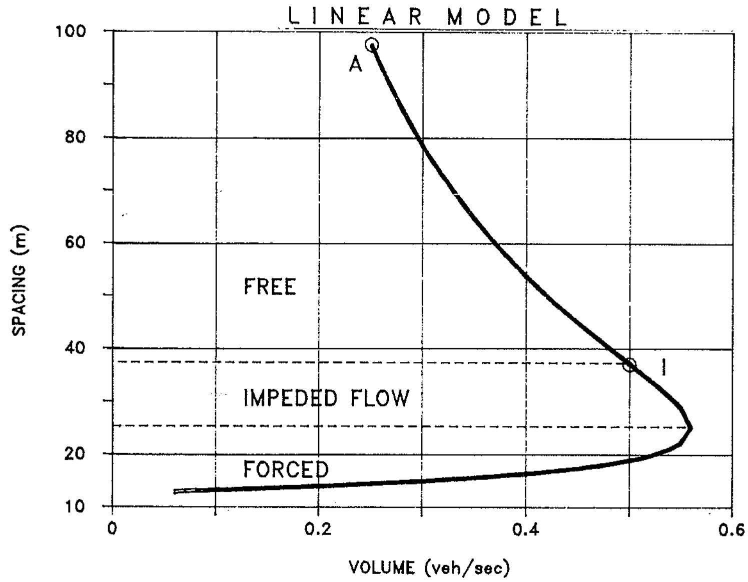


FIGURE 3 Volume-spacing relationship (linear model).

density. In the following section, an explanation for this division into three zones is attempted and at the same time a rational explanation for friction and corresponding platoon dispersion is given.

In order to analyze driver behavior with respect to volume changes, let us study the volume-spacing relationship (fig. 3).

For the purpose of example, this figure represents the linear macroscopic traffic model with a free speed of $v_f = 28$ (m/s) and a jam density of $k_j = 0.08$ (veh/m). Attentive analysis of this figure reveals that there is a point of inflection in the curve at a certain spacing; that is, the driver changes behavior at this point. The situation becomes much clearer when one analyzes the rate of change of spacing with respect to a uniform increase of volume. This relation is shown in figure 4.

Starting from a point A on the free-flow side of the curve, the rate of change of spacing decreases faster and faster with increasing volume up to a point, whereas the rate of change remains nearly constant (at the point of inflection I). For more clarity and in order to better understand the changing behavior of the driver, let us consider figures 5 and 6, which represent the rate of change of speed and headway with respect to volume.

Near the point I, the rate of change of spacings is constant while the rate of change of speeds is increasing, and the rate of change of headways is nearly constant. Because headways remain nearly constant at volumes near the capacity, spacings and speeds will change rapidly when volumes increase beyond the point of inflection. At volumes higher than those observed at the point of inflection (q_I) drivers behave differently with respect to the car they follow than at lower volumes. In figure 4 one can distinguish three zones: free flow, impeded flow, and forced flow. This change in driver behavior influences platoon dispersion. Dispersion increases with increasing volumes up to or near the point of inflection and then decreases to near zero value at capacity. The notion of platoon dispersion at densities higher than those observed at capacity seems to be meaningless.

MATHEMATICAL INVESTIGATION

The location of the point of inflection on the volume-density curve was studied in more detail for the macroscopic traffic stream models defined by Drew (5). In his notation, $n = -1$ represents the exponential model, $n = 0$ the parabolic model, and $n = 1$ the linear model.

The formulae relating spacing s to volume q for these models are as follows. For $n = -1$,

$$q = \frac{1}{s} u_m [\ln(sk_j)]$$

$$\frac{dq}{ds} = \frac{1}{s^2} u_m [1 - \ln(sk_j)]$$

$$\frac{dq}{ds^2} = \frac{1}{s^3} u_m [2\ln(sk_j) - 3]$$

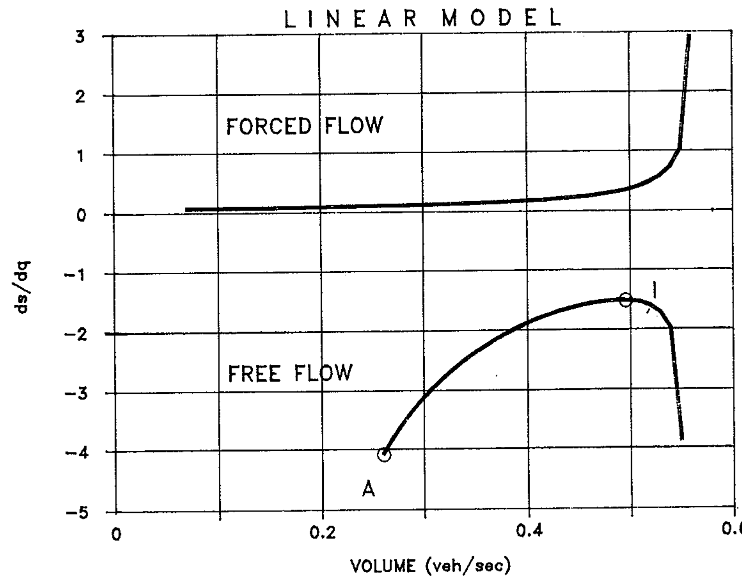


FIGURE 4 Rate of change of spacing versus volume.

For $n \geq -1$:

$$q = \frac{u_f}{s}[1 - (1/sk_j)^{(n+1)/2}]$$

$$\frac{dq}{ds} = u_f\left[\frac{(n+3)}{2}s^{-(n+5)/2}(k_j)^{-(n+1)/2} - s^{-2}\right]$$

$$\frac{dq}{ds} = u_f\left[2s^{-3} - \frac{(n^2+8n+15)}{4}(k_j)^{-(n+1)/2}s^{-(n+7)/2}\right]$$

Table 1 gives a comparison between the spacings sI at the point of inflection and the spacing sQ at maximal volume as a function of the jam spacing sj. Spacings at the point of inflection or of changing driver behavior are 1.5 times longer than the spacings at capacity, and the volume is 10% less than the maximum volume.

It is interesting that the point of inflection is very near or at the same point as the maximum kinetic energy described by Drew (*6*). Rewriting the energy equation as a function of spacing we have, for $n = -1$,

$$E = \frac{(u_m)^2}{s}[\ln(sk_j)]^2$$

For $n \geq -1$,

$$E = \frac{(u_f)^2}{s}[1 - 2(sk_j)^{-(n+1)/2} + (sk_j)^{(n+1)}]$$

The curve relating spacing to kinetic energy is given, as an example, for the linear model in figure 7.

A comparison of spacings at the point of maximum kinetic energy and at the point of inflection is given in table 2.

Drew et al. (*6*) stated that the kinetic energy of a traffic stream is equal to the sum of energies of its constituent particles and will be a maximum when internal friction caused by vehicular interaction is a minimum. In the case of the linear model, the point of inflection and maximum energy or theoretical minimum of internal friction fall together. One can conclude that there is a point where driver behavior changes with respect to spacing, and this point occurs at lower densities and at lower volumes than the capacity. When the volume approaches a certain value, drivers reduce their spacings less and less in reaction to a unit increase of volume. At the point of inflection this tendency is sharply reversed. The actual location of this point of change of driver behavior depends on the type of facility under study and on the model chosen. One might compare this point of inflection to the transition between laminar and turbulent flow in hydraulics.

OTHER CRITERIA FOR REPRESENTING INTERNAL FRICTION

Lee et al. (*7*) developed a criterion for internal energy or friction based on observations. They found that acceleration noise, as proposed by Drew, does not accurately represent internal friction or the internal energy of the traffic stream. Acceleration noise, at least in their observations, did not show a clear relationship to increasing density. They proposed as a good measure for internal friction the coefficient of variation of speed σ/v, also called the constant of diffusion. As spacings decrease, this measure increases up to a point (the point of maximum energy or the point of inflection); then it decreases to nearly zero at capacity. Figure 8 shows this indicator for the values found by Lee et al. as a function of density.

Although there was no indication in their paper about the volume-density relationship, one can see that the relationship follows the one postulated in the preceding paragraphs for the relation between dispersion and volume.

The measure of friction developed by Helly et al. (*8*) is the mean velocity gradient, which represents the acceleration noise divided by the average velocity. The measure seems to have some merits, but there are not enough observations available to confirm them.

In the light of what was said, one can hypothesize that dispersion depends on service volumes, or, stated more simply, on volumes divided by the saturation flow. In order to confirm the hypothesis stated in the preceding paragraphs, a

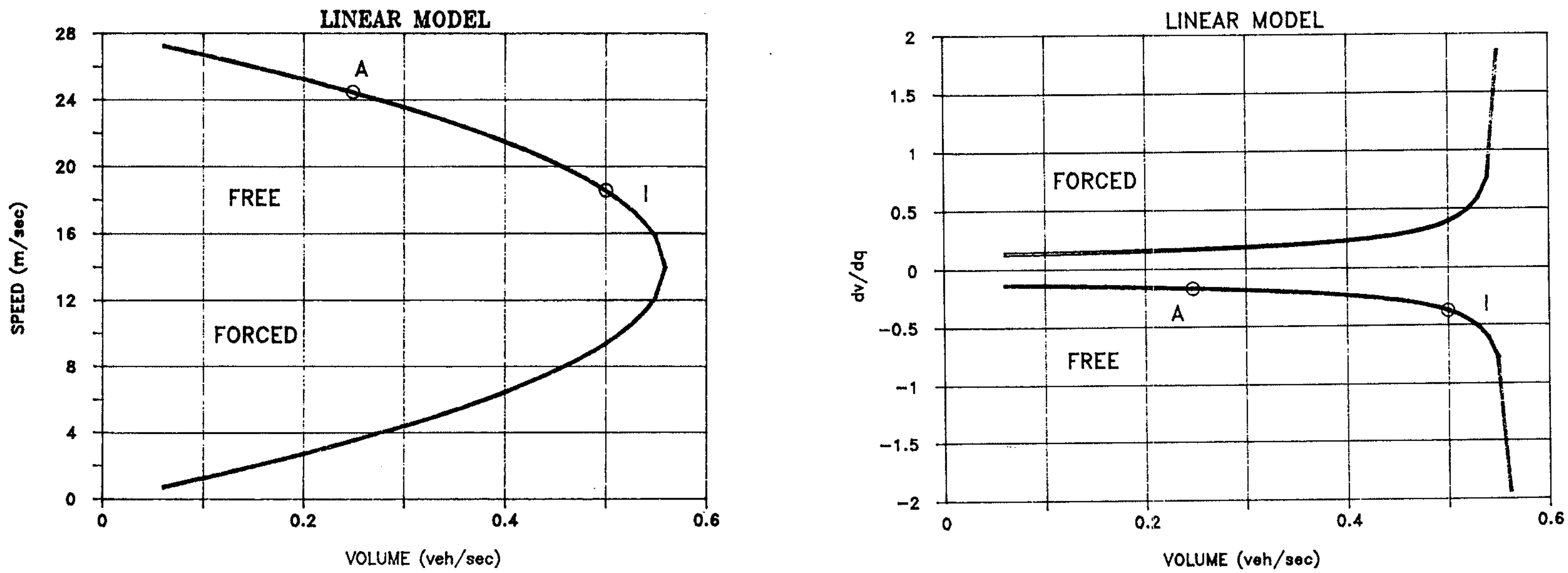


FIGURE 5 *Left*: speed-volume relationship. *Right*: relationship of rate of speed change to volume.

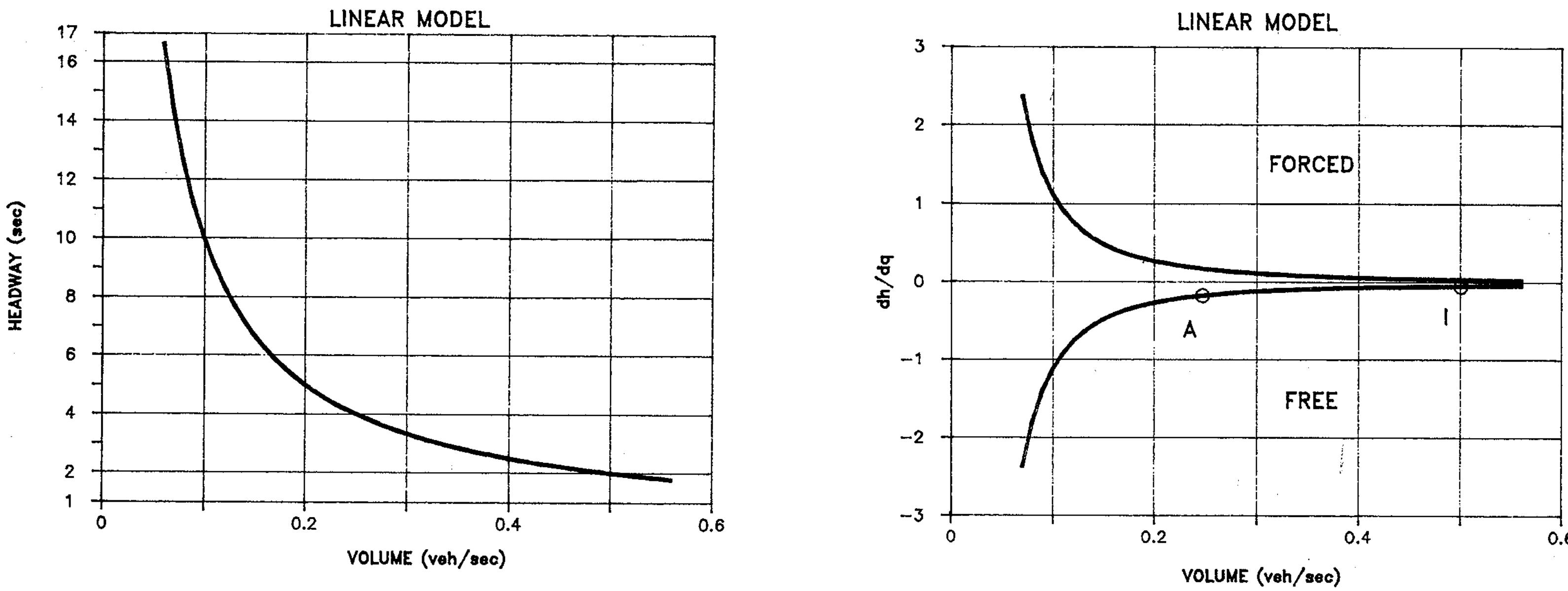


FIGURE 6 *Left*: headway-volume relationship. *Right*: relationship of rate of headway change to volume.

TABLE 1 FLOW AND SPACING AT POINT OF INFLECTION AND AT CAPACITY

Model	n	Spacing at point of inflection	Spacing at point of max. q	s_I/s_Q	Volume at point of inflection	Volume at point of max. q	q_I/q_Q
exponential	n=-1	$4.48s_j$	$2.72s_j$	1.65	$0.33u_m k_j$	$0.37u_m k_j$	0.91
parabolic	n=0	$3.52s_j$	$2.25s_j$	1.56	$0.13u_f k_j$	$0.15u_f k_j$	0.9
linear	n=1	$3s_j$	$2s_j$	1.5	$0.22u_f k_j$	$0.25u_f k_j$	0.9

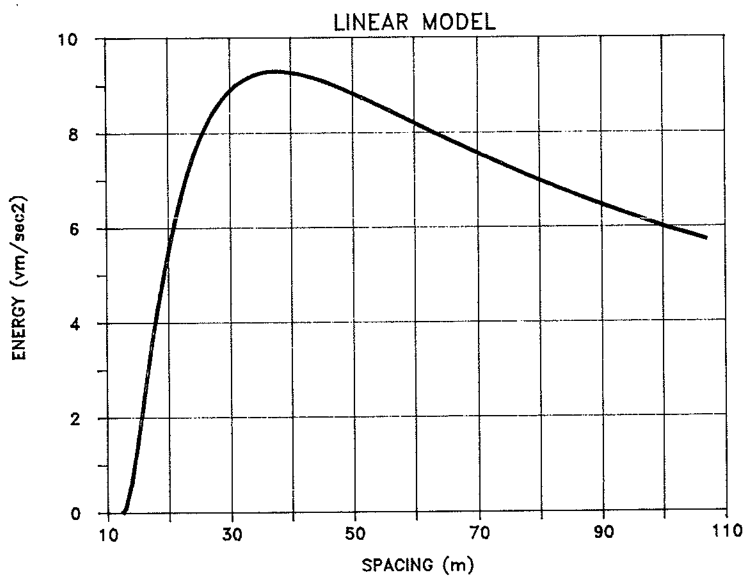


FIGURE 7 Spacing versus kinetic energy for the linear model.

TABLE 2 COMPARISON OF SPACING AT POINT OF INFLECTION AND AT POINT OF MAXIMUM KINETIC ENERGY

Model	n	Spacing at point of inflection	Spacing at point of max. E	s_I/s_E
exponential	n=-1	$4.48s_j$	$7.39s_j$	0.61
parabolic	n=0	$3.52s_j$	$4s_j$	0.88
linear	n=1	$3s_j$	$3s_j$	1

field study was carried out on two straight and level three-lane urban arterials located in an intermediate-type area in the city of Montreal.

FIELD STUDY

The arterials were chosen so as to minimize interferences and marginal and intersectional friction. The distance to the downstream intersection was about 800 meters so that the traffic flow was unaffected by the downstream traffic control devices. The observations were made at the intersection (stop line plus the width of the cross street) and at 100 meters and 200 meters from that point. Both arterials had high volumes during peak hours, with platoons ranging from 16 to 90 vehicles. The small platoons were observed at the beginning and at the end of the peak hours. Approximately 150 platoons were observed at each arterial.

These observations were carried out with three hand-held microcomputers (Tandy 200) with external disk drive and synchronized internal clocks. A program written in BASIC allowed the identification of buses, trucks, and passenger cars. The five keys that were activated on the keyboard were identified by stickers as B (bus), T (truck), P (passenger), CB (beginning of the cycle), and CE (end of the cycle). The data on vehicle types, passage times, and beginning and end of the cycle were stored on the portable disk drive. This procedure was extremely efficient. Only 2% of all platoons had to be eliminated in the validation phase, mostly because of data-saving problems. The commercially available hand-held computers with standard keyboard can be programmed for any

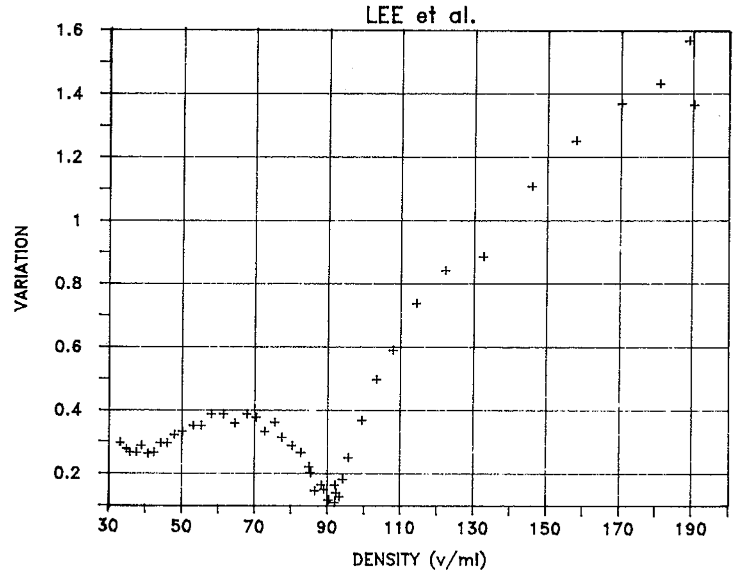


FIGURE 8 Diffusion constant versus density (*7*).

TABLE 3 NUMBER OF OBSERVATIONS AT TWO STUDY LOCATIONS

no of platoons	Papineau Ave		Laurentian Bld	
observed	338		130	
eliminated	7		4	
	platoon size	no of platoons	platoon size	no of platoons
	16	22		
	19	54		
	27	33		
	36	60		
	43	13	46	7
	50	13	55	12
	54	26	60	17
	61	53	67	19
	65	20	73	27
	68	27	75	12
	74	10	78	18
	76	7	81	18

data-acquisition problem by means of simple BASIC programs (see Baass (*9*) and Bonsall (*10*)). Even with the Tandy 200's small memory of 32K, this new technology provides an efficient way of acquiring traffic data.

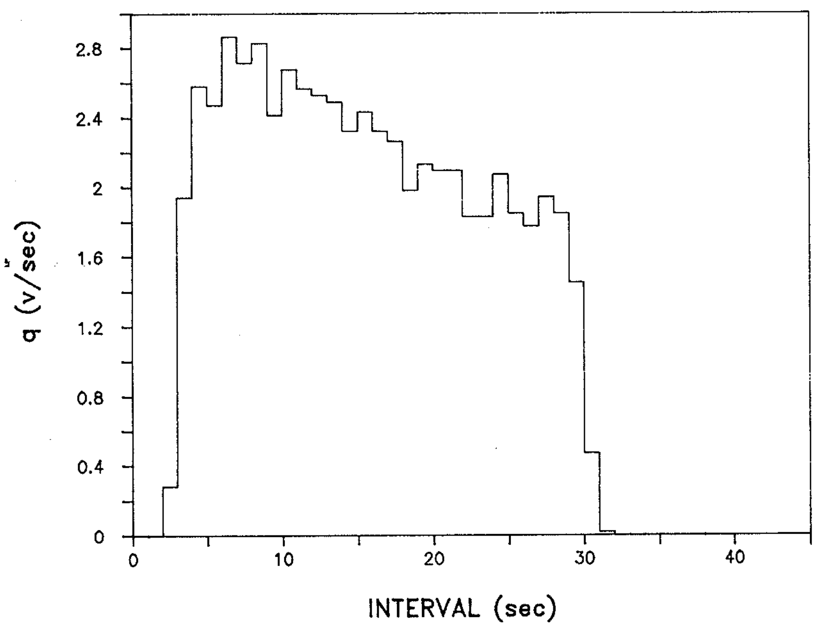
The data were then transferred easily into an IBM PC-AT, where they were treated directly through a spreadsheet (Symphony). Table 3 shows how the validated platoons were classified into groups of similar sizes and average platoons were derived for each platoon size. Figure 9 shows such a platoon at 100 meters and at 200 meters from the stop line.

A special procedure designed on the spreadsheet was then used to simulate the platoon dispersion based on the Robertson model for different trial values of *K*. The parameter β was considered as a constant value of 0.8 in all calibrations because of the complexity of a dual calibration of α and β at the same time (see Axhausen et al. (*11*)). The best fitting *K* was obtained for each average platoon using an indicator similar to the χ^2 by comparing observed and simulated platoon histograms. Therefore the best *K* values remained nearly constant for all stations.

The best *K* values that were obtained in this way are given in table 4, and the relations between *K* and the platoon size are given in figure 10. Lefebvre (*12*) described the experiment and the calibration procedure.

Calculating the saturation flow or level of service *E* at the study sites and using the cycle lengths observed permitted the drawing of figure 11, which gives the best *K* with relation to the volume/capacity ratio.

In both the observed cases, the observations and calibrated *K* values came close to the hypothesized relationship. Further analysis of the data is necessary to explain the relation between cycle duration, platoon-size, and platoon dispersion *K*.



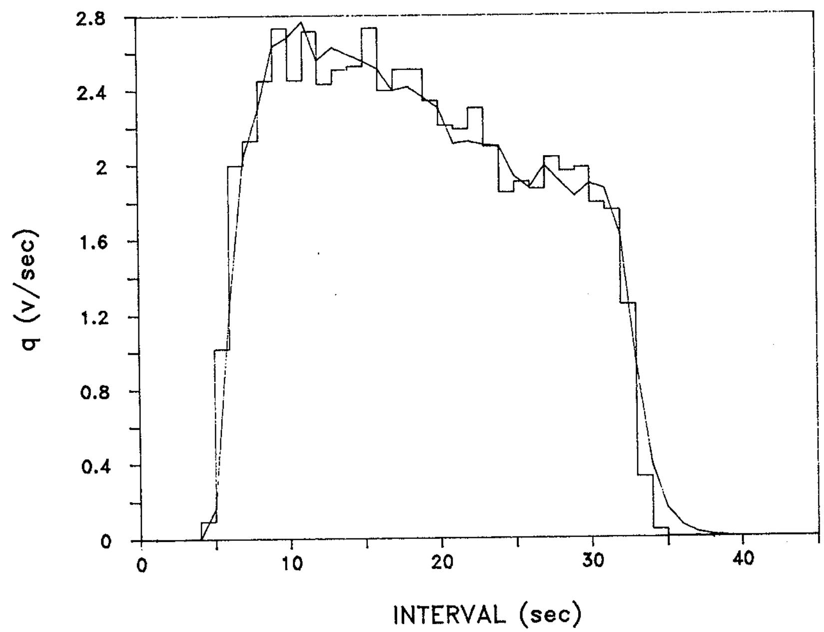


FIGURE 9 Histogram of a large platoon (68 vehicles) at 100 meters (*left*) and 200 meters (*right*) from the stop line.

TABLE 4 RESULTS OF CALIBRATION OF *K* IN MODEL OF ROBERTSON

Papineau Ave		Laurentian Bld	
platoon size	best K	platoon size	best K
16	0.17		
19	0.325		
27	0.35		
36	0.405		
43	0.39	46	0.13
50	0.33	55	0.18
54	0.325	60	0.16
61	0.315	67	0.17
65	0.31	73	0.13
68	0.32	75	0.13
74	0.26	78	0.1
76	0.255	81	0.13

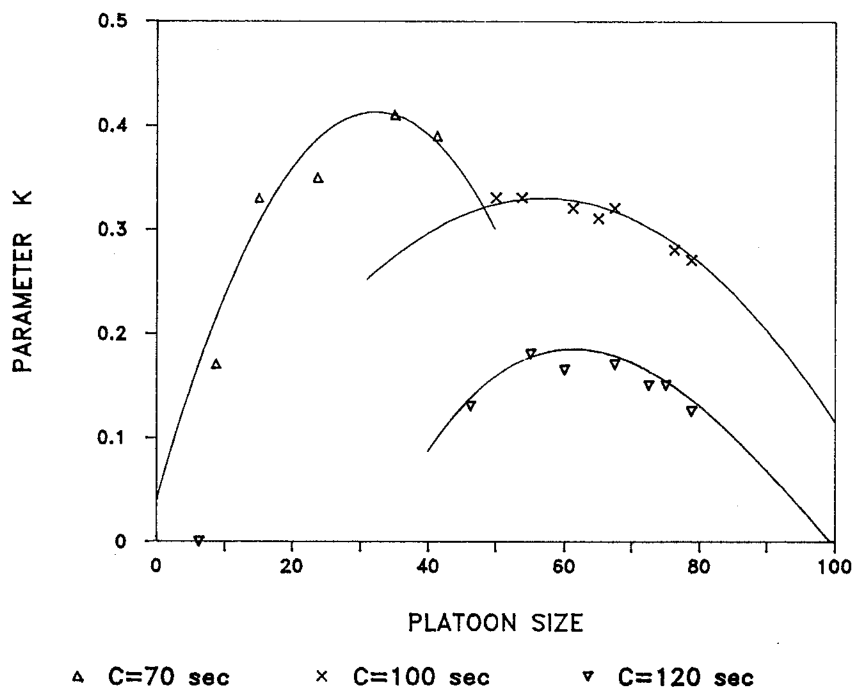


FIGURE 10 Parameter *K* for the two study sites in relation to platoon size.

MICROSCOPIC SIMULATION

Microscopic simulations were carried out on a hypothetical three-lane arterial in order to further verify the postulated relationship. This arterial was chosen because it differed from the arterial streets on which the field study was conducted. Manar (*13*) developed a microscopic traffic simulation model based on the car-following model described by Fox et al. (*14*). Manar introduced into the existing program multilane simulation with lane-changing behavior (see Seddon (*15*)) and increased the platoon sizes to up to 110 vehicles on three lanes.

Different sizes of average platoons were obtained with the simulation model. The histograms describing the platoons were calculated for the average platoons at the stop line and at 100 meters and 200 meters from the stop line. Platoon sizes in different runs are slightly different because this is also a random variable in the model. The 13 different platoons were then treated in the same way as the observed platoons; the best *K* values obtained in this way are presented in table 5.

These values are represented in figure 12 as a function of platoon size. The shape of the curve is similar to the one obtained in the field study (fig. 11); the different characteristics chosen in the simulation account for the different maximum values of *K*.

The simulation program produces several useful outputs. One of these is the macroscopic relationship derived from the car-following model. Figure 13 gives an example for this out-

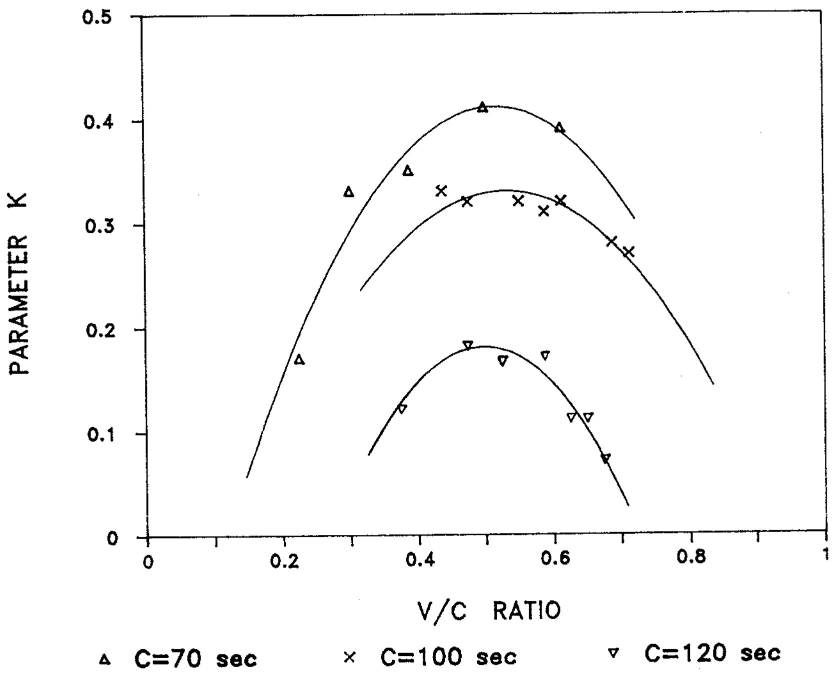


FIGURE 11 Parameter *K* for the two study sites in relation to volume/capacity ratio.

TABLE 5 BEST *K*-VALUES OBTAINED BY SIMULATING DIFFERENT PLATOON SIZES

Simulation	
platoon size	best K
22	0.23
28	0.14
33	0.20
38	0.17
43	0.26
48	0.31
53	0.27
62	0.48
70	0.63
75	0.70
84	0.74
87	0.48
90	0.42

put, which could be used for further analysis of the platoon dispersion.

Other important outputs are several statistics that were calculated to help in the interpretation of the simulated platoons. One of these is the number of lane changes (from left to right and from right to left). Figure 14 illustrates this variable for the simulated platoons.

As volumes increase, drivers lose the ability to maintain their desired speeds. Drivers have to reduce their speed to match the slower vehicles or, if possible, change lanes. The lane-changing possibility diminishes after a maximum value if volumes are further increased. The maximum value may be interpreted as the point at which drivers become influenced by the presence of vehicles ahead of them. The number of overtaking and passing opportunities can be viewed as an index for friction. Even if the 1950 Highway Capacity Manual (*16*) gives only an example for two-lane highways, the form of the relationship in figure 15, which is reproduced here from the manual, may also be valid in multilane arterial analysis.

Two other variables produced by the simulation model are of interest—the velocity gradient defined by Helly (*8*) and the acceleration noise. The values of these variables are illustrated in figures 16 and 17.

The simulated platoons behaved in a way that corresponds

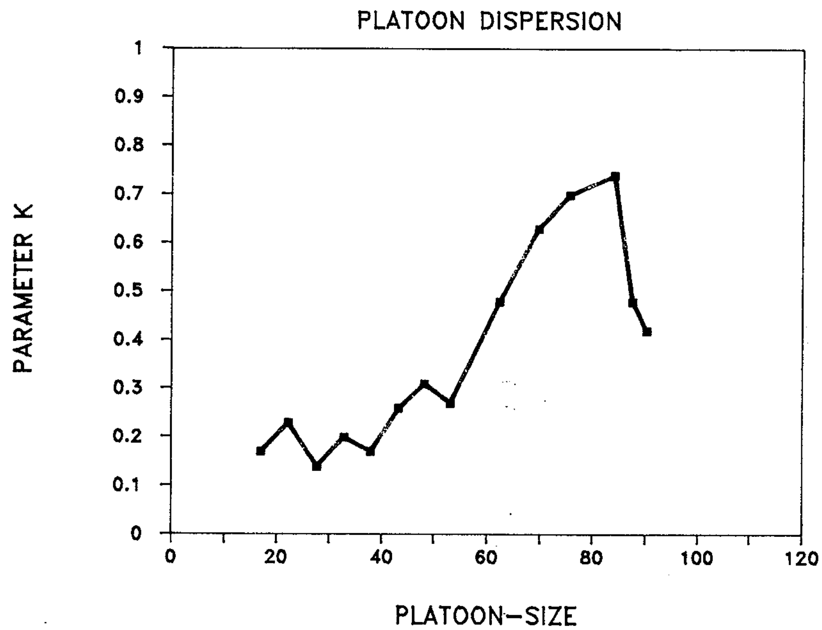


FIGURE 12 Parameter K for the simulated platoons.

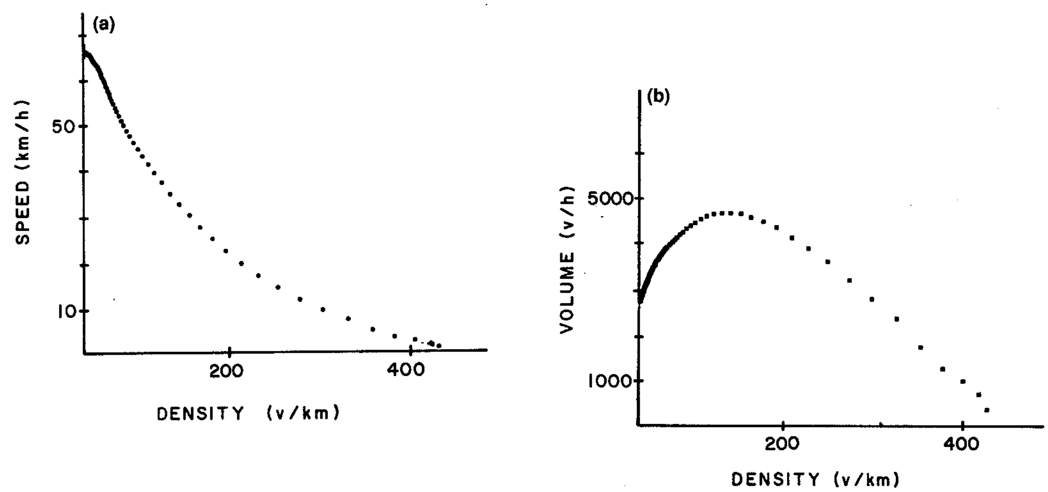


FIGURE 13 Macroscopic relationship derived from car-following model.

fairly well to the proposed relationship between platoon dispersion and traffic volume; these variables show an increase as volume increases, followed by a decrease to a minimum value near capacity.

CONCLUSION

The degree of platoon dispersion depends on internal and external friction. The external friction is contained in the saturation flow concept. Because the internal friction depends on traffic volumes, the platoon dispersion also has to depend on service volumes. Observed and simulated platoons showed low dispersion at low traffic volumes. External friction being the same, dispersion increases as volumes increase, whereby the actual value of K depends on the kind of arterial studied. Dispersion increases further up to a maximum value as traffic volumes increase. This point of maximum dispersion is observed at volumes of 0.6 to 0.8 of the capacity and is near the point of maximum energy. The diminishing dispersion after further volume increases can be explained by changing driver behavior at or near the point of inflection in the volume-spacing curve.

Further study should be devoted to more platoon observations at different locations and for different lane numbers and cycle lengths in order to define (if possible) a typical curve relating service volumes to dispersion. This relationship could have the following form, where q represents the volumes:

$$F = \frac{1}{1 + (a + bq - cq^2)\,t}$$

We would thus be able to consider platoon dispersion link by link and the results of the traffic signal coordination models would be improved.

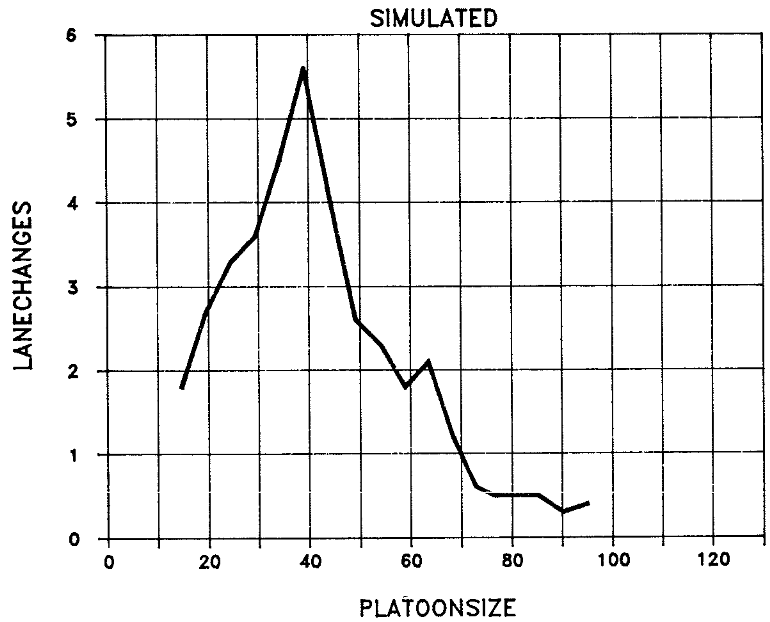


FIGURE 14 Number of lane changes versus platoon size.

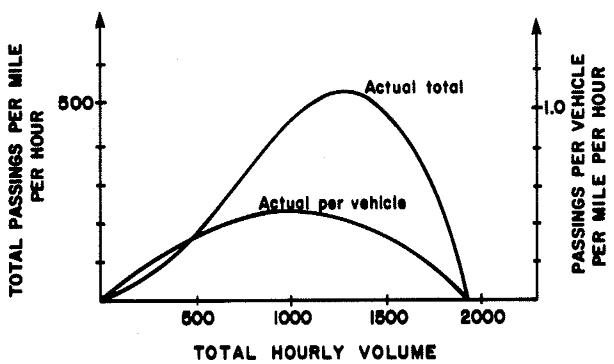


FIGURE 15 Passing opportunities on a two-lane highway (*16*).

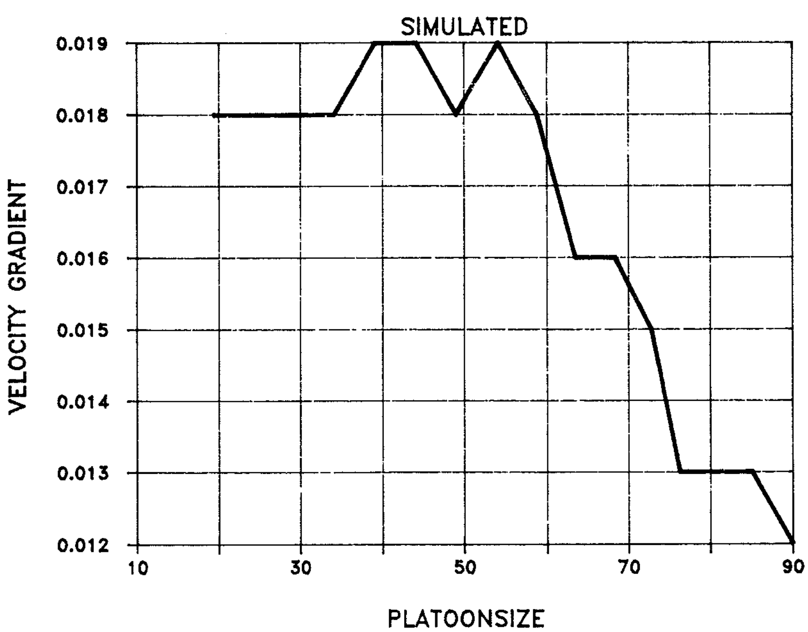


FIGURE 16 Velocity gradient.

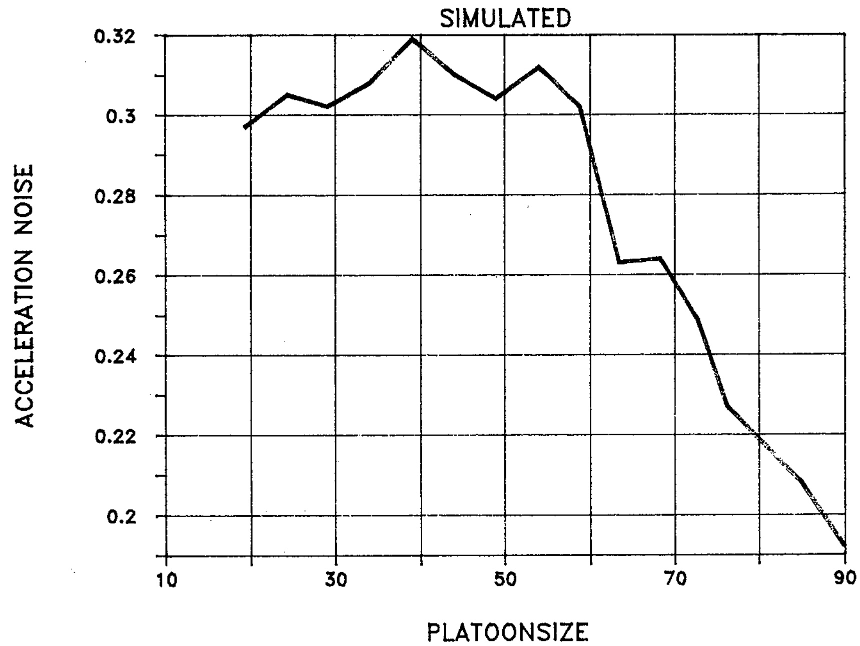


FIGURE 17 Acceleration noise.

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