Development of Limiting Velocity Models for the Highway Performance Monitoring System

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A study was performed for the Federal Highway Administration to increase the efficiency of vehicle speed models for the Highway Performance Monitoring System analytical process. Probabilistic and deterministic models developed by the World Bank were adapted for conditions in the United States. These models estimate vehicle average travel speed as a function of relevant road and traffic characteristics. This is done by evaluating a set of constraining speed models that consider the influence of vertical grades, horizontal curves, roughness, traffic congestion, and highway type. These models were adapted to conditions in the United States using engineering judgment and limited available data. Although further research is needed to refine these models, the models produce reasonable results and are recommended for use in planning models as a basis for computation of road user costs. More research is needed in this general area from the engineering community. Input from experts in vehicle mechanics, dynamics, and human factors would be particularly helpful in determining driver reactions and behavior and further developing speed prediction models as a function of road characteristics and vehicle class.

The Federal Highway Administration (FHWA) uses a set of approximately 92,000 annually monitored sample pavement sections across the United States to assess the condition of the nation's highways and road network. This system is called the Highway Performance Monitoring System (HPMS). One part of the HPMS analysis package is used for planning purposes to study the impacts of different funding scenarios on highway users. A complex, time-consuming, computer algorithm is presently used to estimate vehicle speeds on each sample section from which travel time, fuel consumption, and vehicle operating cost impacts are computed. This paper summarizes the development of an efficient speed prediction model for use in the HPMS analytical process (1) and discusses problems encountered in development of the model.

LIMITING VELOCITY MODELS

Limiting velocity models developed by the World Bank were chosen for adaptation to United States conditions (2). Using the results of past studies and engineering judgement, the following limiting velocity models were formulated. One model relates a vehicle speed to the minimum of five constraining speeds:

\[
V_{ss} = \min (V_{DRIVE}, V_{BRAKE}, V_{CURVE}, V_{ROUGH}, V_{DESIR})
\]

where:

- \( V_{ss} \) = steady state speed,
- \( V_{DRIVE} \) = maximum possible driving speed,
- \( V_{BRAKE} \) = maximum allowable braking speed on downgrades,
- \( V_{CURVE} \) = maximum allowable speed on horizontal curves,
- \( V_{ROUGH} \) = maximum allowable ride severity speed
- \( V_{DESIR} \) = desired speed.

This model is called the Minimum Limiting Velocity Model (MLVM).

The second model treats each constraining speed as a random variable. This model, called the Probabilistic Limiting Velocity Model (PLVM), is:

\[
V_{ss} = \exp \left( \frac{S^2}{2} \right) \left( \frac{1}{V_{DRIVE}} \right)^{1/B} + \left( \frac{1}{V_{CURVE}} \right)^{1/B} + \left( \frac{1}{V_{BRAKE}} \right)^{1/B} + \left( \frac{1}{V_{ROUGH}} \right)^{1/B} + \left( \frac{1}{V_{DESIR}} \right)^{1/B}
\]

where:

- \( S^2 \) = variance associated with unmeasured vehicle, road, and speed measurement characteristics,
- \( B \) = a constant parameter for each vehicle class.

The PLVM has several interesting features. When two or more speeds become equally dominant, the probabilistic speed drops below the deterministic speed by a larger amount. Also, as more speeds begin to lower the probabilistic speed, they do so at a diminishing rate. Thus, the stochastic nature of driver perception is modeled such that as the driver reacts to a greater number of speed constraints, he or she will drive slower than the minimum of the constraining speeds.

MODEL PARAMETERS FOR UNITED STATES CONDITIONS

The parameters of the limiting velocity models that were examined for adjustment to U.S. conditions are:
The constraining speeds for each class, including VDRIVE, V BRAKE, V CURVE, VR OUGH, and VDESIR;
- The exponential parameter B for each vehicle class;
- The variance term S², which represents the errors associated with speed predictions for each vehicle class.

Ideally, calibration of these models for United States conditions should be based on direct field measurements. To best model the effects of various road characteristics, actual sites that have only one dominating characteristic must be selected. Vehicle spot speeds should be measured and the model fitted to the data. Unfortunately, no suitable data representative of conditions in the United States could be found.

**Maximum Possible Driving Speed (VDRIVE)**

The maximum possible driving speed is the speed a vehicle travels when all the available driving power is used. VDRIVE becomes a constraint on the speeds of vehicles on positive vertical grades. The force balance and power relationship used to find VDRIVE is

\[ A v(VDRIVE)^{3} + mg(\text{GR} + \text{CR})VDRIVE = 375 \text{ HPDRIVE} \]  

where

\[ \text{VDRIVE} = \text{maximum possible driving speed, mph}; \]
\[ \text{HPDRIVE} = \text{maximum available driving power, horsepower}; \]
\[ m = \text{vehicle mass}; \]
\[ g = \text{acceleration due to gravity}; \]
\[ \text{CD} = \text{aerodynamic drag coefficient}; \]
\[ A v = \text{frontal area of vehicle}; \]
\[ \text{GR} = \text{vertical gradient}; \]
\[ \text{CR} = \text{rolling resistance coefficient} \]

The terms HPDRIVE, m, CD, Av, and CR are vehicle parameters. VDRIVE is the maximum possible driving speed that a vehicle can achieve when all available power is used. The equation above balances the forces acting on the vehicle and relates them to the available power, vehicle mass, and other parameters.

### TABLE 1 1985 U.S. VEHICLE FLEET MODELING PARAMETERS

<table>
<thead>
<tr>
<th>Vehicle Class</th>
<th>Area Sq. Ft.</th>
<th>Weight lb.</th>
<th>Drag Coef.</th>
<th>CR = RC1 + RC2 (v)^n</th>
<th>HPDRIVE</th>
<th>VDRIVE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small Auto</td>
<td>22.8</td>
<td>2,720</td>
<td>0.42</td>
<td>.0125</td>
<td>6.5E-07</td>
<td>93</td>
</tr>
<tr>
<td>Medium Auto</td>
<td>25.9</td>
<td>3,780</td>
<td>0.45</td>
<td>.0125</td>
<td>6.5E-07</td>
<td>140</td>
</tr>
<tr>
<td>Large Auto</td>
<td>28.7</td>
<td>4,560</td>
<td>0.49</td>
<td>.0125</td>
<td>6.5E-07</td>
<td>200</td>
</tr>
<tr>
<td>Pickup</td>
<td>30.8</td>
<td>5,000</td>
<td>0.59</td>
<td>.016</td>
<td>6.5E-07</td>
<td>175</td>
</tr>
<tr>
<td>2A-SU Truck</td>
<td>36.9</td>
<td>12,000</td>
<td>0.70</td>
<td>.0076</td>
<td>9.0E-05</td>
<td>230</td>
</tr>
<tr>
<td>3A-SU Truck</td>
<td>55</td>
<td>35,000</td>
<td>0.70</td>
<td>.0076</td>
<td>9.0E-05</td>
<td>275</td>
</tr>
<tr>
<td>2S-2 Semi</td>
<td>90</td>
<td>50,000</td>
<td>0.80</td>
<td>.0076</td>
<td>2.0E-05</td>
<td>325</td>
</tr>
<tr>
<td>3S-2 Semi</td>
<td>90</td>
<td>62,500</td>
<td>0.80</td>
<td>.0076</td>
<td>2.0E-05</td>
<td>325</td>
</tr>
</tbody>
</table>

(1) **NOTE:** v = speed, mph

n - 2 for autos and pickups
n - 1 for trucks and semi
dependent. The values of these parameters developed for the U.S. vehicle fleet are shown in Table 1. All terms but HPDRIVE are from automotive industry literature. HPDRIVE is the available horsepower after accounting for internal power losses. To find this parameter, HPDRIVE was back-calculated from Equation 3 by inserting the observed top speeds of various vehicles on 0-% grades. For automobiles it was found that the calculated HPDRIVE approximated the SAE net braking horsepower, which includes the effects of internal power losses. This quantity is recommended as HPDRIVE for automobiles.

A different approach to the estimate of HPDRIVE was required for trucks with diesel engines. The best source of information found for U.S. trucks was a 1970 study by the Western Highway Institute (WHI) (3). The WHI recommended multiplying the rated horsepower of a diesel engine by a factor varying from .78 to .85, depending on the number of axles, gear range, and engine size. This equation is suspect since the truck population has changed significantly since 1970. Although the relationship is used in this study, these results should be reviewed in future studies to determine its suitability.

Maximum Allowable Braking Speed (VBRAKE)

On steep downgrades, a maximum constraining speed, or braking crawl speed, has been observed (4, 5). The braking crawl speed is believed to be related to vehicle braking capability resulting through use of the retardation power of the engine (downshifting) and the brakes. In general, only large vehicles have been observed to slow down on steep down grades. Limiting crawl speeds on downgrades are not generally found on grades less than 4% or shorter than 3,000 feet.

Although large trucks may have braking speeds, little information examining this effect was found. The 1985 Highway Capacity Manual (6) indicates that very few studies have been performed to analyze the impact of heavy vehicles on traffic flow on downgrades. Due to the lack of information on this behavior, this term is not included in this model.

Maximum Allowable Curve Speed (VCURVE)

Most drivers decrease their speed to negotiate sharp horizontal curves. The effect of curves on vehicle speed has been widely studied. The World Bank model (2) related vehicle speed on a horizontal curve to the “maximum perceived friction ratio,” called FRATIO. FRATIO is defined as the ratio of lateral forces on a vehicle to the normal force on the vehicle. The vehicle speed on the curves, with simplifying assumptions can be written as

\[ V_{CURVE} = [(F_{RATIO} + SP) g RC]^{0.5} \]

where:

- \( V_{CURVE} \) = maximum allowable speed on curves,
- \( F_{RATIO} \) = maximum perceived friction ratio,
- \( SP \) = superelevation of curve,
- \( RC \) = radius of curvature.

The \( F_{RATIO} \) value is used to characterize different vehicle classes. A \( F_{RATIO} \) value of 0.155 was found to provide a good fit to the speed-curve model used in the present HPMS for automobiles, pickups and single unit trucks. For large trucks and semitrailer units, a value of .103 was used, based on the relationship between cars and trucks determined from the World Bank study.

Other forms of this model could be used; however, the PLVM requires that this model predict high speeds on curves with large radii to avoid interaction with other terms in the PLVM that would falsely decrease the predicted speeds. Field studies of the performance of trucks on curves and the suitability of this model for U.S. conditions appear warranted.

Maximum Allowable Ride Severity Speed (VROUGH)

It is a common observation that road roughness influences vehicle speed. Few studies, however, relate vehicle speed to the roughness measures used in the United States or in the HPMS. A model is needed that explains differences in vehicle type and road type and accounts for limiting roughness thresholds and minimum speeds at maximum roughness levels.

Based on the information developed in Brazil and the speed-roughness model used in the current HPMS analytical process, the following equations were developed:

\[ V_{ROUGH} = 1.00 \]

\[ \text{ (.0250} - \text{.00275(PSR)) automobiles} \] (5)

\[ V_{ROUGH} = 0.90 \]

\[ \text{ (.0255} - \text{.00333(PSR)) large trucks} \] (6)

where

\[ V_{ROUGH} = \text{ride severity speed, mph;} \]

\[ PSR = \text{present serviceability rating, (0-5).} \]

More work is needed on speed-roughness relationships. The above equations are primarily based on engineering judgment. Relationships derived from direct measurements and based on common roughness measures used in the U.S. are needed to extend the accuracy and usefulness of these speed prediction models.

Desired Speed of Travel (VDESIR)

The desired travel speed is the speed at which drivers travel when they are not constrained, typically less than the maximum possible speed a vehicle can attain. This speed is governed by subjective considerations of safety, speed law enforcement, fuel cost, and vehicle wear. For the purposes of the HPMS, the term should also be sensitive to the effects of traffic congestion and traffic control devices.

A limited nationwide source of information on VDESIR is the annual free flow speed tables published by the FHWA (7). Average, median, and 85th-percentile speeds on highways on which the 55-mph speed limit is the primary speed constraint are published.

To incorporate the effects of traffic congestion and traffic control devices into the model, tables of average speed as a function of highway type, traffic control, number of lanes, and speed limit were developed (1). These tables were devel-
oped from tables of initial running speed contained in the current HPMS model and are based on and extrapolated from the general speed-volume capacity relationships shown in the 1965 and 1985 Highway Capacity Manuals.

Due to the generalized information from which these speeds were developed, more work is needed to relate the desired speed constraint to physical road characteristics. A separate speed constraint term related to a simple measure of traffic congestion such as volume capacity ratio should also needs be developed, particularly for signalized urban streets.

**B and S^2 Parameters**

The exponential B parameter and variance parameter S^2 are part of the probabilistic limiting velocity model (PLVM). They are included to account for the stochastic nature of observed vehicle speeds. B and S^2 are primarily used to reduce the predicted speed when two or more constraining speeds become dominant. The B parameter acts similarly to the coefficient of the standard deviation of a normal distribution typically used to determine confidence levels. The smaller the value of B, the closer the probabilistic speed is to the minimum constraining speed. Meanwhile, the S^2 parameter is associated with errors in speed prediction, due to variations in vehicle and road characteristics, and other errors, due to speed measurements and quantification of road attributes.

These parameters are properly determined using a nonlinear least-squares regression analysis between observed speeds and speed constraint terms. Because there is no U.S. data base from which to develop these parameters, the values of B = 0.1 and S^2 = 0.01 were selected to cause the model to predict speeds that are less than the minimum speed constraint. The choice of these values causes the MLVM and PLVM models discussed in this paper to produce essentially the same results.

**SUMMARY**

The minimum and probabilistic forms of the limiting velocity model offer an excellent method to predict vehicle speeds as a function of relevant constraining speeds due to curvature, gradient, road roughness, braking capability, and other road features. The models presented here can be implemented into planning models for use in predicting user impacts. As discussed, further development of these models is required in order to better define the interrelations between road characteristics and vehicle speed.

The recommended method of further refinement of these models is through a structured nationwide study of vehicle speeds. This study would consist of spot speed studies on sections selected to study a particular speed constraint term.

A full statistical analysis similar to that performed by the World Bank should be performed on this data base. In the face of a more limited study, the authors feel that the speed models presented here could best be improved by studies into the following topics in the following order:

1. Effects of signalization and traffic control
2. Effects of traffic congestion
3. Effect of roughness
4. Large truck performance on downgrades and horizontal curves

Input from experts in vehicle mechanics, dynamics, and human factors is also important in better defining the needed speed estimation models.

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**REFERENCES**


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