Three-Dimensional Analysis of Slab on Stress-Dependent Foundation

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Research described in this paper constitutes the final stage of a multicomponent project, which examined current computerized analysis techniques for slabs on grade. For a more realistic representation of a pavement system, an existing three-dimensional finite element program (GEOSYS) was modified. This can be used to analyze flexible or rigid pavements, thereby validating conclusions reached on the basis of conventional two-dimensional analysis. In the first part of this study, many runs were conducted to develop user guidelines for the fruitful utilization of the GEOSYS model. Effects considered included, among others, mesh fineness, vertical and lateral subgrade extent, boundary conditions, and stress extrapolation from computer results. Practical applications of the three-dimensional approach are presented in the second part of the paper. The three fundamental loading conditions, namely, the interior, edge, and corner of a slab resting on a stress-dependent, elastic, solid foundation are examined. Two typical single-wheel and multiwheel U.S. Air Force aircraft (F-15 and C-141) are considered. An iterative scheme is introduced to account for subgrade stress dependence, and the effect of stress softening, typical of cohesive soils, is evaluated and discussed. Results from this program are compared to those from conventional two-dimensional analyses, employing finite element, finite difference, and numerical integration techniques.

In a recent report for the U.S. Air Force Office of Scientific Research (*1*), an exhaustive examination was presented of existing tools that may be applied to the analysis of slab-on-grade pavement systems, within the context of two-dimensional plate theory. Findings from this research clearly indicated that no current procedure can model fully the behavior of a slab of finite size, supported by a stress-dependent cohesive subgrade extending beyond the slab edges. In a follow-up study (*2*), an existing three-dimensional finite element model (GEOSYS) was adapted to provide a more realistic representation of a pavement system. The model can be used to establish baseline structural response data for flexible or rigid pavements, representative of complex boundary and foundation support conditions, thereby validating and reinforcing conclusions drawn on the basis of two-dimensional analysis.

Results from more than one hundred three-dimensional, finite element runs from that study examining a slab on grade are interpreted and discussed in this paper. In the first part, guidelines for the fruitful utilization of the GEOSYS model are developed, to account for such effects as the sensitivity of the model to mesh fineness, vertical and lateral subgrade extent, boundary conditions, and so forth. Investigations for both interior and edge loading conditions have been conducted (*2*). Only the former are discussed in this part, however, because they have been found to be adequately representative.

In the remainder of the paper, an examination is presented of a slab on grade (rigid pavement) subjected to the three fundamental loading conditions, that is, interior, edge, and corner, using typical single-wheel and multiwheel U.S. Air Force aircraft. An iterative scheme is introduced to account for subgrade stress dependence. The effect of stress softening, typical of cohesive soils, is evaluated and discussed.

GEOSYS PACKAGE

GEOSYS was originally developed in the early 1970s by a group of engineers, members of the technical staff of Agbabian Associates, in El Segundo, California (*3*). For the purpose of this study, the linear, isoparametric, three-dimensional hexahedral brick element was employed. This has eight nodes with three degrees of freedom per node (the displacements in each of the x, y, and z directions). The subgrade is modeled as an elastic solid foundation. A typical input file for GEOSYS consists of several hundred lines, each formatted according to a strict pattern. Since data for different slab-on-grade analysis runs are generally similar in structure, a preprocessor, called "GEZIN" (GEOSYS Easy INput), was coded early in this study. This automatically prepares the data in the required format. Input for GEZIN includes fewer than ten data cards, the format of which is similar to the one used for ILLI-SLAB (*1*). Thus, data preparation is reduced to an almost trivial task, and the probability of errors during this stage is practically eliminated. Several post-processing programs were also coded during this investigation to assist interpretation of computer results. These postprocessors are used in conjunction with an iterative scheme introduced to account for the stress-dependent behavior of fine-grained soils.

SELECTION OF STRESS EXTRAPOLATION METHOD

The problem of a slab on grade may be investigated in three dimensions using a finite element mesh similar that shown in Figure 1. The slab rests on a cube of soil, carved out of the Boussinesq half space and maintained intact by the assumption of boundary conditions on the four vertical sides and on the base. Taking advantage of symmetry, where it exists, allows only a portion of the system to be modeled.

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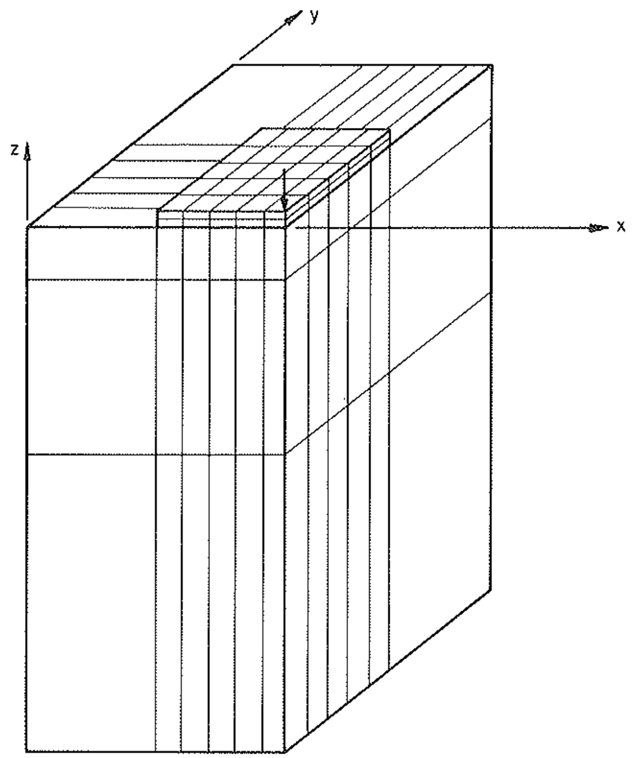

FIGURE 1 Typical three-dimensional finite element mesh.

For the linear solid (brick) element employed, displacements are calculated at each of the eight nodal points, but stresses (and strains) are determined only at the centroid of the element. It is, therefore, often necessary to extrapolate from calculated values to obtain the required stresses at the critical locations. Results from several runs suggested that orthogonal linear extrapolation (employing four known stresses in each determination) should be used in analyzing bending stress results (*2*). This method may also be used with subgrade stress data. When the stress state is known to be axisymmetric, for example, subgrade stress due to an interior load, diagonal linear extrapolation (employing three known stresses in each determination) is also appropriate.

VERTICAL AND LATERAL SUBGRADE EXTENT

Several runs were performed to determine the depth to which the subgrade should be modeled, as well as the size of the soil cube in the horizontal direction, so that boundary effects are eliminated, and computer storage required remains within the available limits. A typical plot of maximum responses obtained is shown in Figure 2. It is observed that both maximum bending and subgrade stresses (σ_i and q_i) converge to a constant value fairly quickly. Increasing the subgrade depth, Z, beyond 20 to 25 feet (or five to seven times the radius of relative stiffness, l_e) will have no effect on these stresses.

The behavior of deflection, however, requires more attention. Maximum deflection, δ_i, increases with subgrade depth as expected, but does not converge to a constant value, even for a depth of 35 ft (9 l_e). This is due to the presence of lateral boundaries at a finite distance, X, from the slab edges. As a result, vertical strains in the subgrade reach a level where they remain constant, rather than decrease as in a truly semi-infinite elastic solid, since they are not allowed to be distributed beyond the model boundaries. Further investigations revealed that the depth at which vertical strain decreases to a constant value, as well as this value itself, are both influenced by the lateral extent of the subgrade (*2*). Therefore, for a given lateral subgrade extent, the finite element model will overestimate vertical deformation, if the subgrade extends vertically beyond the constant vertical strain depth, z_{cvs}. Although

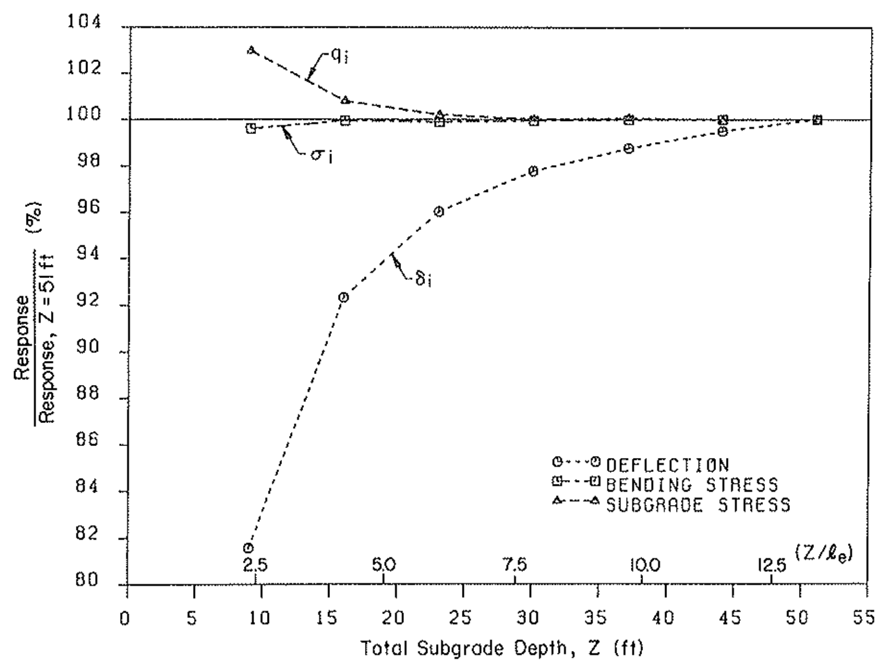

FIGURE 2 Effect of subgrade vertical extent.

this is influenced by the lateral extent of the subgrade, a subgrade depth of about 40 ft (10 l_e) may be used as a typical value. A lateral extent between 25 and 35 ft (or 7–9 l_e) is recommended.

BOUNDARY CONDITIONS

In order to examine the behavior of the two main types of lateral boundaries (i.e., free or on rollers), deflection basins from four runs are compared in Figure 3. These confirm that the system response is more sensitive to the boundary conditions used when the lateral boundaries are closer to the load. Furthermore, maximum deflection, δ_i, was once again affected much more significantly than the other two maximum responses, σ_i and q_i (*2*). Based on these results, neither boundary condition appears to have an advantage over the other. Consideration of radial strains in the surface subgrade layer, however, led to the adoption of roller-type lateral boundaries. The bottom boundary is also assumed to be on rollers, so that elements can move laterally and distribute their load by deforming.

VERTICAL DIVISION OF SLAB AND SUBGRADE

Results indicated that although there is a slight improvement in accuracy as the number of layers used in modeling the pavement slab increases, adequately reliable maximum responses can be determined even using only two slab layers (*2*).

Additional analyses led to the following conclusions with respect to the subgrade. The soil cube may be divided into three portions in the vertical direction. The upper portion should extend from 0 to 4 feet (0 to 1 l_e) and should consist of layers not more than 1 to 2 feet thick (0.25 to 0.5 l_e). The middle portion should extend from 4 to 15 feet (1 to 4 l_e) or half the constant vertical strain depth, z_{cvs}, if known, and should be divided into at least two layers. Finally, the lower portion should cover the remainder of the soil cube and may be divided into one or more layers.

HORIZONTAL SLAB AND SUBGRADE MESH FINENESS AND ELEMENT ASPECT RATIO

Three series of GEOSYS runs were conducted to examine the influence of horizontal slab mesh fineness on the response of the three-dimensional slab on grade. The trends observed in Figure 4 are similar to those noted in earlier two-dimensional finite element studies (*1, 4*), inasmuch as all three maximum responses converge from below. Deflection and subgrade stress appear to be less sensitive to horizontal mesh fineness, and adequate accuracy may be expected as the mesh fineness ratio ($2a/h$) of the (short) side length of the finite element (plan view) to the slab thickness approaches the value of 0.8. This is similar to the value determined from the two-dimensional studies. Bending stress appears to be more sensitive to this effect, requiring values of ($2a/h$) less than 0.8 for convergence.

It was also found that the solution generally deteriorates as the maximum slab element aspect ratio, α_{max}, increases. This is defined as the ratio of the element's long side, $2b$, to its short side, $2a$ (in plan view). The impact of this factor is limited if α_{max} is kept below 4.

The overriding importance of mesh fineness was first identified and quantified in previous University of Illinois studies, using two-dimensional models (*1, 4*). A major conclusion reached was that a more stringent mesh fineness criterion is required under the loaded area than elsewhere in the finite element mesh (*5*). This counterbalances the approximation involved in discretizing applied distributed loads. A corollary

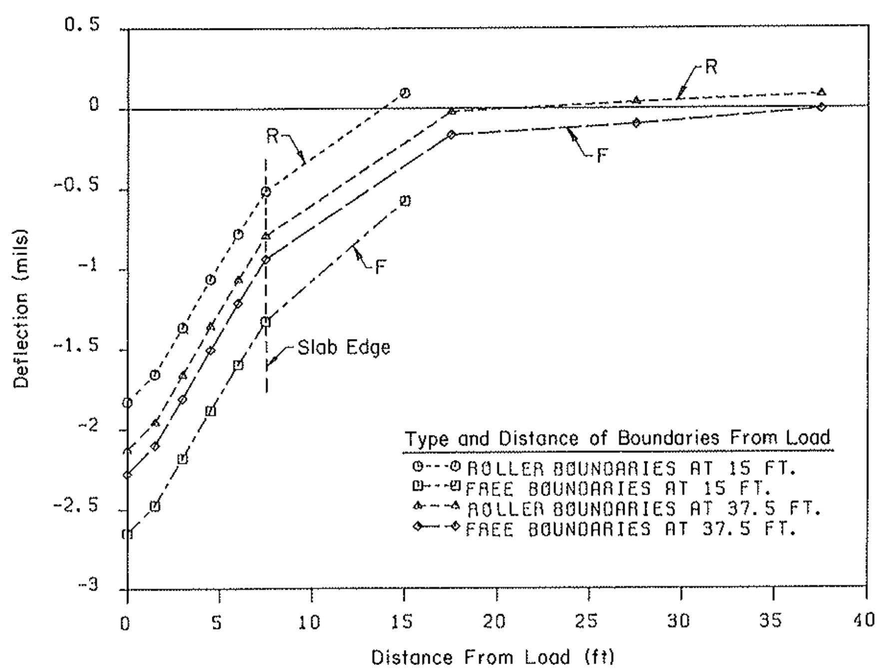

FIGURE 3 Effect of boundary conditions on subgrade surface deflection profile.

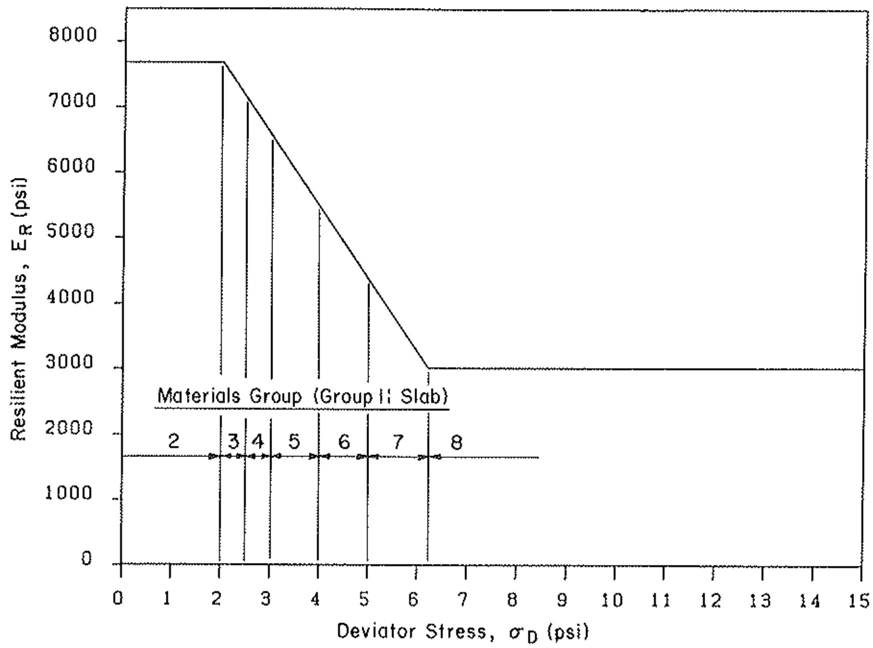

FIGURE 4 Effect of horizontal slab mesh fineness (interior loading).

TABLE 1 INVESTIGATION OF INTERIOR LOADING: F-15 SWL

SOLUTION	δ_i		q_i		σ_i	
	mils	%	psi	%	psi	%
GEOSYS-3D (Cycle 1)	32.15	90	5.031	104	609.55	96
ILLI-SLAB-2D	37.26	104	5.099	106	697.18	110
CLOSED-FORM	35.92	100	4.832	100	636.63	100

Notes:

$E = 4 \times 10^6$ psi $\mu = 0.15$ h = 8 in.
E_s = 7682 psi ('SOFT') $\mu_s = 0.45$ ℓ_e = 33.10 in.
Slab: 15 ft x 15 ft (L/ℓ_e = 5.44)
Load: 30 kips @ 355 psi, converted to 4 work equivalent loads (c = 9.193 in.)

-GEOSYS Mesh (Double symmetry - slab extends between underlined coordinates):
x-coordinates: 0; 30; 35; 36.5; 38; 39; 40; 41; 41.75; 42.5 ft
y-coordinates: 42.5; 12.5; 7.5; 6; 4.5; 3.5; 2.5; 1.5; 0.75; 0 ft
z-coordinates: 0.6; 0.3; 0; -0.5; -1.0; -2.5; -6.5; -17.5; -40 ft
In slab: $(2a/h)_{min}$=1.125; χ=2.0; α_{max}=2.0; (c/2a)=0.511.

-All responses are at interior of slab, under load:
δ_i : at top of slab;
q_i : at surface of subgrade, by diagonal extrapolation;
σ_i : at bottom of slab, by orthogonal extrapolation of σ_x values.

-ILLI-SLAB-2D: (2a/h) = 0.75; square elements.
CLOSED-FORM: Equations by Losberg (13), for infinite slab.

of this, which has serious implications with respect to recent efforts to model non-uniform pressure distributions (*6, 7*), may be stated as follows: in view of the conversion of external distributed loads into nodal components (which inevitably leads to at least some approximation, especially in the case of partially loaded elements), refinement of the applied pressure distribution, for the purpose of simulating observed deviations from simplistic uniformity, without due consideration and reciprocal improvement of mesh fineness, will be self-defeating, leading only to an illusion of improved accuracy.

Based on additional results obtained (*2*), it is recommended that the lateral extent of the subgrade beyond the slab edges, X, be divided into two elements, one 4 feet in size near the slab (1 l_e) and another 26 feet for the remainder (7 l_e).

PRACTICAL APPLICATIONS

In the analyses presented in the remainder of this paper, the following two fundamental questions are addressed:

1. How do three-dimensional analysis results compare with those from two-dimensional programs, such as ILLI-SLAB (*1, 4*), FIDIES (*8*), H51ES (*9, 1*), CFES (*5*), and others? A corollary to this is whether three-dimensional finite element analysis is necessary, and what the implications of its results are on the routine application of two-dimensional models.
2. How important is the effect of introducing stress-dependent resilient subgrade behavior? This effect was found earlier to depend on the placement and severity of the applied loads, using the two-dimensional resilient subgrade in ILLI-SLAB (*10*).

Simplified versions of the curves of subgrade resilient modulus, E_R, versus repeated deviator stress, σ_D, developed by Thompson and Robnett (*11*), are incorporated into the three-dimensional GEOSYS analysis to account for subgrade stress dependence. This is achieved through an iterative procedure implemented by a postprocessor, a program that receives as input results from GEOSYS. This iterative procedure is similar to that used for two-dimensional ILLI-SLAB analysis (*12, 10*). The modified E_R versus σ_D relation for the "soft" subgrade as employed in this study is shown in Figure 5. The deviator stress was defined in this case as the maximum difference between the three principal stresses determined by GEOSYS. Based on the σ_D level, the subgrade elements are classified into seven groups, and an average E_R is assigned to each.

FINITE ELEMENT MODEL FOR SINGLE-WHEEL INTERIOR LOADING

In a previous study, it was shown that subgrade stress dependence is not important when slab-on-grade pavements are loaded by a single tire print at the interior (*10*). To magnify any subgrade stress dependence effects, this study considered a relatively high load on a thin slab (8 inches thick), resting on the "soft" subgrade. The load applied at the interior is an F-15 single-wheel load (SWL) of 30 kips at 355 psi. Other pertinent information for this case is given in Table 1. This table also presents the maximum responses from GEOSYS, ILLI-SLAB, and the closed-form solutions (*13*). In view of symmetry, only one-quarter of the system needs to be modeled.

Effect of Subgrade Stress Dependence

A good indicator of the effect of subgrade stress dependence is the number of elements exceeding the 2 psi limit in σ_D, below which constant modulus behavior is assumed. For this GEOSYS run, only 5 out of 54 subgrade elements (or 9 per-

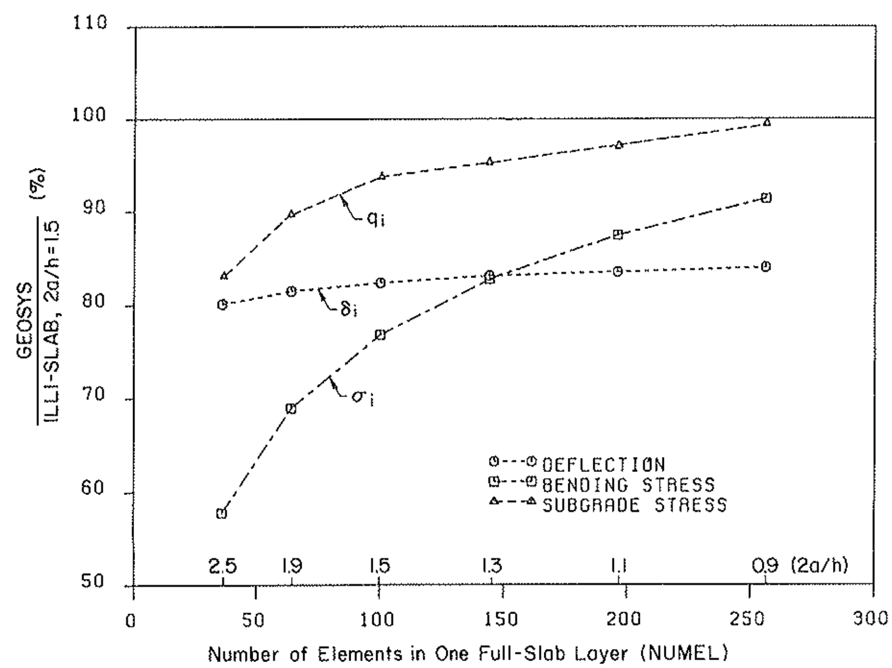

FIGURE 5 Simplified E_R vs. σ_D relation for "soft" subgrade as used in GEOSYS.

cent) had a σ_D above 2 psi. The range of σ_D values in these elements was 2.059 to 2.270 psi, with an average of 2.135 psi. The five overstressed elements (which in a subsequent iteration would change subgrade group) were located in the upper 2.5 feet of the subgrade, underneath the applied load. The updated E_R value for these would be 7,533 psi, a decrease of less than 2 percent from the initial 7,682 psi assumed for the constant, low-stress foundation modulus. It is evident that additional runs would not be very useful, since such a small change cannot be expected to affect the system significantly.

Comparison with Two-Dimensional Results

In Table 1, maximum interior deflection, subgrade and bending stress (δ_i, q_i, σ_i) from the three-dimensional GEOSYS run are compared to those from a similar two-dimensional ILLI-SLAB run. Values predicted by the closed-form solutions (*13*) are also tabulated. The ILLI-SLAB run employed a rather fine mesh. One-quarter of the slab was divided into 225 square elements, compared to only 49 elements used in the GEOSYS model. According to the results of extensive investigations using ILLI-SLAB (*1, 4*), such a fine mesh is necessary for results of adequate accuracy. The coarseness of the GEOSYS mesh is considered to be the prime source of the discrepancy between the finite element results and the closed-form solutions. The investigations conducted in the first part of this study indicate that only about 5 percent of this discrepancy may be attributed to an overall mesh fineness effect. The remainder is probably due to the fact that the mesh near the load needs to be even finer than elsewhere. In the mesh used, the load only partially covered the central element; this leads to some loss of accuracy when the applied load is converted to four work equivalent nodal loads.

The necessity for an even finer mesh under the load than elsewhere was confirmed in a previous study (*5*). This additional mesh fineness requirement over the area of applied load can also explain the high ILLI-SLAB results. All three responses may be expected to converge from above as the tire-print is subdivided into more elements. This assertion is reinforced by preliminary results obtained using the CRAY X-MP/24 supercomputer (*14*). Both GEOSYS and ILLI-SLAB results are also affected by the finite size of the slab in these analyses (compared to the infinite slab assumed in the closed-form solutions). This factor may partially explain why ILLI-SLAB δ_i and q_i are higher than the corresponding closed-form solutions, since these responses converge from above as slab size increases (*4, 5, 8*). Bending stress, however, converges from below, so the relatively high σ_i obtained by ILLI-SLAB cannot be attributed to the slab size factor. The primary source of this discrepancy, therefore, is the mesh fineness effect related to the size of the loaded area.

EDGE LOAD CASE

In this investigation, an F-15 SWL is applied to a relatively thin slab on a weak subgrade. The same slab-and-subgrade system used for interior loading is retained here for comparison. The pertinent characteristics of this system have been presented in Table 1. The finite element mesh was modified appropriately, according to the recommendations formulated in the first part of this investigation. Bending stresses developing in such a slab would be excessive in practice and are only considered here so that the effect of subgrade stress dependence is magnified.

Convergence Criteria

The iterative scheme introduced above may be used to account for the stress-dependent behavior of the subgrade. Figure 6 shows maximum responses for each of five cycles performed for the single-wheel edge loading case, normalized with respect to the corresponding average values from the second and third iterations. A uniform E_s of 7,682 psi is assumed in the first iteration, and its results correspond to those from a conventional linear elastic analysis. It is observed that the maximum responses oscillate about a value to which, presumably, they would eventually converge, if enough cycles were conducted. The amplitude of oscillation becomes progressively smaller as more cycles are performed.

The oscillation between maximum responses from the fourth and fifth iterations is only of the order ±2 percent. This indicates that for most practical purposes, five cycles are more than adequate to achieve convergence. Furthermore, the average of the values of maximum responses obtained from the second and third iterations consistently give an estimate within ±2 percent of the projected converged values. It is, therefore, recommended that three iterations be performed and that the average of the responses from the second and third cycles be adopted.

Effect of Subgrade Stress Dependence

A direct way to evaluate the effect of subgrade stress dependence for the F-15 SWL at the slab edge is to compare the maximum responses from the first iteration (which correspond to those from a conventional linear elastic analysis using a uniform, low σ_D soil modulus) to the average of those from

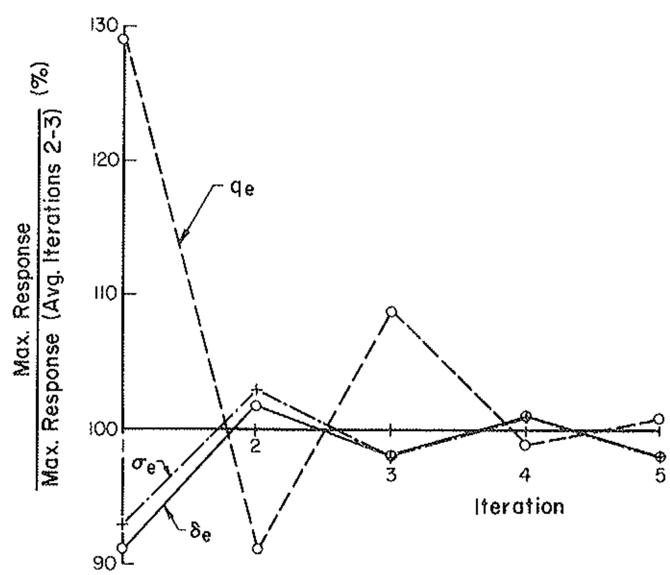

FIGURE 6 Maximum responses normalized with respect to average of second and third iterations.

the second and third iterations. Such a comparison shows in this case that maximum deflection and bending stress increase by 10 percent and 8 percent, respectively. This change is comparable to that observed with the two-dimensional K_R model in ILLI-SLAB (*10, 12*). On the other hand, maximum subgrade stress appears to be much more sensitive to stress dependence, decreasing by about 23 percent. It will be shown below that this is largely due to the development of high subgrade stress concentrations near the loaded edge. Subsequent iterations redistribute these stresses, thereby modeling local yielding, which would occur in a real soil subjected to these high stresses.

Results from this study indicate that the overall behavior of the slab is quite similar for the constant modulus and the stress-dependent subgrades. Furthermore, the differences in behavior that do exist, occur in and around the location of the subgrade elements with decreased resilient moduli. Only about 8 percent of all the subgrade elements experience a decrease in E_R. These elements are located in the upper 6.5 feet of the subgrade. Additional bending stress and deflection accumulate in elements in and under the slab as more iterations are performed, due to the decrease in subgrade support. The surface deflection basin observed after five iterations is, therefore, deeper than the one obtained after the first iteration.

A different phenomenon is observed beyond the slab, in the subgrade adjacent to the loaded edge. There, the cycle 5 profile is generally shallower than the cycle 1 profile. In addition, deflection of the stress-dependent subgrade (cycle 5) is reduced to 50 percent of the maximum value within only 6 inches from the loaded slab edge. The structural contribution of the subgrade beyond the slab is less significant for the stress-dependent model than for the constant modulus subgrade. Therefore, it can be seen that stress dependence tends to move the elastic solid model toward the direction of the dense liquid idealization, in which the contribution of the subgrade adjacent to the slab edges is altogether neglected.

Comparison with Two-Dimensional Results

Results obtained using GEOSYS may be compared to those from a number of two-dimensional models available, in order to establish the relative adequacy of the two approaches. In this section, results from programs FIDIES (*8*), H51ES (*1, 9*), and ILLI-SLAB (*4*) will be considered. FIDIES is a two-dimensional finite difference solution, employing square elements throughout the slab. The external loads are converted to point loads, applied at the center of each element. Responses are calculated at these points alone. Thus, an extrapolation, similar to that used with GEOSYS results, is necessary to obtain the edge responses. H51ES may only be used to calculate maximum bending stress at the edge, but its results are akin to a closed-form solution. Thus, they are not a function of user-specified parameters, such as mesh fineness, and element aspect ratio. These considerations can be crucial in the case of programs such as FIDIES and ILLI-SLAB. On the other hand, the latter two programs can account for the finite size of the slab and can determine the spatial distribution of all three responses, rather than just the maximum value of one of them.

Table 2 presents results obtained using these programs. The grid employed in the FIDIES run consisted of 441 square elements (no symmetry capability exists in FIDIES at this time). Previous studies (*1, 8, 15*) suggest that this grid may be slightly coarse, but it is dictated by computer memory limitations. In any case, the approximation involved in representing the applied load as a point load, located a few inches from the edge, is the overriding consideration here. Unfortunately, this approximation cannot be avoided. The ILLI-SLAB run was conducted using a mesh found to produce results of adequate accuracy (*15*). The other results are normalized in Table 2 with respect to the ILLI-SLAB responses.

The value of σ_e from ILLI-SLAB is about 11 percent higher than the closed-form solution from H51ES. This discrepancy cannot be explained by reference either to an inadequate mesh fineness or to the small slab size, since increasing these parameters would tend to increase the ILLI-SLAB value of σ_e. A similar discrepancy was observed with the dense liquid foundation as well, and was related to the size of the loaded area. For the (c/l_e) value of 0.28 used, the discrepancy was 10 percent (*1*), which is close to that observed here. Preliminary results using the CRAY X-MP/24 (*14*) indicate that this discrepancy disappears when the mesh under the loaded area is refined further.

On the other hand, FIDIES values of δ_e and σ_e are higher than the corresponding ILLI-SLAB ones, while q_e is lower. Again, overall grid fineness and slab size considerations cannot explain the discrepancy observed. It is considered that the major sources of this are the conversion of the external load to a point load acting at the center of an edge element, and the extrapolation involved in obtaining the values in Table 2 from the FIDIES output.

In Table 2, GEOSYS results from the first cycle are also presented. Comparison with those from ILLI-SLAB suggests that three-dimensional analysis gives q_e and σ_e values that are lower than those from ILLI-SLAB by about 60 percent and 30 percent, respectively. Maximum edge deflections are in relatively better agreement, the ILLI-SLAB value being only 9 percent higher than the corresponding one from GEOSYS. Part of these discrepancies (5 to 10 percent) may be attributed to the coarse mesh used with GEOSYS. The remainder of the discrepancy is probably due to the fact that both GEOSYS σ_e and q_e are extrapolated from calculated values at the centroid of each brick element. This is particularly important in the case of subgrade stresses. The contour plot of subgrade stresses in Figure 7 indicates that high subgrade stresses occur in a narrow zone along the loaded edge. Such a drastic increase in subgrade stress right at the edge of the slab has also been observed under interior loading (*5*), and is similar in nature to the infinite reactions predicted by Boussinesq's theory at the edge of a rigid punch. The linear extrapolation used to determine q_e from GEOSYS results is unable to reproduce the high stress gradients in this area.

In the narrow region immediately adjacent to the loaded edge, local yielding of the soil will occur. Thus, the theoretical value of the subgrade stress at the physical edge of the slab may not be of practical significance. A more meaningful value may be the subgrade stress developed a few inches inside from the edge, for example, at a distance of $0.2\,l_e$. A previous study showed that this value is relatively insensitive to mesh fineness (*5*). Additional iterations have the effect of redistributing the stress away from highly stressed elements. Thus, introducing subgrade stress dependence enables the user to model local

TABLE 2 COMPARISON OF TWO- AND THREE-DIMENSIONAL SOLUTIONS: F-15 EDGE SWL

SOLUTION	δ_e		q_e		σ_e	
	mils	%	psi	%	psi	%
GEOSYS-3D (Cycle 1)	60.32	91	24.63	42	826.0	70
FIDIES-2D (Linear extrap.)	73.21	111	30.93	53	1518.1	129
FIDIES-2D (Quadr. extrap.)	73.61	111	35.36	60	1605.8	136
H51ES	-	-	-	-	1055.9	89
ILLI-SLAB-2D (23x13 Mesh)	66.09	100	58.84	100	1180.8	100

Notes:

-For slab and subgrade characteristics, see Table 1.

-GEOSYS Mesh (Symmetry about x-axis employed - slab extends between underlined coordinates):
x-coordinates: 0; 18.5; 37; 41; 45; 47; 48; 49; 49.5; <u>50</u>; 50.9375; 51.875; 53.750; 55.625; 57.5; 60; 62.5; <u>65</u>; 95 ft
y-coordinates: 42.5; 12.5; <u>7.5</u>; 5.625; 3.75; 1.875; <u>0</u> ft
z-coordinates: <u>0.6</u>; 0.3; <u>0</u>; -0.5; -1.0; -2.5; -6.7; -19.27; -40 ft
In slab: $(2a/h)_{min}$=2.81; χ=2.0; α_{max}=2.0; $(c/2a)$=0.20.

FIDIES mesh: (Δ/h) = 1.07; 21x21 square elements;
ILLI-SLAB mesh (symmetry about x-axis employed):
(23x13): $(2a/h)_{min}$=0.575; χ=3.0; α_{max}=2.18;
H51ES: 50 points used to define outline of applied load.

-All responses at intersection of loaded edge and centerline of load:
δ_e : at top of slab;
q_e : at surface of subgrade, by orthogonal extrapolation;
σ_e : at bottom of slab, by orthogonal extrapolation of σ_y values.
FIDIES results extrapolated using 2 (linear) or 3 (quadratic) elements along the slab centerline;
No extrapolation is involved in ILLI-SLAB and H51ES results.

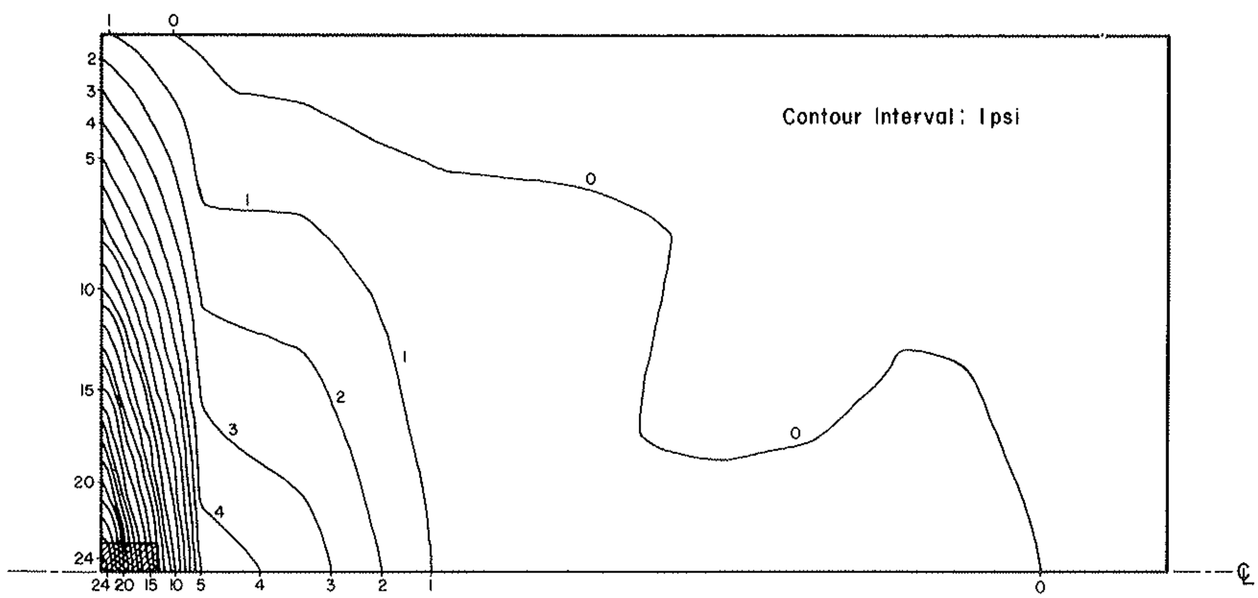

FIGURE 7 Surface subgrade stress contours from GEOSYS under slab: constant modulus subgrade (cycle 1).

yielding that would occur in a real subgrade. Notwithstanding the differences in their maximum responses, ILLI-SLAB and GEOSYS produce similar response distributions outside the critical edge region. The plots in Figure 8 reinforce the validity of the comments made above.

Similar conclusions were also reached from an investigation of multiwheel edge loading (MWL), using one landing gear from a C-141 aircraft. The effect of accounting for subgrade stress dependence is quite significant in this case. Maximum deflection increases by about 30 percent, and maximum bending stress by more than 20 percent. On the other hand, maximum subgrade stress decreases by more than 30 percent. Furthermore, the extent of the region of reduced resilient moduli obtained with the C-141 load is much larger compared to the F-15 case. The subgrade stress distributions in Figure 9 indicate that comparing only the maximum q_c values from the two- and three-dimensional models may be misleading. The overall system responses are much closer to each other than the individual maximum values would suggest. This is especially true when fine meshes are used.

CORNER LOAD CASE

To complete the series of three-dimensional investigations using GEOSYS, the corner loading condition is examined in this section, with an F-15 single-wheel load. Unfortunately, the lack of symmetry along either of the two major coordinate axes of the slab dictates the use of a full mesh in the finite element idealization. This results in prolonged execution times (in excess of 12 CPU hours on the HARRIS 800-2 virtual memory computer), even using a rather coarse mesh, which gives rise to less accurate results. General trends, however, may still be observed and these can be useful despite any limitations in the accuracy of individual response values.

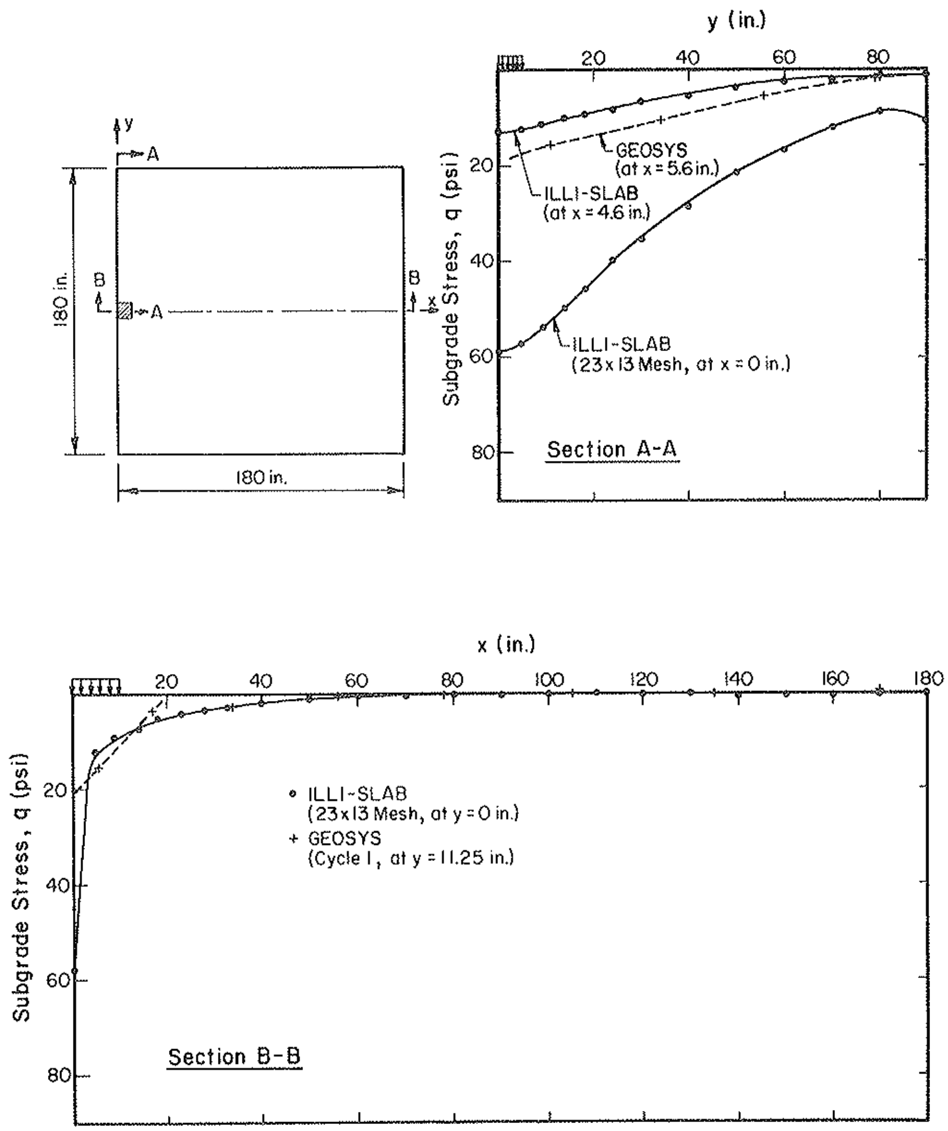

FIGURE 8 Comparison of GEOSYS and ILLI-SLAB surface subgrade stress distributions for F-15 edge SWL, under slab.

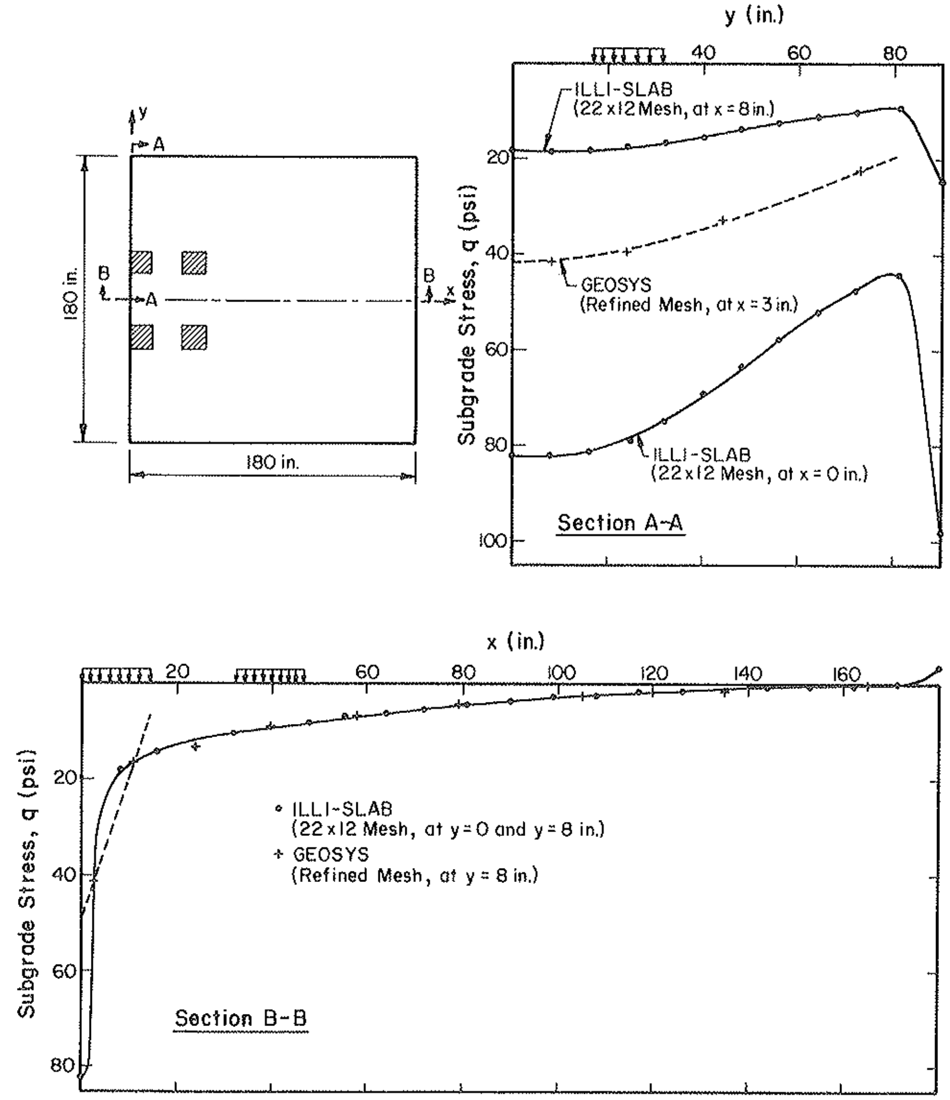

FIGURE 9 Comparison of GEOSYS and ILLI-SLAB surface subgrade stress distributions for C-141 edge MWL, under slab.

Effect of Subgrade Stress Dependence

Table 3 presents a summary of maximum responses obtained form three iterations performed for this case. Maximum deflection, δ_c, and subgrade stress, q_c, occur at the corner of the slab. On the other hand, maximum (tensile) bending stress, σ_c, occurs at the top of the slab, some distance from the corner. Preliminary results using ILLI-SLAB (*15*) suggest that a slab resting on an elastic solid also develops a high tensile bending stress at the bottom fiber under the load. This cannot be confirmed at this time using GEOSYS, however, since a much finer mesh would be required.

Subsequent iterations in Table 3 lead to a maximum deflection that is almost 20 percent higher than the value obtained from cycle 1. Similarly, an increase of more than 10 percent is observed in the maximum bending stress developing in the slab. On the other hand, a dramatic decrease in the value of the maximum subgrade stress predicted by the first iteration is obtained as subgrade stress dependence is considered. This reflects the redistribution of high subgrade stresses concentrated at the slab edges, observed earlier. Local yielding of soil in the corner region prevents these high stresses from developing, and this is accounted for by the iterative procedure used with GEOSYS. The maximum subgrade stress occurring at the slab corner itself, therefore, may not be a meaningful response. The subgrade stress developing under the center of the load, q_c^*, for example, may be a more realistic indication of subgrade response. This will be substantially lower than the value at the corner and, therefore, less sensitive to stress dependence. The oscillation of q_c^* in Table 3 is of smaller amplitude than that of q_c itself.

It is interesting to observe that about 20 percent of the subgrade elements are affected by stress dependence, compared to only 8 percent for the F-15 edge load. Subgrade stress dependence is predictably a more serious consideration under corner than under edge loading. The high values of σ_D are limited to the upper 5.5 feet of the subgrade, below which σ_D is less than 2 psi.

TABLE 3 EFFECT OF SUBGRADE STRESS DEPENDENCE: F-15 CORNER SWL

ITERATION	δ_c		q_c		q_c^*		σ_c	
	mils	%	psi	%	psi	%	psi	%
1	72.42	100	42.93	100	31.27	100	312.1	100
2	87.67	121	28.16	66	19.95	64	358.5	115
3	82.99	115	22.16	52	19.86	64	343.1	110
Avg. 2-3	85.33	118	25.21	59	19.81	64	350.8	112

Notes:

$E = 4\times10^6$ psi $\mu = 0.15$ $h = 12$ in.
$E_s = 7682$ psi ('SOFT') $\mu_s = 0.45$ $\ell_e = 49.64$ in.
Slab: 15 ft x 15 ft $(L/\ell_e) = 3.63$
Load: 30 kips @ 355 psi, converted to four work equivalent loads ($c = 9.193$ in.)

-Mesh (Slab extends between underlined coordinates):
x-coordinates = 0; 27.5; 29.167; <u>30</u>; 30.833; 32.5; 35; 40; <u>45</u>; 60 ft
y-coordinates = 60; <u>45</u>; 40; 35; 32.5; 30.833; <u>30</u>; 29.167; 27.5; 0 ft
z-coordinates = <u>1</u>; 0.5; <u>0</u>; -0.5; -1; -2.5; -6.5; -17.5; 40 ft

In slab: $(2a/h)_{min}=0.833$; $\chi=1.0$; $\alpha_{max}=6.0$; $(c/2a)=0.92$.

-Location and method of extrapolation of responses:
δ_c : at slab corner, at top of the slab;
σ_c : tension at top of slab, at some distance from corner: $\sigma_c = \sigma_1$, from values of σ_x, σ_y, τ_{xy}, extrapolated to the top, using linear orthogonal extrapolation;
q_c : compressive, at surface of subgrade under corner of slab: extrapolated up to surface of subgrade from values under corner region of slab, using linear orthogonal extrapolation.
q_c^* : compressive, at surface of subgrade, at x=5 in., y=5 in. from corner, under slab, extrapolated like q_c.

Comparison with Two-Dimensional Results

In view of the relatively coarse mesh employed in the GEOSYS runs, considerable discrepancies exist between three- and two-dimensional responses. GEOSYS gives a δ_c which is about 10 percent lower than the one predicted by ILLI-SLAB using a fine mesh. Differences in σ_c and q_c are even higher (+10 percent and −70 percent, respectively). According to the criteria developed above, GEOSYS results may be expected to be too low by less than 5 percent. Possible causes of the remainder of the gap between the two- and three-dimensional results were identified above as the coarseness of the GEOSYS mesh in the area of the load and the extrapolation involved in obtaining maximum stress values from the GEOSYS output. The latter is more important in the case of subgrade stress, in view of the high stress gradients existing near loaded slab edges and corners.

Figure 10 shows the distribution of subgrade stresses predicted by ILLI-SLAB and GEOSYS. These confirm that the two models are in good agreement, except in a region about 8 inches wide (or 0.2 l_e) along the slab edges. Pronounced subgrade stress gradients develop very close to the edges of the slab under all three fundamental loading conditions.

DISCUSSION AND CONCLUSIONS

Three-dimensional finite element analysis is shown in this paper to be a feasible and viable tool in pavement studies. Although this sophisticated numerical technique is unlikely to become a routine procedure in the near future, results presented herein indicate that it is possible to use a three-dimensional model to account for subgrade stress dependence, soil yielding, among other factors, as well as to accommodate multiwheel gears situated anywhere on the pavement surface. The GEOSYS model described is currently the only program that can be employed for analyzing both rigid and flexible pavements. Thus, it will be indispensable in future

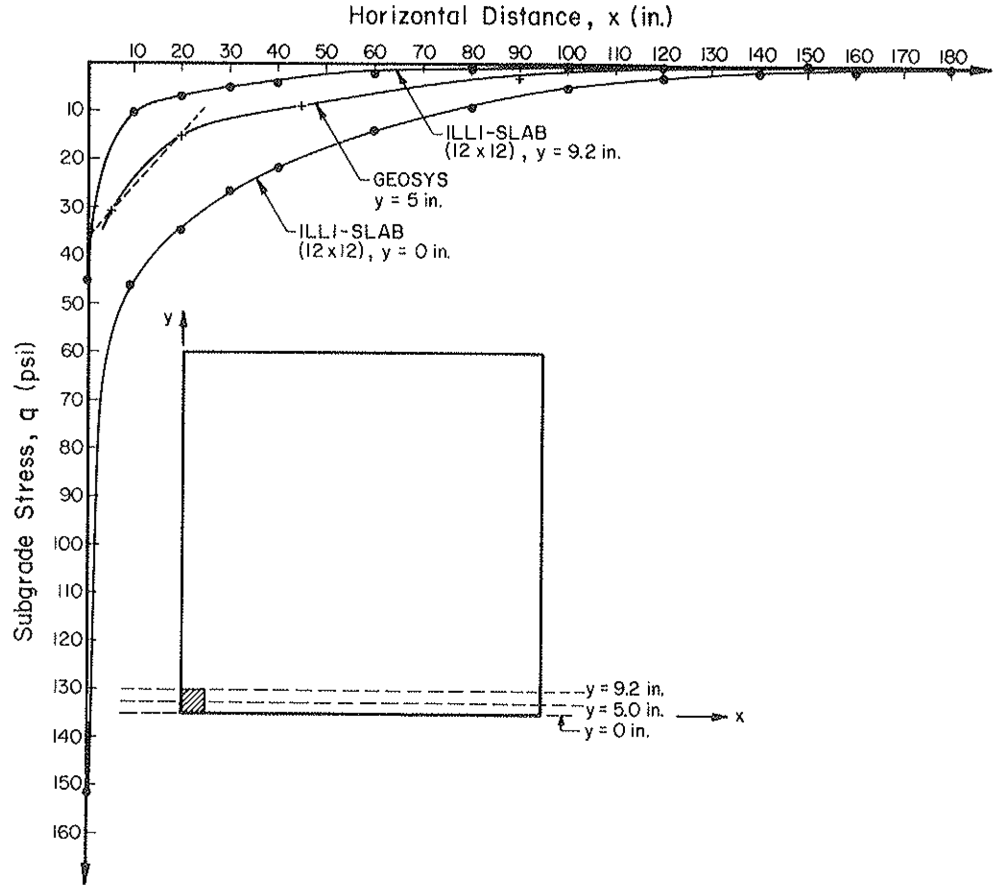

FIGURE 10 Comparison of GEOSYS and ILLI-SLAB surface subgrade stress distributions for F-15 corner SWL, under slab.

efforts to establish the elusive "unified" approach and to develop a generalized mechanistic design procedure.

A most significant conclusion reached, however, is that the three-dimensional investigations reinforce the validity and desirability of conventional two-dimensional analysis. In view of the relatively limited amount of three-dimensional results generated during this investigation, the accuracy of these results was often assessed by reference to the corresponding two-dimensional values. Where these disagreed, probable causes were considered and were usually found to be due to the coarse three-dimensional mesh used in these analyses.

This is not to deny the desirability of three-dimensional analysis as a means of checking and validating two-dimensional results. It does suggest, however, that for a meaningful utilization of the much more complex and demanding three-dimensional approach, adequate computer resources must be available. The rapid advances of computer technology in general, and the introduction of supercomputers in particular, provide reasons for optimism in this respect. Results from this study will be invaluable when considering the implementation of such a model on the mammoth machines anticipated in the near future.

From a more practical viewpoint, this study has shown that subgrade stress dependence may sometimes be important, primarily when considering heavy edge and corner loads. Subgrade stress dependence affects the maximum subgrade stress to a much greater extent than the maximum deflection or bending stress. When the load becomes more severe, for example, when it is placed near a corner rather than near an edge, the difference between the first and last iterations becomes much greater. These conclusions are similar to those reached using the two-dimensional K_R model (*10, 12*).

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