Direct Calculation of Maximum Curvature and Strain in Asphalt Concrete Layers of Pavements from Load Deflection Basin Measurements

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Deflection measurements under impulse load carried out with the Falling Weight Deflectometer (FWD) are usually processed by using elastic layer analysis programs, back-calculating layer moduli from measured deflections, and then forward-computing critical performance parameters such as stresses and strains at locations under the load. Presented here is a direct, and more reliable way, of computing the important value of horizontal strain at the bottom of the asphaltic layer. The measured deflections from an FWD test, the radius of load distribution, and the thickness under the load are the only data needed to calculate, first, the curvature, and then, the strains in the top (asphaltic) layer of a pavement structure, in the center under the load. Corresponding stresses can also be calculated but additional information is needed, namely, the elastic stiffness of the first layer, Poisson's ratio, and the load-induced vertical stress. It is shown that strains and stresses in the immediate vicinity of the load position as computed with elastic layer analysis methods via back-calculated moduli are not as reliable as this new proposed strain criterion. At this time, the proposed new method of calculating strain directly is derived, presented, and discussed as a theory, without experimental field verification.

To determine the strength and predict the performance of asphalt pavements from some kind of deflection measurement has been the subject of major efforts in research for many years. The Falling Weight Deflectometer (FWD) test is only one of many methods to measure deflections of pavements; however, it is a very recent one and it is widely accepted in the U.S. The FWD test accurately measures a set of deflections of a deflection bowl under an impulse load of circular distribution, simulating the transient load of a passing wheel (*1*, *2*). In this method there is a choice of several levels of impulse load, corresponding to variations in axle weight of actual vehicles. Deflections are measured at various distances from the load, including at the load axis itself. In operation, the test equipment is fast, economical, and very simple to use.

The state-of-the-art in processing data from FWD measurements is as follows. Asphalt concrete pavements with known layer thicknesses are analyzed by using a set of deflections measured along the wheel path, typically in the center underneath the circular loading plate and at six adjacent locations outside the plate, spaced over 1.5 to 2 m. These deflections (together with known layer thicknesses) are used to back-calculate the elastic stiffness (Young's modulus) of all pavement layers, including the subgrade, using certain computer programs for elastic layer analysis. Once the elastic stiffnesses are known, any stresses, strains, and deflections can be computed by the same or similar compatible programs in forward mode. This paper is concerned with the maximum horizontal strains and stresses in the first, usually asphaltic layer, of a multi-layered pavement structure. It is customary to regard strains or stresses at certain locations under wheel loads as critical. One of these locations is at the bottom of the asphaltic layer underneath the load, where horizontal stresses and/or strains are regarded as critical distress parameters, being related to fatigue strength (*3*, *4*). In this context it can be shown that there is a more direct and simpler way to calculate the horizontal strains and stresses, which can supplement the aforementioned methods.

For reasons of pure geometry, the horizontal strains in any top layer of a pavement structure can be calculated directly from measured deflections, based on the theory of elastic plates, using the (partial) second derivatives or radii of curvatures directly obtained from the deflection basin. This basin is given by discrete values from the FWD testing. Through these points of measurement a suitable curve must be fitted. However, this approach is hampered by a lack of data points in the immediate vicinity of the load axis. The critical maximum curvature is underneath the load, and the difficulty is to find a sufficiently accurate value of it, representing the deflection basin in the center under the circular contact pressure area. From such a curvature value (and the given thickness) the strains in the asphalt layer can be calculated without knowing the modulus of elasticity or Poisson's ratio. The thickness of the asphaltic layer underneath the load must be known, but need not be so widely uniform as assumed in elastic layer analysis methods. In turn, corresponding stresses can be calculated from those strains, if additional information is available, such as vertical stresses, elastic modulus of the asphaltic layer, and Poisson's ratio.

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STRESSES AND STRAINS FOR CIRCULARLY DISTRIBUTED LOADS

For such a single load, well inside a (non-cracked) pavement, the second derivative or curvature of the deflection basin under the load axis can be assumed to be a maximum, and equal in all directions because of the circular contact pressure area. The following equations apply:

Plate stiffness

$$K = \frac{EH^3}{12(1 - \mu^2)} \tag{1}$$

Bending moment

$$M = -KW''(1 + \mu) \tag{2}$$

Horizontal stress

$$S = 6M/H \tag{3}$$

Horizontal strain

$$e = S(1 - \mu)/E \tag{4}$$

where

K = bending stiffness of plate,
E = elastic stiffness of asphaltic layer(s),
H = (total) thickness of asphaltic layer(s),
μ = Poisson's ratio,
W = deflection as a function of distance,
W'' = second derivative of deflection at zero distance,
M = bending moment under the load at zero distance,
S = maximum stress at bottom of asphaltic layer, and
e = maximum strain at bottom of asphaltic layer.

Note: The second derivative is negative. Bending moments, stresses and strains are equal in all directions.

Without a knowledge of the material properties of the first layer (the modulus, E, and Poisson's ratio), the strain, e, can still be calculated from pure geometrical conditions:

either

$$e = W''H/2 \tag{5}$$

or

$$e = H/(2R) \tag{6}$$

The stress is then

$$S = eE/[(1 - \mu)(1 + S_v/S)] \tag{7}$$

where

R = radius of curvature in mm, and
S_v = vertical stress in MPa.

At this stage, in order to calculate corresponding stresses from strains, additional information is required, namely the elastic stiffness (E) of the first asphaltic or concrete layer, the Poisson's ratio of this layer, and the vertical stress (S_v) at the particular point. This vertical component of stress must be obtained by an elastic layer analysis program; however, a simple version based on equivalent layer thickness would be sufficient for an estimate of such stress.

The equations quoted above are based on the concept of regarding the asphaltic layer as an elastic plate. This plate, as a free body, deforms or deflects under two forces, the applied wheel load or falling weight impulse load acting more or less concentrated from above, and the corresponding reactions of soil pressure from the layers underneath, acting onto the plate from below in a more distributed fashion (Figure 1a). For a given load, the deflection bowl, its depth and largeness, depends on the stiffness of the plate and the relative stiffness or strength of the base or soil layers underneath.

Equations 5, 6, and 7 constitute a direct calculation of horizontal strain and stress in the A.C. layer. However—and this is the difficult part—it is now necessary to find an expression for the curvature or second derivative from an accurate deflection function.

CURVATURE CALCULATION INSIDE THE CONTACT PRESSURE AREA

The maximum value of the second derivative (W'') or curvature ($1/R$) of the asphaltic layer, for a circular, uniformly distributed load (p), is located in the center of this circular contact pressure area. The value, W'', can be derived in accordance with elastic plate theory. For the derivation, the following general assumptions should be noted:

1. Single tire loads and impulse loads from the FWD test are uniformly distributed over a circular area.
2. The resultant load per unit area on the circular part of the asphaltic layer "plate" has a paraboloidal distribution, for the following reason. Within the loaded area, from $-a$ to $+a$, the tire or plate contact pressure must be combined with the soil pressure acting from below, resulting in a reduced diagram of parabolically distributed load as shown in Figure 2. The resultant area load (i.e., load per unit area) on the asphaltic layer plate in this region is actually assumed to be distributed in the form of a square paraboloid. (The derivation of the exact solution for this case from the differential equation of circular plates is not presented here.) The result for maximum curvature is:

$$W''(0) = -\frac{2(Y_1 - Y_a)}{a^2} - \frac{a^2(8p_o + p_a)}{288K} \tag{8}$$

where

$W''(0)$ = second derivative in the center, in 1/mm;
Y_1 = maximum deflection in the center, in mm;
Y_a = deflection at the edge of the loaded area, in mm;
a = radius of the loaded area, in mm;
p = circular contact pressure from load, in MPa;
p_o = resultant pressure in the center, in MPa;
p_a = resultant pressure at the edge, at a, in MPa;
K = plate stiffness (Equation 1); and
r = distance from the center, in mm.

A better understanding of the curvature function or second derivative, W'', within the circular contact pressure area can be achieved by studying W'' as a function of distance, r:

$$W(0) = -\frac{2(Y_1 - Y_a)}{a^2} - \frac{p_o}{32K} \times (a^2 - 6r^2) - \frac{p_a - p_o}{288Ka^2}(a^4 - 15r^4) \tag{9}$$

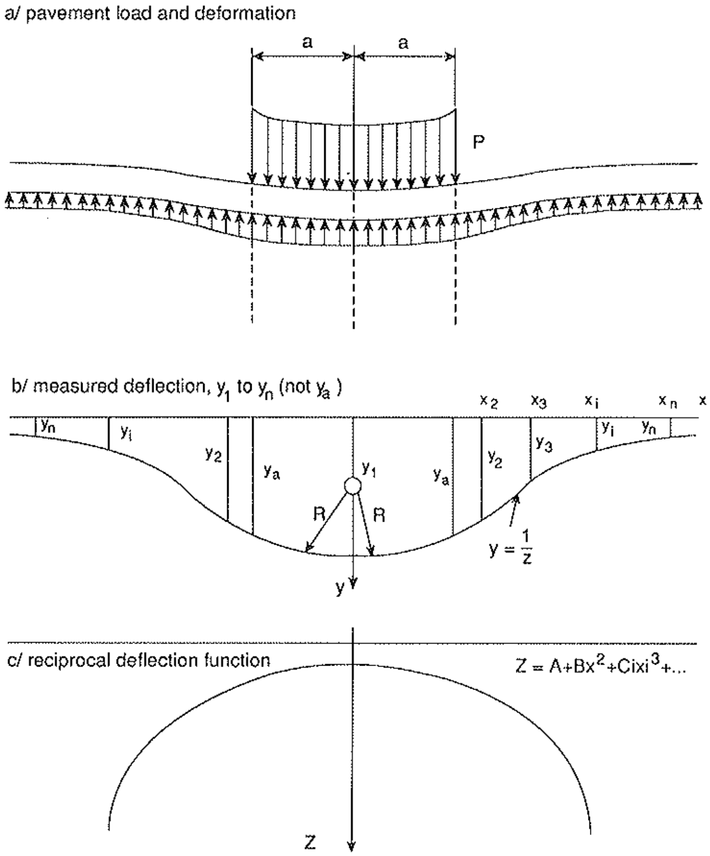

FIGURE 1 Deflection basin geometry.

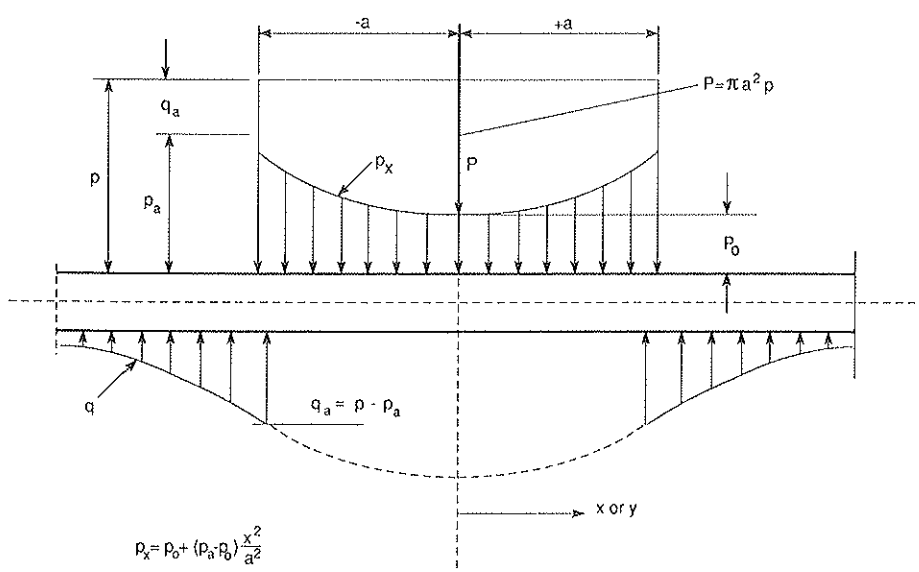

FIGURE 2 Load distribution between $-a$ and $+a$.

Equation 9 is plotted for a typical example as illustrated in Figure 3. Note, the maximum curvature is at $r = 0$, namely $5.422*10^{-6}$(1/mm). The first term of Equation 8, by itself, would lead to a value of $4.764*10^{-6}$ (1/mm), which can be interpreted as an average within a distance, r, of about 100 mm. This slightly lower value of average curvature agrees fairly well with values obtained from manual curvature calculations using simulated Chevron deflection output, as shown below. Thus, the first term of Equation 8 or 9 constitutes an approximate calculation of curvature compatible with elastic layer analysis programs, a smaller value than the actual maximum. This first term would be an exact solution for a hypo-

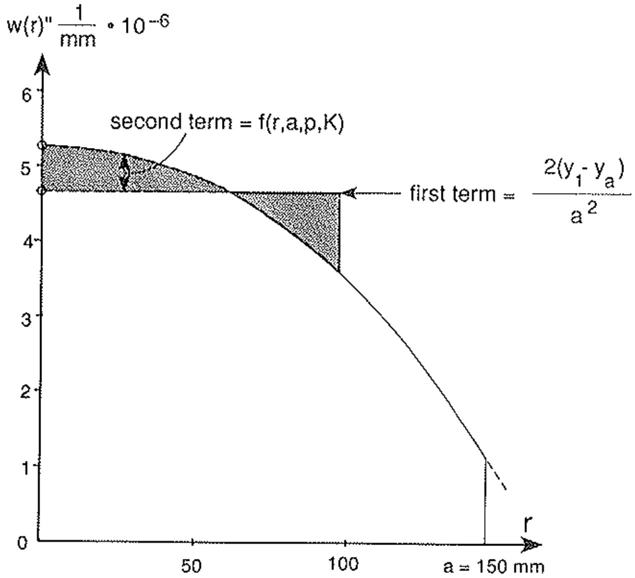

FIGURE 3 Radial curvature in the asphaltic layer.

thetical case in which the soil pressure, q, and the contact pressure, p, were removed, and the plate were loaded via a circular ring of radius, a.

Unfortunately the deflection, Y_a, cannot be measured, but must be calculated by interpolation using the other measured deflections. The ensuing error is positive, i.e., in the direction of higher values, being closer to the theoretical maximum (tables 1 and 2). Further, the deflection basin consists of two different parts, i.e., of two curves not continuous in all derivatives. The inner part ($-a$ to $+a$) has been accurately derived, resulting in Equation 9. The outer part only is empirically given by the measurements of outside sensors.

The curve-fitting function chosen is a reciprocal polynomial (Z) (Figure 1). It is used only to calculate the very influential and important value of the deflection, Y_a, at the edge of the loaded area (i.e., at $x = a$) by interpolation.

The curve-fitting and interpolation procedure to compute an approximate substitute for the theoretical value of Y_a becomes more reliable, thus leading to less erratic results, when the first outside sensor (Y_2) is as close to the circular disk as possible and when the center point deflection is included as part of the curve (as illustrated in Figure 1b and 1c), in

TABLE 1 COMPARISON OF STRAIN *R* WITH CHEVRON/ELSYM5

DESCRIPTION	CURVATURES * 10^{-6} 1/mm		STRAIN * 10^{-6}		
	DIRECT	ELSYM5/ CHEVRON	DIRECT	ELSYM5/CHEVRON bottom	average
Typical 3-layer case a = 150 mm, H = 100 mm	-5.160	-4.80	258.0	170.4	193
Typical 4-layer case a = 150 mm, H = 100 mm	-5.426	-5.28	271.3	185.7	255
Overload case, 2.5x a = 150 mm, H = 100 mm	-14.46	-13.8	642.6	423.0	520
Deep strength 140 mm on weak soil	-4.334	-4.53	302.7	278.2	278
Reduced area a = 102 mm high pressure 400 psi	-26.97	-26.75	1198.9	786.4	860
Wider area, a=203 mm normal pressure	-8.753	-8.66	389.0	243.2	341
Typical 3-layer case average deflection	-5.066	-5.18	253.3	208.2	204
Very soft layer inserted under asphalt	-8.438	-8.74	421.9	383.8	385

Note: In the last column, "average" means the average of the absolute value of the top and bottom strains of the first layer from the output listing of CHEVRON or ELSYM5. These values in the last column ought to be close to or equal to H/2 times the curvature values in the second column. This identity check would establish consistency within the CHEVRON or ELSYM5 programs themselves, not involving the STRAINR model. Note that the check fails except in the case of a very soft inserted layer in the last line.

TABLE 2 CURVATURE BY SECOND DIFFERENCES VERSUS EQUATION 8

CASE (see also Table 1)	SECOND DIFFERENCES $[1/mm]*10^{-6}$	EQUATION 8 $[1/mm]*10^{-6}$
1/ Typical 3-layer case	-4.80	-5.475
2/ 4-layer case, example	-5.28	-6.018
3/ Overload case, 2.5 x	-13.78	-15.535
4/ Deep strength, weak soil	-4.53	-4.603
5/ Reduced area, high pr.	-26.75	-28.931
6/ Wide area, normal pr.	-8.66	-9.722
7/ 3-layer case, av. defl.	-5.182	-5.562
8/ Soft 2nd layer inserted	-8.744	-8.875

spite of the fact that it belongs to the inner part which is mathematically different. Nevertheless, a useful solution has been established which has been verified as follows:

The first term of both Equations 8 and 9, using the accurate value of Y_a, leads to practically the same curvatures as obtained by using the method of second differences, for the same data from Chevron or ELSYM5 simulation examples.

Comparative computations show that the exact calculation of curvature by Equation 8 results in larger values (up to 15 percent) than the second differences from Chevron or ELSYM5. These results from second differences compare much better with the first term of Equations 8 or 9, omitting the second term. Thus there is a valid argument to dispense with this term which requires additional information about the asphaltic layer. Thus, the following Equation 10 can be used to calculate the minimum radius of curvature, R:

$$R = -\frac{(Y_1 - Y_a)^2 + a^2}{2(Y_1 - Y_a)} \qquad (10)$$

where

Y_1 = maximum deflection in the center, in mm;
Y_a = deflection at the edge of the load disk, in mm;
a = radius of the circular loading area, in mm; and
R = radius of curvature, in mm.

Note that the first term in the numerator is very small and can be set to zero; then $1/R$ becomes exactly the first term of the second derivative, W'', in Equations 8 and 9.

A small computer program can be written to calculate curvatures and strains of the asphaltic layer, based on Equation 10, using a substitute value of Y_a found by interpolation of the deflection basin function shown in Figure 1, which is curve-fitted from relevant FWD measurements.

Alternatively, one can take into account the second term of Equation 9. This solution could ultimately be useful in conjunction with a simplified elastic layer analysis program based on Odemark, using the concept of equivalent layer thickness.

EXAMPLE

The following example illustrates one of the verification tests of the proposed method, comparing the new proposed calculations (based on Equation 11) with those from a corresponding case calculated by the Chevron program. For running the small program of the proposed method, calculated Chevron deflections were selected as input, simulating feasibly-spaced FWD measurements, in the center and at four other points outside the circular load area. In addition to these measurements, the following data are needed to run the new program:

Radius of load area: a = 150 mm; layer thickness: H = 100 mm

The deflections chosen from the Chevron example output, used as simulated input into the new program, are as follows:

at 0.0 m 0.4903 mm

at 0.2 m 0.3951 mm

at 0.3 m 0.3394 mm

at 0.4 m 0.2949 mm

at 0.5 m 0.2585 mm

The following values were calculated by the proposed method:

Deflection at the edge of the loaded area: Y = 0.4293 mm (the actual Chevron value is 0.4305, an inevitable error in curve-fitting and interpolation)
Radius of curvature at the center: $R = -184.3$ m
Second derivative at center: $W'' = -5.43 * 10^{-6}$ (1/mm)
Strain at bottom of asphaltic layer: $e = +271 * 10^{-6}$

More information was needed to run the Chevron program, namely:

Load: 40.01 kN, tire pressure: 566.00 kPa, load radius: 150 mm
Layer values:

(1) Modulus: 3000 MPa, Poisson's ratio: 0.35, Thickness: 100 mm
(2) Modulus: 500 MPa, Poisson's ratio: 0.35, Thickness: 200 mm
(3) Modulus: 80 MPa, Poisson's ratio: 0.35, Thickness: infinite

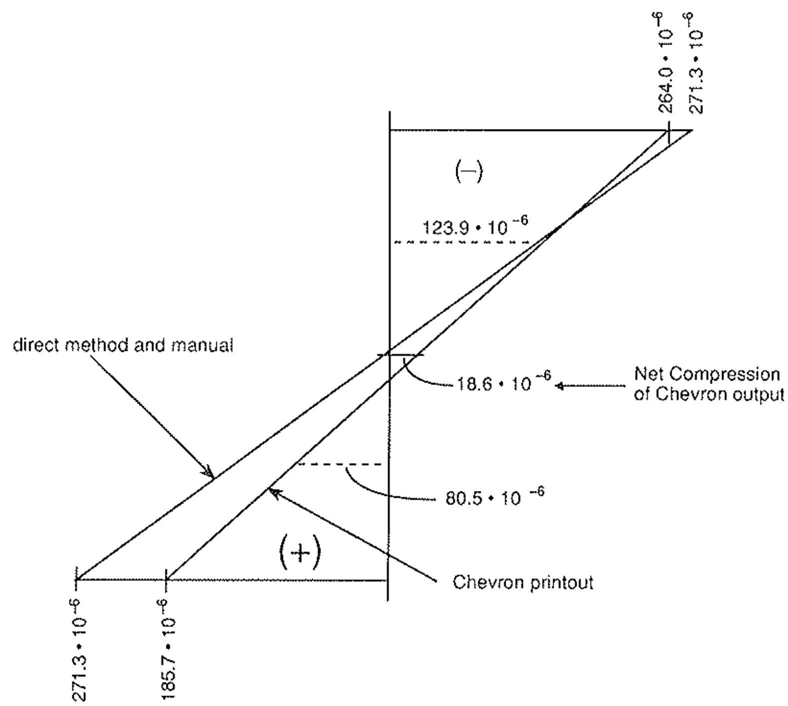

FIGURE 4 Comparison of strain calculation (Chevron example).

Note: (1) = surface course, (2) = base course, (3) = subgrade.

Using these input values for the Chevron program, the deflections were calculated every 50 mm. Then the finite difference method was used to manually compute the second derivatives between $X = 0$ and 150 mm.

The results fluctuate somewhat inconsistently, and the average was found to be −5.37, with a slightly smaller value at $X = 0$ of −5.28. This compares fairly well with the computed value of −5.43 of the output listed above, being slightly larger and therefore closer to the theoretical maximum.

Figure 4 shows a comparison of the corresponding strain calculations. The strain diagram from the direct Chevron printout is slightly curved at the top; it also exhibits a small net compressive strain. The strains of the direct output ($+185.7 * 10^{-6}$ and $-246.0 * 10^{-6}$) are smaller, especially at the bottom (+), and the steeper slope of this strain diagram seems to be inconsistent with the second derivative, manually computed from the same output (−5.37). At least the slope of the printed-out strain diagram of Chevron should concur with the manually calculated curvature from the same printout (calculated by second differences).

With the strain calculated in the printout above, the stress at the bottom of the asphalt layer can be computed by means of Equation 7. Setting $S = 0$, the result would be:

$S = 0.000271 * 3000/0.65 = 1.25$ MPa (tensile stress)

The vertical stress, S, under the load is not zero but rather is negative, a fraction of the vertical contact pressure. Let us estimate the ratio of Sv/S to be −0.5; the value of the last term in the bracket of Equation 7 is then reduced to $0.5 * 0.35 = 0.175$, and the factor $e * E$ must be divided by 0.825 instead of 0.65. The result is:

$S = 0.000271 * 3000/0.825 = 0.99$ MPa

The corresponding value printed out from Chevron is only $S = 0.6975$ MPa

The example illustrates that curvatures or second derivatives ($1/R$ or W'') can be calculated with great confidence and sufficient accuracy. The strain computed either as $e = W'' * H/2$, or as $e = H/2R$ is certainly more reliable than the inconsistent output of the Chevron program. The stress, on the other hand, is dependent not only on layer material constants (E and μ), but also on the vertical stress component, which can be significant in the vicinity of the applied load.

Note: In the last column of Table 1, "average" means the average of the absolute value of the top and bottom strains of the first layer from the output listing of Chevron or ELSYM5. These values in the last column ought to be close to or equal to $H/2$ times the curvature values in the second column. This identity check would establish consistency within the Chevron or ELSYM5 programs themselves, not involving the new direct method. Note that the check fails except in the case of a very soft inserted layer (Table 1).

TESTING OF THE PROPOSED NEW METHOD

Many different cases have been calculated by the Chevron or the ELSYM5 program to verify the new proposed method. The results are listed in Tables 1 and 2. In both tables curvatures were first calculated manually from densely spaced deflection printouts from selected cases computed by elastic layer analysis (Chevron or ELSYM5). The second differences of the deflections were divided by the square of the selected space increments (50 or 25 mm). These manually calculated curvature values are listed under "Chevron/ELSYM5" in Table 1, and under "second differences" in Table 2. Then, another set of deflection printouts was selected in order to simulate FWD measurements, namely the deflection in the center and some deflections outside the circular disk. These were used to calculate curvatures in accordance with Equation 10 for Table 1, and Equation 8 for Table 2. The second term of Equation 8 required the use of further printout values of vertical pressures.

With regard to Table 1, it should be noted that there is a fair agreement between manually calculated curvatures and those calculated by the proposed direct method, listed under "direct". However, the horizontal strains at the bottom of the asphaltic layer do not agree (Table 1, second and third column from the right). Naturally, agreement cannot be expected because the elastic layer analysis programs assume full friction between the asphaltic and granular base layer, whereas the proposed direct method assumes zero friction. However, the slopes of the deflection diagrams ought to agree, concordant with the agreement in curvature.

In order to test this, the last column of Table 1 contains the average of the absolute values of strains from top and bottom, printed out by Chevron or ELSYM5. It shows improvement, but most of these average strains from printouts are still too low. It seems that the elastic layer analysis programs suffer from the phenomenon of quasi-singularity, which results in underestimating the maximum horizontal strain in the first top layer. The assumption of full friction aggravates the situation. The assumption of zero friction in the proposed direct method may not be quite correct either, however, it may still be closer to reality to overestimate the tensile strain in this way because it may counteract neglecting the second term in Equation 8. With regard to Table 2, for the region under the distributed load, from $-a$ to $+a$, the curvature or second derivative has been independently derived by solving the differential equation of elastic circular plates, resulting in Equations 8 and 9. Using Equation 8, we can calculate maximum curvatures and compare them with corresponding results from using the second differences of the deflections printed out by Chevron or ELSYM5. Eight cases, the same as in Table 1, have been calculated and compared in this way, and are listed in Table 2. The comparison shows that the manual calculations of curvature from printouts fall short of the theoretical maxima. This proves the advantage of assuming zero friction in the proposed direct method.

The curvatures calculated by Equation 9 are higher, and the corresponding strains are again higher than the ones printed out by Chevron or ELSYM5.

These verification tests reveal that loads on pavements, distributed over a relatively small circular area, constitute a point of quasi-singularity of the asphaltic "plate." This is the main reason why elastic layer analysis programs cannot catch the true maxima of strain and curvature and why they compute horizontal strains and stresses under the load axis (close to the load) too low.

Better results might be expected by finite element techniques with densely spaced grids around the contact pressure area.

CONCLUSIONS AND RECOMMENDATIONS

For single loads on asphalt pavements, which can be assumed to be distributed over a circular area, the curvature of the asphaltic layer and the horizontal strain at the bottom of this layer can be calculated directly from some measured values of the deflection basin near the center. Other values needed are the radius of the contact pressure area, and the thickness of the asphaltic layer at the load position. Because no other information is needed (about layer materials and thicknesses of lower base layers), the horizontal strain calculated in this particular way is a very potent parameter for pavement design.

The bottom strain in the asphalt layer so calculated is larger than the strain from elastic layer analysis, where full friction between asphaltic and granular layers is assumed, which causes tensile stresses at the top of the granular base.

Contrary to the proposed calculation of strain (based on pure geometry), the corresponding horizontal stresses are affected by the usual uncertainties of determining material parameters such as elastic stiffness, and Poisson's ratio.

The direct calculation of curvature and strain by this new approach has been verified against more detailed computations and output from Chevron or ELSYM5 programs. The second derivatives of the deflections in the center, the maximum curvatures, were found to agree fairly well, but the corresponding printed-out strains were found to be inconsistent with Chevron's or ELSYM5's own deflections and curvatures. The slopes of printed-out strain diagrams from Chevron and ELSYM5 did not concur (as they should have) with the manually computed curvatures from the same runs of the programs; thus, the maximum horizontal strains were much too low. The new method has also been verified through an independently derived formula, based on a solution of the differential equation of elastic plates for the region under the circular load. The variability or function of curvature within the loaded area has been studied and discussed: Chevron and ELSYM5 deflection print-outs, via second differences, seem to closely approximate an average curvature between the maximum value at the load axis and the much lower value at a distance from the axis about equal to the layer thickness. This average curvature is somewhat smaller than the maximum curvature in the center, but sufficiently close to it (Figure 3).

Since the strain parameter is generally conceived to be correlated to the fatigue strength of asphaltic pavement layers, routinely processing data from Falling Weight Deflectometer tests by this new strain criterion is recommended, in addition to other current FWD processing methods.

There is as yet no immediate verification by field experiments of the new method proposed here. Theoretical derivation within an accepted theory of structural analysis and comparison with elastic layer analysis methods can only go so far. Thus carrying out experiments with various sizes of FWD disks and various types and sizes of tires which may not have exactly circular contact pressure areas is suggested. Can we use an equivalent radius for noncircular pressure areas, and what would that radius be? Can we use a similar approach for dual tires? If field experiments should prove too "rough" with respect to quality control, this might be an area for laboratory tests.

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