Airport Gate Position Estimation Under Uncertainty

S. BANDARA AND S.C. WIRASINGHE

The aircraft gate requirement for a planned airport terminal is usually estimated using deterministic methods, although the relevant parameters—aircraft arrival rate, gate occupancy time, and aircraft separation time at a gate—are random quantities. An empirically determined "utilization factor" normally is used as a surrogate variable for separation time. The validity of the utilization factor is questionable because of its dependence on the number of gates available and the existing schedule at the airport at which it is calculated. The mean and variance of the gate requirement can be estimated if the means and variances of the aircraft arrival rate, the gate occupancy time, and the aircraft separation time can be estimated. It is shown that the gate requirement is likely to follow certain probability distributions. The design gate requirement is then chosen to satisfy a given reliability that is defined as the probability that there are sufficient gates to ensure zero delay to aircraft seeking gates. The method is applicable under common and preferential gate use policies, as well as for estimating the required number of remote aircraft stands for use in overflow situations. The gate requirement at Calgary International Airport is analyzed for common and preferential gate use policies.

The gate position requirement at an airport is an essential parameter in terminal planning. The passenger terminal and apron design is governed largely by the gate position requirement. It influences the configuration of the terminal building and the layout of the apron area and affects passenger walking distances and aircraft taxi lengths.

The number of gate positions required to accommodate a given number of flights will depend on the airline schedules, airport operating policy, the type of gates available, and the efficiency with which each gate position is used.

A number of studies have been done to investigate the gate position requirement, gate utilization, and the staging of gate position construction. Horonjeff (*1*) proposed a deterministic model to compute the required number of gate positions, based on the design volume for arrivals and departures in aircraft per hour (C), mean gate occupancy time in hours (T), and a utilization factor (U). The number of gate positions (G) was given by

$$G = CT/U \tag{1}$$

Department of Civil Engineering, The University of Calgary, 2500 University Drive N.W., Calgary, Alberta T2N IN4, Canada.

The aircraft arrival rate at gate positions varies with the hour of the day, day of the week, and month of the year. The gate occupancy time is dependent not only on the aircraft type but also on the type of operation: turnaround, continuing, originating, and terminating. If the aircraft arrival and departure times are known, the gate requirement can be determined exactly. However, exact schedules are not available in the planning stage of a terminal. Even if schedules are available, aircraft do not operate exactly on schedule. Therefore, the preceding parameters must be treated as random quantities.

McKenzie et al. (*2*) used the probability distributions of the preceding two parameters and simulation techniques to study the effect of adding one extra gate to the existing ones. Steuart (*3*) developed a stochastic model, based on empirical information relating actual flight arrivals and departures to the schedule, to study the influence of "bank operation" on the gate requirement. He found that a uniform schedule generates the minimum requirement and that banking tended to increase the number of gates.

In this paper the number of gates (G) required to provide a given reliability is estimated based on the aircraft arrival rate at the gates (A), the gate occupancy time (T), and the aircraft separation (buffer) time (S), considering them as random quantities.

BASIC CONSIDERATIONS

Imagine an idealized situation in which a constant aircraft arrival rate at gate positions (A) and identical gate occupancy times (T) exist. The gate occupancy time is measured from the aircraft's wheel stop time at the gate to the time of moving out from the gate. If all gates are capable of handling any aircraft, a lower bound for the number of gate positions required (G_L) is given by

$$G_L = AT \tag{2}$$

This formulation does not account for the time separation required for maneuvering aircraft between a departure from a gate position and the next arrival; thus it underestimates the gate position requirement.

The estimated lower-bound number of gates can be increased either by introducing a "utilization" factor, as suggested by Horonjeff (1), or by adding a time period that represents the aircraft separation time (buffer time) at a gate (S) to the gate occupancy time, as suggested by Transport Canada (4).

If the utilization factor—which represents the amount of time a gate position is occupied with respect to the total time available—is used and is determined empirically, its validity is questionable because of its correlation with the total number of gates available and the existing schedule at the airport where it is estimated.

The aircraft separation time can be defined as the time between a departure from a gate position and the next arrival; it consists of the push-out or power-out time, the time required by departing aircraft to clear the apron area, and the time required by arriving aircraft to move in from the apron entrance to the gate position. Although the aircraft separation time is influenced by the apron and terminal layouts, it can be estimated in a manner that is independent of the existing schedule. Further, it will be shown that the gate position requirement is less sensitive to the aircraft separation time than to the utilization factor. Hence the aircraft separation time (S) is selected to modify Equation 2, and the modified gate position requirement is given by

$$G = A(T + S) \tag{3}$$

The parameters A, T, and S are random variables. Hence G is a function of three random variables. Simply substituting the mean values of A, T, and S in Equation 3 will provide an estimate of the mean value of G. Designing a terminal for the mean value of G, however, will result in a low level of reliability (approximately 50 percent) since an aircraft queue will form whenever the gate requirement exceeds the mean gate requirement.

STOCHASTIC MODEL

The number of aircraft arrivals varies with the hour of the day, day of the week, and month of the year. The maximum number of aircraft arrivals at gates is partially governed by the airport's runway capacity. In addition, some of the originating flights may come from a hangar, and some terminating flights may not use a gate. Hence the aircraft arrival rate (A) is defined as the hourly aircraft arrivals at gate positions.

The mean and variance of the arrival rate can be obtained either from arrival patterns observed at an existing airport or from arrival patterns generated for the future. The observed values could be used for short-term planning situations, to check the gate requirement of an existing airport, and to study the effects of different gate allocation policies. For long-term planning, the necessary values may be obtained from computer-generated arrival patterns (2, 5) or by increasing the present mean arrival rate in proportion to the expected growth of air traffic and assuming that the variance will not change with time.

Gate occupancy times will vary depending on the aircraft size and the type of flight: originating, terminating, continuing, and turnaround. Available aircraft service facilities also have an effect on the gate occupancy time. None of the aforementioned factors are dependent on the total number of gates available. Further, McKenzie et al. (2) have shown that there is no significant dependence between aircraft arrivals in each hour of the day and the gate occupancy time for those arrivals. Therefore, independently observed gate occupancy times for different sizes of aircraft and types of flights can be used to calculate the mean and variance of the gate occupancy time for a given aircraft mix, if the aircraft service facilities are assumed to remain unchanged. When existing conditions are not applicable, the critical path network analysis method suggested by Braaksma and Shortreed (6) can be used to estimate the mean gate occupancy time.

When excess gates are available, aircraft can stay at a gate position longer than required. For example, a turnaround flight that arrives in the morning and is scheduled to depart in the evening can stay at a gate position if that gate is not required for another aircraft. Otherwise, it can be towed away to an off-terminal stand and reassigned to a gate when it is required. Therefore, if empirical data are used, a maximum on-gate time should be imposed to avoid overestimation of the gate requirement due to aircraft with unnecessarily long on-gate times. Hence the actual gate requirement at a particular time can be defined as the minimum number of gates that would be sufficient to ensure zero delays to all arrivals and departures. The foregoing argument is valid only if the time period over which the data have been considered is large enough to accommodate any reassignment of delayed aircraft.

The aircraft separation time depends on the aircraft type, type of parking (nose in, parallel), taxi-out method (push-out, power-out), and terminal and apron layouts. Strictly speaking, it is necessary to consider the terminal configuration and the apron layout to estimate the aircraft separation time accurately. Since the magnitude of the aircraft separation time is on the order of one-tenth the magnitude of the gate occupancy time, the accuracy of the aircraft separation time will not have a significant effect on the estimated mean gate position requirement. Hence if the taxiing speeds for different aircraft are known, the aircraft separation time can be calculated with respect to an assumed average taxi length. For short-term planning this quantity may be obtained by a sample survey.

Data analysis performed on operational data from Calgary International Airport shows that there is no statistically significant correlation between any of the three input parameters: aircraft arrival rate at the gate position (A), gate occupancy time (T), and aircraft separation time (S). McKenzie et al. (2) and Steuart (3) also have shown the independence between the arrival rate and the gate occupancy time. Hence the three input parameters A, T, and S can be treated as independent random quantities with means $\bar{A}$, $\bar{T}$, $\bar{S}$ and variances σ_A^2, σ_T^2, σ_S^2, respectively.

If the means and variances of the preceding parameters are known, estimates of the mean and variance of G can be obtained using moment generating functions, as given in Appendix A:

$$\bar{G} = \bar{A}(\bar{T} + \bar{S}) \tag{4}$$

$$\sigma_G^2 = \sigma_A^2(\sigma_T^2 + \sigma_S^2) + \bar{A}^2(\sigma_T^2 + \sigma_S^2) + (\bar{T} + \bar{S})^2\, \sigma_A^2 \tag{5}$$

Reliability

If $\bar{G}$, σ_G^2, and the probability distribution of G are known, the number of gates to be provided (g) to satisfy a chosen reliability $(1 - \alpha)$ can be obtained using

$$P(G \leq g) = 1 - \alpha \tag{6}$$

where reliability is defined as the probability that there are sufficient gates to ensure zero delay to aircraft on the apron, in a given time period. Here the given period is the duration of time over which data have been considered in determining $\bar{A}$ and σ_A^2. The level of service provided will depend on the chosen reliability and the time period over which the aircraft arrival rate has been considered.

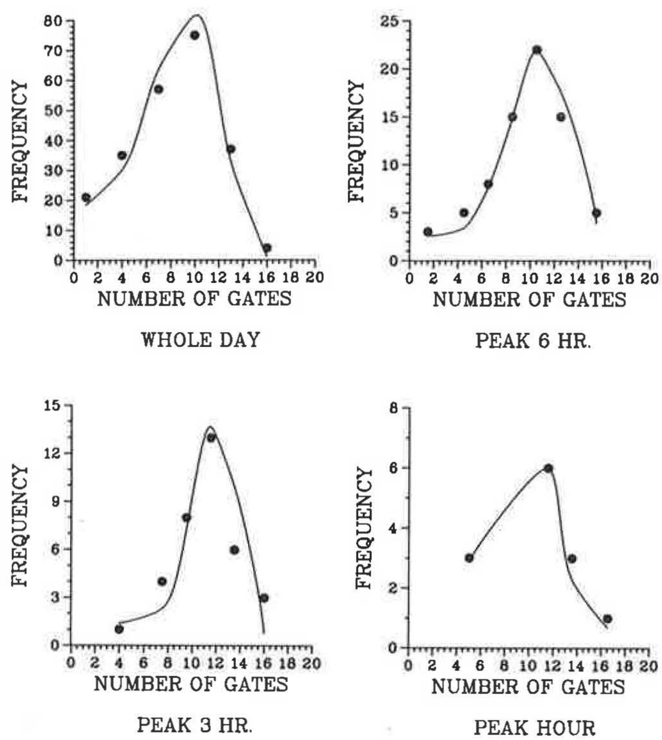
For example, if data from throughout the day for a one-month (30-day) time period are used in determining $\bar{A}$ and σ_A^2, a 95 percent reliability implies delays to some aircraft during 1.2 hours per day on average over the month considered $(30 \times 24 \times 0.05/30)$. Thus delays are likely during each peak hour. If data for the 30 high-activity hours of the year are used, a 90 percent reliability implies delays to some aircraft during 3 hours per year on average $(30 \times 1 \times 0.10/1)$.

Thus the time period should be specified for the reliability to be meaningful, and the level of service is dependent on the expected number of hours in which delays will occur during the specified period.

Probability Distribution of *G*

The probability distribution of G cannot be obtained unless the probability distributions of A, T, and S are known. The distribution of G is related to the time period over which data are collected. For example, if the data for only peak hours are used, the type 1 extreme value distribution of largest values will likely provide a good fit. On the other hand, if all the hours of a month are used, the type 1 extreme value distribution of smallest values will be likely to provide a good fit, since many of the hours will have low arrival rates.

Data analysis performed on operational data from Vancouver International Airport Terminal for a one-week period showed that it is possible to accept the hypothesis that G can be approximated by a type 1 extreme value distribution of smallest values except for the case of peak-hour distribution of gates. For gate requirements based on peak-hour data, type 1 extreme value distribution of largest values was more appropriate (Figure 1).



● Observed Frequency (mean number of 5 m intervals per day in which the gates were in use)

— Theoretical Frequency Distribution

FIGURE 1 Type 1 extreme value distributions.

Preferential Gate Use

In the previous analysis, it was assumed that any gate is available to any aircraft. Preferential gate use is common, however, where certain gates are dedicated to certain airlines and/or aircraft types. The method of analysis proposed can easily be extended to the case of preferential gate use if data for each airline/aircraft group, i, are available a priori for the arrival rate (A_i), gate occupancy time (T_i), and aircraft separation time (S_i). Then the gate requirement, g_i, for the group i is given by

$$P(G_i \leq g_i) = 1 - \alpha_i \tag{7}$$

where α_i is the reliability chosen for group i. The total gate requirement is then given by $\Sigma_i g_i$.

APPLICATION TO CALGARY INTERNATIONAL AIRPORT

Data on aircraft arrivals, gate occupancy times, and aircraft separation times were collected at Calgary International Airport on December 21, 1984, between 5 p.m. and 11 p.m.

At the time of data collection, Calgary International Airport had 23 gate positions under operation, two of which are off-terminal aircraft stands (Figure 2). Most of the gate positions have been allocated for use by specific airlines, but they have the exclusive right of use only for the connecting bridge. While the specific airline has preference over others for a particular gate, the airport manager has the power to assign gate positions to other airlines if required. The remaining gates are common gates. Six

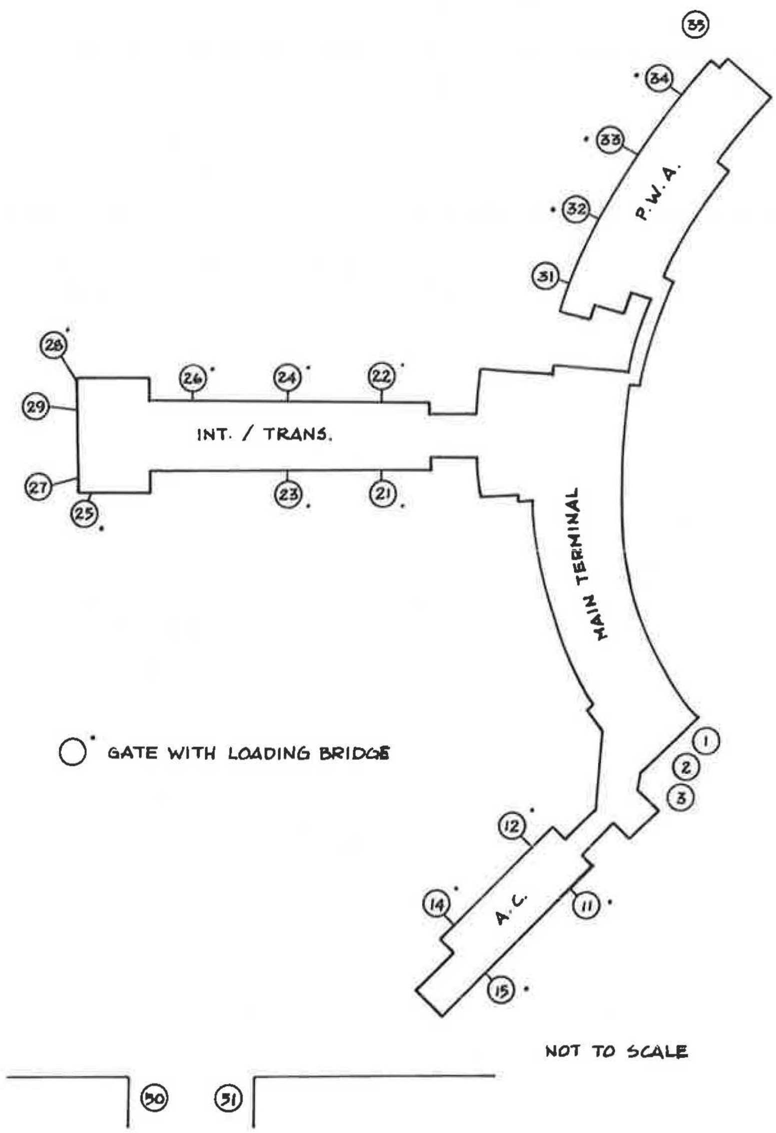


FIGURE 2 Calgary terminal layout plan.

different categories of gates are considered, as given in Table 1.

Table 2 shows the hourly aircraft arrivals at the different gate categories, and Table 3 shows the means and the variances of A, T, and S values observed at Calgary International Airport. The mean and the variance of the gate position requirement calculated using values in Table 3 and Equations 4 and 5 are given in Table 4.

During the period of time that the data were collected, no aircraft was delayed on the apron because of unavailability of gate positions. Hence the actual number of gates occupied during that time can be used to compare the results obtained from the model. Table 5 shows the means and the variances of the actual gate requirements obtained from the actual number of gates that were in use during the period of data collection and the 95 percent probability interval for the mean gate requirement. Five-minute intervals were considered for the preceding calculations. Further, the maximum on gate times suggested by Transport Canada (*4*) were used when calculating the properties of gate occupancy times and estimating the gate usage. It can be seen from Tables 4 and 5 that all the estimated mean values reported in Table 4 fall within the 95 percent probability interval for G.

The gate position requirement for reliabilities of 90 percent and 95 percent, respectively, calculated based on the extreme value distribution of smallest values as well as the normal distribution, as given in Appendix B, and the maximum number of gates that were in actual use at a particular time are given in Table 6. The values within the parentheses show the fraction of time that the actual gate requirement exceeded the estimated number of gates. It can be seen that the normal approximation tends to overestimate slightly the gate position requirement.

Horonjeff (*1*) has suggested two ranges of the utilization factor: 0.6 to 0.8 and 0.5 to 0.6 for use with common and preferential gate use, respectively, when the arrival rate is not available by airline/aircraft group. If the arrival rates are available for each group, however, the gate requirement for each group can be estimated using the common gate use utilization factor, and the total requirement under preferential gate use is estimated by summing the individual group requirements. As shown in Table 7, the two methods do not give consistent results.

Consider that only 13 gates are available at the Calgary International Airport. These gates can serve on average at 90 percent reliability if a common gate use policy is used. If a preferential gate use policy is used, 14 gates are required

TABLE 3 MEAN GATE OCCUPANCY TIMES, ARRIVAL RATES, AND GATE SEPARATION TIMES

Gate Category	Gate Occupancy Time $\bar{T}$ (hrs.)	σ_T	Aircraft Arrival Rate $\bar{A}$ (per hr.)	σ_A	Aircraft Separation Time $\bar{S}$ (hrs.)	σ_S
All	0.69	0.52	8.50	3.78	0.09	0.02
1	0.41	0.36	1.14	0.98	0.04	0.01
2	0.64	0.21	2.00	1.41	0.09	0.02
3	0.61	0.25	0.50	0.55	0.10	0.02
4	0.81	0.50	1.83	0.75	0.10	0.02
5	1.05	0.59	0.33	0.52	0.09	0.02
6	0.59	0.18	2.67	1.37	0.08	0.01

TABLE 1 CALGARY INTERNATIONAL AIRPORT GATE ASSIGNMENT

Category Number	Category Name	Number of Gates Available
1	Time Air	1, 2, 3
2	Air Canada (Domestic)	11, 12, 14, 15
3	C.P. Air	21
4	International & Transborder	22, 23, 24, 25, 26, 28
5	Military and Others	27, 29
6	P.W.A.	31, 32, 33, 34, 35

TABLE 4 MEAN AND VARIANCE OF GATE REQUIREMENT

Gate Category	$\bar{G}$	σ_G^2
All	6.63	32.13
1	0.51	0.48
2	1.46	1.33
3	0.36	0.18
4	1.66	1.45
5	0.37	0.48
6	1.79	1.14

TABLE 2 CALGARY INTERNATIONAL AIRPORT HOURLY ARRIVAL RATE

Gate Category	December 21, 1984 (5 p.m.–11 p.m.)					
	5–6 p.m.	6–7 p.m.	7–8 p.m.	8–9 p.m.	9–10 p.m.	10–11 p.m.
1	1	3	1	0	1	1
2	0	4	1	2	3	2
3	0	1	0	1	0	1
4	1	2	3	2	1	2
5	0	1	0	0	1	0
6	1	3	2	2	3	5
All	3	14	7	7	9	11

TABLE 5 MEAN AND VARIANCE OF ACTUAL GATE OCCUPANCIES

Gate Category	$\bar{G}$	σ^2_G	95% Probability Interval for $\bar{G}$
All	7.18	22.28	6.09–8.27
1	0.67	0.71	0.48–0.86
2	1.59	1.90	1.27–1.91
3	0.36	0.23	0.25–0.47
4	1.72	1.27	1.46–1.98
5	0.40	0.24	0.29–0.51
6	1.89	2.82	1.50–2.28

TABLE 6 NUMBER OF GATES FOR GIVEN RELIABILITY

	Estimated No. of Gates for Given Reliability				
	g_{90}		g_{95}		
Gate Category	Extreme Value	Normal	Extreme Value	Normal	Actual Maximum Number of Gates Required
All	13(0.11)[a]	14	14(0.05)[a]	16	15
1	2(0.07)	2	2(0.07)	2	3
2	3(0.15)	3	3(0.15)	4	4
3	1(0.0)	1	1(0.0)	1	1
4	3(0.10)	4	4(0.0)	4	4
5	2(0.0)	2	2(0.0)	2	1
6	3(0.19)	4	4(0.12)	4	5

[a] The values in parentheses indicate the fraction of time that the actual gate usage exceeded the estimated value.

TABLE 7 GATE REQUIREMENT: HORONJEFF'S METHOD

Gate Category	Gate Requirement Estimate		
	$u = 0.6$	$u = 0.7$	$u = 0.8$
Common Gate Use Policy			
All	16	14	12
1	2	2	1
2	4	4	3
3	1	1	1
4	4	3	3
5	2	2	1
6	5	4	4
	18	16	13
	$u = 0.5$	$u = 0.55$	$u = 0.6$
Preferential Gate Use Policy			
All	20	18	16

to provide the aforementioned reliability. It can be shown that to provide a 99 percent reliability for common and preferential gate assignments, 16 and 17 gates, respectively, are required. For a reliability greater than 99 percent, the gate requirement tends to increase very rapidly for both extreme value and normal distributions. Hence the preceding estimates for 99 percent reliability can be considered the maximum gate requirements. Therefore a 90 percent reliability can be provided for gates, and a 99 percent reliability can be provided for accommodating all the aircraft at a gate or an off-terminal stand, by providing one more gate position in addition to the 13 available gates and three off-terminal aircraft stands.

SENSITIVITY ANALYSIS

One of the reasons for selecting the aircraft separation time rather than a utilization factor is its relatively small influence on the gate requirement estimation for a given reliability. If the aircraft separation time (S) is used for the estimation as shown in Equation 6, for a ΔS error in the estimate of S, the gate requirement estimate will change by an amount of $\bar{A}(\Delta S)$ if the other two parameters remain constant. On the other hand, if a utilization factor is used, as given by Horonjeff (*1*), for a ΔU increase in the value of the utilization factor U, the gate requirement estimate will decrease by an amount of $CT(\Delta U)/U^2$.

It can be seen that for the first case, an error in the gate estimate does not depend on the value of aircraft separation time. For the second case, however, the error is inversely proportional to the square of the utilization factor.

In general, the design hour volume (C) will be greater than or equal to $\bar{A}$, and U will be always less than 1. Since the mean gate occupancy time generally exceeds 0.5 hour, for most situations $\bar{A}$ will be less than CT/U^2.

As an example, even if $C = \bar{A}$, the foregoing will be true if $T = 0.5$ hours and $U \leq 0.7$ or if $U = 0.8$ and $T > 0.64$ hours. Hence the gate requirement estimate is more sensitive to the accuracy of U than to the accuracy of S. Consider a situation where the error in the estimate of S is as high as 20 percent or 2 min. For a unit change in the gate requirement estimate, $\bar{A}$ should be about 30 aircraft per hour. On the other hand, consider a situation where the design hour volume (C) is 30 aircraft arrivals per hour and T is as low as 0.5 hr. Even for a high utilization factor of 0.8, the estimate of gate requirement will change by 1.0, if the estimate of U is changed by an amount of 5 percent or 0.04. Thus the proper value of U is crucial for the use of Horonjeff's method.

Further, the magnitudes of $\bar{S}$ and σ^2_S are small compared to the means and variances of the other parameters, and has no major influence on the magnitude of G. Hence use of a constant value of S, which represents the mean aircraft separation time for the aircraft mix in question, may be sufficient for a reasonable accuracy.

CONCLUSION

The number of gates required at an airport in the future can be estimated for a given reliability during a specified period if the means and variances of the arrival rate, gate occupancy time, and aircraft separation time can be used as inputs.

ACKNOWLEDGMENTS

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APPENDIX A

For a continuous random variable X, suppose that there is a positive number h such that for $-h < t < h$, the mathematical expectation $E(e^{tx})$ exists. The preceding expectation is called the moment generating function of x.

$$M(t) = E(e^{tx}) = \int_{-\alpha}^{\alpha} e^{tx} f(x)\, dx \tag{a}$$

Further, it has been shown (*7*) that

$$M'(0) = E(x) = \bar{x} \tag{b}$$

and

$$M''(0) = E(x^2) = \sigma_x^2 + \bar{x}^2 \tag{c}$$

where $\bar{x}$, σ_x^2 are the mean and the variance of x, respectively.

Let X and Y be two random variables with moment generating functions $M(t_1)$ and $M(t_2)$, respectively. Let

$$Z = XY \tag{d}$$

If X and Y are stochastically independent, the moment generation function for the joint distribution $M(t_1, t_2)$ is

$$M(t_1, t_2) = M(t_1) \cdot M(t_2) \tag{e}$$

that is,

$$M(t_1, t_2) = \int_{-\alpha}^{\alpha} e^{t_1 x} f(x)\, dx \cdot \int_{-\alpha}^{\alpha} e^{t_2 y} g(y)\, dy \tag{f}$$

$$M'(t_1, t_2) = \int_{-\alpha}^{\alpha} x e^{t_1 x} f(x)\, dx \cdot \int_{-\alpha}^{\alpha} y e^{t_2 y} g(y)\, dy \tag{g}$$

and

$$M''(t_1, t_2) = \int_{-\alpha}^{\alpha} x^2 e^{t_1 x} f(x)\, dx \cdot \int_{-\alpha}^{\alpha} y^2 e^{t_2 y} g(y)\, dy \tag{h}$$

Then,

$$M'(0,0) = E(Z) = E(X) \cdot E(Y) \tag{i}$$

and

$$M''(0,0) = E(Z^2) = E(X^2) \cdot E(Y^2) \tag{j}$$

From Equation i,

$$\bar{z} = \bar{x}\, \bar{y} \tag{k}$$

From Equations i, j, and c

$$\begin{aligned} \sigma_z^2 &= E(Z^2) - E(Z)^2 \\ &= E(X^2) \cdot E(Y^2) - E(X)^2 \cdot E(Y)^2 \\ &= (\sigma_x^2 + \bar{x}^2)(\sigma_y^2 + \bar{y}^2) - (xy)^2 \\ &= \sigma_x^2 \sigma_y^2 + \bar{x}\sigma_y^2 + \bar{y}\sigma_x^2 \end{aligned} \tag{1}$$

Letting $T + S = Y$ and $A = X$, the mean and the variance of G are given by

$$\bar{G} = \bar{A}\,(\bar{T} + \bar{S}) \tag{m}$$

and

$$\begin{aligned} \sigma_G^2 = \sigma_A^2 (\sigma_T^2 + \sigma_S^2) + \bar{A}^2 (\sigma_T^2 + \sigma_S^2) \\ + (\bar{T} + \bar{S})\, \sigma_A^2 \end{aligned} \tag{n}$$

APPENDIX B

The cumulative probability density functions of the type 1 extreme value distribution of the largest and smallest values are defined as:

$$F_z(z) = 1 - \exp(-e^{-\alpha(z-u)}) \qquad -\alpha \leq z \leq \alpha \tag{a}$$

and

$$F_z(z) = 1 - \exp(-e^{\alpha(z-u)}) \qquad -\alpha \leq z \leq \alpha \tag{b}$$

respectively. The α and u are two parameters that will be estimated from observed data such that

$$\begin{aligned} \bar{z} &= u + \frac{0.577}{\alpha} \quad \text{for largest values} \\ \bar{z} &= u - \frac{0.577}{\alpha} \quad \text{for smallest values} \end{aligned} \tag{c}$$

and

$$\sigma_z^2 = \pi^2/6\alpha^2 \tag{d}$$

A reduced variate w is defined such that

$$w = (z - u)\alpha \quad \text{for largest values} \qquad \text{(e)}$$
$$w = -(z - u)\alpha \quad \text{for smallest values}$$

and the cumulative distribution of largest values has been tabulated in terms of the reduced variate w by Benjamin and Cornell (*8*). The table can be used for both distributions, as shown in Equation f.

$$F_z(z) = F_w\,((z - u)\alpha) \quad \text{for largest values} \qquad \text{(f)}$$
$$F_z(z) = 1 - F_w\,(-(z - u)\alpha) \quad \text{for smallest values}$$

Consider the estimated gate requirement for the common gate use policy given in Table 4, where $G = 6.63$ and $\sigma_G^2 = 32.13$. The number of gates required to satisfy a reliability of 95 percent, g_{95}, is given by

$$F_G\,(g_{95}) = P(G \leq g_{95}) = 0.95$$

If G is assumed to be represented by a type 1 extreme value distribution of smallest values, from Equation d,

$$\alpha = \frac{\pi}{\sqrt{6}\,\sigma_G} = \frac{\pi}{\sqrt{6 \times 32.13}} = 0.226$$

and from Equation e,

$$u = \bar{G} + \frac{0.577}{\alpha} = 9.18$$

from tables for the type 1 extreme value distribution of largest values, and $w = -1.1$ for $F(w) = 0.05$. From Equation f,

$$1 - Fw\,(-1.1) = 0.95 = F_G\,(g_{95})$$

Therefore g_{95} can be obtained by solving Equation 3 for $w = -1.1$:

$$w = -(g - u)\alpha = -1.1$$

$$g_{95} = \frac{1.1}{0.226} + 9.18 = 14.04 = 14 \text{ gates}$$

If G is assumed to be normally distributed, using normal tables,

$$\begin{aligned} g_{95} &= 1.65\,\sigma_G + G \\ &= 1.65\sqrt{32.13} + 6.63 = 15.98 \\ &= 16 \text{ gates} \end{aligned}$$

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