Linear Programming Model for Pavement Management

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A computer model, CONNPAVE, has been developed for the Connecticut Department of Transportation. The model uses a probabilistic linear programming formulation for optimizing maintenance and reconstruction activities. The objective function is to minimize user costs; the constraints are the budget, production capacity, and the recursive relation, which carries the optimization over the planning period. The ease of running the program permits the examination of numerous budgetary scenarios. It is anticipated that the projections of deterioration and treatment effectiveness, which are central to the model, will be continually updated as routine field surveys monitor pavement performance.

The element of a pavement management system that deals with mathematical optimization through linear programming is examined. This optimization technique is part of a comprehensive system under development by the Connecticut Department of Transportation (ConnDOT).

For several years, ConnDOT has had certain operational elements of a pavement management system. Photologging has been used since 1970, and the Office of Maintenance in the Bureau of Highways has regularly rated roadway pavements (by means of a windshield survey) since 1980. In the summer of 1982, work began at the University of Connecticut (UConn) on the development of an optimization technique. At the outset, it was recognized that, when implemented, the technique should

1. be feasible in terms of both the economic and the personnel resources of ConnDOT
2. make maximum use of appropriate existing data and data acquisition programs
3. contain sufficient flexibility to determine both the optimal use of a given funding level and the optimum funding given a prescribed serviceability level; and
4. allow for continual updating to ensure that maintenance and reconstruction decisions are based on current information.

The development of an optimization technique for this system included the selection of initial model parameters and the testing of the model, called CONNPAVE. Some of these data are included herein.

The decision to develop an optimization technique specific to ConnDOT's needs followed an extensive review of the literature. The various pavement management systems in use by a number of states have been well documented elsewhere (*1*). Although all these systems ultimately produce priority rankings, few of them optimize. Perhaps the system most similar to CONNPAVE is the Network Optimization System (NOS) used by the state of Arizona (*2*). The objective in the NOS is to minimize "preservation" costs while achieving and maintaining minimum pavement standards. Linear programming is used for the optimization.

The objective in CONNPAVE is the minimization of user costs subject to budgetary and other constraints. Although it does not directly minimize treatment costs, CONNPAVE is sufficiently simple and inexpensive to run that numerous realistic budget scenarios can be examined.

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MATHEMATICAL MODEL

General

The model optimizes (i.e., suggests the best distribution of funds), based on minimizing user costs given a sequence of annual budgets. To apply the model, each mile of roadway in the system is considered to be in a certain state, as determined by its physical condition and traffic volume. As output, the model suggests which of several available treatments should be applied, to how many miles in each state, and when they should be applied. Linear programming is used to perform the optimization.

The input required consists of the initial number of miles in each state, minimum and maximum budgets for each year in the analysis period, a maximum total budget over the entire analysis period, minimum and maximum yearly output of each treatment considered, unit treatment costs for each treatment, and unit user costs for each roadway state.

In addition, stochastic models of roadway deterioration and treatment effectiveness must be built. These models are specified by state transition probabilities for deterioration and treatment as a function of roadway state.

Roadway States k, s, and u

Roadway state is variously designated by the subscripts k, s, or u, and is typically defined on the basis of condition, average daily traffic (ADT), rate of change of condition, and environment (Figure 1). These four stages of pavement description are used to describe a pavement-state numbering convention. The lowest stage is pavement condition, which rates from worst to best with increasing state number. The next

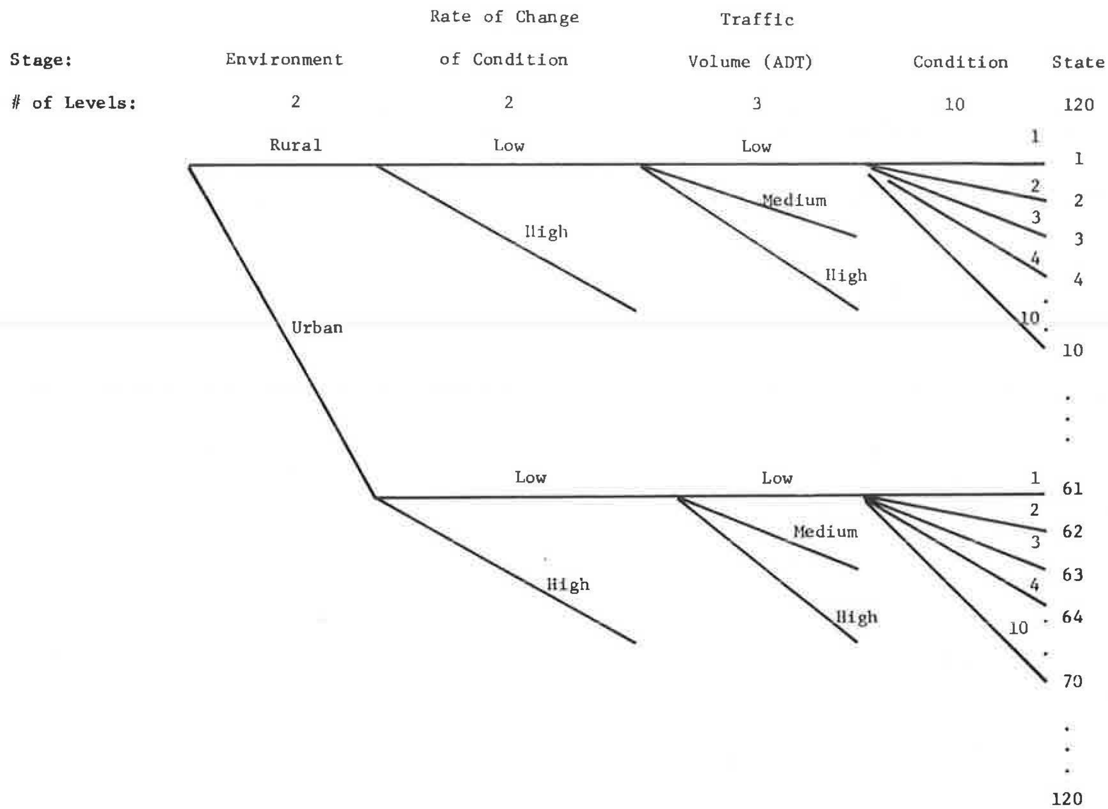

FIGURE 1 State definition: 120-state case.

stage is ADT, which also rates from low to high. Each ADT state contains all pavement condition levels, and the number of states defined at this point is the number of pavement condition levels times the number of ADT levels. The third state is the rate of change of condition, and the fourth stage is environment. At each stage, each level of that stage contains all levels of all stages below it. Thus the total number of states is the product of the number of levels at each stage. The total number of states in a given problem is designated as N_s.

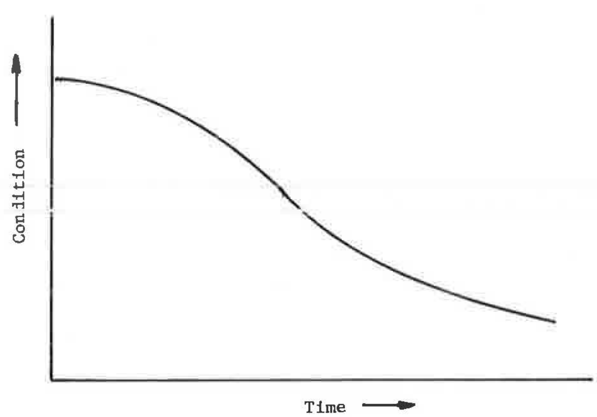

FIGURE 2 Idealized deterioration curve.

Roadway Deterioration, D_{uk}

Central to the model is the ability to predict the deterioration in roadway condition over time. The idealized curve shown in Figure 2 depicts this qualitatively, but the process of deterioration is complex.

As a result, the predictions are probabilistic. This probabilistic feature is incorporated into the model by means of the matrix $\mathbf{D}_{uk}$ that represents the probability that a roadway originally in state u will deteriorate to state k in the course of one year. Obviously,

$$\sum_{K=1}^{N_s} D_{uk} = 1 \qquad \text{for all } u \tag{1}$$

Treatment Effectiveness, E_{stu}

The change in pavement condition brought about by a given maintenance activity is termed "treatment effectiveness." Ideally, the effect of performing a given maintenance activity on a segment of pavement in a given state ought to be completely predictable. Practically, as the result of such things as uncertainty on the condition of the roadway base and local variation in the roadway condition, treatment effectiveness is also expressed as a matrix of probabilities. The matrix $\mathbf{E}_{stu}$ represents the probability that a roadway in state s will be

TABLE 1 TREATMENT UNIT COSTS

Treatment No.	Treatment	Cost ($/2-lane mi)
1	Do nothing	0
2	Seal coat	12,000
3	2½-in. overlay	150,000
4	4-in. overlay	210,000
5	Reconstruct	800,000

transformed to state u if given treatment t. Again, note that

$$\sum_{u=1}^{N_s} E_{stu} = 1 \qquad \text{for all } s \text{ and } t \tag{2}$$

Treatment Costs, D_t

In the mathematical formulation, C_t represents the cost per two-lane mile for treatment t. Generally in N_t treatments, treatment 1 is always the do-nothing or null treatment. By including this null treatment, it is possible to assume that every mile of the roadway network gets treated every year. This assumption is the basis for the network continuity equation given later. To date, five treatments have been considered, and unit costs for these treatments are given in Table 1.

User Costs, G_s

As noted earlier, the optimization is based on the minimization of user costs. Unit costs (dollars per mile per year) associated with each roadway state are rough approximations based on the work of Witczak and Rada (*3*). In the mathematical formulation, G_s represents the annual unit cost to users of a roadway in state s. The unit user costs for a 120-state model are given in Table 2.

Roadway State and Treatment Mileage, $\mathbf{X}_{sty}$

The model proceeds with the optimization based on the initial observed condition of the roadway network. Network condition is defined by the frequency distribution of roadway states. The vector $\mathbf{X}_{sy}$ represents the number of miles of roadway in state s at the beginning of year y. More specifically, $\mathbf{X}_{sty}$ is the number of miles in roadway state s that are given treatment t during year y. As a consequence of including the null treatment,

$$\sum_{t=1}^{N_t} X_{sty} = X_{sy} \tag{3}$$

for any of the N_y years under consideration. It is the values of $\mathbf{X}_{sty}$ that must be determined in the optimization. X_{s1} is the initial network condition.

Continuity Equation

If maintenance activities are performed on a highway pavement, the idealized curve shown in Figure 2 is modified as shown in Figure 3. Note that the performance of the maintenance activity results in an improvement in condition and that this is followed by a general deterioration in condition until maintenance is again performed. This cyclic behavior can be modeled by the following recursive relation:

$$X_{k\ (y+1)} = \sum_{t=1}^{N_t} \sum_{s=1}^{N_s} X_{sty} \sum_{u=1}^{N_s} E_{stu} D_{uk} \tag{4}$$

Letting

$$\sum_{u=1}^{N_s} E_{stu} D_{uk} = H_{stk} \tag{5}$$

it is observed that the transformation H_{stk} is the compound probability that a roadway in state s will be transformed into state k given treatment t followed by its expected deterioration. Finally, by combining equations 3, 4, 5, the following network continuity equation is obtained:

$$\sum_{t=1}^{N_t} X_{kt(y+1)} = \sum_{t=1}^{N_t} \sum_{s=1}^{N_s} X_{sty} H_{stk} \tag{6}$$

which is the basis of the entire modeling process.

Objective Function

As noted earlier, the purpose of the optimization is to minimize user costs on the network for the period affected by the

TABLE 2 USER COSTS

Environment	Rate of Change of Condition	Traffic Volume	Condition 1	2	3	4	5	6	7	8	9	10
Rural	Low	Low	1.11	1.05	1.00	0.91	0.83	0.77	0.73	0.71	0.70	0.68
		Medium	3.34	3.18	3.02	2.73	2.50	2.32	2.21	2.13	2.10	2.13
		High	5.49	5.22	4.93	4.45	4.07	3.80	3.61	3.48	3.43	3.38
	High	Low	1.11	1.05	1.00	0.91	0.83	0.77	0.73	0.71	0.70	0.68
		Medium	3.34	3.18	3.02	2.73	2.50	2.32	2.21	2.13	2.10	2.13
		High	5.49	5.22	4.93	4.45	4.07	3.80	3.61	3.48	3.43	3.38
Urban	Low	Low	2.28	2.00	1.68	1.38	1.22	1.16	1.10	1.09	1.08	1.07
		Medium	5.28	4.62	4.03	3.40	3.00	2.81	2.68	2.62	2.57	2.49
		High	6.93	6.21	5.56	4.91	4.42	4.68	3.84	3.71	3.62	3.54
	High	Low	2.28	2.00	1.68	1.38	1.22	1.16	1.10	1.09	1.08	1.07
		Medium	5.28	4.62	4.03	3.40	3.00	2.81	2.68	2.62	2.57	2.49
		High	6.93	6.21	5.56	4.91	4.42	4.68	3.84	3.71	3.62	3.54

Note: Units are millions of dollars per year per two-lane mile.

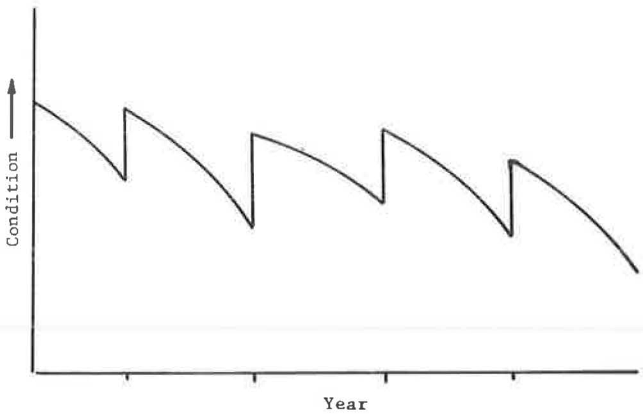

FIGURE 3 Deterioration-restoration cycle.

analysis. Mathematically, the objective is to minimize

$$Z = \sum_{s=1}^{N_s} G_s \sum_{y=2}^{N_y+1} X_{sy} \tag{7}$$

Inequality Constraints

There are several possible constraints on the solution. The first three of these are budgetary:

$$\sum_{y=1}^{N_y} \sum_{s=1}^{N_s} \sum_{t=1}^{N_t} C_t X_{sty} \leq B^* \tag{8}$$

$$\sum_{s=1}^{N_s} \sum_{t=1}^{N_t} C_t X_{sty} \leq B_y^+ \tag{9}$$

and

$$\sum_{s=1}^{N_s} \sum_{t=1}^{N_t} C_t K_{sty} \geq B_y^- \tag{10}$$

where

B^* = total budget,
B_y^+ = maximum budget for the year y, and
B_y^- = minimum budget for the year y.

Equation 8 is the constraint on the overall budget during the analysis period. Equations 9 and 10 are the constraints on each of the yearly budgets. Note in passing that the sum of B_y^+ may be larger than B^*.

Two additional constraints are imposed by production capacity:

$$X_{ty} \leq T_{ty}^+ \tag{11}$$

and

$$X_{ty} \geq T_{ty}^- \tag{12}$$

where T_{ty}^+ and T_{ty}^- are the maximum and minimum production capacities for treatment t in year y, respectively. Equation 11 sets a maximum on the amount of a certain treatment that can be employed in any given year. Equation 12, which sets a minimum on this amount, has been introduced to avoid a solution that calls for extreme shifts in pavement material production from year to year.

Formulation of the Linear Program

It is now possible to write a consistent formulation of the entire linear program in terms of the basic unknown quantities, X_{sty}. Equation 3 or the continuity equation (equation 6) is used as appropriate to express each equation with respect to the proper variables.

The objective is to find

$$X_{sty} \geq 0, \quad 1 \leq s \leq N_s$$
$$1 \leq t \leq N_t$$
$$1 \leq y \leq N_y$$

such that

$$Z = \sum_{k=1}^{N_k} G_k \sum_{y=1}^{N_y} \sum_{t=1}^{N_t} \sum_{s=1}^{N_s} X_{sty} H_{stk} \tag{7a}$$

is a minimum. Subject to:

The total budget constraint

$$\sum_{y=1}^{N_y} \sum_{s=1}^{N_s} \sum_{t=1}^{N_t} C_t X_{sty} \leq B^* \tag{8a}$$

The annual budget constraints

$$\sum_{s=1}^{N_s} \sum_{t=1}^{N_t} C_t X_{sty} \leq B_y^+ \tag{9a}$$

and

$$\sum_{s=1}^{N_s} \sum_{t=1}^{N_t} C_t Y_{sty} \geq B_y^- \tag{10a}$$

The annual production constraints

$$\sum_{s=1}^{N_s} X_{sty} \leq T_{ty}^+ \tag{11a}$$

and

$$\sum_{s=1}^{N_s} X_{sty} \geq T_{ty}^- \tag{12a}$$

And consistent with:

The initial network state

$$\sum_{t=1}^{N_t} X_{st1} = X_{s1} \tag{3a}$$

And annual network continuity

$$\sum_{t=1}^{N_t} X_{kt(y+1)} = \sum_{t=1}^{N_t} \sum_{s=1}^{N_s} X_{sty} H_{stk} \tag{6a}$$

Equation 6a is used for $2 < y \leq N_y - 1$, and equations 9a, 10a, 11a, and 12a are used for $1 \leq y \leq N_y$.

IMPLEMENTATION

The system is now operational on the UConn IBM 3081 computer system located on the main campus at Storrs. It has been successfully accessed remotely from the Rocky Hill office of ConnDOT. An interactive EXEC program called PAVEMENT has been prepared to make the optimization system quite easy to use. The system includes an annual output of expenditures for each treatment, the miles of roadway in each state proposed for each treatment, the predicted network condition, and the estimated user cost. The program has been used for a number of cases and has been found to give reasonably quick and inexpensive results. For example, a case with 120 states, 5 treatments, and a 5-year analysis period runs in less than 100 sec and costs $50.

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