Estimation of Trip Matrices: Shortcomings and Possibilities for Improvement

RUDI HAMERSLAG AND BEN H. IMMERS

Since the early 1970s several techniques have been developed for estimating origin–destination (O-D) matrices using all kinds of data. This paper presents a survey of four different O-D matrix estimation techniques. It is shown that some techniques can be applied only in certain restricted conditions and, even worse, may lead to considerable loss of information. To alleviate this problem, it is suggested that elastic, instead of fixed, constraints be used. It is also possible to overcome this problem by using an explicit stochastic estimation technique (binary calibration). Because of the absence of time and space-dependent coefficients, this technique can also be used for making forecasts.

Origin–destination (O-D) matrices contain trips between a number of origins and destinations per time period, means of transport, and the like. O-D matrices—an essential part of the basic information on transport demand—are used for several purposes—for example:

- transport planning and design: the calculation of traffic flows and the prediction of bottlenecks in road networks;
  - evaluation of alternatives: sensitivity analyses;
- simulation of traffic flows in networks, design of traffic control devices, and specification of signal settings for controlled intersections.

Depending on the objective of the study, the information stored in the O-D matrix can be specified with respect to:

- size of study area: entries and exits of an intersection or zones in a regional transport study;
- means of transport: per mode or combination of modes;
- time period (time of the day): 24 hours, peak hour, 15-minute intervals, etc.;
- date: past, present, or future situation;
- purpose: home-based work, recreation, shopping, etc.

If the O-D matrix contains information on the present situation, it is called "base year matrix." Such a matrix

R. Hamerslag, Departments of Civil Engineering and of Mathematics and Information Technology, Delft University of Technology, P.O. Box 5048, 2600 GA Delft, The Netherlands. B. H. Immers, Department of Civil Engineering, Delft University of Technology, P.O. Box 5048, 2600 GA Delft, The Netherlands.

can be estimated using all kinds of (available) traffic and transport data, such as:

- Complete data: all trips are observed. (Because of organizational and financial constraints, this procedure will never be applied.)
- Incomplete, direct, and indirect data. Incomplete data comes from taking a sample. Direct data results from observing O-D trips. Indirect data is a product of calculating O-D trips using other information sources, such as traffic counts, route assignment, etc. Examples of these kinds of data follow:

  - Household surveys: trips of only those people living in the survey area are observed.
  - Road questionnaires: Trips of only those people passing through are observed. Double counts as well as long-distance trips may bias the estimates.
  - Ticket sales: season tickets and special tickets may lack O-D information.
  - Cordon/screenline/traffic counts: only traffic volumes on a road section are observed.
  - Route choice information.
  - Old O-D matrices.

Problems arise, however, in expanding these data. For example, because all data sources are incomplete, a great number of trips (and O-D pairs) are not observed. Also, the data sources show considerable overlap, which may result in apparently contradictory O-D information (caused by the stochastic properties of the data).

To give a complete and consistent picture of the trip (O-D) pattern, O-D matrix estimation techniques are used. Although the underlying assumptions as well as the applied optimization techniques may differ, the objective of all base-year matrix estimation procedures is to obtain an optimum fit between the estimate and the available set of surveyed data.

Since the early 1970s several techniques have been developed using different approaches with respect to:

- Formulation of the objective function in combination with the use of an underlying distribution model—for example, maximum likelihood (Hamerslag and Huisman [1]), minimum of variance (Smit [2,3]), constrained least

squares (Hendrickson and McNeil [4]), information minimization (Van Zuylen [5], Van Zuylen and Willumsen [6]), entropy maximization (Willumsen [7]), and others (e.g., Nickesen et al. [8]).

- Use of different data sources (direct and indirect data) and a priori information as well as the incorporation of constraints in the calibration process of the model (for example, see Van Zuylen [9,10], Van Zuylen and Branston [11], Willumsen [12], Cremer and Keller [13], Bell [14,15]).
- Incorporation of a stochastic component (as data from observations are stochastic) and assumptions about the distribution pattern of the observed data (for example, see Hamerslag and Huisman [1], Smit [2], Hamerslag et al. [16], Hendrickson & McNeil [4], Willumsen [17]).
- Influence of congestion on route choice (for example, see Nguyen [18], Willumsen [17], Gur [19], and Turnquist and Gur [20]).

All kinds of O-D matrix estimation techniques are being used in the transportation planning process. The objective of this paper is to analyze the qualities and possibilities for applying some of the matrix estimation techniques that are most frequently used.

In this paper four different techniques are discussed:

1. The weighted Poisson estimator in combination with partial matrix estimation techniques,
2. Entropy maximizing and information minimizing techniques,
3. Information minimizing technique with elastic constraints,
4. Explicit stochastic estimation technique (the binary calibration model).

This paper does not deal with techniques that allow for alternative route choice assumptions (e.g., LINKOD; see Gur [19]). Description of the model in terms of all-or-nothing assignment implies that comparisons between observed and calculated link flows should preferably be made on screenline level. Incorporation of alternative route choice assumptions and, consequently, comparison of observed and calculated link flows per link may induce errors introduced in trip generation and the trip distribution model that are being corrected for in the assignment model.

A second reason for exclusion of the LINKOD model is that application of the model is possible only if volume counts on all links are available (which is hard to realize in practical situations). Alleviation of this severe constraint by Gur (19) results in a model based on partial volume counts. However, this model also requires quite some additional information—that is, total vehicle-hours of travel and capacities and free travel times for all links.

The paper is organized as follows. First the weighted Poisson estimator is presented. The weighted Poisson estimator can be used for estimating deterrence functions as well as O-D matrices. The estimator can be used even if only part of all O-D pairs has been observed.

Next both entropy and information optimization techniques are discussed. Based on a practical application, the entropy and information optimizing techniques are shown to generate very poor results and to induce a considerable loss of information. Therefore a new technique is introduced (information optimizing model with elastic constraints) that produces considerably better results. This is the next topic described.

Then a description is presented of an explicit stochastic estimation technique. This technique is based on the likelihood estimation theory. The coefficients of a transportation forecasting model are calibrated in such a way that an optimum fit is obtained to all available data. Simultaneously a base-year matrix and the coefficients of a forecasting model are estimated.

Finally, some major conclusions are drawn about the possible applications of all the techniques discussed. A summary of the particular qualities of the techniques appears in the paper's closing table.

WEIGHTED POISSON ESTIMATOR

The weighted Poisson estimator is based on two assumptions. First, it is assumed that all interzonal volumes are independent and Poisson distributed with some expected value. Second, the expected number of trips is multiplicative—the product of some independent variables. Maximizing the likelihood yields the estimation equations.

The following well-known model (see, e.g., Wilson [21]) is an example of a forecasting model with a multiplicative form:

$$\hat{T}_{ijm} = Ca_iO_ib_jD_jF_m(c_{ijm}) \quad \forall i, j, m \tag{1}$$

with

$$\sum_{jm} \hat{T}_{ijm} = G_i \quad \forall i \tag{2}$$

and

$$\sum_{im} \hat{T}_{ijm} = A_j \quad \forall j \tag{3}$$

where:

T_{ijm} = the estimated number of trips from i to j with mode m
C = constant term
a_i, b_j = balancing factors
O_i, D_j = polarities (generation and attraction power) of zone i resp. j
$F_m(c_{ijm})$ = deterrence function for mode m
c_{ijm} = generalized cost for O-D pair ij and mode m
G_i = the generation of trips in zone i (origins)
A_j = the attraction of trips in zone j (destinations).

Let us assume

$$a_i O_i = o_i \qquad b_j D_j = d_j \tag{4}$$

and

$$F_{mk} \stackrel{\text{def}}{=} F_m(c_{ijm}) \tag{5}$$

where c_{ijm} belongs to generalized cost class k.

Substitution of formulas 4 and 5 in formula 1 yields

$$\hat{T}_{ijm} = C o_i d_j F_{mk} \quad \forall i, j, m \tag{6}$$

The probability of observing T_{ijm} trips can be given by the equation

$$\Pr[T_{ijm} \mid \hat{T}_{ijm}] = \exp(-\hat{T}_{ijm})(\hat{T}_{ijm})^{(T_{ijm})}/T_{ijm}! \tag{7}$$

where T_{ijm} = estimate of T_{ijm}.

The number of trips T_{ijm} is assumed to be independent for all combinations of $i,j,$ and m. As a result the value of the log-likelihood function (L^*) becomes

$$L^* = \ln(L) = \sum_{ij} \sum \ln(\Pr[T_{ijm} \mid \hat{T}_{ijm}]) \tag{8}$$

The coefficients in formula 1 should be chosen in such a way that the log-likelihood has a maximum value. Substitution of formulas 1 and 7 in 8 yields the log-likelihood function:

$$L^* = \sum_{ij} \sum \{(-\hat{C}\hat{o}_i \hat{d}_j \hat{F}_{mk})$$
$$+ T_{ijm}[\ln(\hat{C}) + \ln(\hat{o}_i) + \ln(\hat{d}_j) + \ln(\hat{F}_{mk})]$$
$$- \ln(T_{ijm}!)\} \tag{9}$$

The maximum value of the log-likelihood is found by setting the first partial derivatives to zero:

$$\frac{\delta L^*}{\delta \hat{o}_i} = 0 \quad \forall i \tag{10}$$

$$\frac{\delta L^*}{\delta \hat{d}_j} = 0 \quad \forall j \tag{11}$$

$$\frac{\delta L^*}{\delta \hat{F}_{mk}} = 0 \quad \forall m, k \tag{12}$$

$$\frac{\delta L^*}{\delta \hat{C}} = 0 \tag{13}$$

The result is a set of nonlinear equations with which the coefficients can be determined using an iterative method, such as the Gauss-Seidel principle.

$$\hat{o}_i = \frac{\sum_{jm} \sum T_{ijm}}{\sum_{jm} \sum \hat{C}\hat{d}_j \hat{F}_{mk}} \quad \forall i \tag{14}$$

$$\hat{d}_j = \frac{\sum_{im} \sum T_{ijm}}{\sum_{im} \sum \hat{C}\hat{o}_i \hat{F}_{mk}} \quad \forall j \tag{15}$$

$$\hat{F}_{mk} = \frac{\sum_{ij} \sum T_{ijm}}{\sum_{ij} \sum \hat{C}\hat{o}_i \hat{d}_j} \quad \forall m, k \tag{16}$$

and

$$\hat{C} = \frac{\sum_{ijm} \sum \sum T_{ijm}}{\sum_{ijm} \sum \sum \hat{o}_i \hat{d}_j \hat{F}_{mk}} \tag{17}$$

This set of equations can be solved only if there are no inconsistencies in the data. In general this condition will not be fulfilled if data from more than one home interview or from various cordon interviews are used.

The weighted Poisson estimator was applied for the first time in the early 1970s. At first the model was used for estimating deterrence functions (Evans and Kirby [22], Hamerslag [23]). Since then, the model has also been used for estimating O-D matrices, even if only a part of all O-D pairs has been observed. For that reason, the model is especially suited for making estimates using survey data from cordon interviews. An estimate of unobserved O-D pairs in the matrix can be obtained using partial matrix techniques (Neffendorf and Wootton [24]). This application of the model, however, is not always without problems (see, for example, Day and Hawkins [25], Kirby and Murchland [26]). Information from road traffic counts or public transport ridership cannot be used as it lacks O-D information. The weighted Poisson estimator is also used for the analysis of multidimensional matrices (Hamerslag et al. [27]).

ENTROPY MAXIMIZING AND INFORMATION MINIMIZING MODEL

According to information theory, the most likely trip matrix is a matrix that can arise in the greatest number of ways and that satisfies whatever constraints are placed on the system. The entropy maximizing (EM) and information minimizing (IM) models are equivalent; the only difference between them is that the latter makes use of some initial knowledge about the likely trip matrix (an a priori matrix). The IM model is a generalization of the EM model.

If only the total number of trips in the system is known, the entropy maximizing method will distribute these trips evenly over all cells of the matrix. Adding some constraints might result in a more realistic picture. Wilson (28), for example, subjected the entropy to the following constraints: the total number of origin and destination trips per zone and a general function of distance (e.g., generalized cost). The result is a conventional, double-constrained gravity model with an exponential deterrence function. It is also possible to incorporate some extra information into the model—for example, the observed link flows (link counts V_a per direction). It should be noted, however, that each link count (V_a) adds an extra constraint to the estimate that results in an extra Lagrange multiplier and, consequently, an extra coefficient in the model. Of course, it is also possible to make an estimate of a trip matrix without using information that is essential in the conventional gravity model (e.g., the average distance traveled).

The EM model for the estimation of trip matrices from traffic counts was introduced by Willumsen (7). Van Zuylen (9) derived the IM model. An attractive feature of Van Zuylen's model is the possibility of incorporating extra information (an a priori trip matrix) that might result in a more realistic estimate of the actual trip matrix. Later the possibility of incorporating extra a priori information is also introduced in the EM model. The better the a priori matrix, the better the estimate will be. An old O-D matrix might be well suited for use, but it is also possible to use an O-D matrix estimate from the weighted Poisson model (see preceding section) or a Wilson type of model as a first guess.

Formulation of the Information Minimizing Model

According to the information minimizing theory (Van Zuylen [9]), the most likely trip matrix T_{ij} satisfies the following equation:

$$L = \min_{T_{ij}} \sum_i \sum_j \left[\hat{T}_{ij} \ln\left(\frac{\hat{T}_{ij}}{T_{ij}}\right) \right] \tag{18}$$

subject to

$$\sum_i \sum_j (\hat{T}_{ij} d_{ij}^a) = R_a \quad \forall a \tag{19}$$

where

  L = "distance" between matrices
  $\hat{T}_{ij}$ = the number of trips from i to j (to be estimated a posteriori)
  R_a = constraint a (e.g., traffic count, total number of arrivals per zone, trip length distribution, etc.)
  d_{ij}^a = the fraction of trips ij subjected to constraint R_a (e.g., the number of trips ij that make use of link a)
  T_{ij} = a priori matrix. This matrix is based on some initial knowledge (old O-D matrix or model estimate).

If $T_{ij} = 1$, Equation 18 is equivalent to the entropy maximizing formulation.

Minimizing the information of Equation 18 subject to the constraints of 19 gives an estimate for T_{ij}. The solution can be derived by minimizing the Lagrangian (if the Kuhn–Tucker conditions are fulfilled)

$$L = \min_{\lambda_a, \hat{T}_{ij}} \left\{ \sum_i \sum_j \left[\hat{T}_{ij} \ln\left(\frac{\hat{T}_{ij}}{T_{ij}}\right) \right] \right. \tag{20}$$
$$\left. - \sum_a \left(\lambda_a \left[\sum_i \sum_j (\hat{T}_{ij} d_{ij}^a) - R_a \right] \right) \right\}$$

The solution is found by setting the partial derivatives to zero

$$\frac{\delta L}{\delta \hat{T}_{ij}} = 0 \quad \forall ij \tag{21}$$

$$\frac{\delta L}{\delta \lambda_a} = 0 \quad \forall a \tag{22}$$

Equation 21 yields

$$\frac{\delta L}{\delta \hat{T}_{ij}} = 1 + \ln(\hat{T}_{ij}) - \ln(T_{ij}) - \sum_a \left(\lambda_a \sum_i \sum_j d_{ij}^a \right) = 0 \tag{23}$$

Suppose

$$\ln(X_o) = -1 \qquad \ln(X_a) = \lambda_a \sum_i \sum_j d_{ij}^a$$

Substitution in Equation 23 yields

$$\hat{T}_{ij} = T_{ij} x_o \prod_a X_a \quad \forall i, j \tag{24}$$

where

$$\sum_i \sum_j (\hat{T}_{ij} d_{ij}^a) = R_a \quad \forall a \tag{19}$$

Equation 24 shows that the a priori information (O-D matrix) is multiplied by a number of coefficients X_a. Each constraint (R_a) introduces an extra coefficient X_a into the model. An important difference between the IM and EM models is that in case of more observations (N) per O-D pair, there is a positive correlation between the bias of the EM estimate and the number of times the flow is counted; the IM estimate is free of bias (see Maher [29]).

The model described earlier has some specific characteristics:

• The amount of traffic (T_{ij}) is estimated using a model with coefficients that are time- and place-dependent. A change in the volume counts will result in a change of the coefficients (Lagrange multipliers) of the model.

• Equation 18 is not defined in case $T_{ij} = 0$. All relations of the trip matrix T_{ij} that are not observed cannot be used in the estimation process. This may become a severe problem if an observed a priori O-D matrix is used that (in most cases) contains, for the greater part, zero values. All O-D pairs that are not observed should be estimated first, for example, by using the weighted Poisson model (see preceding section).

• If the set of constraints in Equation 19 contains inconsistencies, no feasible solution will be found. To be able to solve this problem, it is necessary to remove all inconsistencies from the data and to estimate the trip matrix from the consistent flows (see Van Zuylen [9], Van Zuylen and Willumsen [6]). This problem often occurs, especially if cordon surveys are used in which O-D pairs might be observed several times. Therefore, some techniques have been developed to cope with this problem (see, e.g., Roos [30], Ellis and Van Ammers [31], and Smit and Te Linde [32]).

To improve the estimation technique, the entropy maximizing model has been extended in several ways—for example, the incorporation of a route information component (Van Maarseveen and Ruygrok [33], Van Maarseveen et al. [34], Willumsen [12]), as well as the introduction of time-dependent trip matrices (Willumsen [17]).

Loss of Information

Van Vuren [35] used the IM model to estimate an O-D matrix of bicycle trips in Delft. The a priori information consisted of an O-D matrix of partly observed, partly modeled trips; bicycle counts were added as extra information. As can be seen from Figure 1, the model estimate (a posteriori matrix) differed considerably from the a priori matrix, which indicates that quite some a priori information got lost.
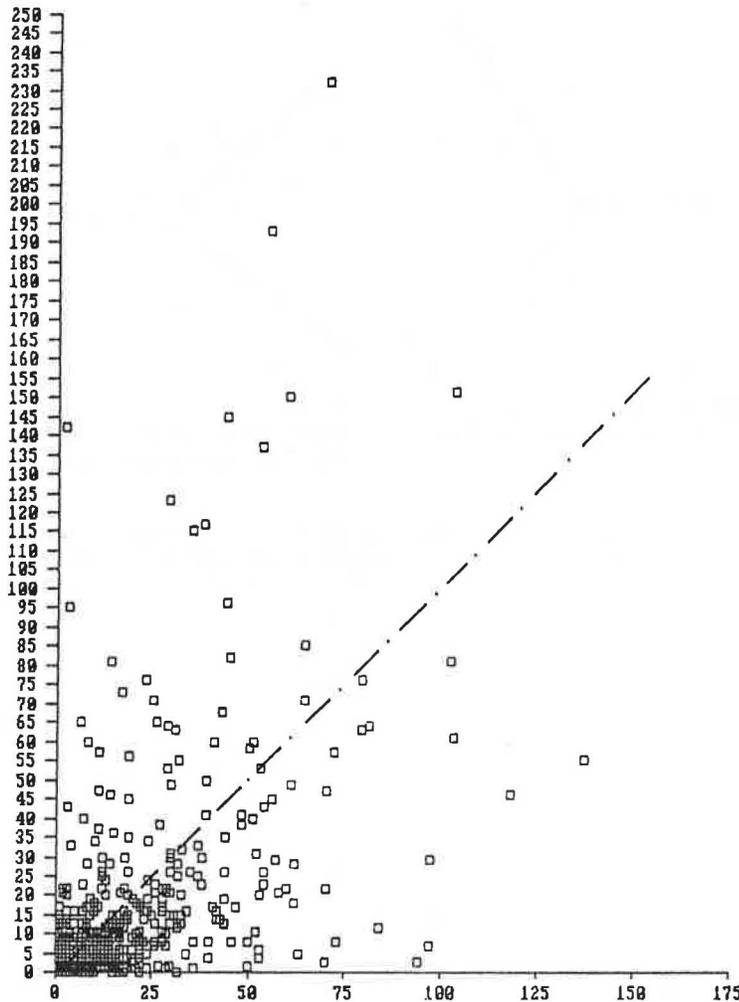
FIGURE 1 Random sample of the a priori and a posteriori values of O-D pairs estimated using the information minimizing model (35).

The tremendous change in the trip matrix might have been caused by the following factors:

- incorrect a priori matrix,
- specific properties of the information minimizing model,
- inconsistent data.

Unfortunately, it is not possible to trace the effects of all factors. However, an indication of the effects of the specific properties of the model can be given using the following example in which the model as been applied to a small network. The network is shown in Figure 2. Trips are generated between the zones 1, 2, 3, and 4 only. The observed trip pattern (T_{ij}) is shown in Table 1. This trip matrix will be used as a priori information. The assignment of all trips to the network results in link flows as indicated with C (calculated) in Figure 2.

The observed link flows are indicated with O. There is some dissimilarity between the calculated and observed flows, but the deviations are kept within bounds. As is usual in practice, only the network flows on a limited number of links are observed.

The results of the estimation of the O-D matrix using the information minimizing model are also shown in Table 1; the calculated flows are shown in Figure 3. As can be seen from Figure 3, the observed and calculated network flows are practically identical, which implies that there are no inconsistencies in the data. Otherwise, no feasible solution would have been obtained. A closer look at Table 1, however, makes it evident that the estimated O-D matrix has changed considerably.

This example clearly illustrates that because of the necessary adjustments that had to be made to meet the link flow constraints, the information stored in the a priori matrix has changed considerably.

In fact, the entropy maximizing model as introduced by Willumsen (7) will render almost the same results. The introduction of constraints with regard to the total number of incoming and outgoing flows per zone as well as the total distance traveled will result in the entropy model as formulated by Wilson. This very model, however, can also be considered an a priori matrix. The incorporation of extra constraints will modify the results of the estimation process in the same way as has already been indicated (except for the adaptation to the incoming and outgoing

FIGURE 2   Observed (O) and calculated (C) link flows. Calculated link flows are based on assignment of observed O-D matrix (Table 1).
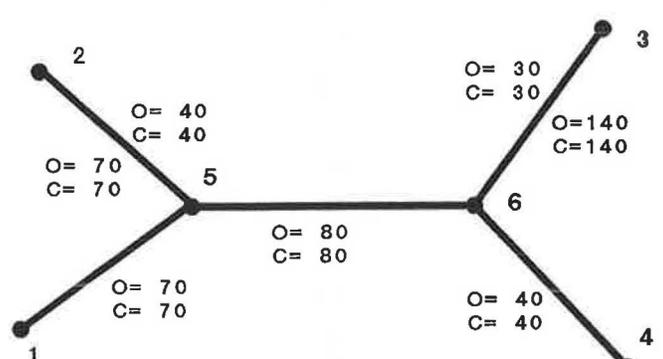
FIGURE 3   Observed and calculated link flows using the information minimizing model.

TABLE 1   OBSERVED (A PRIORI MATRIX T_{ij}) AND ESTIMATED ($\hat{T}_{ij}$) O-D MATRIX USING THE INFORMATION MINIMIZING MODEL

from \ to	1		2		3		4			
	T_{ij}	$\hat{T}_{ij}$	T_{ij}	$\hat{T}_{ij}$	T_{ij}	$\hat{T}_{ij}$	T_{ij}	$\hat{T}_{ij}$	$\sum_j T_{ij}$	$\sum_j \hat{T}_{ij}$
1	–	–	10	27	40	30	10	11	60	68
2	10	31	–	–	40	28	10	10	60	69
3	10	6	10	5	–	–	10	19	30	30
4	10	10	10	8	40	81	–	–	60	99
$\sum_i T_{ij}$; $\sum_i \hat{T}_{ij}$	30	47	30	40	120	139	30	40	210	266

flows per zone). Figure 1, being the result of a practical application of the model, clearly shows which adjustments in the a priori matrix were necessary to cope with all constraints. Actually the model formulation places less importance or less confidence in the a priori information. The concept of giving weights (e.g., in accordance with the reliability of different types of information) might be more appropriate.

INFORMATION MINIMIZING MODEL WITH ELASTIC CONSTRAINTS

The example in the preceding section clearly indicates that application of the information minimizing model may lead to considerable loss of information. The reason for this (unexpected) result can be found in the omission of the stochastic properties of the constraints (traffic counts and the like). All constraints that are incorporated into the model behave deterministically. To solve this problem of "information loss," it is suggested that the model be reformulated using elastic instead of fixed constraints (analogous to the transportation model with elastic constraints; Hamerslag [23]).

Model Specification

Application of elastic constraints in the optimization problem yields the following equations:

$$\hat{T}_{ij} = T_{ij}X_o \prod_a X_a \quad \forall i, j \tag{24}$$

$$\sum_{ij}\sum (\hat{T}_{ij}d_{ij}^a) = S_a \quad \forall a \tag{25}$$

$$S_a = X_a^{-g_a}R_a \quad 0 \leq g_a \leq \infty \tag{26}$$

So Equation 19 is replaced by Equations 25 and 26. But in case $g_a = 0$, S_a equals R_a.

In case of a constraint b, substitution of Equations 24 and 25 in 26 yields the following equation:

$$X^{-g_b}R_b = X_b \sum_{ij}\sum \left(T_{ij}X_o \prod_{\substack{a \\ a \neq b}} (X_a)d_{ij}^b\right) \tag{27}$$

which, after some conversion results in:

$$X_b = \left\{\frac{R_b}{\sum_{ij}\sum\left[T_{ij}X_o \prod_{\substack{a \\ a \neq b}} (X_a)d_{ij}^b\right]}\right\}^{1/(1+g_b)} \tag{28}$$

The exponent $[1/(1 + g_b)]$ in Equation 28 is defined as the elasticity of constraint b. If the elasticity $[1/(1 + g_b)]$

equals 0, then $X_b = 1$, which implies that constraint b does not have any influence on the model estimate. If the elasticity $[1/(1 + g_b)]$ equals 1, however, the model estimate will correspond completely with constraint b. In the latter case, the information minimizing model is being dealt with.

An interesting feature of the model with elastic constraints is that (choosing correct values of the elasticities) the loss of a priori information that will occur has been reduced considerably. This can easily be demonstrated using the example given earlier. The results of an application of the model using an overall value of the elasticities of 0,5 are shown in Figure 4 and Table 2.

As can be seen from Table 2 the estimated O-D matrix is very similar to the a priori matrix; this implies that the information stored in the a priori matrix influences the model estimate (no loss of information). Still some differences between the observed and calculated link flows will remain. This phenomenon is completely in accordance with the nature of observed link flows, however, showing quite a bit of variation over time. Therefore the amount of extra information added to the problem by observed link flows (traffic counts) is limited; consequently, the application of fixed constraints would be erroneous.

The estimate of the O-D matrix strongly depends on the values of the elasticities (as is shown in Table 3), especially for values close to 1. Although it is not yet possible to give a clear indication of which values should be given to the elasticities of the constraints, it is obvious that two factors are predominant:

- the additive amount of O-D information and
- the reliability of the information.

Apart from these factors, it is obvious that the values of the elasticities should be chosen in such a way that the sum (or weighted sum) of chi^2 (for flows and O-D pairs) has a minimum value. (See Figure 5 and Table 4.)

As has been shown, the application of the information-minimizing model with elastic constraints might improve
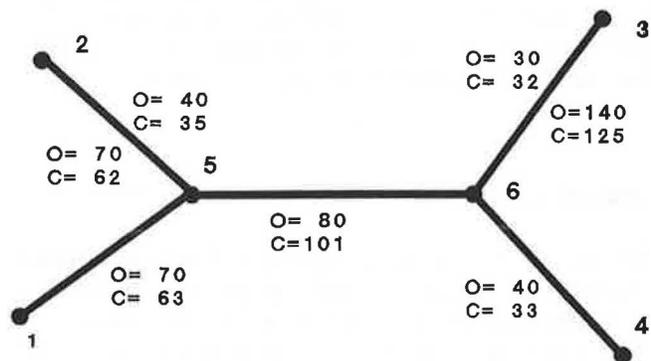
FIGURE 4 Observed and calculated link flows using the information minimizing model with elastic constraints.

TABLE 2   OBSERVED (A PRIORI MATRIX T_{ij}) AND ESTIMATED ($\hat{T}_{ij}$)
O-D MATRIX USING THE INFORMATION MINIMIZING MODEL
WITH ELASTIC CONSTRAINTS

from to	1		2		3		4			
	T_{ij}	$\hat{T}_{ij}$	T_{ij}	$\hat{T}_{ij}$	T_{ij}	$\hat{T}_{ij}$	T_{ij}	$\hat{T}_{ij}$	$\sum_j T_{ij}$	$\hat{\sum}_j T_{ij}$
elasticity	–	0,5	–	0,5	–	0,5	–	0,5		
1	–	–	10	13	40	40	10	11	60	64
2	10	11	–	–	40	40	10	11	60	62
3	10	9	10	11	–	–	10	11	30	31
4	10	10	10	11	40	45	–	–	60	66
$\sum_i T_{ij}; \hat{\sum}_i T_{ij}$	30	30	30	35	120	125	30	33	210	223

considerably the estimate of an O-D matrix using a priori information and traffic counts. Nevertheless, some shortcomings (some of them indicated in the section on the weighted Poisson estimator) will remain:

- Each observation is formulated as a constraint and consequently results in the incorporation of an extra coefficient in the estimation model. These coefficients are time- and place-dependent, which precludes the possibility of using the model for predicting medium- to long-term changes.
- The estimated O-D matrix depends heavily on the values of the elasticities (see Table 3). If those values equal 0, the a priori matrix will not change. A value of the elasticities equal to 1 will result in the information minimizing model.
- The available information from traffic counts is used to fit only those O-D pairs that pass along the observed links. All other O-D pairs will not change at all. This problem can be overcome by using additional O-D information—for example, trip distance distribution.

BINARY CALIBRATION MODEL

The use of traffic data for estimation (calibration) purposes requires some caution. The continuous variation of traffic flows, the availability of data from various sources (surveys, counts, ticket sales), as well as differences in sample size and observed population groups may result in a considerable amount of stochasticity. Especially if various kinds of survey data are used because of this stochasticity, some inconsistencies in the data may show up (the data seem to be contradictory). In general, use of various data sources does not raise any problems; it is also possible (especially in case of inconsistencies), however, for no feasible solution to be found. The latter is especially true for the entropy type of model. Estimation models do exist, however, that can deal with the stochastic properties of the observations. Like the entropy model, all coefficients are calibrated in such a way that an optimum fit will be obtained between the observed data and the estimated trip matrix. One essential difference, in contrast to the entropy type of model, is that no additional coefficients are introduced to meet all constraints.

Hamerslag and Huisman (1) formulated the binary calibration model, which is based on the assumption that the observed data are independently Poisson distributed.

Smit (2) and Hendrickson and McNeil (4) use a normal distribution. Use of both latter models may raise some problems because they allow for negative flows (especially in case of relatively small flows).

Model Formulation

Suppose T_{ij} is the available a priori information (e.g., an old, observed O-D matrix). Instead of using an observed O-D matrix, it is preferable to use estimates of T_{ij} (e.g., an estimate of $\hat{T}_{ij}$ using the weighted Poisson model).

An advantage of the use of estimates is that the O-D matrix T_{ij} will not contain zero values. Analogous to

TABLE 3 ESTIMATE OF O-D MATRIX FOR DIFFERENT VALUES OF THE ELASTICITIES

from	to	1	2	3	4
	elasticities				
1	0.0		10	40	10
	0.5		13	40	11
	0.8	-	17	37	11
	0.9		20	34	11
	1.0		27	30	11
2		10		40	10
		11		40	11
		15	-	38	11
		19		35	11
		31		28	10
3		10	10		10
		9	11		11
		8	10	-	13
		8	8		15
		6	5		19
4		10	10	40	
		10	11	45	
		10	12	55	-
		10	11	63	
		10	8	81	

Equation 6 such a matrix can be formulated as follows:

$$\hat{T}_{ij} = C o_i d_j F_k \quad \forall i, j \tag{29}$$

where

$\hat{T}_{ij}$ = number of trips from i to j (model estimate or a priori matrix)

C = constant term

$o_i = a_i O_i$ = product of balancing factor and polarity of zone i

$d_j = b_j D_j$ = product of balancing factor and polarity of zone j

F_k = value of deterrence function for a generalized cost class k and $c_{ij} \in k$.

Let us further assume

Y_a = information from observations (a) (e.g., departures, arrivals per zone, traffic counts, trip distance distribution, etc.)

d_{ij}^a = a binary variable indicating whether O-D pair ij belongs to observation a ($d_{ij}^a = 1$) or not ($d_{ij}^a = 0$)

and

$$Y_a \overset{\text{def}}{=} \sum_i \sum_j (\hat{T}_{ij} d_{ij}^a) \tag{30}$$

The relation between the estimate of the base year matrix ($\hat{T}_{ij}$) and the a priori matrix can be represented by

—— Flows ---- O-D matrix --- Sum
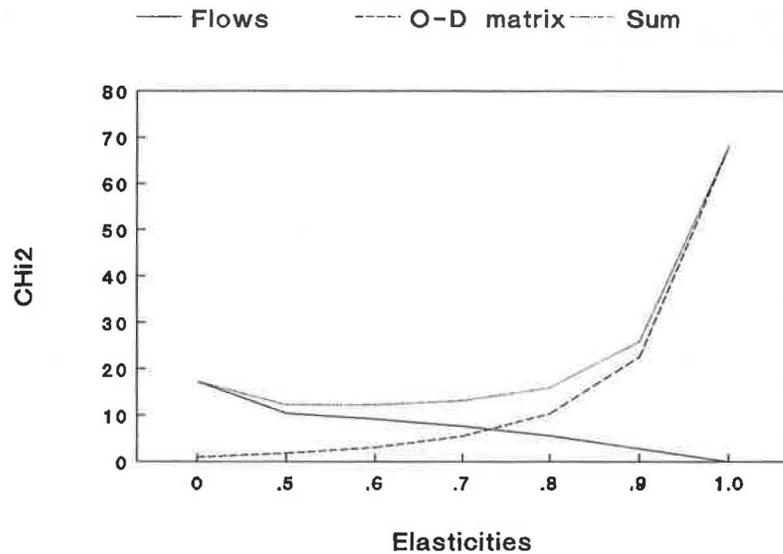
FIGURE 5 Chi-square for different values of the elasticities.

TABLE 4 OBSERVED (A PRIORI MATRIX T_{ij}) AND ESTIMATED ($\hat{T}_{ij}$) O-D MATRIX USING THE BINARY CALIBRATION MODEL

from to	1		2		3		4			
	T_{ij}	$\hat{T}_{ij}$	T_{ij}	$\hat{T}_{ij}$	T_{ij}	$\hat{T}_{ij}$	T_{ij}	$\hat{T}_{ij}$	$\sum_j T_{ij}$	$\sum_j \hat{T}_{ij}$
1	–	–	10	10	40	40	10	12	60	62
2	10	10	–	–	40	40	10	12	60	62
3	10	10	10	9	–	–	10	11	30	30
4	10	11	10	11	40	40	–	–	60	62
$\sum_i T_{ij}; \sum_i \hat{T}_{ij}$	30	31	30	30	120	120	30	35	210	216

the following equation:

$$\hat{T}_{ij} = Co_I d_J F_K T_{ij} \tag{31}$$

where $i \in I$, $j \in J$, and $k \in K$.

The preceding formulation implies that sets of balancing factors as well as discrete values of the deterrence function are modified; however, the general form of the model remains unchanged.

Assuming that the observations (Y_a) are independent and Poisson distributed, the likelihood (L) can be determined as follows:

$$\Pr[Y_a \mid \hat{Y}_a] = \exp(-\hat{Y}_a)(\hat{Y}_a)^{(Y_a)}/Y_a! \tag{32}$$

$$L = \prod_a (\Pr[Y_a \mid \hat{Y}_a]) \tag{33}$$

The logarithm of the likelihood yields

$$\ln(L) = L^* = -\sum_a (\hat{Y}_a) + \sum_a [Y_a \ln(\hat{Y}_a)] \\ - \sum_a [\ln(Y_a!)] \tag{34}$$

An optimum adjustment between $\hat{Y}_a$ and Y_a can be determined by maximizing L^* or as

$$\sum_a [\ln(Y_a!)] = \text{constant}$$

by maximizing

$$L_1^* = -\sum_a (\hat{Y}_a) + \sum_a [Y_a \ln(Y_a)] \tag{35}$$
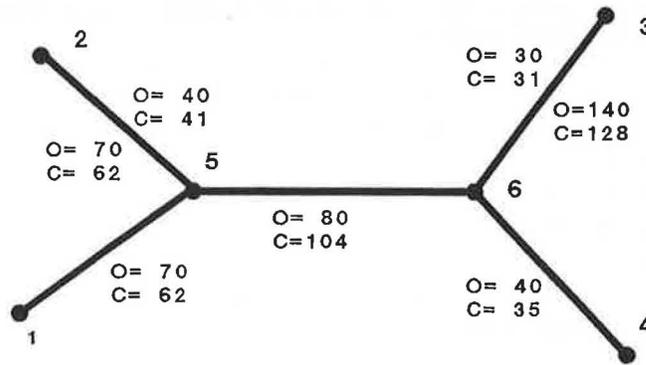
FIGURE 6 Observed and calculated link flows using the
binary calibration model.

which yields the following equations:

$$\sum_a \left[\left(\frac{Y_a}{\hat{Y}_a} - 1 \right) \frac{\delta Y_a}{\delta o_I} \right] = 0 \quad \forall I \tag{36}$$

$$\sum_a \left[\left(\frac{Y_a}{\hat{Y}_a} - 1 \right) \frac{\delta Y_a}{\delta d_J} \right] = 0 \quad \forall J \tag{37}$$

$$\sum_a \left[\left(\frac{Y_a}{\hat{Y}_a} - 1 \right) \frac{\delta Y_a}{\delta F_K} \right] = 0 \quad \forall K \tag{38}$$

$$\sum_a \left[\left(\frac{Y_a}{\hat{Y}_a} - 1 \right) \frac{\delta Y_a}{\delta C} \right] = 0 \tag{39}$$

The values of o_I, d_J, F_k, and C will be determined itera-
tively (analogous to the Gauss-Seidel principle). Inserting
those values in Equation 31 yields the estimates for the
base year matrix $\hat{T}_{ij}$.

Application of the Model

The results of a practical application of the binary calibra-
tion model using the same example as before (network, a
priori information, and link traffic counts) are shown in
Figure 6 and Table 4.

It can be noted from Table 4 that the trip matrix estimate
($\hat{T}_{ij}$) is practically equivalent to the observed a priori
matrix. It can also be noted from Figure 6 that all link
traffic flow estimates are not equivalent to the link traffic
counts, but there is a great similarity. Both results corre-
spond to the very nature of the data used in the estimation
process (see also the preceding section on information
optimizing with elastic constraints).

The advantage of using the binary calibration model,
especially compared to the information minimizing model
with elastic constraints, is that no extra coefficients are
incorporated in the model that allows use of the model for
making medium- and long-term forecasts.

CONCLUSIONS

This paper deals with four models that can be used for
estimating an O-D matrix:

- the weighted Poisson estimator
- the entropy maximizing and information minimizing
model
- the information minimizing model with elastic con-
straints
- the binary calibration model

The weighted Poisson model is especially well suited for
estimating deterrence functions and a base year matrix
using information from household or cordon surveys (only
O-D information). This base year matrix estimate is espe-
cially appropriate for use in the information minimizing
model with elastic constraints or the binary calibration
model (the base year matrix estimate is treated as a priori
information).

The entropy maximizing and information minimizing
models show some severe shortcomings. First, neither
model allows for inconsistent information. Second, all
coefficients in the model are time- and place-dependent,
which precludes the possibility of using the models for
making medium- and long-term forecasts. And, last but
not least, use of both models induces loss of information.

The latter problem can be resolved using a model with
elastic instead of fixed constraints. As has been shown, the
values of the elasticities strongly influence the estimate
and, consequently, the loss of information. Besides, the
available information from traffic counts is used only to
fit those O-D pairs that pass along the observed links. All
other O-D pairs will not change at all.

The binary calibration model does not show any of the
shortcomings just mentioned. The model, however, has a
rather complex structure and has been successfully applied
in the Netherlands (see, e.g., Hamerslag et al. [16] and
Heere and Huisman [36]).

Table 5 presents a summary of the main characteristics
of the models dealt with in this paper.

TABLE 5 SUMMARY OF MAIN CHARACTERISTICS OF FOUR O-D ESTIMATION TECHNIQUES

characteristic	model type weighted Poisson	entr.max. inf.min. estimator	inf.min. elastic model	binary calibrat. constraints
estimation unobserved O-D pairs	yes	no	no	yes
apparently contradictory information permitted	no	no	yes	yes
possibility of using traffic counts	no	yes	yes	yes
change of O-D pairs	-	only observed O-D pairs	only observed O-D pairs	all O-D pairs
loss of information	no	yes	dependent on value elasticities	no
time and place dependency of coefficients (if yes, it is not possible to use the model for medium and long term forecasts)	no	yes	yes	no
complex structure of model	no	no	no	yes

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