Updating Ride Checks with Multiple Point Checks

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A procedure is described for estimating ride checks (ons and offs by stop) by updating old ride checks with recent multiple-point-check data (on, off, and load at selected points). The procedure involves synthesizing an origin-destination (O-D) matrix method and bringing this matrix into agreement with the point-check observations using multiproportional adjustments. Testing on several Los Angeles bus lines indicates how estimation accuracy varies with number of points checked, number of days checked, and length of time period of aggregation. Period-level estimates for periods as small as 20 min are found to have reasonably good accuracy for total boardings, passenger-miles, and maximum load. The procedure can be an economical way to derive ride-check data.

Ride checks, which provide a record of ons and offs by stop along a transit route, are the most complete set of route-level data normally collected by a transit agency. They reveal (a) the total boardings on route and on route segments, (b) passenger-miles, (c) location of the peak load point, (d) maximum load per trip, and (e) average load at the peak load point or any other point of interest. Because of this wealth of information, ride checks are valuable for route and schedule planning, particularly on long or heavy-volume lines, which are conducive to scheduling options such as short-turning, alternating deadheading, zoning, and offering limited stop service (*1*). However, ride checks are expensive to conduct and, consequently, are done infrequently. Ride-check data available to a typical route or schedule planner generally consist of a single day's sample and may be several years old.

Point checks are less expensive to conduct than ride checks. For example, at the Southern California Rapid Transit District (SCRTD), ride checking the entire weekday schedule (covering each trip once) requires 3,350 checker-days, whereas point checking the weekday schedule at peak-load points during 12 daytime hours requires two checkers per point at 132 points, or about 400 checker-days. The passenger use information that point checks provide is limited to ons and offs at the checkpoint and arriving or departing loads. Because they are less expensive, point checks can be measured more frequently, providing the planner with recent and statistically sound estimates. A natural question, then, is how to combine rich but outdated ride-check information with limited but recent point-check information in order to estimate recent ride-check measures. In practice, planners often do the "mental gymnastics" of fitting an old ride check to recent point data. This exercise is extremely difficult to do well, and an updating methodology can provide a mechanism to reconcile these different sources of data into a useful profile. Even if a ride check is recent, it may be suspect if based on a single day's measurement, and combining point-check information from several other days should improve accuracy.

When point checks measure only load at a single point, updating an old ride check is straightforward; the ride check is simply factored up or down to agree with the recently measured load. But if load is measured at several points, or if on and off information is to be incorporated as well, an updating method is not obvious. Simply factoring by an average increase in loads over multiple points presents two problems. First, the resulting estimates will not agree with measured load at any of the points. Second, if three or more points are averaged, they should not necessarily be weighted equally, because if two points are close together their loads will be highly correlated.

MODELING APPROACH

Underlying the on, off, and load information of a ride check is a stop-to-stop origin-destination (O-D) matrix. Ons and offs are row and column totals of the O-D matrix. Likewise, loads are represented in the O-D matrix by rectangular blocks of cells extending to the northeast corner of the matrix, as shown in Figure 1. Although Figure 1 illustrates through load, arriving or departing load could be used instead. O-D volumes can be more easily manipulated than ride-check data, because O-D volumes are independent of one another and, therefore, intrinsically do not need to balance (as do total ons and total offs) or show serial correlation (as do loads). Methods for updating an O-D matrix with summary information such as row and column totals have been widely studied and reported in the literature, having been applied to such areas as updating a bus route O-D matrix with ride-check data (*2*), updating a matrix of intersection turning movements with inflow and outflow totals (*3–5*), updating automobile trip tables with segment flows (*6–8*), and updating regional input-output matrixes with forecasts of regional input and output (*9*). When such a method is used, an O-D matrix can be adjusted to make its row totals, column totals, and block totals agree with observed ons, offs, and loads from point-check data. The modeling approach is, therefore, to estimate or synthesize the O-D matrix underlying the original ride check, to update this matrix to agree with point-check measurements of load and (if available) ons and offs at the checkpoint, and then to reduce the

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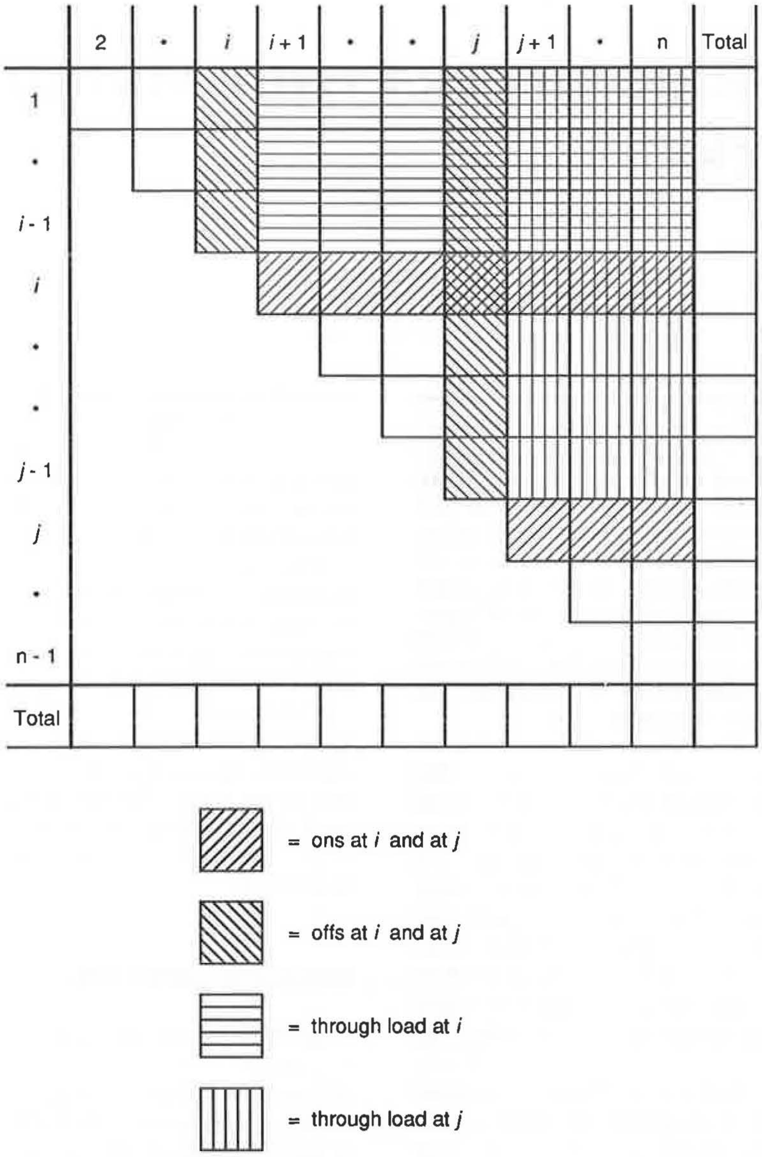

FIGURE 1 Updating with two checkpoints.

updated matrix to its row and column totals, yielding an updated ride check.

Because of space limitations, only an overview of the models will be presented. O-D matrix synthesis is done using a method developed by Tsygalnitzky (*10*). This method was tested with favorable results on two SCRTD lines and is described by Simon and Furth (*11*). The matrix is updated using iterative multiproportional adjustments, as described and tested by Ben-Akiva et al. (*2*) and McNeil and Hendrickson (*12*), using the scalar adjustment derived by Bell (*6*) to make the results constant with respect to scaling of the original seed. Intrasegment travel volumes, which are unaffected by matrix updating (because they are not observed by the point checks), are scaled up or down in proportion to the change in the intersegment volumes beginning or ending in the corresponding segment. More detail is given in the project report (*13*).

APPLICATION TO SCRTD

The updating methodology was applied to SCRTD Lines 30, 45, 53, 92, 117, 152, 200, 209, and 260. Included were heavy-volume lines (peak headway under 5 min) and lighter-volume lines. Existing SCRTD ride-check and line-description computer files were used without modification. Old ride-check data ("seed data") were used to generate seeds. Point-check data were simulated by extracting on, off, and load at designated checkpoints from a set of ride checks taken 1 year later ("new data"). For each line, checkpoints were chosen by SCRTD staff in order of priority so that point checks done at one, two, or three or more stops could be simulated. Because complete ride checks for the new data were available, the accuracy of the updating procedure could be assessed by comparing the estimated ride-check profile to the true profile.

Implementing the updating procedure required resolving some technical issues, which are discussed here. First, it became obvious that each route variation, or branch (as it is called at SCRTD), needs its own seed. A line might have several variations such as the main route, a short line, and minor branching variations. Second, to make the seeds reflect changing travel patterns throughout the day, a separate seed was created for each branch/time-period combination. The day was broken into time periods with boundaries at 6 a.m., 9 a.m., 2 p.m., and 6:30 p.m. The seed matrix for a branch/period combination was found by first accumulating on and off totals by stop from the seed data for that branch and period and then generating an O-D matrix from this period profile.

When a branch/period combination has only a few trips in the seed data, it may be unwise to rely exclusively on those few trips for the seed matrix. Therefore, a method was devised to incorporate information from trips on other branches that served many of the same stops as the branch in question. An O-D matrix was generated for every trip in the seed data, and these O-D matrixes were accumulated by period, summing over all branches. The period O-D matrixes were then normalized to yield the passenger flow between each O-D pair per bus trip serving that O-D pair. When a branch/period combination contained too few trips, its seed matrix was generated by extracting from that period's normalized O-D matrix the cells served by that branch. Branch/period combinations with no trips in the seed data were not analyzed.

EVALUATION PROCEDURES

To evaluate the accuracy of the estimated ride checks, three summary measures were compared to the true values: total boardings, maximum load, and passenger-miles. These items were analyzed separately, recognizing that the updating procedure might estimate some items more accurately than others and that greater accuracy might be desired for some items than for others.

It should be emphasized that "maximum load" is the greatest load on a trip, regardless of where it occurs, and differs from "peak load," which is the load at the point of highest average volume. We did not assess the accuracy of measuring peak load since the point checks are nearly always done at the peak-load point, and so peak load will be estimated without estimation error.

The summary measures were compared at both the trip level and the period level. Because most scheduling and planning decisions are based on time-period averages, rather than on individual trip measures, the research goal was to achieve good agreement between actual and estimated measures at the period level.

MEASURES OF ACCURACY

The measure of error for trip-level quantities is the relative standard error. A relative standard error is calculated for each line/direction/time period as the ratio of the standard error to the mean true value. The standard error is the square root of the average squared difference between the estimated value (of, say, boardings) and the true value.

For period-level quantities there is only one estimated-to-true comparison for each item, and so there is no standard error as such. Therefore, the reported error measure is the relative error, which is the actual error divided by the true value.

To provide summaries of the error measurements for a given item (e.g., boardings), the error measures were averaged over the many line/direction/time-period combinations. The general rules used in aggregating are as follows. To get average relative standard error, standard errors and means are averaged separately, and then the average standard error is divided by the average mean value. To get average relative error, errors and means were likewise averaged separately and then divided. To get average relative absolute error, absolute errors and means were likewise averaged separately and then divided.

NUMERICAL RESULTS

Comparisons by line, direction, and time period were made for all nine lines, using one, two, and three points of point-check data. A typical result, displayed here as Table 1, is that for Line 53, Time Period 2 (6 a.m. to 9 a.m.), which encompasses 23 trips. The Line 53 seeds were taken from 1984 ride checks, whereas the new data are from 1985 ride checks. As shown in Table 1, the estimates of average boardings (127, 119, and 115, using one, two, and three points) show good agreement with the observed mean (121). The prediction of average maximum load is consistently about 5 percent low. The comparison of average passenger-miles, which indicates how well load all along the route is estimated, shows the advantage of using multiple points. Using one, two, and three points, the relative absolute errors drop from 12 percent to near 0 percent.

The trip-level relative standard errors for boardings, using one, two, and three points, are 26 percent, 22 percent, and 16 percent, showing a good deal of estimation error. Trip-level relative standard errors for maximum load are 9 percent, 8 percent, and 8 percent. These rather accurate estimates suggest that maximum loads for this line occur at or very near the checkpoints at which load is observed. The trip-level relative standard errors for passenger-miles are 24 percent, 16 percent, and 7 percent for one, two, and three points, showing very good predictive accuracy for the three-point estimate.

A summary of accuracy statistics from all the line/direction/time-period combinations is shown in Table 2. Of primary importance are the relative errors. To the degree they differ significantly from zero, they indicate an overall tendency in the method to underestimate or overestimate. For route boardings, the relative errors using one, two, and three points are 3 percent, 3 percent, and 2 percent, indicating almost no bias. For passenger-miles, the relative errors again show a slight tendency to overestimate, and improve with each additional point used for the estimation.

The relative errors for maximum load, however, show a small negative bias. This phenomenon is expected since the updating procedure predicts the "most likely" route profile for each trip and, thus, tends to avoid high peaks that randomly occur. Users of this updating procedure should recognize this phenomenon and perhaps compensate by inflating

TABLE 1 ACCURACY OF RIDE CHECK UPDATING—LINE 53, A.M. PEAK

MEASURE	NO. POINTS	BRDGS	PASS MILES	MAX LOAD
MEAN	OBS	121.09	384.62	64.57
	1	126.65	430.15	62.09
	2	119.45	405.23	61.32
	3	114.72	384.36	61.27
RELATIVE ERROR	1	.05	.12	-.04
	2	-.01	.05	-.05
	3	-.05	.00	-.05
RELATIVE ABSOLUTE ERROR	1	.05	.12	.04
	2	.01	.05	.05
	3	.05	.00	.05
RELATIVE STANDARD ERROR	1	.26	.24	.09
	2	.22	.16	.08
	3	.16	.07	.08

TABLE 2 RIDE CHECK UPDATING—SUMMARY

NO. TRIPS: 1501

MEASURE	NO. POINTS	BRDGS	PASS MILES	MAX LOAD
MEAN	OBS	90.58	298.90	40.06
	1	93.65	308.09	36.59
	2	93.59	308.35	37.98
	3	91.69	300.22	37.79
RELATIVE ERROR	1	.03	.03	-.09
	2	.03	.03	-.05
	3	.02	.02	-.05
RELATIVE ABSOLUTE ERROR	1	.12	.10	.12
	2	.08	.06	.07
	3	.06	.05	.07
RELATIVE STANDARD ERROR	1	.31	.27	.25
	2	.22	.17	.16
	3	.18	.13	.13

the estimates slightly. Average load at a peak point, however, should not be biased in this way.

Relative absolute errors in the estimates of period-level averages are also displayed in Table 2. For route boardings, the relative absolute errors using one, two, and three points are 12 percent, 8 percent and 6 percent. The passenger-mile and maximum-load errors are comparable. These results suggest that the updating procedure is quite accurate at estimating time-period-level averages.

Also displayed in Table 2 are the relative standard errors of trip-level items. For route boardings, aggregated over all lines and time periods, these errors are 31 percent, 22 percent, and 18 percent, using one, two, and three points. Passenger-mile and maximum-load results are comparable. These results show that it would be improper to place much confidence in a ride check estimated for a single trip. Indeed, accuracy at this level of detail cannot be expected from any updating procedure using only 1 day of observation because of the high day-to-day variability in passenger activity at the trip level. However, by doing point checks on several days and averaging the results, it may be possible to obtain a reliable estimate of trip-level activity at moderate cost.

The same comparisons were performed using peak-period data only. The results are similar, appearing to be a little bit better on the whole. Line/direction/time periods were then grouped according to their average trip boardings (below 50, above 100, and in between) to see if any one group was estimated with better or worse accuracy. Little significant difference was found. Details of these analyses are given in the project report (*13*).

Because SCRTD's Scheduling Department bases headways on 20-min averages, estimates were also made for 20-min periods between 6 a.m. and 9 a.m. in both directions of Line 45. No trips were observed in one direction in one of the 20-min periods, yielding a total of 17 periods, encompassing 57 trips. Updating was tested using one, two, three, and four points of point-check data. Two of these cases are summarized in Table 3. The quantity of primary concern, maximum load, is estimated very well with three points. The maximum error is 13 percent, and all but two periods had errors below 10 percent. Passenger-miles are estimated almost as well; boardings are estimated a little worse, with a few periods having errors above 15 percent. Estimates based on a single point check, by contrast, are extremely unreliable, with errors above 10 percent being more the rule than the exception. Maximum load for one period was estimated at 29.9 when the true value (averaged over two trips) was 67, indicating that information from a single point was insufficient to detect an unusual crowding pattern in this period. Line 45's need for multiple, as opposed to single, point checks is reasonable because it goes through the downtown, with heavy loads on both sides of downtown.

ACCURACY VERSUS NUMBER OF POINTS

Estimation accuracy was tested on Line 30 using one to nine points of point-check data. The points were selected in order of priority by SCRTD staff. The data encompass 315 trips over an entire day in both directions. Figure 2 shows that the overall estimation bias is small (within 5 percent) for all three quantities of interest for any number of points and that there is little improvement after the fourth point. Oddly, the passenger-mile bias worsens beyond four points; however, it never exceeds 5 percent in magnitude.

Figure 3 shows how trip-level standard errors improve with the number of points. The biggest gain is in the first four points, although improvement continues until about the seventh point, where the standard errors are between one-half and one-fifth of the size of the standard errors based on a single point.

TABLE 3 ACCURACY OF 20-MIN-PERIOD ESTIMATES—LINE 45

	Periods With Error < 10% (out of 17)	Periods With Error < 15% (out of 17)	Worst Case Error
a. Using 3 points			
Boardings	11	14	23%
Pass.-mi	15	17	14%
Avg Maximum Load	16	17	13%
b. Using 1 point			
Boardings	4	6	42%
Pass.-mi	7	11	36%
Avg Maximum Load	11	12	55%

These two figures together indicate that the systematic component of estimation error is rather small, whereas the random component, which tends to balance out when averaged over many trips, can be large but reduces with more information about each trip.

It is difficult to generalize these results into guidance for how many points ought to be counted. To prevent significantly sized markets from going unobserved, it would seem reasonable to station checkers 4 to 5 mi apart, because the average unlinked trip length on the system is about 3 mi. On route segments with average trip distance smaller, and passenger activity more variable in its distribution, closer spacing is warranted. On route segments with average trip distance larger and passenger activity less variable in its distribution, farther spacing is warranted.

The lower cost of point checks suggests the possibility of doing them for several days and averaging the estimates made for those days. Because the desired measure for planning and scheduling is *average* passenger activity, even a single day's ride check is only an estimate, and so the possibility exists that a multiday estimate based on point checks may be more reliable than a single day's ride check. Averaging together samples taken on n days will reduce random error and variability components inversely with the square root of n; however, systematic error or bias components will be unaffected.

Much of the error in a period-level estimate can be attributed to systematic error, which arises because all of the trips in a given period are estimated with the same seed. A reasonable and conservative judgment is to attribute 80 percent of the period-level squared error to systematic error. The same degree of systematic error applies to trip-level estimates. The balance of the estimation error is considered random.

Besides estimation error, another source of error is day-to-day variation. Because multiday ride checks were not available for this study, this type of variation could not be measured. However, other studies indicate a route/direction/time-period day-to-day variation (i.e., coefficient of variation) in trip-level boardings of 20 percent or higher, whereas the day-to-day variation of period-level boardings is around 8 percent. The same figures can be applied to passenger-miles.

Based on these assumptions, and using the average estimation errors reported in Table 2, expected standard errors based on multiple days of updating with point checks using two and three points were calculated for selected items and are displayed in Tables 4 and 5. For comparison, the standard errors expected from using a day of ride checks as a daily average are shown as the day-to-day variation.

These results show that a single day's estimate using the updating procedure is, naturally, worse than a single day's measurement using a full-ride check. Using three points, standard errors are about 25 percent greater (e.g., a relative standard error of 10 percent instead of 8 percent); using two points, errors are about 38 percent greater. There is, therefore, a small but significant loss in accuracy from substituting

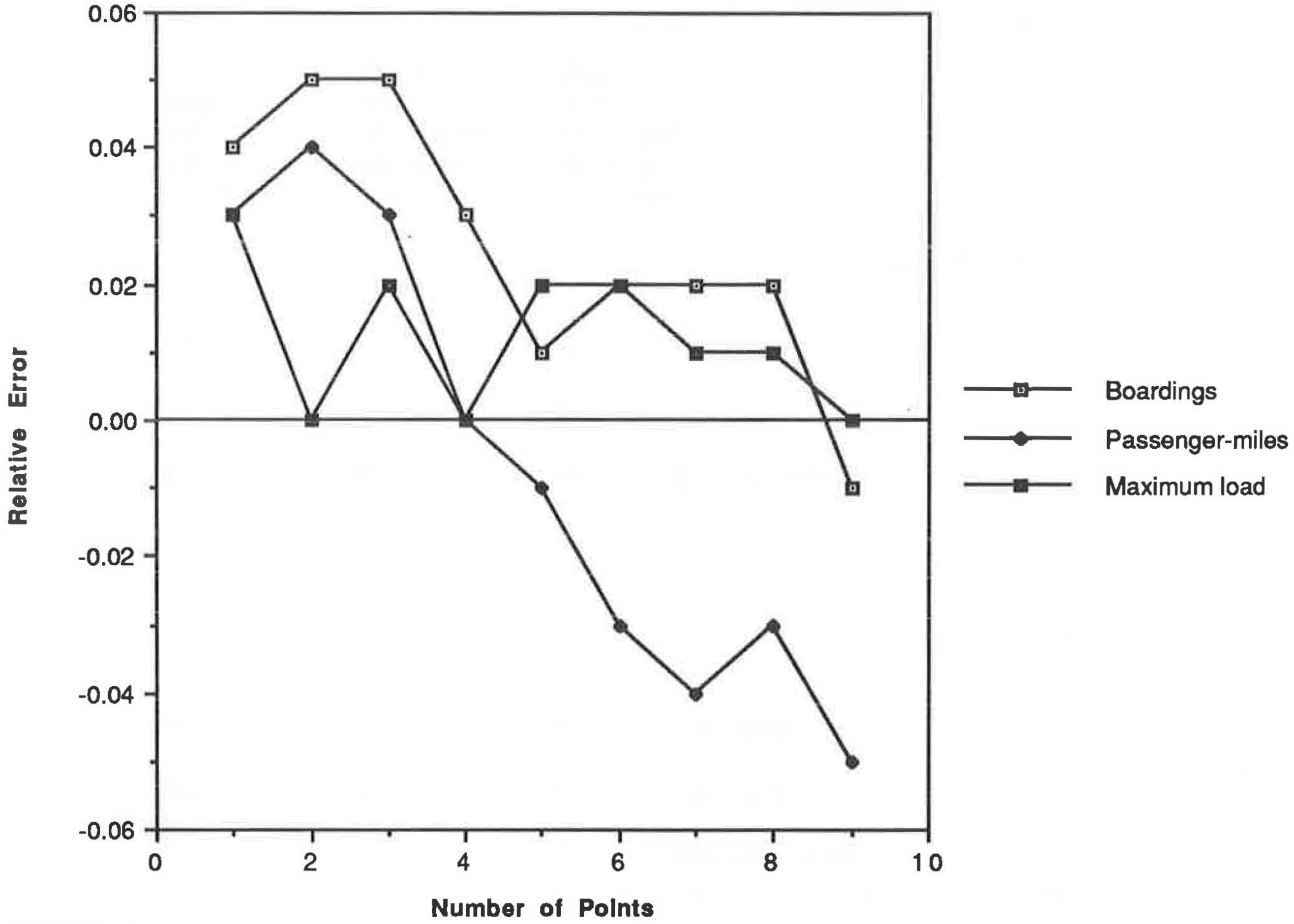

FIGURE 2 Relative error versus number of points.

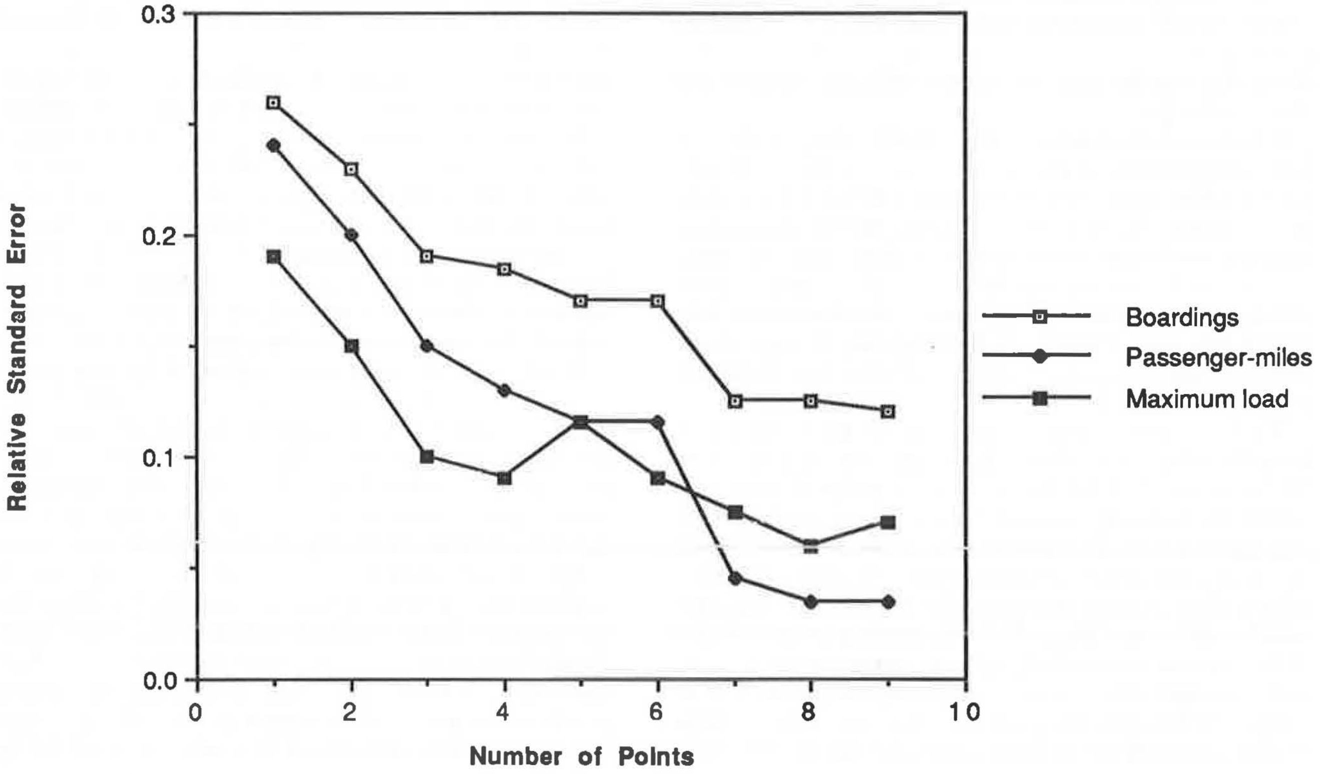

FIGURE 3 Relative standard error versus number of points.

TABLE 4 MULTIPLE-DAY COUNT OF RELATIVE ERRORS USING THREE POINTS

	Day-to-day Variation	Random Est'n Error	Total Random Variation	Sytematic Est'n Error	Relative standard error		
					1 day	2 days	3 days
Period Boardings	0.080	0.027	0.084	0.054	0.100	0.080	0.072
Trip Boardings	0.200	0.172	0.264	0.054	0.269	0.194	0.161
Period Pass.-Mi	0.080	0.022	0.083	0.045	0.094	0.074	0.066
Trip Pass.-Mi	0.200	0.122	0.234	0.045	0.239	0.172	0.142

TABLE 5 MULTIPLE-DAY COUNT OF RELATIVE ERRORS USING TWO POINTS

	Day-to-day Variation	Random Est'n Error	Total Random Variation	Sytematic Est'n Error	Relative standard error		
					1 day	2 days	3 days
Period Boardings	0.080	0.036	0.088	0.072	0.113	0.095	0.088
Trip Boardings	0.200	0.208	0.289	0.072	0.297	0.216	0.181
Period Pass.-Mi	0.080	0.027	0.084	0.054	0.100	0.080	0.072
Trip Pass.-Mi	0.200	0.161	0.257	0.054	0.262	0.189	0.158

a 1-day multiple-point check for a 1-day full ride check. However, since much of error is day-to-day variation (which affects both ride and point checks), additional days of point checks lower the standard errors of the estimates. With 2 days of point checks using three points, with ride checks estimated for each day separately and then averaged together, the resulting standard errors are less than those standard errors associated with 1 day of full-ride checks. With 3 days of point checks, standard errors are smaller still. Using two points instead of three is not as accurate, especially in estimating period boardings where even with 3 days of point checks the error is still worse than with a single day of ride checks.

EFFECT OF MEASUREMENT ERROR

One problem with a point check-based methodology is that measurement errors can be significant. In Table 6 the expected levels of accuracy of using a three-point check have been revised to account for measurement error as well as the error sources used in Table 4. The assumptions underlying this adjustment are as follows:

- Standard error for load measurement of an individual trip is 13 percent (as reported in an internal SCRTD study).
- Averaged over many trips and all checkers, loads are systematically undercounted or overcounted; the direction of the bias is unknown (otherwise SCRTD could simply adjust load figures accordingly). A magnitude of 3 percent was used.
- Each individual checker, averaged over many trips, has a different systematic error. An average systematic error of 6 percent was assumed.
- When a point check is done at *m* points for *n* days, it can be assumed that the same checker will be assigned to a point for the *n* days. (The alternative assumption, i.e., that checkers change location from day to day, would lead to better accuracy estimates.)

TABLE 6 RELATIVE ERRORS USING THREE POINTS, ACCOUNTING FOR MEASUREMENT ERROR

	overall systematic measurem't error	individual systematic measurem't error	individual total error	day-to day variation	Relative standard error from n days of updating		
					1 day	2 days	3 days
Period Boardings				0.08	0.12	0.10	0.09
Trip Boardings	0.03	0.06	0.13	0.20	0.27	0.20	0.17
Period Pass.-Mi				0.08	0.12	0.09	0.08
Trip Pass.-Mi	0.03	0.06	0.13	0.20	0.26	0.19	0.16

- Error components for a given checker are assumed independent.

The net effect of measurement error, combined with all the other sources of variation, is to raise relative errors somewhat, making it more important that estimates be based on several points and on several days if possible.

IMPLEMENTATION ISSUES

Incorporating ride-check updating into a systematic data collection program requires a thorough review of a transit system's data needs followed by the design of a statistically sound program that most economically meets these needs. Difficult-to-quantify trade-offs are often necessary. Data needs beyond the ridership profiles discussed in this paper must be addressed, such as route revenue and running time data, which can better be determined using a ride check than using multiple-point checks with ride-check updating. The general problem may be framed thus: for a given amount of data collection resources, how to allocate these resources to best meet the transit agency's objectives using available data collection and analysis techniques. Should ride check updating be done every other year, or 2 years out of 3, or 9 years out of 10 (with real ride checks done in the other years)? Should ride check updating be done on all lines, or only some lines? at all times of day, or only daylight times or peak periods? How many points should be checked per line, and should multiple-day point checks be done? Some design questions cannot be properly addressed without more research. For instance, seeds that are more than 1 year old have not yet been tested. The method should also be tested in other cities and on different types of routes. Various sampling strategies can also be tested.

Some trade-offs can be made using the results reported in this paper. For example, if it were determined, based on the Table 6 results, that a three-point check must be conducted on 2 days to provide sufficient accuracy to substitute for a full-ride check (i.e., riding every trip on a line for 1 day), would this substitution be economical? A full ride check requires one checker per bus. This requirement is the same whether a ride check is desired for one direction or both. The checker requirement for point checks depends on the extent to which a checker can monitor multiple lines and two directions. Because of the width of Los Angeles streets and the heavy bus volumes in many locations, it is common for checkers to monitor only one direction. With the conservative assumption made that only one line will be monitored at a time, point checks require one checker per point per day per direction. To do point checks at three points on 2 days will, therefore, require up to 6 checkers for a peak-direction ride check and up to 12 for ride checks in both directions. A margin should be added to account for analysis costs and complications. Estimates based on point checks will therefore yield a cost savings if the line uses more than 15 buses during the time period of interest or 8 buses if only a peak direction profile is needed. On lines and during time periods when fewer buses are operated, a ride check will be more cost-effective. At times and locations where a checker's safety is of serious concern, ride checks can continue to be the main source of data.

Some objections to using such a methodology can be overcome creatively. For example, if running time data are needed over the entire line, adding point checks at the route endpoints can meet this need. Also, because multiple-point checks are blind to many incidents such as detours and accidents that cause traffic delays, it may be wise to have an occasional ride checker verify whether the running time data collected with multiple point checks are valid. These additional costs must be reckoned with in designing the data collection program, of course.

A preliminary implementation study has been conducted for SCRTD. Validation testing is now under way within the agency to assess the procedure's accuracy on several additional lines. Efforts are also being considered to see whether point-check measurement error can be materially reduced. At the same time, efforts are under way to automate point-check data collection using hand-held devices. If successful, these efforts will make updating a more attractive option. Full implementation, if approved, will require a significant amount of computer programming and documentation to provide the flexibility of dealing with different forms of data input, data error checking, minor routing changes, and so on. Testing on different routes may also be needed to provide more guidance in the selection of checkpoints.

CONCLUSIONS

A methodology for updating ride checks using multiple-point checks has been developed and tested. Its accuracy when using a 1-day, 1-year-old ride check for the seed is not as good as taking a new ride check, but if point checks can be repeated on several days, the estimates can be of comparable accuracy. Ride-check updating can offer an economical way to acquire ride-check information on high-frequency lines, as well as an inexpensive way to get better use out of point-check data that may now be routinely collected. There are significant implementation and research issues still to be resolved before this methodology is adopted as a regular part of a data collection program, but it seems to offer many benefits.

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