Dynamic Optimization Model for Bridge Management Systems

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The systematic procedures of a bridge management system provide bridge managers with tools for making consistent and cost-effective decisions related to maintenance, rehabilitation, and replacement of bridges on a systemwide basis. An optimization model was developed as part of an effort to develop a comprehensive bridge management system for the Indiana Department of Highways (IDOH). The techniques of dynamic programming, integer linear programming, and Markov chain were applied in the model. The use of dynamic programming, in combination with integer linear programming and Markov chain, provides bridge managers with an efficient tool for system optimization and budget allocation in managing bridge systems. The model can be used to plan bridge maintenance, rehabilitation, and replacement activities for a given bridge budget and program period. The application of dynamic programming assures that the results are optimal not only for a program period but also for the subperiods.

The Federal Highway Administration (FHWA) has recently encouraged states to develop comprehensive bridge management systems. Several states, including Pennsylvania, North Carolina, Virginia, Nebraska, and Kansas, have developed relatively comprehensive systems (*1*). All these systems, however, are based on priority ranking techniques for selecting bridge improvement projects; these techniques do not usually guarantee optimal solutions. Mathematical techniques of optimization have not yet been effectively used in bridge management systems.

This paper describes an optimization model developed for a comprehensive bridge management system for the Indiana Department of Highways (IDOH). The model applies dynamic programming and integer linear programming to select projects, while the effectiveness or benefit of a bridge system is maximized subject to the constraints of available budgets over a given program period. Markov chain transition probabilities of bridge conditions are used in the model to predict or update bridge conditions at each stage of the dynamic programming.

The use of dynamic programming, in combination with integer linear programming and Markov chain, makes it possible to manage efficiently a system with hundreds of bridges. The model can be used to plan bridge maintenance, rehabilitation, and replacement activities for given available budgets and program period. The application of dynamic programming assures that the results are optimal not only for a program period but also for the subperiods.

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PERFORMANCE CURVES AND MARKOV PREDICTION MODEL

The performance curves and the Markov chain prediction model of bridge conditions were incorporated into the optimization model. The detailed description of performance curves and the Markov prediction model can be found elsewhere (*2*). The following is a description of the performance analysis that is relevant to the optimization model.

There are about 5,400 state-owned bridges in Indiana. All of these bridges have been inspected every 2 years beginning in 1978. The inspection includes the rating of individual components, such as deck, superstructure, and substructure, as well as of the overall bridge condition. According to the FHWA bridge rating system, condition ratings range from 0 to 9; 9 is the rating of a new bridge (*3*).

The objective of developing performance curves was to find the relationship between condition rating and bridge age. The performance curves of bridge components for concrete and steel bridges on interstate highways, as well as on noninterstate highways, were developed separately. A third-order polynomial regression model was used to obtain performance function, as shown below.

$$Y_i(t) = \beta_0 + \beta_1 t_i + \beta_2 t_i^2 + \beta_3 t_i^3 + \varepsilon_i$$

where $Y_i(t)$ is the condition rating of a bridge at age t, t_i is the bridge age, and ε_i is the error term.

Figure 1 presents an example of the performance curves,

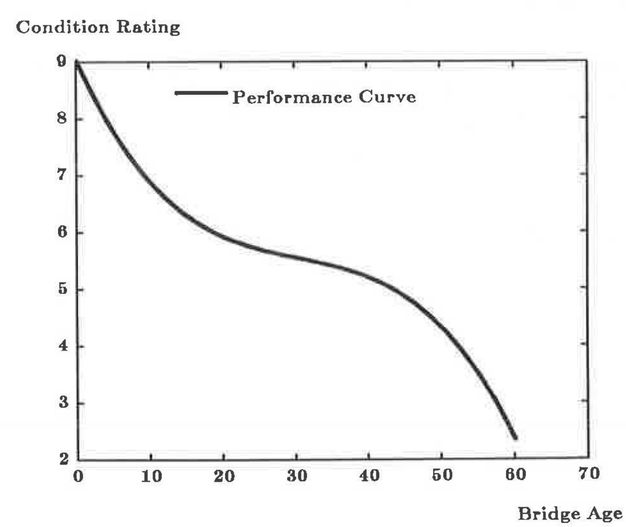

FIGURE 1 Performance curve of deck condition of concrete bridges.

TABLE 1 CORRESPONDENCE OF CONDITION RATINGS, STATES, AND TRANSITION PROBABILITIES

		R=9	R=8	R=7	R=6	R=5	R=4	R=3
		S=1	S=2	S=3	S=4	S=5	S=6	S=7
R=9	S=1	$p_{1,1}$	$p_{1,2}$	$p_{1,3}$	$p_{1,4}$	$p_{1,5}$	$p_{1,6}$	$p_{1,7}$
R=8	S=2	$p_{2,1}$	$p_{2,2}$	$p_{2,3}$	$p_{2,4}$	$p_{2,5}$	$p_{2,6}$	$p_{2,7}$
R=7	S=3	$p_{3,1}$	$p_{3,2}$	$p_{3,3}$	$p_{3,4}$	$p_{3,5}$	$p_{3,6}$	$p_{3,7}$
R=6	S=4	$p_{4,1}$	$p_{4,2}$	$p_{4,3}$	$p_{4,4}$	$p_{4,5}$	$p_{4,6}$	$p_{4,7}$
R=5	S=5	$p_{5,1}$	$p_{5,2}$	$p_{5,3}$	$p_{5,4}$	$p_{5,5}$	$p_{5,6}$	$p_{5,7}$
R=4	S=6	$p_{6,1}$	$p_{6,2}$	$p_{6,3}$	$p_{6,4}$	$p_{6,5}$	$p_{6,6}$	$p_{6,7}$
R=3	S=7	$p_{7,1}$	$p_{7,2}$	$p_{7,3}$	$p_{7,4}$	$p_{7,5}$	$p_{7,6}$	$p_{7,7}$

Note: R = Condition Rating

S = State

$p_{i,j}$ = Transition Probability from State i to State j

representing the deterioration of concrete bridge decks on noninterstate highways over years. The Markov chain, as applied to bridge condition prediction, is based on the concept of defining states in terms of bridge condition ratings and obtaining the probabilities of bridge condition transiting from one state to another. These probabilities are represented in a matrix form called the transition probability matrix or, simply, transition matrix of the Markov chain. Knowing the present state of bridge conditions, or the initial state, the future condition can be predicted through multiplications of the initial state vector and the transition matrix.

Seven bridge condition ratings were defined as seven states, with each condition rating corresponding to one of the states. Thus, condition 9 was defined as state 1, rating 8 as state 2, and so on. Without repair or rehabilitation, the bridge condition rating decreases as the bridge age increases. Therefore, there is a probability of condition transiting from one state, say i, to another state, j, during a 1-year period, which is denoted by $p_{i,j}$. Table 1 shows the correspondence of condition ratings, states, and transition probabilities. Because the lowest recorded rating number in the database was 3, indicating that the bridges are usually repaired or replaced at a rating not less than 3, condition ratings less than 3 were not included in the transition matrix.

An assumption was made that the bridge condition rating would not drop by more than 1 in a single year. Thus, the bridge condition would either stay in its current rating or move to the next lower rating in 1 year. The transition matrix has, therefore, the form:

$$P = \begin{matrix} p(1) & q(1) & 0 & 0 & 0 & 0 & 0 \\ 0 & p(2) & q(2) & 0 & 0 & 0 & 0 \\ 0 & 0 & p(3) & q(3) & 0 & 0 & 0 \\ 0 & 0 & 0 & p(4) & q(4) & 0 & 0 \\ 0 & 0 & 0 & 0 & p(5) & q(5) & 0 \\ 0 & 0 & 0 & 0 & 0 & p(6) & q(6) \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{matrix}$$

where $q(i) = 1 - p(i)$. The correspondence between $p(i)$ and $p_{i,i}$ and between $q(i)$ and $p_{i,i+1}$ can be seen in Table 1. Therefore, $p(1)$ is the transition probability from rating 9 (state 1) to rating 9, and $q(1)$ from rating 9 to rating 8, and so on.

It should be noted that the entry 1 in the last row of the matrix indicates that state 7 (rating 3) is an absorbing state. That is, state 7 does not transit to another state unless the bridge is repaired.

By Markov chain, the state vector for any time, t, $Q_{(t)}$, can be obtained by the multiplication of initial state vector $Q_{(0)}$ and the transition probability matrix P raised to the power of t (*4*):

$$Q_{(1)} = Q_{(0)} * P$$

$$Q_{(2)} = Q_{(1)} * P = Q_{(0)} * P^2$$

$$\vdots$$

$$Q_{(t)} = Q_{(t-1)} * P = Q_{(0)} * P^t$$

Since the transition matrix can be estimated (*2*), the future condition of a bridge at any time t can be predicted.

USE OF INTEGER LINEAR PROGRAMMING

Zero-one integer linear programming (*5*) was used in this model. This technique is a well-defined procedure and can be used to maximize benefit or minimize cost, subject to a number of constraints. In developing the bridge management system, three major rehabilitation activities—deck reconstruction, deck replacement, and bridge replacement—were considered. Each activity of a bridge was defined as a zero-one decision variable. When the value of a decision variable is 1, the corresponding activity is selected; otherwise, routine maintenance is assumed for the bridge. The objective function of the integer linear programming was to maximize the system's effectiveness in each year.

INTRODUCTION TO DYNAMIC PROGRAMMING

Dynamic programming is a particular approach to optimization. It is not a specific algorithm in the sense that Simplex algorithm is a well-defined set of rules for solving a linear programming problem. Dynamic programming is a way of looking at a problem that may contain a large number of interrelated decision variables so that the problem is regarded as if it consisted of a sequence of problems, each requiring the determination of only one (or a few) variables (*6*).

The dynamic programming approach substitutes n single variable problems for solving one n variable problem, so that it usually requires much less computational effort. The principle that makes the transformation of an n variable problem to n single variable problems possible is known as the principle of optimality, which is stated as "every optimal policy consists only of optimal subpolicies" (*6*).

An important advantage of dynamic programming is that it determines absolute (global) maxima or minima rather than relative (local) optima. Also, dynamic programming can easily handle integrality and nonnegativity of decision variables. Furthermore, the principle of optimality assures that dynamic programming results not only in the optimal solution of a problem but also in the optimal solutions of subproblems. For example, for a 10-year program period, dynamic programming gives the optimal project selections for the entire 10-year period, as well as the optimal project selections for any period less than 10 years. These optimal solutions of the subperiods are often of interest to bridge managers.

The key elements of dynamic programming are stages, states, decision, and return (*6*). A bridge system can be considered to progress through a series of consecutive stages; each year is viewed as a stage. At each stage, the system is described by states, such as bridge condition and available budget. Decisions (project selections) are made at each stage by optimizing the returns (system benefit). The bridge conditions are predicted and updated by Markov chain technique, and the system proceeds to the next stage.

A major limitation of dynamic programming is that if there are too many state variables and decision variables, then there are computational problems relating to the storage of information as well as the time it takes to perform the computation.

THE OPTIMIZATION MODEL

The proposed optimization model for the Indiana Bridge Management System requires that it handle about 1,000 bridges, with about 3,000 decision variables, if only three improvement alternatives are considered (deck reconstruction, deck replacement, and bridge replacement). Furthermore, each bridge has a number of associated factors, such as condition rating, traffic safety index, community impact index, and so on. Because of the size of the problem, it was not possible to use only dynamic programming to optimize such a large system. Therefore, integer linear programming was used in combination with dynamic programming to optimize the project selections on a statewide basis.

The dynamic programming divides the federal and state budgets of each year into several possible spending portions, and the integer linear programming selects projects by maximizing yearly system effectiveness subject to different budget spendings. The dynamic programming chooses the optimal spending policy, which maximizes the system effectiveness over a program period by comparing the values of effectiveness of these spendings resulting from the integer linear programming.

In terms of dynamic programming, each year of the program period is a stage. The federal and state budgets are state variables. Each activity of a bridge is a decision variable of the dynamic programming as well as of the integer linear programming. The effectiveness of the entire system is used as the return of the dynamic system.

At each stage, a decision must be made about the optimal solution from stage 1 to the current stage. When a decision is made, a return (or reward) is obtained and the system undergoes a transformation to the next stage. The bridge conditions are updated for the next stage by the Markov transition probabilities obtained through the performance model described by Jiang et al. (*2*). Figure 2 is a flowchart of the optimization model that illustrates the optimization process. For a given program period, the objective of the model is to maximize the effectiveness of the entire system. The formulation of the model, along with the definition of system effectiveness, is discussed as follows.

FORMULATION

The effectiveness of a bridge improvement activity was defined as follows:

$$E_i = ADT_i * \Delta A_i(a) * (1 + C\text{safe}_i) * (1 + C\text{impc}_i)$$

where

E_i = effectiveness gained by bridge i if activity a is chosen;
a = improvement activity;
a = 1, deck reconstruction;
a = 2, deck replacement;
a = 3, bridge replacement;
ADT_i = average daily traffic on bridge i; and

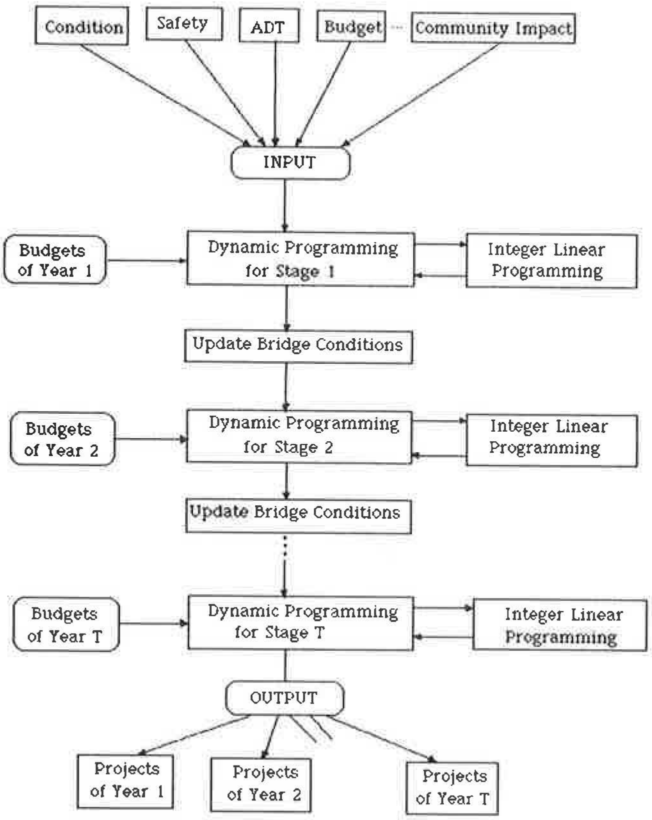

FIGURE 2 Flowchart of the optimization model.

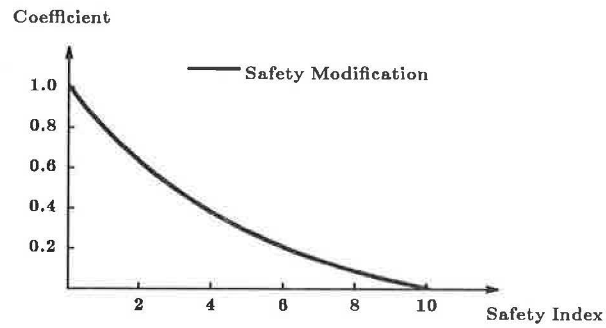

FIGURE 4 Coefficient of safety condition.

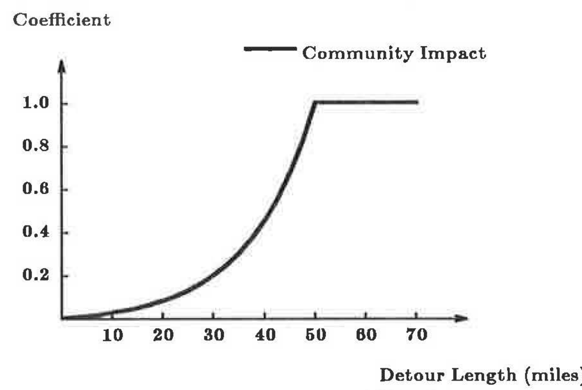

FIGURE 5 Coefficient of community impact.

$\Delta A_i(a) = f_1\Delta b_{i1} + f_2\Delta b_{i2} + f_3\Delta b_{i3}$, representing the average area under performance curves of components of bridge i obtained by activity a, where f_j's are the frequencies of the corresponding component being repaired in activity a, Δb_{ij}'s are the areas of the component gained by activity a, with j = 1, 2, and 3 corresponding to deck, superstructure, and substructure, respectively. Figure 3

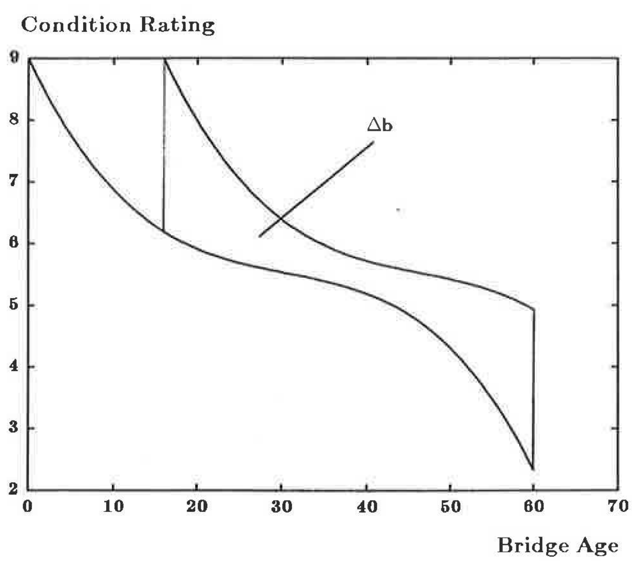

FIGURE 3 Area of performance curve obtained by rehabilitation.

shows an example of Δb_{11}—that is, the area obtained under the performance curve of deck condition.

$Csafe_i$ = transformed coefficient of the traffic safety condition (primarily based on bridge geometrics) of bridge i, as shown in Figure 4. The safety index ranges from 1 to 10, with 10 being the index of "perfect" safety condition.

$Cimpc_i$ = transformed coefficient of community impact of bridge i in terms of detour length, as shown in Figure 5.

Considering that budgets can be carried over from year to year, the mathematical model for maximizing the overall effectiveness of various activities over a program period T was formulated as follows:

$$\max \sum_{t=1}^{T} \left[\sum_{i} \sum_{a} X_{i,t}(a) * E_i * d_i(t) \right] \quad (1)$$

Subject to the following constraints:

Available federal budget,

$$\sum_{t=1}^{T} \left[\sum_{i} \sum_{a} X_{i,t}(a) * c_i(a) * F_i \right] \leq C_{BF} \quad (2)$$

Available state budget,

$$\sum_{t=1}^{T} \left[\sum_{i} \sum_{a} X_{i,t}(a) * c_i(a) * (1 - F_i) \right] \leq C_{BS} \quad (3)$$

One activity cannot be undertaken more than once on one bridge in T years,

$$\sum_{t=1}^{T} X_{i,t}(a) \leq 1 \tag{4}$$

Constraints in Equations 5 to 9 correspond to an integer linear programming:

Maximize system effectiveness of year t,

$$\max \sum_{i} \sum_{a} [X_{i,t}(a) * E_i * d_i(t)] \tag{5}$$

Spending constraint of year t for federal budget,

$$\sum_{i} \sum_{a} [X_{i,t}(a) * c_i(a) * F_i] \leq \eta_{tF} \tag{6}$$

Spending constraint of year t for state budget,

$$\sum_{i} \sum_{a} [X_{i,t}(a) * c_i(a) * (1 - F_i)] \leq \eta_{tS} \tag{7}$$

No more than one activity can be chosen on one bridge in year t,

$$\sum_{a=1}^{3} X_{i,t}(a) \leq 1 \tag{8}$$

Decision variable,

$$X_{i,t} = 0 \text{ or } 1 \tag{9}$$

Update bridge conditions by Markov chain transition probabilities if bridge i is not selected in year t,

$$R_{i,t+1} = R_{i,t} * p_i(R,t) + (R_{i,t} - 1) * (1 - p_i(R,t)) \tag{10}$$

Improvement of bridge condition if bridge i is selected in year t for activity a,

$$R_{i,t+1} = R_{i,t} + \Delta R_i(a) \tag{11}$$

where

$X_{i,t}(a)$ = 1, if bridge i is chosen for activity a;
= 0, otherwise;
$d_i(t)$ = the absolute tangent value on performance curve of bridge i at time t, as shown in Figure 6;
C_{BF} = total available federal budget for the program period;
C_{BS} = total available state budget for the program period;
F_i = federal budget share of bridge i;
$1 - F_i$ = state budget share of bridge i;
$c_i(a)$ = estimated cost of bridge i, activity a;
η_{tF} = spending limit of federal budget in year t;
η_{tS} = spending limit of state budget in year t;
$R_{i,t}$ = condition rating of bridge i in year t;
$p_i(R,t)$ = Markov condition transition probability of bridge i with condition rating R in year t; and
$\Delta R_i(a)$ = condition rating gained by bridge i for activity a.

SOLUTION TECHNIQUE

Equations 1 through 11 constitute a dynamic programming that includes an integer linear programming (Equations 5 to 9) as part of the constraints. The model's objective is to obtain optimal budget allocations and corresponding project selections over T years so that the system effectiveness can be maximized. For example, suppose that T equals 2 years and the available budgets for both year 1 and year 2 are $100 million. If the possible spendings for year 1 are $50, $60, $70, $80, $90, and $100 million, then the possible spendings for year 2 could be $150, $140, $130, $120, $110, and $100 million, respectively. That is, the remaining budget for year 1 can be carried over to year 2; therefore, the spendings of the two years are interrelated. The task of the dynamic programming is to determine the optimal spendings among these possible combinations of spendings (i.e., 50, 150; 60, 140; 70, 130; 80, 120; 90, 110; and 100, 100) and to obtain the corresponding optimal project selections. Similarly, if T is larger than 2, say 10, the model can determine the optimal spendings from year 1 to year 10 and give the corresponding project selections.

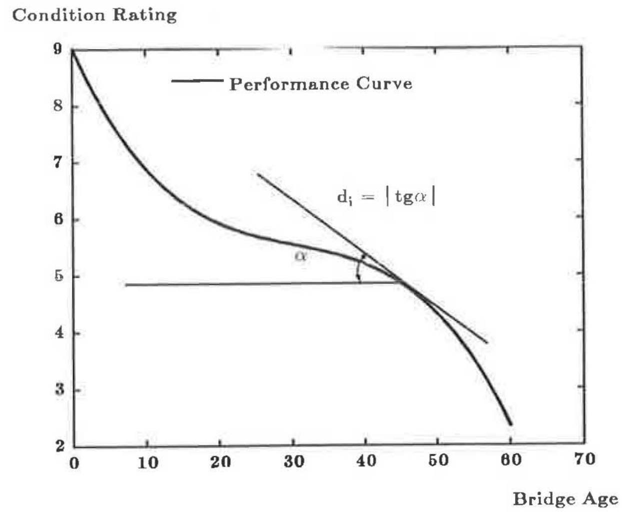

FIGURE 6 Slope of performance curve.

Let us denote the number of spending combinations by N, the number of possible spendings of each year by s, and the program period by T; then N can be expressed by s and T, $N = s^{T-1}$. When T is large, the number of possible spending combinations becomes so large that the search for the optimal path of spendings from year 1 to year T requires great effort and computation time.

Dynamic programming is an efficient technique to search for the optimal path among the combinations of spendings. Rather than examining all the paths, dynamic programming looks at only a small part of these paths. According to the principle of optimality, at each stage the programming finds the optimal subpath up to the current stage, and only this subpath is used to search for the optimal subpath up to the next stage. The paths that do not belong to the optimal subpath are abandoned as the search goes on, which makes the search efficient and saves a great deal of time.

The search for the optimal path can easily be performed by expressing the problem as recurrence relations (*6*). In doing so, Equations 1, 2, and 3 are rewritten as follows,

$$\max \sum_{t=1}^{T} \Phi_t(Y(t)) \tag{12}$$

$$s \cdot t \sum_{t=1}^{T} Y_F(t) \leq C_{BF} \tag{13}$$

$$\sum_{t=1}^{T} Y_S(t) \leq C_{BS} \tag{14}$$

where

$$\Phi_t(Y(t)) = \sum_i \sum_a [X_{i,t}(a) * E_i * d_i(t)]$$

$$Y_F(t) = \sum_i \sum_a [X_{i,t}(a) * c(a) * F_i] \leq C_{BF}$$

$$Y_S(t) = \sum_i \sum_a [X_{i,t}(a) * c(a) * (1 - F_i)] \leq C_{BS}$$

$$Y(t) = Y_F(t) + Y_S(t)$$

The state variable is defined as

$$\lambda_t = \lambda_{t+1} - Y(t + 1) \tag{15}$$

The optimal return function is defined as

$$g_1(\lambda_1) = \max \Phi_1(Y(1)), \qquad 0 \leq Y(1) \leq \lambda_1 \tag{16}$$

$$g_2(\lambda_2) = \max [\Phi_2(Y(2)) + g_1(\lambda_2 - Y(2))], \quad 0 \leq Y(2) \leq \lambda_2 \tag{17}$$

$$g_t(\lambda_t) = \max [\Phi_t(Y(t)) + g_{t-1}(\lambda_t - Y(t))], \quad 0 \leq Y(t) \leq \lambda_t \tag{18}$$

By the recurrence relations of Equations 16, 17, and 18, the dynamic programming process starts at year 1, or stage 1, and $g_1(\lambda_1)$ can be obtained for all the possible spendings of year 1. Then the bridge conditions are updated by Equation 10 or Equation 11 according to the project selections corresponding to $g_1(\lambda_1)$; and $g_2(\lambda_2)$ can be solved based on the information of $g_1(\lambda_1)$ as well as the updated bridge conditions. This forward recursion is performed for every successive year of the program period until $g_T(\lambda_T)$ is obtained; therefore, the optimal spending policy and project selection from year 1 to year T are obtained.

The value of $\Phi_t(Y(t))$ can be obtained by solving the integer linear programming (Equations 5 to 9). The value of the objective function (Equation 5) of the linear programming equals $\Phi_t(Y(t))$ if η_{tT} and η_{tS} of Equations 6 and 7 are substituted by possible spending limitations of year t.

A SAMPLE APPLICATION

A computer program of the optimization model was coded in Fortran 77. XMP package (*7*) was used in the programming to solve the integer linear programming. Branch-and-bound method (*5*) was applied to solve the integer linear programming, which is essentially a direct enumeration technique that excludes from consideration a large number of possible integer combinations; it therefore makes possible the solution of a problem with hundreds of decision variables. The input of the problem includes the following:

1. Condition ratings of bridge components;
2. Bridge age;
3. Bridge type;
4. Highway type;
5. Safety index;
6. Detour length;
7. Average daily traffic;
8. Available federal and state budgets;
9. Federal budget share for bridge projects by highway type;
10. Recommended activity and timing by engineers;
11. Estimated rehabilitation cost; and
12. Program period.

The output of the program is a list of selected bridges and activities and the corresponding federal and state costs for each year of the program period.

To demonstrate the application of the model, a sample problem is presented as follows. Table 2 gives the general information on 20 bridges. It includes a description of each bridge and the activities and timings recommended by bridge inspectors or engineers. A 5-year program period is used (that is, $T = 5$). Suppose the available federal and state budgets for each year are \$2,250,000 and \$460,000, respectively, and the bridges being considered are eligible for a 90 percent federal budget share (F_i) on interstates and an 80 percent federal budget share on noninterstate highways. Taking this information as input, the program yields the output shown in Table 3. The output provides project selections for each year of the program period and the corresponding costs. The plan of optimal spendings from year 1 to year 5, therefore, is given as: \$1,696,000; \$3,636,000; \$1,673,000; \$2,913,000; and \$2,891,000.

CONCLUSIONS

The use of dynamic programming in combination with integer programming and the Markov chain provides bridge managers with an optimization tool for managing bridge systems. The model selects projects by maximizing the effectiveness of the entire system over a given program period subject to budget constraints. Therefore, for any available budget, the model always gives a project selection that maximizes system effectiveness for the given budget. That is, the model always offers optimal solutions to decision makers. The priority ranking methods as used in some bridge management systems, however, do not usually guarantee optimal solutions because they are based solely on the comparison of rankings. In a ranking procedure the following two important ingredients may be missing (*8*): evaluation of interproject trade-offs in selecting projects and selection of strategies that are guaranteed to adhere to existing budget limitations.

The principle of optimality assures that dynamic programming results in the optimal solution not only for the program period T but also for any period less than T. These optimal solutions for the subperiods are important to bridge managers in scheduling bridge activities. Furthermore, these solutions are guaranteed by the principle of optimality to be absolute optima rather than relative optima.

The optimization model has a simple structure and a powerful capability for handling a system with hundreds of bridges. Decision makers can use it to gain maximum return by effectively spending the limited bridge budgets within both short-term and long-term planning horizons.

TABLE 2 INFORMATION PERTAINING TO 20 SAMPLE BRIDGES

Bridge No.	1	2	3	4	5	6	7	8	9	10
Bridge Age	35	30	35	25	40	40	48	51	52	30
Bridge Type	S	S	S	C	S	C	C	S	S	C
Highway Type	N	I	N	N	N	N	N	N	N	I
ADT (x100)	20	90	30	15	45	19	27	38	42	150
Detour Length (miles)	8	14	9	5	10	6	7	11	13	17
Safety Index	5	7	2	4	8	3	1	5	6	7
Deck Condition Rating	6	6	5	6	5	5	5	5	5	6
Superstructure Condition Rating	4	7	5	6	4	5	5	5	3	5
Substructure Condition Rating	5	7	7	7	4	4	5	4	3	5
Recommended Activity	3	1	2	1	3	3	3	3	3	3
Recommended Activity Year	1	2	1	1	1	1	3	2	1	3
Last Year Activity Can Be Deferred	5	6	4	6	4	5	6	4	2	5
Estimated Cost ($1000)	558	150	500	80	1082	542	542	1004	1063	789

Note: S = Steel Bridge C = Concrete Bridge

I = Interstate Highway N = Non-interstate Highway

TABLE 2 (Continued)

Bridge No.	11	12	13	14	15	16	17	18	19	20
Bridge Age	30	35	50	48	45	50	45	35	54	38
Bridge Type	C	S	S	S	C	S	C	C	C	C
Highway Type	N	N	N	N	I	N	N	N	I	N
ADT (x100)	7	30	12	15	133	40	12	14	100	35
Detour Length (miles)	10	5	7	8	14	15	9	6	15	15
Safety Index	2	9	4	5	7	6	2	1	8	5
Deck Condition Rating	6	5	5	5	6	5	6	5	5	6
Superstructure Condition Rating	6	5	3	3	5	4	4	5	3	5
Substructure Condition Rating	6	7	3	3	4	3	4	6	3	5
Recommended Activity	1	2	3	3	3	3	3	2	3	3
Recommended Activity Year	2	2	1	1	2	1	1	1	1	3
Last Year Activity Can Be Deferred	6	5	2	2	5	2	3	5	3	5
Estimated Cost ($1000)	100	400	1102	317	1004	633	589	400	1063	891

Note: S = Steel Bridge C = Concrete Bridge

I = Interstate Highway N = Non-interstate Highway

TABLE 3 PROJECTS SELECTED BY THE OPTIMIZATION MODEL

Year	Bridge Number	Activity	Federal Cost ($1,000)	State Cost ($1,000)	Total Cost ($1,000)
1	16 19	Bridge Replacement Bridge Replacement	1,463	233	1,696
2	2 9 13 14 15	Deck Reconstruction Bridge Replacement Bridge Replacement Bridge Replacement Bridge Replacement	3,024	612	3,636
3	4 8 17	Deck Reconstruction Bridge Replacement Bridge Replacement	1,338	335	1,673
4	3 5 7 10	Deck Replacement Bridge Replacement Bridge Replacement Bridge Replacement	2,409	504	2,913
5	1 6 11 12 18 20	Bridge Replacement Bridge Replacement Deck Reconstruction Deck Replacement Deck Replacement Bridge Replacement	2,313	578	2,891

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