An Analytical Model for Driver Response

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An analytical model using the lognormal probability density function is applied to published driver response time measurements. Close agreement is obtained when this function is fitted to the measured responses of drivers to the onset of the amber signal as they approach signalized intersections. Statistical parameters associated with three sets of measured data are presented. Parametric estimates of 1.1 to 1.2 sec for the median, 1.3 to 1.4 sec for the mean, and 0.55 to 0.75 sec for the standard deviation of the distribution function appear to give the best fit to the experimental values.

The statistical distribution of driver response time is an important consideration in traffic engineering and highway design. AASHTO uses a design reaction time value of 2.5 sec to determine stopping sight distances (*1*). The *Transportation and Traffic Engineering Handbook* (*2*) suggests a value of 1.0 sec for computing the yellow clearance interval at a signalized intersection. Parsonson, in a discussion to a paper by Wortman and Matthias (*3*), indicates that computation of the yellow clearance interval was a controversial issue during the preparation of the Handbook.

It is the purpose of this paper to present an analytical model that estimates the measured statistical distributions of three sets of published response time measurements. The need to estimate different percentile ranges of driver response times has been suggested recently by Shapiro et al. (*4*). They suggest that future editions of *The Manual on Uniform Traffic Control Devices* should include a section on design driver criteria. Design driver response times for braking should be broken down by the 50th, 85th, and 95th percentile categories of the driver population. They also suggest breaking down response times by key driver groups, such as by age or other significant driver characteristics.

Presented in this paper are key percentile estimates for response times associated with three sets of experimental data to which the lognormal probability density function was fitted.

LOGNORMAL PROBABILITY DENSITY MODEL

The lognormal probability density model has been applied to represent driver reaction times measured under anticipatory conditions (*5*). The lognormal function was shown to give an acceptable fit, within the 5 percent level of significance, to reaction time measurements published by Moss and Allen in 1925; Johansson and Rumar in 1971; and by Gazis et al. in 1960 (*5*). In the first two investigations, drivers were forewarned that their responses were to be measured. In the third investigation, drivers were tested involuntarily.

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The lognormal function will be used to model the measured values of the surprise response times of drivers. The equation $f(t)$ of this probability density function is given by

$$f(t) = \frac{1}{\sqrt{2\pi}\,\xi t} \exp\left[-\tfrac{1}{2}\left(\frac{\ln t - \ln \lambda}{\xi}\right)^2\right] \qquad (1)$$

where λ equals the median value of response time. If μ equals the mean value and σ equals the standard deviation, then the measure of dispersion ξ is given by the formula

$$\xi^2 = \ln\left(1 + \frac{\sigma^2}{\mu^2}\right) \qquad (2)$$

The cumulative probability distribution function $F(t)$ is given as

$$F(t) = \int_0^t f(\tau)\, d\tau \qquad (3)$$

The properties of this function are discussed by Ang and Tang (*6*).

FITTING THE MODEL TO MEASURED DATA

In 1983, Wortman and Matthias published the results of measured response times of 839 drivers to the onset of the amber signal at signalized intersections (*3*). This is one of the largest samples of surprise response time measurements available in the literature.

The tests were conducted at six intersections in Arizona. Daytime values were obtained at all six intersections and nighttime response times were recorded at two of the six intersections. All drivers were tested involuntarily. Response time measurements appeared to be a function of the locations tested. For example, the mean values ranged from 1.09 to 1.55 sec and the 85th percentile estimates varied between 1.5 and 2.1 sec at the different locations. For the entire sample, the mean value was 1.3 sec and the standard deviation was 0.60 sec. The 85th percentile value was reported to be 1.8 sec. Using Equations 1 and 2, the model parametric values were determined to be $\lambda = 1.14$ sec and $\xi = 0.439$. The estimates of the statistical distribution from this model are shown in Figure 1 and summarized in Table 1.

The model was fitted also to data provided by Chang et al. (*7*). The 579 drivers in this study also were responding to the

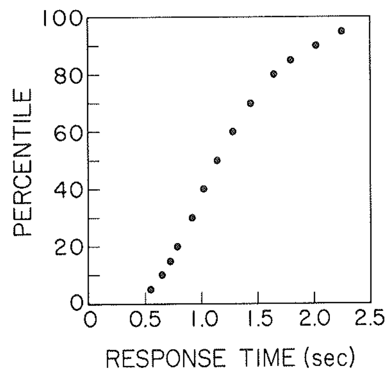

FIGURE 1 Response times from Wortman and Matthias (3).

onset of the amber traffic signal and were unaware that their response times were being measured. These measurements were recorded under diverse circumstances, including daylight and evening conditions, on both wet and dry roadways, and at peak and offpeak hours. The response time measurements therefore reflect the variability in lighting and weather conditions under which the investigation was conducted. The reported distribution times were

Median value λ = 1.1 sec
Mean value μ = 1.3 sec
85th percentile = 1.9 sec
95th percentile = 2.5 sec

The computed parameter estimates for these data were

Standard deviation = 0.736 sec
Dispersion parameter = 0.527

The third set of data to which this model was fitted was published by Sivak et al. (*8*). The response values presented in this study were measured during car-following experiments in which the distance between a lead car and a follower car was varied from approximately one to five car lengths. The responses of the driver of the follower car to the appearance of the brake lights of the lead car were carefully recorded. These measurements were recorded under clear and sunny daytime conditions. The 87 drivers in this third sample therefore were tested under nearly ideal driving conditions. The reported values were 1.38 sec for the mean and 0.56 sec for the standard deviation. The computed parametric estimates were λ = 1.19 sec for the median and ξ = 0.390 for the dispersion parameter.

COMPARISON OF THE PROBABILITY DISTRIBUTIONS

The median, mean, 85th, 90th, and 95th percentile estimates for the three sets of experimentally measured response times are given in Table 2. The median values fall between 1.1 and 1.2 sec, the mean values are between 1.3 and 1.4 sec, and the standard deviations are between 0.55 and 0.75 sec. There is

TABLE 1 DISTRIBUTION OF RESPONSE TIMES (*3*)

Cumulative Distribution	Response Time (Sec)
0.05	0.55
0.10	0.65
0.15	0.72
0.20	0.79
0.30	0.91
0.40	1.02
0.50	1.14
0.60	1.28
0.70	1.44
0.80	1.64
0.85	1.80
0.90	2.01
0.95	2.25

TABLE 2 PERCENTILE RESPONSE TIME DISTRIBUTION ESTIMATES FROM EXPERIMENTAL MEASUREMENTS IN SECONDS

Source	Median	Mean	Standard Deviation	Percentile 85th	90th	95th
Wortman (*3*)	1.14	1.30	0.60	1.80	2.01	2.35
Chang et al. (*7*)	1.10	1.30	0.74	1.90	2.16	2.50
Sivak et al. (*8*)	1.19	1.38	0.56	1.78	1.96	2.26

close agreement between the data of Wortman and Matthias (*3*) and the results of Sivak et al. (*8*) in the 85th to 95th percentile range.

The 85th to 95th percentile response times in Table 2 indicate that the shortest estimates are computed from Sivak's measurements, while the longest estimates are derived from Chang's data. These results can be explained by the fact that Sivak's testing was done under daylight conditions, while Chang's investigation was conducted in the evening and during the day and in both clear and inclement weather.

It should be mentioned that Chang et al. postulated the existence of a driver lag time interval between perception time and brake reaction time (*7*). The median value of driver lag time was estimated to be approximately 0.2 sec, with the 85th percentile estimate between 0.6 and 0.7 sec. The existence of driver lag time is not addressed in this paper. Further experimental study is required to clarify the issue. Sivak (*9*) has written an excellent survey article on driver reaction times in car-following situations.

CONCLUSIONS

It has been shown that the lognormal function can be used to model the probability density function of the surprise response times of drivers. The median driver should respond within 1.2 sec, the mean driver within 1.4 sec, and the 85th percentile driver within 1.9 sec, under surprise braking conditions.

The reaction time of 1.0 sec suggested for use in the amber clearance interval formula (*2*) may be insufficient. A value between 1.2 to 1.8 sec, corresponding to the 85th percentile estimate, is suggested. The possibility of lengthening this design parameter should be investigated.

The AASHTO design value of 2.5 sec may correspond to the response time of the 95th percentile driver. The stopping sight distance design driver assumption is satisfactory at the present time.

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