A Sensitivity Analysis of the Application of Dynamic Programming to Pavement Management Systems

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There has been a concerted research effort to update the prediction and optimization capabilities of the Micro PAVER pavement management system. An approach uniting the Markov probability prediction curves and dynamic programming optimization has been proposed. This paper briefly outlines the background work done on both. It examines the sensitivity of the dynamic programming output to changes in input parameters. Data from three existing databases and a fourth, formulated database are used in the analysis. Output values from dynamic programming are also compared with results obtained from deterministic analysis using best-fit curves for prediction and cost versus condition data. It is concluded that the dynamic programming results are reasonable. A multizone dynamic programming approach was seen to give more consistent results than a one-zone approach. It was found that for life-cycle lengths greater than 15 years, there was little change in optimal decision or cost. Low interest rates favored more expensive, longer-lasting solutions. The minimum Pavement Condition Index (PCI) level specified affected the results appreciably, in general. These results were consistent through all four databases examined. It was concluded that the Markov/dynamic programming approach was functioning satisfactorily, and was suitable and appropriate for use at the microcomputer level.

This paper describes a series of analyses performed on the dynamic programming optimization package developed for the Micro PAVER pavement management system (*1*) in which the sensitivity of the output to changes in input parameters is documented. The outputs are also compared with a "traditional," deterministic life-cycle analysis to determine how well the dynamic programming algorithm is performing. The major advantage in using this dynamic programming approach is that the generation of the output is extremely fast and efficient on a microcomputer. The system is directly applicable to any pavement management system that uses a condition index to indicate overall condition.

This work is part of an overall effort to improve the prediction, optimization, and budget allocation capabilities of Micro PAVER. A necessarily brief background on the Markov process and dynamic programming is included. Further details are available in the literature cited (*2–6*).

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MARKOV PREDICTION MODEL

It is not possible to describe the functioning of the dynamic programming algorithm without first describing the prediction model used. More detailed and comprehensive descriptions have already been published elsewhere (*2,3*). Much of the terminology used in this description also is used in dynamic programming.

The Pavement Condition Index (PCI) range of 0 to 100 is divided into ten states, each state being 10 PCI points wide. A pavement is modeled as beginning its life in near-perfect condition (a PCI of 100 being perfect) and deteriorating as it is subjected to a sequence of duty cycles. A duty cycle is defined as 1 year's imposition of the effects of weather and traffic. A state vector indicates the probability of a pavement section being in each of the ten states in any given year. Figure 1 shows the schematic representation of state, state vector, and duty cycle.

The sections are grouped into families of sections having common characteristics, such as pavement type, traffic, and so forth. All of the sections are grouped into one of the ten states at any age. It is assumed that all of the pavement sections are in state 1 (PCI of 90 to 100) at an age of 0.

It is necessary to identify the Markov probability matrix to model the deterioration process of the pavements. The assumption made is that the pavement condition will not drop by more than one state (10 PCI points) in a single year. Thus, the pavement will either stay in its current state or transit to the next lowest state in 1 year. The probability transition matrix has a diagonal structure as shown in Figure 2.

The state vector for any duty cycle t, $S(t)$, is obtained by multiplying the initial state vector $S(0)$ by the transition matrix P raised to the power of t. Thus

$$S(1) = P * S(0),$$
$$S(2) = P * S(1) = P^2 * S(0), \text{ and}$$
$$S(t) = P * S(t-1) = P^t * S(0).$$

If the transition matrix probabilities can be estimated, the future condition of the road at any duty cycle (age) t can be predicted.

The probabilities are estimated using a nonlinear programming approach. Probability values are found that, when inserted into the Markov chaining process, match the actual (PCI, age) data points as closely as possible. It has been found that this approach can accurately model the pavement deteriora-

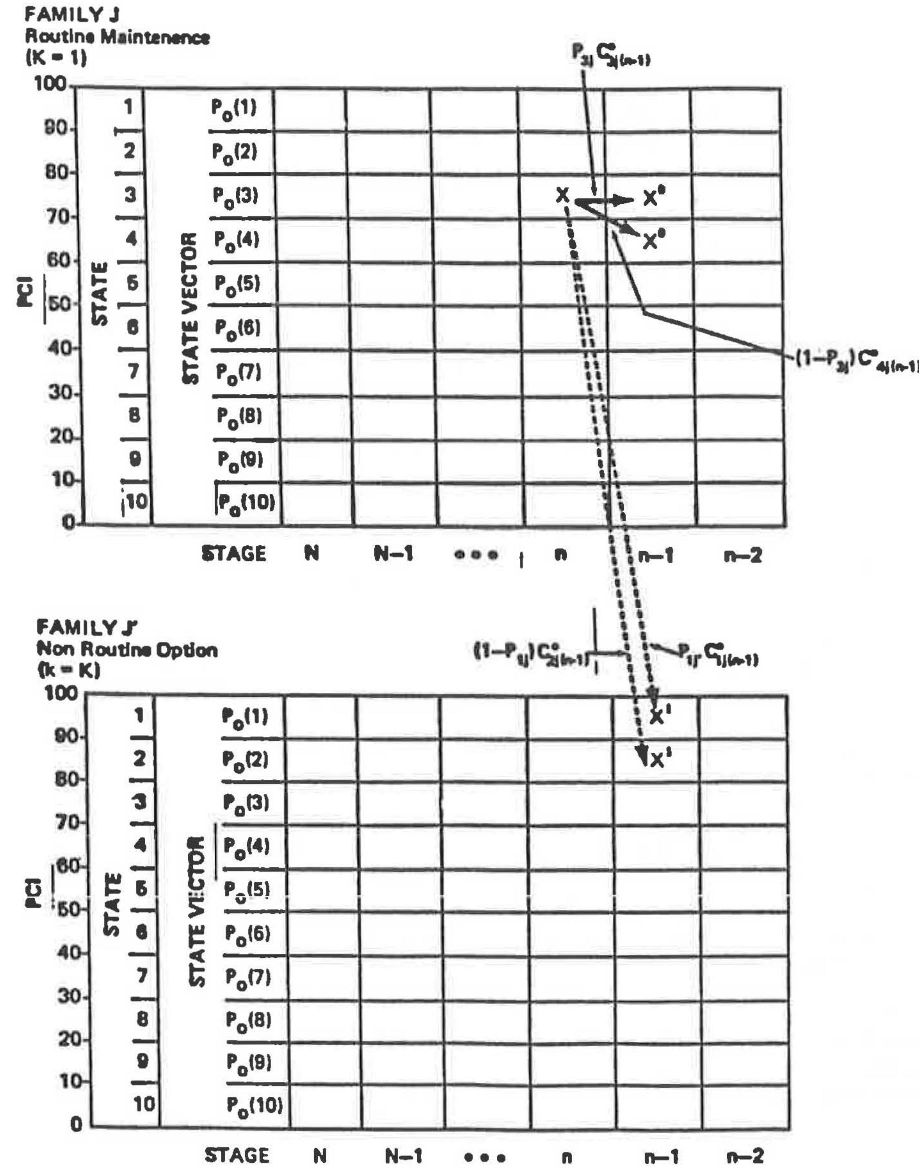

FIGURE 1 Diagram of state, state vector, and duty cycle.

$$P = \begin{bmatrix}
p(1) & 1-p(1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & p(2) & 1-p(2) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & p(3) & 1-p(3) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & p(4) & 1-p(4) & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & p(5) & 1-p(5) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & p(6) & 1-p(6) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & p(7) & 1-p(7) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & p(8) & 1-p(8) & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p(9) & 1-p(9) \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}$$

FIGURE 2 Probability transition matrix structure.

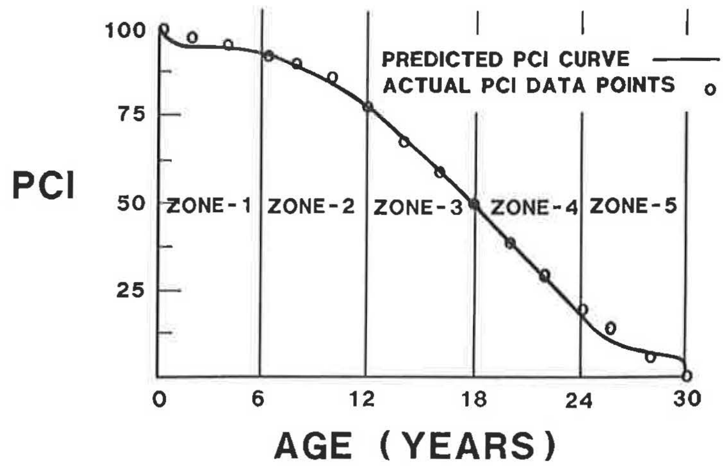

FIGURE 3 Sample Markov prediction curve.

tion over time. A sample output of this program is shown in Figure 3.

INTRODUCTION TO DYNAMIC PROGRAMMING

Dynamic programming is an approach to optimization. It is not based on a well-defined, consistent algorithm such as the simplex algorithm used in linear programming. Instead, it seeks to take a single, complex problem and break it down into a number of smaller constituent problems. It is hoped that the solution of these smaller problems will arrive much faster and require considerably less computational power. If set up properly, dynamic programming is guaranteed to find the global optimal solution(s).

This is a major advantage over almost all classical optimization techniques. A general dynamic programming approach is also extremely robust in that it can handle integrality, negativity, and discreteness of variables very easily. It also, by its nature, produces the solution to all of its constituent subproblems.

The major constraint upon the use of dynamic programming is that the proposed problem to be solved must be able to be formulated in terms of subproblems. If that is possible, however, as in the present case, dynamic programming provides an extremely fast and efficient optimization tool.

Structure of Dynamic Programming

The basic components of dynamic programming are states, stages, decision variables, returns, and transformation or transition functions (*7*). A physical system is considered to progress through a series of consecutive stages. In pavement performance terms, each year is viewed as a stage.

At each stage, the system must be capable of being fully described by the state variables or state vector. In the present case, as described earlier, each state is a 10-PCI bracket for every pavement family, and the condition of the pavement at any year (stage) can be defined as being in one of the ten states.

At each stage, for every possible state, there must be a set of allowable decisions. The decisions being made in the dynamic programming model are *which repair alternative to implement in each state at every stage.*

Finally, there is the transformation or transition function. If a process is in a given state and a feasible decision is made, there must be a function that determines the new state to which the process should move. In general, dynamic programming transformation functions can be deterministic or stochastic. In this particular case, the transition function is defined by the Markov probability matrix derived in the curve-fitting process described earlier and, hence, is a stochastic process.

In summary, the problem set up for this dynamic programming formulation is:

Minimize: Expected cost over a specified life-cycle length subject to keeping all sections above a defined performance standard.

The dynamic programming parameters are:

States: Each bracket of 10 PCI points in a family.
Stages: Each year in the analysis period.
Decision Variables: Which M&R treatment to apply.
Transformation Function: The Markov transition probability matrix defines the transformation.
Return: Expected cost if a particular decision is made in each state at each stage.

Dynamic Programming Output

The output from the dynamic programming program consists of

1. A file containing the optimal maintenance alternative in every year (stage) for every family/state combination;
2. The discounted present worth costs expected to accrue over the life cycle specified if the optimal decisions are implemented;
3. The expected effectiveness accrued as a result of following the optimal decisions calculated for every family/state combination; and

4. The calculated effectiveness/cost ratio for every family/state combination.

INTRODUCTION TO ANALYSIS

The data analysis is made up of two main sections. First, an analysis of the sensitivity of the dynamic programming results to changes in input parameters is performed for four databases. These databases were selected because relevant condition performance and condition/cost data were available.

Second, a comparison of the dynamic programming results with those obtained from a deterministic analysis using best-fit performance curves is performed.

DESCRIPTION OF PARAMETER SETTINGS

The parameters considered in the sensitivity analysis of the dynamic programming solution were

1. Minimum allowable state,
2. Combined zone versus multizone probability values,
3. Effective interest rate, and
4. Length of life-cycle analysis.

The variable levels used in each parameter were as follows:

1. *Minimum allowable state (MAS):* The minimum allowable state, as the name implies, is the worst PCI level that the pavement manager will allow a pavement to reach before it must be repaired. In terms of feasible maintenance alternatives, the implication is that routine maintenance is not allowed in the minimum allowable state; the only feasible alternatives are those that will raise the PCI above the present condition state. It differs from a trigger point in that a nonroutine maintenance action can be taken above the MAS if it is economically advantageous to do so. Thus, with a specified MAS of 5 (minimum PCI of 55), if dynamic programming recommends a surface treatment at a PCI of 75, this nonroutine maintenance action will be taken. Three levels of this parameter were chosen: minimum allowable states of 3, 5, and 7.

2. *Zone approach:* There were two levels for this variable: the combined zone approach and the multizone approach. In the approach used to obtain the Markov probabilities that define the deterioration of condition over time, the life span of each family is divided into a number of zones. A separate set of Markov transition probabilities is obtained for each zone.

The multizone approach uses all of these sets and uses all of the transition probabilities within each set to define the transition from one state to another as a function of time. The combined zone approach identifies the states in which most of the data points of that zone are located. The state transition probabilities corresponding to those states *only* are chosen. Thus, for example, if the majority of the data points in zone 2 are located in the PCI range of 70 to 90, the combined zone approach will take the transition probabilities corresponding only to states 2 and 3 (PCIs of 80 to 90 and 70 to 80, respectively). In this way, a single representative transition probability matrix is compiled for use in the dynamic programming algorithm.

3. *Interest rate:* Two levels of effective interest rates were used in the analysis: 5 percent and 15 percent. The interest rate is used to discount future expenditures to present worth costs, the most equitable basis of comparison for strategies having different expenditures at differing times.

4. *Life-cycle analysis period:* Three analysis periods were investigated: 5, 15, and 25 years.

AVAILABLE DATABASES

Four databases were prepared for use by the dynamic programming programs:

1. Fort Eustis,
2. Tulsa,
3. Great Lakes Naval Center, and
4. A sample formulated database.

Information on cost and performance for the three actual databases was obtained from the CERL report, "The Relationship of Pavement Maintenance Costs to the Pavement Condition Index" (*8*). This report contains detailed cost information and best-fit performance curves for five locations. The three selected have the most complete data and more maintenance alternatives from which to choose.

The formulated database has hypothetical performance and cost relationships. Some of these relationships were chosen specifically to attempt to show how optimal alternatives may change depending upon state, interest rates, life-cycle length, and other factors.

The maintenance alternatives available for each database are shown in Table 1. Sample best-fit constrained least-squares performance curves are shown in Figures 4 and 5 for the Great Lakes base. Markov probability values were obtained to model these curves. Examples are shown in Figures 6 and 7. Some sample PCI/cost relationships are shown in Figures 8 through 10 for both initial cost and routine maintenance cost. In the CERL report, cost values are given for every 20-PCI-point bracket. These values were taken to be centered at the midpoint of the bracket, and a best-fit curve was obtained for the entire PCI range using these points.

PROCEDURE UTILIZED FOR SENSITIVITY ANALYSIS

The basic procedure used follows. For each of the four databases, 12 dynamic programming runs were performed. (This number is made up of two interest rate levels times three minimum allowable state levels times two zone approaches used in estimating Markov probability values.) A life-cycle analysis period of 25 years was always specified. Because of the nature of the dynamic programming formulation, the 25-year solution set also contains the optimal solutions for every year fewer than 25 years; thus the 5- and 15-year optimal decisions and costs are readily obtainable. Forty-eight dynamic programming runs were performed.

TABLE 1 MAINTENANCE ALTERNATIVES FOR EACH DATABASE

	FORT EUSTIS	TULSA	GREAT LAKES	SAMPLE DATA
ROUTINE MAINTENANCE	YES	YES	YES	YES
SURFACE TREATMENT	YES	YES	YES	YES
THIN OVERLAY	YES	YES	YES	YES
THICK OVERLAY	YES	YES	YES	YES
RECONSTRUCTION	NO	YES	NO	YES

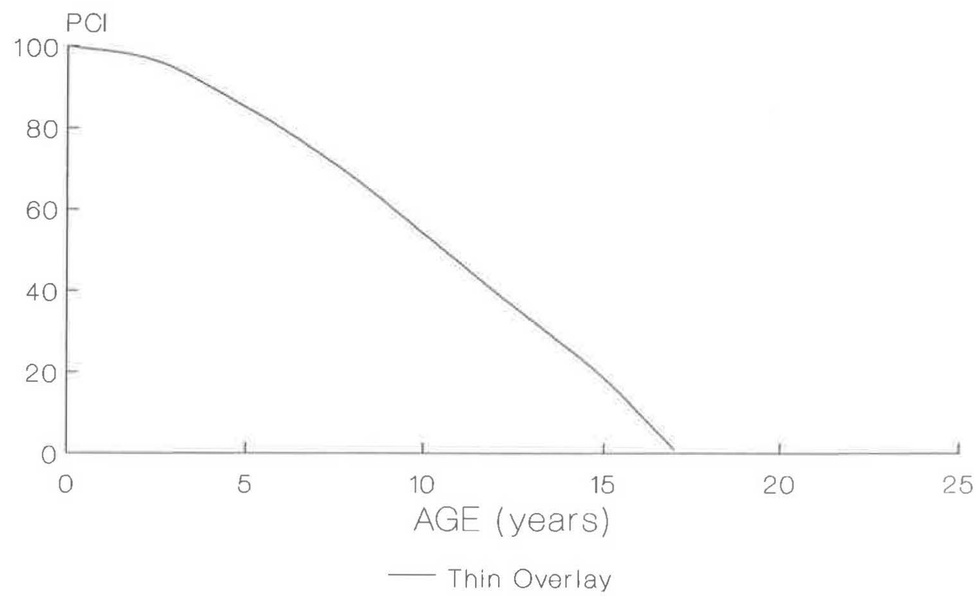

FIGURE 4 Thin overlay at Great Lakes.

It is not possible to obtain the variance of the expected cost from dynamic programming. Consequently, a random simulation was performed after each dynamic programming run. The purpose of this was to obtain a mean estimated cost and variance for each family/state combination. The estimated cost can be compared with the expected cost obtained from dynamic programming. These values will not usually be the same but should be reasonably close in magnitude. Twenty-five simulations were performed for each family/state combination. Thus, the final products for all such combinations were

1. Optimal decisions from dynamic programming,
2. Expected costs from dynamic programming,
3. Estimated mean cost from simulation runs, and
4. Estimated variance of the mean cost from simulation runs.

All of the data were entered into data spreadsheets, one for each database. This was done for ease of data manipulation. Confidence limits were drawn about the mean cost using the estimated variance. An example from the Great Lakes database is shown in Table 2. It can be seen that, in general, these limits are reasonably tight, especially given that a 95 percent confidence level is used. In almost all cases, the

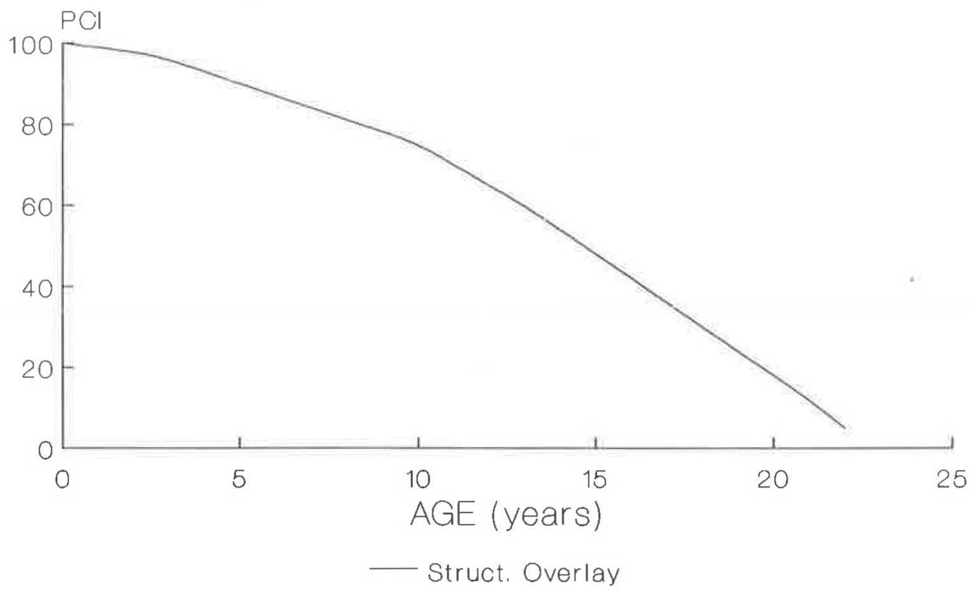

FIGURE 5 Structural overlay at Great Lakes.

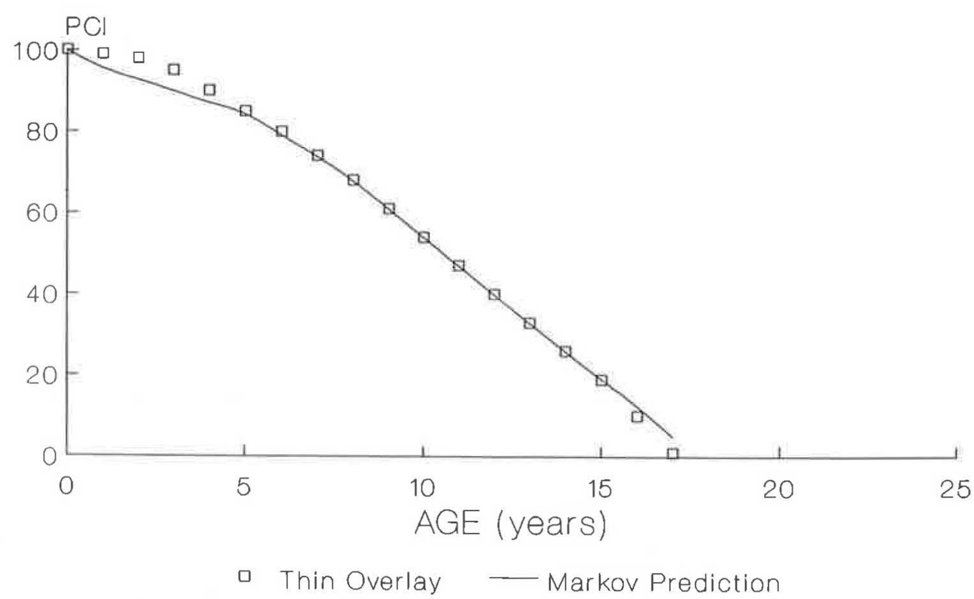

FIGURE 6 Markov prediction for thin overlay.

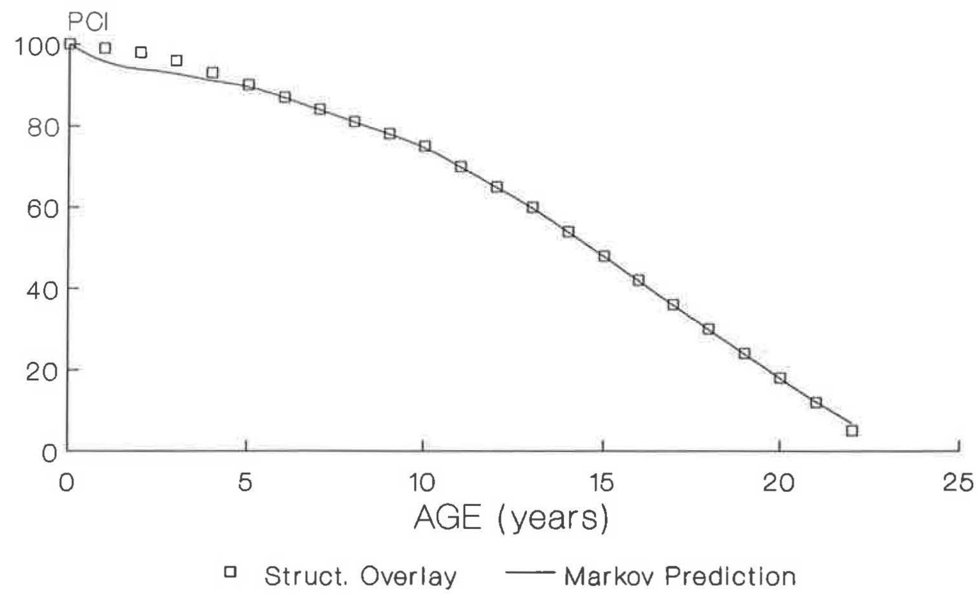

FIGURE 7 Markov prediction for structural overlay.

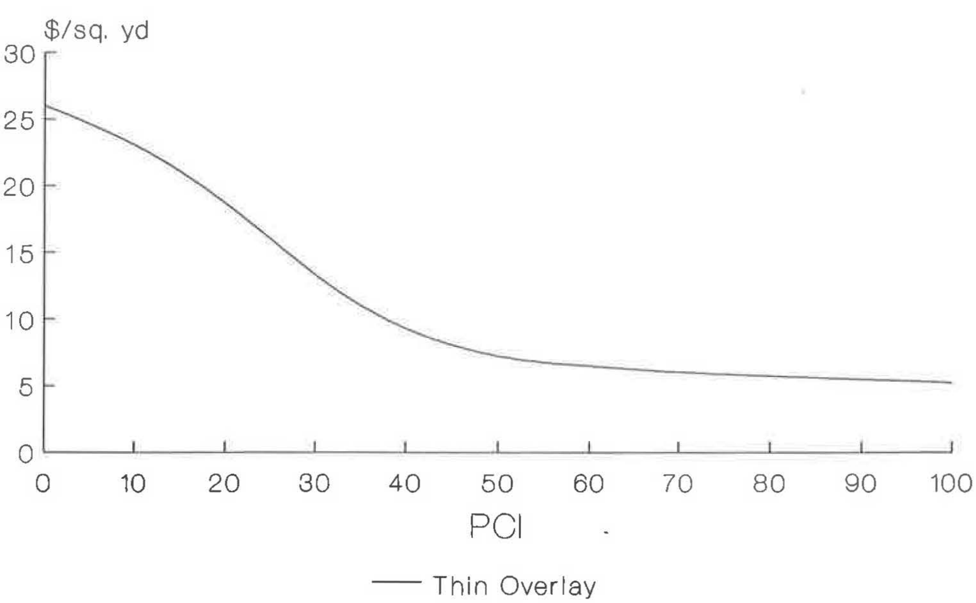

FIGURE 8 Initial cost of thin overlay.

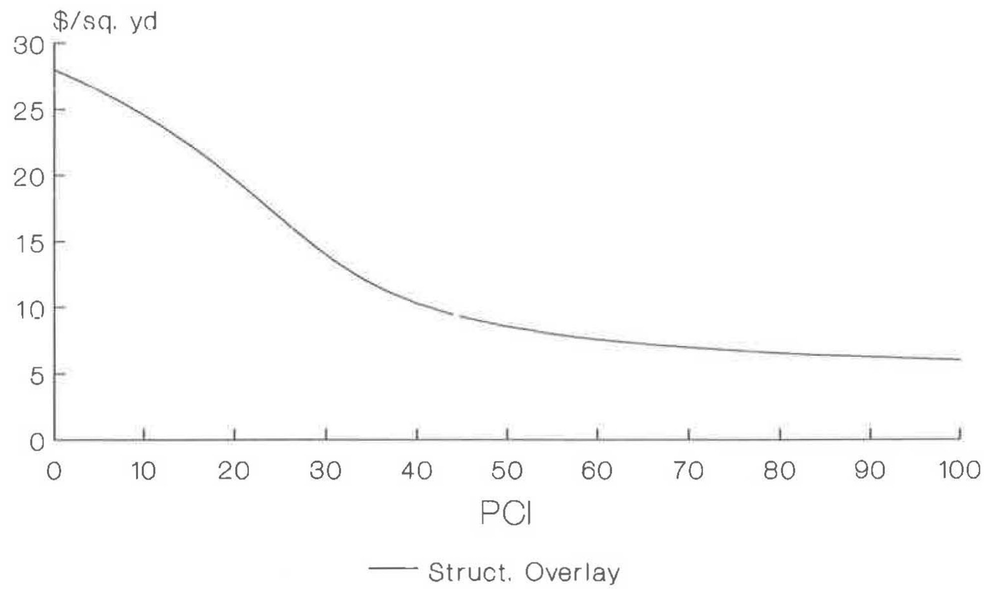

FIGURE 9 Initial cost of structural overlay.

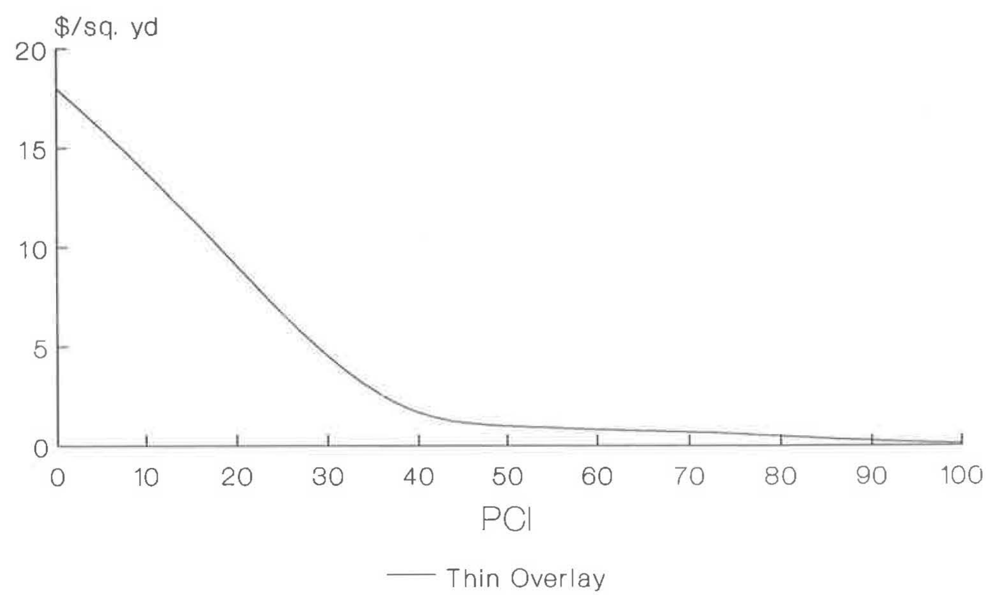

FIGURE 10 Maintenance cost of thin overlay.

TABLE 2 GREAT LAKES EFFECTIVENESS/COST RATIOS

INTEREST RATE: 5%

FAMILY	STATE	STRAT. A	STRAT. B	STRAT. C	STRAT. D	COMB. ZONE	MULTI ZONE
1	1	121.1	118.99	109.08	112.26	200.2	160.6
1	4	81.8	77.99	65.08	87.62	111.8	90.9
1	7	67.46	62.27	59.59	66.67	89.6	71.3
2	7	64.13	59.32	56.71	63.43	149.2	121.5
3	1	115.49	109.68	95.35	106.16	89.6	71.3
3	4	78.15	78.06	64.24	88.57	117.5	90.9
4	1	141.57	113.89	109.62	109.38	252.8	163.3

INTEREST RATE: 15%

FAMILY	STATE	STRAT. A	STRAT. B	STRAT. C	STRAT. D	COMB. ZONE	MULTI ZONE
1	7	93.98	90.64	83.81	91.99	106	94.7
2	1	243.25	219.29	172.59	225.48	294.8	258.3
2	7	87.63	84.52	78.23	85.93	106	94.7
3	1	266.59	259.9	216.55	249.53	326.2	297.6
3	7	92.18	88.91	90.32	82.27	106	94.7

estimated mean cost from simulation falls within a 95 percent confidence bracket around the dynamic programming mean. This is encouraging and indicates that the costs obtained are reasonable and the programs are functioning correctly.

For the purposes of analysis, the data were extracted from the database in a number of structured ways. As there were four parameters of interest, data were extracted so that the effect of changing the parameter level of one variable while holding the others constant could be examined.

SUMMARY OF RESULTS

A brief description of the trends seen in all four databases for each of the input parameters follows.

Interest Rates

The predominant observed trend was that low interest rates tended to favor the alternatives with more expensive initial costs and lower subsequent maintenance costs. This pattern is consistent with basic economic theory. There was an interaction between interest rate and time. As the analysis period was increased, differences between costs using the 5 percent and 15 percent interest rates became more substantial, both in absolute terms and in percentage terms for any family/state combination. This pattern was observed regardless of the minimum allowable state (MAS) specified.

Minimum Allowable State

Costs decreased as the MAS increased. This is again in line with the expected pattern, as specifying an MAS is basically equivalent to putting a constraint on the feasible optimal decisions; and the greater the MAS, the less confining the constraint is. Consequently, the optimal costs would be expected to decrease as the MAS is increased. For any family/state combination, the cost difference between an MAS of 3 and one of 5 was greater than the difference between an MAS of 5 and one of 7. Consequently, it was concluded that the optimal solutions were reaching a steady-state level at about an MAS of 5; and further loosening of the constraint to an MAS of 7 had little further influence.

Varying Zone Analysis

Few consistent patterns were obvious in comparing the optimal decisions and costs for the two zone approaches specified. Generally, it was found that the 5 percent interest rate caused differences in optimal decisions between the two approaches much more than the 15 percent rate did. There were no consistent trends other than this. For any given family/state combination, the optimal decisions and costs varied considerably from database to database. Sometimes the combined zone approach gave higher costs; other times the multizone costs were higher. The analysis later in this paper comparing the dynamic programming results to the deterministic results was used to determine which approach was more realistic.

Varying Life-Cycle Length

The present worth costs always increased with increasing life-cycle length, as expected. In general, when the present worth costs were expressed as equivalent uniform annual costs (EUACs), the EUAC decreased with increased life-cycle length. This was believed to be because the costlier but longer-lasting M&R alternatives were getting an insufficient length of time to "justify" their cost over the 5-year life-cycle length.

Occasionally, this pattern was reversed in states 1 and 2 when the EUACs for the 5-year life cycle were lowest. Routine maintenance was always the optimal decision in those states when this pattern was observed, and it is believed that this was reflecting the fact that routine maintenance is a short-lived activity that does not require a lengthy life cycle to justify its expenditure.

It was also observed that there was much less of a drop in EUAC from 15 to 25 years than from 5 to 15 years. Again, this was believed to be because the life cycles of most of the M&R alternatives were greater than 5 years but shorter than 25 years. Thus, the 5-year life cycle was not long enough to "prove" the worth of the costlier alternatives, while life-cycle lengths greater than 15 years basically reproduced a repeat of the 15-year optimal decision cycle with little or no further reduction in EUAC.

COMPARISON WITH DETERMINISTIC RESULTS

The dynamic programming algorithm has as its output the optimal maintenance decision to make for every state of every family in each year of the life cycle considered. The costs and benefits associated with this decision are also produced. These results are obtained on a probabilistic basis and represent the mean minimum cost in each case.

Certain simplifying assumptions are made in defining the dynamic programming setup for network-level optimization. The major assumption is in the state definition concept that essentially assumes that all sections in each 10-PCI-point bracket of each family behave in the same way. The Markov assumption hypothesizes that the performance curve for any family can be represented through the Markov transition probability matrix values. Information on both initial cost and routine maintenance cost is used on a state basis, again assuming that all family sections in each 10-PCI-point bracket will have the same maintenance costs.

As a result of these simplified assumptions, there may be some doubt as to the veracity of the "optimal" decisions chosen. To obtain some idea of how good the dynamic programming solutions are, it is necessary to compare the solutions with those obtained from a different analysis of the available data. The alternative analysis used is the "strategy" algorithm, which contains PCI versus age and PCI versus dollar curves into a life-cycle cost analysis. Figure 11 shows the sample strategies considered.

By experimenting with different strategies for a given life cycle and interest rate, it is possible to come close to the optimal combination of alternatives. This program's approach is deterministic, assuming that future condition can be predicted precisely. Thus, it is not connected theoretically or practically with the dynamic programming approach. By comparing the outputs of these programs, it should be possible to

1. Confirm or reject the optimality of the dynamic programming solution,
2. Confirm or reject the validity of the cost and cost/benefit ratios output by dynamic programming (the magnitude of these values should be comparable for both programs), and
3. Investigate the sensitivity of the dynamic programming solution to changes in input values.

ANALYSIS APPROACH USED

It was not realistic to attempt to run the deterministic cost analysis package on every combination of parameter variables for every database, as the number of such combinations is much too large. It was decided to specify certain variable-level combinations and then randomly choose a number of these combinations for analysis.

It was decided to fix the life-cycle analysis length at 25 years, as it was believed that this would yield the most valuable insights into the behavior of the one-zone and multizone dynamic programming approaches. It was decided to examine the results using both the 5 percent and 15 percent effective interest rates. It was further decided that all four families in each database should be candidates for analysis and that all four databases should be examined. Also, it was decided to limit the initial candidate starting states to states 1, 4, and 7.

Thus, for each database, the potential number of combinations of parameters for deterministic analysis is

$$3 \text{ states} * 2 \text{ interest rates} * 4 \text{ families} = 24 \text{ combinations}$$

It was anticipated that there would be four trial strategies run deterministically for each such combination, leading to almost 100 runs for each of the four databases. It was decided to select randomly 12 combinations from the 24 to reduce the number of required runs to a manageable figure.

After random selection, the list of combinations selected was as follows:

1. Family 1, state 1, 5 percent interest rate;
2. Family 1, state 4, 5 percent interest rate;
3. Family 1, state 7, 5 percent interest rate;
4. Family 2, state 7, 5 percent interest rate;
5. Family 3, state 1, 5 percent interest rate;
6. Family 3, state 4, 5 percent interest rate;
7. Family 4, state 1, 5 percent interest rate;
8. Family 1, state 7, 15 percent interest rate;
9. Family 2, state 1, 15 percent interest rate;
10. Family 2, state 7, 15 percent interest rate;
11. Family 3, state 1, 15 percent interest rate; and
12. Family 3, state 7, 15 percent interest rate.

ANALYSIS OF DETERMINISTIC RESULTS

The deterministic analysis cost package was run for each of the family/state combinations chosen in each database. The results, the present worth cost and total effectiveness, were entered on spreadsheets for ease of analysis.

The effectiveness/cost ratio for each of the strategies chosen was tabulated with those predicted by the dynamic programming approaches. These results are given in Tables 3 to 6. The remainder of this analysis is concerned with the examination of these tables in an effort to determine the reason-

STATE 1 OF EACH FAMILY:

A: SURFACE TREATMENT WHEN PCI = 70, REPEAT SURFACE TREATMENT WHENEVER PCI FALLS TO 70.

B: THIN OVERLAY WHEN PCI = 50, REPEAT THIN OVERLAY WHENEVER PCI FALLS TO 50.

C: STRUCTURAL OVERLAY WHEN PCI = 40, REPEAT STRUCTURAL OVERLAY WHENEVER PCI FALLS TO 40.

D: SURFACE TREATMENT WHEN PCI = 70, THEN APPLY THIN OVERLAY WHEN PCI = 50, REPEAT SURFACE TREATMENT WHEN PCI = 70.

STATE 4 OF EACH FAMILY:

A: SURFACE TREATMENT WHEN PCI = 55, REPEAT SURFACE TREATMENT WHENEVER PCI FALLS TO 70.

B: THIN OVERLAY WHEN PCI = 50, REPEAT THIN OVERLAY WHENEVER PCI FALLS TO 50.

C: STRUCTURAL OVERLAY WHEN PCI = 40, REPEAT STRUCTURAL OVERLAY WHENEVER PCI FALLS TO 40.

D: SURFACE TREATMENT WHEN PCI = 65, THEN APPLY THIN OVERLAY WHEN PCI = 50, REPEAT SURFACE TREATMENT WHEN PCI = 70.

STATE 7 OF EACH FAMILY:

A: SURFACE TREATMENT WHEN PCI = 35, REPEAT SURFACE TREATMENT WHENEVER PCI FALLS TO 70.

B: THIN OVERLAY WHEN PCI = 35, THEN APPLY THIN OVERLAY WHENEVER PCI FALLS TO 50.

C: STRUCTURAL OVERLAY WHEN PCI = 35, REPEAT STRUCTURAL OVERLAY WHENEVER PCI FALLS TO 40.

D: SURFACE TREATMENT WHEN PCI = 35, REPEAT SURFACE TREATMENT WHENEVER PCI = 70.

FIGURE 11 Symbol key for database LCCST comparisons.

ableness of the dynamic programming results vis-à-vis the deterministic results, and to establish which of the dynamic programming zone approaches gives more reasonable results.

EFFECTIVENESS/COST RATIO COMPARISONS

In general, the effectiveness/cost (E/C) ratios predicted by the deterministic approach and those given by dynamic programming were certainly comparable in magnitude. It was not anticipated that the figures would be exactly the same since the dynamic programming approach uses cost figures on a state-by-state basis, assigning the same cost for each PCI in each 10-PCI-point bracket. The deterministic approach, on the other hand, is more detailed and can calculate the cost for any PCI point between 0 and 100.

It was also anticipated that, in general, the dynamic programming E/C ratios would be higher as these solutions should be global optimal in comparison with the strategies selected for the deterministic analysis. In fact, in many cases, it was found that the best strategy in the deterministic analysis mirrored the dynamic programming strategy almost completely, leading to almost identical E/C ratios under both analyses.

There are a total of 48 family/state combinations to be analyzed, spread over the four databases of interest. In 37 of the 48 cases, the multizone dynamic programming approach was close to the best deterministic E/C result. In 6 cases, the one-zone and multizone results were equally close to the deterministic results, and in 5 cases, the one-zone dynamic programming results were closer. Of the 37 cases in which multizone was closer, 34 of the E/C ratios were higher than the deterministic values, an expected result in the context of global and local optima.

In the Great Lakes analysis, the multizone results were closer to the deterministic results in 11 of the 12 comparisons. The magnitudes of the results are generally very close, especially for the multizone results. The only obvious difference is in family 2, state 7, under a 5 percent interest rate, where

TABLE 3 TEST EFFECTIVENESS/COST RATIOS

INTEREST RATE: 5%

FAMILY	STATE	STRAT. A	STRAT. B	STRAT. C	STRAT. D	COMB. ZONE	MULTI ZONE
1	1	140.87	145.7	119.98	144.64	242.5	229.8
1	4	67.29	68.8	58.52	63.6	136.2	104.7
1	7	79.35	66.73	86.19	111.68	113.2	90.7
2	7	78.55	66.18	85.54	110.81	113.2	90.7
3	1	120.17	76.58	98.91	95.42	218.3	139.8
3	4	64.84	63.69	59	76.26	136.2	104.7
4	1	167.1	112.63	125.51	134.17	195	178.9

INTEREST RATE: 15%

FAMILY	STATE	STRAT. A	STRAT. B	STRAT. C	STRAT. D	COMB. ZONE	MULTI ZONE
1	7	127.77	113.61	140.99	164.65	158.3	132.3
2	1	243.83	163.27	183.6	198.44	426.5	270.43
2	7	125.72	111.89	139.27	162.56	158.3	132.3
3	1	283.63	203.27	223	239.44	466.6	325.2
3	7	124.57	110.93	138.26	161.34	158.3	132.3

TABLE 4 FORT EUSTIS EFFECTIVENESS/COST RATIOS

INTEREST RATE: 5%

FAMILY	STATE	STRAT. A	STRAT. B	STRAT. C	STRAT. D	COMB. ZONE	MULTI ZONE
1	1	379.32	225.49	150.92	241.33	3066.53	922.3
1	4	126.09	116.88	91.1	127.84	260.7	231.3
1	7	105.19	97.53	92.48	67.92	142.5	134.3
2	7	109.38	104.11	70.07	95.81	111.3	106.6
3	1	257.72	265.69	178.16	240.72	1307	786.3
3	4	144.55	134.83	103.99	133.55	231.3	209
4	1	360.17	256.66	170.58	244.9	2479.9	1020.9

INTEREST RATE: 15%

FAMILY	STATE	STRAT. A	STRAT. B	STRAT. C	STRAT. D	COMB. ZONE	MULTI ZONE
1	7	128.33	126.81	91.83	110.65	151.6	148.3
2	1	563.68	477.1	538.6	407.36	1280.4	860.5
2	7	134.77	133.18	95.79	115.45	116.4	114.9
3	1	704.55	734.48	609.26	700	3324.4	1908.9
3	7	131.4	129.84	93.72	112.94	116.4	114.9

the one-zone and multizone analyses have substantially higher E/C ratios.

In the Fort Eustis analysis, the same pattern is again evident. However, there are more family/state combinations where the dynamic programming E/C results are substantially greater than the deterministic results. Generally, these differences occur in state 1 of the various families, especially with a 5 percent interest rate being used. An examination of present worth costs shows that the dynamic programming costs are much lower than those given by the deterministic analysis.

It is believed that the costs projected through dynamic programming in this case are, in fact, too low. The value is low because the Markov transition probability value selected by the Markov program for state 1 is well above 0.9, thus encouraging a pavement section to be retained in state 1 for longer than it would normally be expected to remain. Naturally, this results in a lower life-cycle cost. Modifications to the Markov program since this analysis was performed have resulted in more reasonable state 1 values being chosen consistently.

The test and Tulsa databases show patterns similar to those

TABLE 5 TULSA EFFECTIVENESS/COST RATIOS

INTEREST RATE: 5%

FAMILY	STATE	STRAT. A	STRAT. B	STRAT. C	STRAT. D	COMB. ZONE	MULTI ZONE
1	1	232.38	185.66	115.16	178.25	472	253.5
1	4	159.23	124.08	87.5	120.46	294	156.3
1	7	122.84	103.25	66.83	108.79	159.1	114.3
2	7	119.77	101.09	65.77	106.6	121.1	92.2
3	1	221.53	140.99	105.76	167.25	484.4	240.9
3	4	149.08	124.07	83.27	120.46	280	160.3
4	1	234.77	174.13	114.05	180.2	472.7	259.2

INTEREST RATE: 15%

FAMILY	STATE	STRAT. A	STRAT. B	STRAT. C	STRAT. D	COMB. ZONE	MULTI ZONE
1	7	165.61	150.47	106.35	140.46	187.6	139.6
2	1	490.99	318.07	230.64	393.68	969.8	456.9
2	7	160.31	145.93	103.69	136.66	134.9	103.9
3	1	483.76	341.69	263.54	387.78	1073.5	528.3
3	7	162.8	148.27	105.06	138.62	139.1	106.9

TABLE 6 COMPARISON OF DYNAMIC PROGRAMMING MEAN WITH SIMULATION MEAN

FAMILY	STATE	DYN. PROG. MEAN	SIMULATION MEAN	UPPER BOUND	LOWER BOUND
1	1	7.7	7.02	8.39	7.01
	2	9.65	10.43	10.65	8.65
	3	12.42	11.55	12.95	11.89
	4	13.01	12.94	13.76	12.26
	5	14.15	14.32	14.96	13.34
	6	15.2	15.48	15.96	14.44
	7	16.7	16.95	17.41	15.99
2	1	9.98	9.74	10.57	9.39
	2	10.92	11.09	11.83	10.01
	3	11.97	11.83	12.57	11.37
	4	13.01	13.18	13.68	12.34
	5	14.15	14.86	14.92	13.38
	6	15.2	15.3	15.89	14.51
	7	16.7	16.62	17.33	16.07
3	1	9.13	8.87	9.8	8.46
	2	10.61	10.49	11.29	9.93
	3	11.86	11.39	12.61	11.11
	4	13.01	13.12	13.68	12.34
	5	14.15	14.34	14.76	13.54
	6	15.2	15.48	15.92	14.48
	7	16.7	17.03	17.41	15.99
4	1	7.66	7.44	8.35	6.97
	2	11.22	11.36	12.01	10.43
	3	12.41	12.28	13.03	11.79
	4	13.23	13.2	13.83	12.63
	5	14.38	14.49	15.16	13.6
	6	15.23	15.56	16.18	14.28
	7	16.18	15.85	16.81	15.55

observed in Great Lakes. In general, the E/C values for the optimal deterministic strategy and the multizone dynamic programming approach are very close, especially considering the differences in exactness of cost estimation and PCI prediction. In the test database, for a 15 percent discount rate, three of the five family/state combinations result in ties between the multizone and one-zone approaches in terms of closeness to the optimal deterministic result.

It is interesting to note that the structural overlay option is found to be most cost-effective in the deterministic approach, as generally the thin overlay option is favored in state 7 for most of the databases. In summary, the E/C ratios predicted by the multizone dynamic programming approach are in good agreement with those predicted by a deterministic approach.

SUMMARY

This paper describes the results of an experimental analysis performed on four databases where condition and cost data were available. The effect of varying parameter-level inputs for these databases was investigated and reported. The effect was measured both in terms of change and expected cost. In general, the anticipated changes were actually reflected in the outputs. How the formulation is sensitive to changes in input values and which parameters in particular affected the results in a substantial way were revealed.

In general, the longer life-cycle analysis periods tended to favor more costly initial alternatives with higher initial cost and greater life expectancy. Lower interest rates also tended to favor these alternatives. Changes in the minimum allowable state produced much greater differences as the state was lowered from 3 to 5 than when it was lowered from 5 to 7. The effect of using the Markov probability values in two different ways in the dynamic programming analysis was also considered.

This was also seen in the latter part of the paper, where the outputs for the two zoning approaches were compared with the results from a deterministic analysis. Based on these comparisons, it was concluded that the approach using all the Markov values in every zone was superior to the alternative of using the Markov values for each state from the zone that contains the deterioration curve in that state. It was also concluded that the dynamic programming and deterministic analysis results were certainly of the same magnitude and selected similar optimal maintenance decisions.

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