# Research Program for Predicting the Frictional Characteristics of Seal-Coat Pavement Surfaces

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Numerous factors, including aggregate characteristics, construction variables, traffic volume, and environment, are believed to affect the frictional performance of highway pavements. In this research, the effects of these factors on the field frictional resistance of seal-coat surfaces are investigated. The investigation reported here involved establishing seal-coat test sections in different climatic regions in the state of Texas with various aggregate types and sources and under different traffic volumes. Samples of the aggregates used were examined in the laboratory to determine their physical properties, polish and wear characteristics, resistance to weathering, resistance to impact and abrasion, and petrographical and mineralogical qualities. Field tests for measuring friction and texture are being performed on the surface of test sections twice a year at random intervals. Probabilistic prediction models resulting from the study will provide an engineering solution whereby the frictional life of a seal-coat surface can be predicted or the characteristics of the aggregate required to maintain a given level of frictional resistance can be determined during the design phase.

The lack of skid resistance of highway pavements, particularly wet pavements, is a serious problem of increasing concern to highway engineers and researchers. As traffic speeds and average daily traffic (ADT) continue to rise, the chances of skidding accidents and their attendant consequences are growing at an alarming rate with each passing year (1,2).

Many variables contribute to skid resistance. These include pavement surface friction, pavement microtexture and macrotexture, construction variables, drainage properties of the surface, traffic volume, environment, highway geometrics, vehicle speed and load, tire-tread depth and inflation pressure, driver experience, and rainfall intensity.

Lack of pavement surface friction has long been recognized as the primary factor in skidding (3-5). The use of polish-resistant coarse aggregates or other aggregates with good frictional performance has always been considered a useful remedy. The Materials and Tests Division (D-9) of the Texas State Department of Highways and Public Transportation (SDHPT) employs the polish-value (PV) test (6), whereby an aggregate undergoes accelerated polishing to establish the polish susceptibility of coarse aggregates incorporated in

pavement work. The skid resistance test (7) is used by D-9 to measure the frictional resistance of pavement surfaces, expressed as the skid number (referred to in this paper as the friction number (FN)). Minimum laboratory PVs of coarse aggregates have been established and used in Texas for years for the purpose of providing acceptable pavement friction. Normally, high-traffic-volume roads require aggregates with high resistance to polish and wear, whereas low-traffic-volume roads may operate with lower polish-resistant aggregates. The current PV requirements based on ADT are as follows:

ADT	PV		
Greater than 5,000	32		
5,000 to 2,000	30		
2,000 to 750	28		
Less than 750	None		

#### **OBJECTIVES OF THE STUDY**

This research is one phase of a study whose overall objective is to investigate and develop design criteria to provide and maintain adequate pavement friction. Specifically, these objectives are

- To develop a comprehensive, long-range strategic research plan which addresses all aspects of pavement friction, and
- To investigate the relationship between laboratory frictional properties of coarse aggregates (i.e., PV) and the frictional resistance (FN) of roads built with these aggregates.

Implied in the second objective is an investigation of the factors that help predict the friction number; the PV test by itself, or with other laboratory tests performed on the coarse aggregate, may predict the FN with a certain confidence. Investigation of the effects of traffic, environment, and other factors on any possible relationships is also included in the scope of the second objective.

# SCOPE OF THIS RESEARCH

In general, providing skid-resistant surfaces for highway pavements involves developing guidelines for skid resistance and incorporating the guidelines into the design of new pavements or into the process of maintaining and rehabilitating existing

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pavements. These research efforts should be directed more toward improving the frictional resistance of existing pavements since a huge highway network already exists in Texas.

Many pavement rehabilitation methods (8-13), including seal-coat and hot-mix asphalt concrete (HMAC) overlays, have been used in Texas to improve the frictional resistance and other surface characteristics of the highways. In this phase of the study, the frictional resistance of seal-coat overlays is being investigated; that of HMAC will be investigated in a later phase. A seal-coat overlay is a rehabilitation method in which asphalt and aggregate are applied to a roadway surface in a layer usually under 1 in. thick; seal coats can be put on pavements of all classes, from low-volume roads to Interstate highways, but they are used mostly on rural highways.

This investigation included gathering and assimilating the pertinent literature, surveying nine selected districts in Texas, establishing seal-coat test sections with various coarse aggregates and traffic volumes, performing laboratory tests on the obtained samples and field tests on the test sections, and designing the layout of the analysis to be performed on the data.

#### **Survey of Texas Districts**

Nine Texas districts (Districts 2, 3, 4, 5, 15, 16, 18, 23, and 25) were surveyed to find out what laboratory evaluations of coarse aggregates and aggregate frictional performance were made and whether any problems were encountered. Although this research is directed toward investigating the frictional resistance of seal-coat surfaces only, the district surveys covered the use of aggregates in HMAC as well as seal-coat surfaces and the frictional resistance of the aggregates. The researchers believed that they could better rank and refine the study objectives with this broader understanding of testing policies and problems. Information sought included requirements for the PVs of aggregates used, other methods for laboratory evaluation of coarse aggregates, FNs obtained, correlation between PV and FN where applicable, visual inspection of typical sections for high and low polish values and high and low frictional resistance, and personal observations.

#### **Findings**

The following information was gathered.

- All surveyed districts use the PV test to evaluate the polish susceptibility of coarse aggregates in pavements. In seven of the surveyed districts, the four-cycle magnesium soundness test is used along with the PV test (and is preferred to the PV test in Districts 5 and 25). The minimum PV requirements in most of the surveyed districts were 32 for high-volume roads and 28 for low-volume roads. The maximum allowable loss in the magnesium soundness test (MSS) was 30 percent.
- The soundness of the aggregates is an important characteristic affecting the frictional resistance of pavement surfaces. Aggregates that had high PVs but were inadequate in

soundness did not have good frictional performance on the roads.

- Districts 5, 15, and 25 allow the use of aggregates that do not meet the PV requirements only if the aggregates have good frictional performance history. District 2 preserves friction data for many seal-coat and HMAC projects, along with laboratory information on the aggregates used in those projects.
- Districts 15, 16, and 18 have set up seal-coat and HMAC test sections for investigating frictional resistance. A study conducted years ago in District 25 revealed little correlation between PV and FN but did find some correlation between sand equivalency of the surface texture and FN.
- Districts 2 and 23 do not like to use aggregate blends; District 18 does. District 2 personnel believe that although the initial FN is improved for blends, the FN eventually drops and tends to decrease to the PV of the poorer material. District 23 personnel prefer to use the low-PV aggregate and then apply a sealant of lightweight aggregate (high PV) when the FN drops below acceptable limits. However, District 18 personnel believe that blending aggregates with different PVs (e.g., 30 and 34) gives better performance than using one aggregate with a PV of 32.
- District 3 reported that an aggregate need not meet a high PV requirement because, usually, the road is resurfaced for other rehabilitation purposes before the FN drops below the acceptable limit. However, for low-volume roads, where low-PV aggregates can be used, the relationship between the drop in FN and accumulated traffic was reported to be of value.
- Factors mentioned as important for frictional resistance were stripping of aggregates in the wheel paths (District 4), flushing of asphalt in the wheel paths (District 5), and slipperiness of pavement surfaces right after rainfall (District 5). District 18 reported that the outside lanes have lower FNs than the inside lanes because traffic is heavier on the outside lanes.
- Finally, only District 2 incorporated the FN in its pavement management rehabilitation system. However, District 5 personnel believe that safety regulations will eventually call for the inclusion of FN in such a system.

# OBSERVATIONS FROM OBTAINED FRICTION DATA

Friction data, collected over the past 6 to 8 yr, were obtained from several districts supplemental to those surveyed. Data from three districts were combined to yield information for four selected sources of aggregates used in HMAC surfaces with various traffic volumes. Besides FN, the data included laboratory information on PV and MSS. Graphs of the FN versus accumulated traffic per lane (14) are plotted in Figure 1. The aggregate types and the laboratory information are shown in Table 1.

Obviously there are performance differences among the aggregates. First, the overall performance of the sandstone aggregate, which had a high PV of 47 and an MSS in the range of 9 to 14 percent, was markedly better than that of the limestone aggregates. A comparison between the PV and

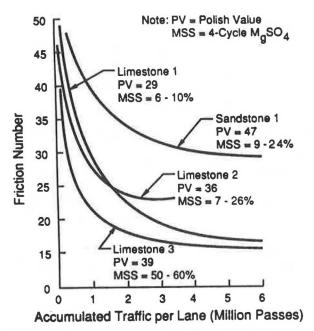


FIGURE 1 Frictional resistance of HMAC surfaces constructed with four aggregates in three Texas districts (14).

TABLE 1 AGGREGATES CONSIDERED IN THE ANALYSIS OF OBTAINED FRICTION DATA

	Aggregate Properties			
Aggregate Type	PV	MSS, %		
Sandstone	47	9-24		
Limestone 1	29	6-10		
Limestone 2	36	7-26		
Limestone 3	39	50-60		

MSS values for the sandstone and Limestone 1 aggregates suggests that, because the MSS is less for the limestone aggregate, the sandstone aggregate's markedly better performance is most likely due to its higher PV of 47. Second, up to 2 million passes, the limestone aggregates exhibited a dramatic decrease in FN compared with the rather flattened decrease exhibited by the sandstone aggregate. Third, Limestone 1, which had the lowest MSS, maintained an FN higher than those of the other two limestone aggregates up to about 2 million accumulated passes, after which the FN of Limestone 2, with a PV of 36 and an MSS value in the range of 7 to 26 percent, flattened and remained constant. Fourth, Limestone 3, in spite of its good PV of 39, had the worst performance throughout the life of the road because it was inadequate in soundness (MSS = 52 percent). Last, the terminal FNs of Limestone 1 and Limestone 3 were about the same at 6 million passes. Yet, had the roads been resurfaced when the FN dropped below 20, Limestone 1 could have sustained twice as much traffic. In summary, good polishing and soundness characteristics both are essential to good frictional performance.

Please note that these observations must be interpreted

cautiously because they are tentative and based on plots that represent best-fit curves. Although the trend of the decrease in FN was traceable, the data of each curve suffered such large, unexplained variations that overlapping between some of the data resulted. These variations could be attributable to factors unknown to the researchers.

# RESEARCH METHODOLOGY

Many coarse aggregate types and sources have been used for placing seal coats on Texas highways. The major types, or categories, include crushed limestone, crushed sandstone, and crushed siliceous gravel. Other types include lightweight aggregate, limestone rock asphalt, traprock, granite, and rhyolite. Differences in field frictional resistance of these aggregates have been observed over the years (15–21). Numerous factors, including aggregate characteristics, construction variables, traffic volume, and environment, are believed to be major contributors to these performance differences. The objective of this phase of the study was to investigate the effects of these factors on the field frictional resistance of coarse aggregates used in seal-coat surfaces.

The methodology followed in this investigation involved preparing seal-coat test sections in different climatic regions of Texas, using as many as possible of the affordable aggregates predominant in the areas. A construction survey was made for each test section. The survey included construction variables such as design application rates of asphalt and aggregate, asphalt and aggregate type, weather condition, type and condition of existing pavement, and type of construction forces. Aggregate samples were obtained from the job sites and examined in the laboratory to determine physical properties, polish and wear characteristics, resistance to weathering, resistance to impact and abrasion, and petrographical and mineralogical qualities. Field tests continue to be made on the test sections twice a year at random intervals. Testing involves measuring surface friction and texture. Finally, annual and periodic data on average temperature, total precipitation in inches, and total inches of snow are gathered for each test section.

All information is being stored in the database being created on an IBM PC-AT. In-depth statistical analysis of the data will lead to probabilistic models that incorporate the effects of the involved variables on the friction of seal-coat surfaces.

#### **TEST SECTIONS**

# **Environmental Considerations**

Figure 2 is a map that shows the six different climatic regions of the United States and the environmental characteristics associated with each (22). Texas lies within four of these regions (I, II, IV, and V), as shown in Figure 3. The environmental characteristics of each respective region are wet and no freeze, wet and freeze-thaw cycling, dry and no freeze, and dry and freeze-thaw cycling.

Seal-coat test sections have been established in all four climatic regions. Ideally, one source of each major aggregate category would have been used in all four regions so that the

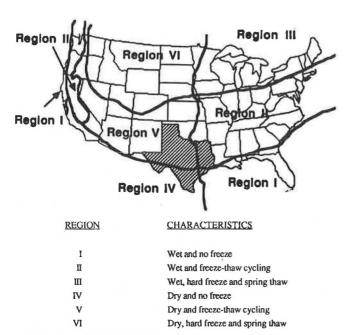


FIGURE 2 The six climatic regions of the United States (22).

effect of climate on that aggregate category could be evaluated. No one aggregate is currently used in all four regions, however, and hauling an aggregate to distant locations is neither feasible nor practical. Instead, and to make the experimental design representative of current practice, sections were constructed using different qualities of the main aggregate categories that were locally available.

# **Statistical Considerations**

To make the experimental design statistically sound, some requirements were established.

- Sections must be at least 1,000 ft long, to allow five friction or texture values to be measured.
- For each major aggregate type, as many as four sections with aggregates obtained from four different sources are to be constructed in each environmental region. However, the total number of test sections must be kept to a level that allows effective handling.
- Replications of sections should be established wherever possible to test for a constant variation in field responses (friction and texture) under various experimental conditions. Expected variations can result from the following:
  - Time. The quality of an aggregate pit can change over time.
  - Traffic count. The ADT is provided for a divided highway as a total figure for both directions. Replications built in both directions may clarify whether traffic is divided equally. Also, in the case of more than one lane per direction, replications are built in all lanes to account for different traffic volume in different lanes.
  - Less importantly, construction practices. Two sections are placed a few miles apart on the same lane to evaluate variations caused by construction equipment, such as changes in application rates of asphalt or aggregate.

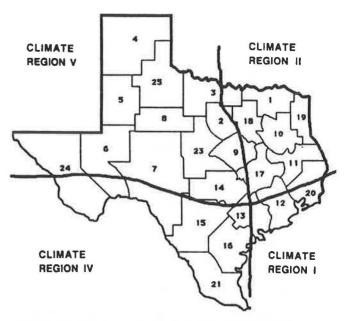


FIGURE 3 Location of the four climatic regions in Texas (22).

Please note, however, that replications were constructed for only a few sections, where circumstances permitted.

#### Criteria for Selection

Test sections were chosen according to the following criteria.

- Have all sections be tangent sections.
- Consider only sections with a minimal slope, up to 2 percent.
  - Have no major intersections within or between sections.
- Ideally, have as many sections end-to-end as the number of different aggregates used.
- Ideally, select two sections for each aggregate, one constructed by the maintenance forces and the other by a contractor.
- In the case of divided highways, have sections in only one direction, preferably the direction of heavier traffic. Where possible, make replications, as discussed earlier.

#### **Selected Test Sections**

Sixty-two seal-coat test sections were established in nine districts of the four environmental regions; fourteen of them are replications. Various aggregate types and sources were used, as shown here.

Aggregate Type	No. of Sources			
Crushed limestone	10			
Limestone rock asphalt	1			
Crushed sandstone	4			
Crushed siliceous gravel	8			
Lightweight	5			
Traprock	1			
Rhyolite	2			

More than 50 percent of the aggregates were non-coated; the

rest were precoated. Different aggregate grades were also considered, in view of the effect gradation may have on pavement surface texture and friction.

The region, district, county, and highway designation where each aggregate was placed as well as the ADT to which the aggregate is exposed, the aggregate type, material, grade, producer, and pit were documented.

#### DATA COLLECTION

Field, laboratory, and weather data are being collected. The field data and the weather data are related to the established test sections and their geographical locations, respectively. The laboratory data involve the aggregates used in establishing the test sections.

#### Field Data

The field data constitute information obtained by monitoring the sections since the date of construction. Specifically, the data for each test section consist of a survey of construction variables performed at the construction sites, results of field testing obtained twice a year, and evaluations of a visual condition survey done concurrently with field testing.

#### Construction Survey Data

The construction survey consists of information on the location of each section, condition and type of existing pavement, personnel contacted at the site, type of construction forces, coarse aggregate material and asphalt type used, design application rates of aggregate and asphalt, weather conditions, and traffic volume.

#### Field Testing Data

The skid resistance test (7) and the British pendulum test (23) for measuring friction and the sand patch test (24) for measuring macrotexture are being performed twice a year on the surfaces of test sections.

The original plan, based on the literature (25-28), was to conduct the tests after long periods of dryness, when the pavement surface is expected to show the minimum frictional resistance and texture depth. Later, in order to detect and understand the effects of long-term seasonal variations caused by long periods of wetness or dryness, tests were performed on a random basis. The season in which the tests are undertaken is now viewed as a variable that can help explain variations in the obtained measurements.

# Visual Condition Survey Data

The visual condition survey is made to determine the condition of a test section at the time of field testing. Three types of distress affect the friction and texture measurements of seal coat surfaces: poor aggregate retention, inadequate aggregate

embedment, and bleeding or flushing of asphalt (29,30). If any of these types of distress is observed to be severe, the results of field testing will be questionable in that they might not properly represent the frictional properties of the surface aggregate. Only if a test section displays a differential or discontinuous type of distress along a wheel path may field testing still be considered, and then only on parts of the wheel path where the distress is minimal. Otherwise, monitoring of the distressed section must be terminated.

#### **Laboratory Data**

Two types of laboratory data are being collected on the aggregate samples obtained from the job site of test sections. The first type concerns the aggregates' physical properties, which are determined from the results of numerous tests performed at the engineering laboratories. The other type deals with aggregate mineralogy and petrographic characteristics gathered from examinations at the geology laboratories.

# Data of Aggregate Physical Properties

Four groups of tests were found applicable for measuring the degree of deterioration an aggregate may exhibit when placed in field service. Most of the selected tests are performed on prepared samples in conformity with the test methods described in the *Manual of Test Methods* of the SDHPT. Most of these methods are modifications of standard ASTM test methods; some are procedures identical to those prescribed by ASTM. Two of the selected tests, the insoluble residue test and the aggregate durability index, are performed in accordance with ASTM standards.

#### Group One: Testing for Basic Properties

Sieve Analysis The test is used to determine the particle distribution, that is, gradation, of the obtained aggregate samples (31). The gradation of an aggregate is the main determinant of the texture of a seal-coat surface and thus affects surface friction.

**Specific Gravity and Absorption Test** The test is performed (a) according to Test Method Tex-403-A (32) to determine the saturated-surface/dry specific gravity and water absorption of natural aggregates and (b) according to Test Method Tex-433-A (33) to determine the dry bulk specific gravity and absorption of lightweight aggregates. Specific gravity and absorption together indicate the porosity of an aggregate. Both characteristics importantly influence aggregate frictional properties.

**Decantation Test** This test is performed in conformity with Test Method Tex-406-A (34). During the test, the amount of aggregate material finer than the No. 200 sieve is removed by washing and its percentage by weight is calculated. The

amount of material removed may be related to the relative stability of aggregate particles in seal-coat surfaces (adhesion between aggregate particles and asphalt) and to the amount of asphalt needed to ensure a desired stability. This measure, along with some of the construction variables, may explain some of the problems pertaining to surface texture, particularly to dislodgement or loss of aggregates from seal-coat surfaces.

Group Two: Testing for Polish and Wear Characteristics

Accelerated Polish Test Test Method Tex-438-A (6) is employed. An aggregate is subjected to accelerated polishing to evaluate its polish susceptibility when incorporated in pavement work. The PVs of aggregates could be a helpful tool in predicting the frictional characteristics of aggregates placed in field service. The idea is based on the concept that the limiting PV of an aggregate, which is reached after 9 hr of polishing, may match or correlate well with the terminal frictional resistance of a roadway after exposure to a certain volume of traffic.

Insoluble Residue Test for Carbonate Aggregates This test is conducted in accordance with ASTM D-3042 (35). It estimates the amount of noncarbonate (insoluble) material in carbonate aggregate and involves a grain-size distribution of these insoluble particles. The theory is based on the concept that the frictional resistance of carbonate aggregates is related to the differential hardness of the minerals that make up the structure of the aggregate (26). According to this concept, when a carbonate aggregate is subjected to polish, the softer minerals will wear away at a faster rate than the harder ones. The result will leave the wearing surface of the aggregate with a rough, uneven texture, which increases or maintains the friction properties of the carbonate aggregate.

Crushed Particles in Gravel Aggregates This test is performed in accordance with Test Method Tex-413-A (36) and is used to determine the percentage by weight of crushed particles in aggregates. This characteristic is of interest because the asperities of the texture of crushed particles, as opposed to the smooth texture of noncrushed particles, greatly affect an aggregate's polish susceptibility.

Group Three: Testing for Resistance to Disintegration Due to Weathering Action

Four-Cycle Sodium or Magnesium Sulfate Soundness Test In this test, the aggregates are examined according to Test Method Tex-411-A (37) to estimate their soundness when subjected to weathering action. The aggregate samples are repeatedly immersed in saturated solutions of sodium or magnesium sulfate, which is followed by oven-drying to partially or completely dehydrate the salt precipitated in permeable pore spaces. The dehydration of salt upon reimmersion causes internal

expansive forces, which simulate the expansion of water on freezing.

Coarse Aggregate Freeze-Thaw Test As in the four-cycle soundness test, the aggregates are tested to judge their soundness when subjected to weathering action. This is accomplished, as Test Method Tex-432-A (38) dictates, by subjecting the aggregates to 50 cycles of freezing and thawing in the presence of water. The internal expansive forces created by repeated freezing of water in the pore spaces cause the aggregate to disintegrate. This action is supposed to simulate what happens to the aggregates when they are placed in regions characterized by freeze-thaw cycling.

The soundness and freeze-thaw losses are believed to be indicative of the strength and hardness of the cementing material that holds the crystal grains of aggregate particles together.

Group Four: Testing for Resistance to Degradation Due to Abrasion, Impact, and Grinding

Los Angeles Abrasion Test The Los Angeles test provides a measure of degradation resulting from a combination of actions, including abrasion or attrition, impact, and grinding. This is done, in accordance with Test Method Tex-410-A (39), by placing the aggregate in a rotating steel drum with a specified number of steel spheres, the number depending upon the gradation of the test sample.

Aggregate Durability Index This test is performed in compliance with ASTM D-3744 (40). The test establishes the durability index—an empirical value indicative of the aggregate's relative resistance to generating detrimental claylike fines when subjected to mechanical degradation by agitation for 10 min in a mechanical washing vessel containing water.

Aggregate Degradation Test This test was developed as a part of Center for Transportation Research Research Project 3-9-85-438 for the SDHPT (14). Its purpose is to determine the resistance of aggregates to degradation in HMAC and seal-coat surfaces. The procedure is to subject an aggregate to mechanical degradation by agitation in the wet ball-mill apparatus in the presence of water. Degradation ensues from interparticle impact, abrading, and grinding actions.

The mechanical degradation an aggregate undergoes in this group of tests is expected to simulate (a) the impact of axle loads on aggregate particles in the wearing surface of a roadway and (b) the abrasive and grinding actions created when fines and grit accumulated on the roadway surface come between rubber tires and aggregate particles.

Data of Aggregate Mineralogy and Petrographic Examinations

The feasibility of performing petrographic analyses on the obtained aggregate samples is being investigated. The follow-

ing discussion outlines the purposes, preliminary procedures, and significance of these analyses.

# Purposes Petrographic examinations would be made

- To determine by petrographic methods the physical properties of an aggregate that have a bearing on the performance of the aggregate in seal-coat surfaces;
- To identify, describe, and classify the constituents of the aggregate sample; and
- To determine the relative amounts of the constituents when they differ significantly in a property, such as hardness, that may influence the frictional behavior of the aggregate when it is used in pavement surfaces.

Summary of Procedure A systematic petrographic examination of each aggregate is made under a polarizing microscope to determine the percentages of mineral constituents. The percentages are then used to determine the approximate percentage of hard mineral content (i.e., minerals harder than 5 on Moh's scale) in each aggregate. The results suggest the relationship between aggregate performance and mineralogy.

The microscopic examination also reveals information on the size and shape of crystal grains, the ground mass formation, and the grain distribution of different minerals. This information is supported by photomicrographs taken for later comparison of the aggregates.

In addition, other relevant features of the aggregate are described during the examination. These include particle surface texture and particle shape. Particle surface texture is assessed with a binocular microscope to determine the degree of roundness or grittiness the aggregate particles possess. Particle shape is evaluated by roundness and sphericity of particles. Roundness is concerned with the curvature of the corners of a particle; six classes, from very angular to well rounded, are distinguished. Sphericity is a measure of how closely the particle shape approaches that of a sphere.

Significance of Findings The results can be used in many different ways. First, by correlating or regressing the percentages of hard mineral content with the respective FNs of the aggregates, a conclusion might be reached as to whether a relationship between the two exists. Another possible finding could be what the optimum compositional proportion of hard to soft minerals should be for an aggregate to have highly favorable skid resistance. Aggregates having mixed composition of hard and soft minerals are expected to have higher skid resistance than do aggregates consisting predominantly of minerals of the same type or of the same hardness (41,42). The concept is that the soft ground mass wears away relatively quickly, exposing the hard grains and providing a sandpaperlike surface. Before the asperities of these hard grains lead to enough wearing action to cause them to polish, the matrix has been worn down to where it can no longer hold the hard particles, allowing them to be dislodged to expose fresh, unpolished particles. This continuous renewal of the pavement surface is believed to yield highly favorable skid resistance properties. It should be noted, however, that the influence of the compositional proportion might be modified by the effects of other features, such as size, shape, and distribution of the hard grains.

Second, the photomicrographs of two aggregates grouped in the same classification (e.g., sandstone) and with approximately the same percentages of hard mineral may reveal markedly different grain sizes. The more angular and the larger the mineral grains or crystals in individual aggregate particles, the higher the expected skid resistance of aggregate particles incorporated in pavement surfaces. Also, the coarser and more angular the hard mineral grains, and the more uniform their distribution in the softer mineral matrix, the higher the expected skid resistance.

Finally, the results of particle surface texture and particle shape may turn out to be valuable indications of micro- and macrotexture of the surfaces where aggregates are placed.

#### Weather Data

Climatological data from many recording weather stations in Texas are published. The primary components of the climatic description furnished by the majority of these stations include precipitation, snowfall, snow on ground, temperature, evaporation, and wind.

Annual averages of the climatic components are being compiled for each test section from the publications of the nearest recording weather station. Detailed climatological data for the periods before and during field testing are also sought. Specifically, the data include the length of the last rainfall period, the number of days between the last rainfall that occurred in that period and the day of field testing, and the total inches that fell in that period. This information will help explain the effects of short-term weather variations, caused mainly by localized showers, on the frictional properties of roadway surfaces (43,44).

The season, dry or wet, in which field testing is undertaken is a variable that affects the obtained field measurements and may account for long-term seasonal variations in pavement surface frictional properties caused by long periods of dryness or wetness. To properly define this weather-caused variable, detailed information on climatic, precipitation-related patterns in the state of Texas was sought (45,46). The state was found to have 10 climatic subdivisions formed by blocks of counties with similar amounts of rainfall. Monthly precipitation data based on the averages for the 30-year period 1951–1980 were obtained for the 10 climatic subdivisions. After manipulation, the data suggested the appropriate segmentation of a year into wet and dry seasons.

# DATABASE AND STATISTICAL METHODS

All data are being stored and manipulated in the database being created on an IBM PC-AT using the Statistical Analysis System (SAS) program. In-depth statistical analysis will be performed on the data to formulate multivariable probabilistic models for predicting the frictional resistance or performance of seal-coat surfaces. The literature review revealed that under the effects of the long-term seasonal changes, the magnitude of which depends on traffic volume and aggregate type (28), the curves of frictional performance in numerous studies showed no consistent upward or downward trend for the annual min-

imum levels after about 2 yr of exposure to traffic (21,44, 47–52). Accordingly, a preliminary analysis may be performed on the data after the 2-yr friction measurements are obtained. The analysis will involve analysis of covariance (ANCOVA), which combines the analysis of variance (ANOVA) and regression analysis with multivariable regression analysis.

#### STATISTICAL ANALYSIS

The friction and texture measurements or performance responses of test sections are the dependent variables—the criterion variables. Performance is measured by FN, British Pendulum Number (BPN), and average texture depth (ATD), all of which are quantitative dependent variables. These responses will be dealt with, one by one, to evaluate how much their variations can be explained by independent variables (construction, laboratory, weather, and traffic variables). All of the performance measurements obtained for each test section and their associated accumulative traffic volumes will be used when an analysis is performed. This will be done so that the effect of traffic on changes in friction and texture over time can be better estimated.

The normality and homoscedasticity assumptions will be tested using the five readings obtained for each response and the established replications, respectively. It may happen that one of these assumptions is not satisfied. For example, the response variances may not be homogeneous. This situation can sometimes be remedied by transforming the response measurements (53-55). That is, instead of using the original response measurements, their square roots, logarithms, or some other function of the response might be used. Similarly, transformation will be done if the normality assumption of

the response measurements is not satisfied. In fact, transformations that tend to stabilize the variance of a response have been found to make the probability distribution of the transformed response more nearly normal (56).

# **Analysis of Covariance**

Design of the Experiment for the Environmental Effect

The design of this experiment is aimed toward better understanding of the effect of environment on the frictional resistance of seal-coat surfaces. In this experiment, the field responses—FN, BPN, and ATD—are the criterion, or dependent, variables, while climate and aggregate type are the main predictors considered. The layout of this experiment is shown in Table 2. The number of sections built in each region with each of the aggregate categories is shown inside the cells. These numbers actually mean the numbers of observations obtained for each of the criterion variables, with each observation being a set of five readings.

It happened that test sections for some of the cells in the table could not be established. The researchers will attempt to employ some of the approaches suggested by Dodge (57) to solve this problem. However, if the designed experiment is found to lose balance and not all of the usual parametric functions can be estimated, testing for the climatic effects will be incorporated only in the multiple regression analyses. This will be done by obtaining the total annual freeze-thaw cycles and total annual precipitation recorded in the weather station closest to each test section. These two weather variables will then be considered as covariables in the regression analyses.

TABLE 2 DESIGN OF THE EXPERIMENT FOR THE ENVIRONMENTAL EFFECT

Aggregate Type		Climatic Zones						
	Ī		п		IV		v	
	PV < 30	1p		\$ <b>*</b> 0		1 <sup>b</sup>		1
Limestone	PV > 33			2 <sup>a</sup>		-		2 <sup>b</sup>
Sandstone	PV ~ 40	98		4a		*		#
Siliceous Gravel PV < 3	0 1				,		1	
Limestone Rock Asphalt		4¢		2 <sup>c</sup>		3 <sup>c</sup>		-
Lightweight		5°		*				-

a Traffic count and/or construction replication(s)

b Time or pit replication(s)

c Replications that comply with both "a" and "b"

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#### Formulation of Model

The model generated by applying the ANCOVA technique has the following form:

$$Y = AGE + ADT + WTV + CSV + CLR$$
  
+  $AGT + CLR * AGT + R(CLR AGT)$ 

where the criterion variable Y could be  $Y_1$  (FN),  $Y_2$  (BPN), or  $Y_3$  (ATD). The predictors are CLR (climatic regions) and AGT (aggregate type). CLR \* AGT is the interaction term between climatic regions and aggregate type. R(CLR AGT) is a term used to account for the fact that replications exist for some aggregates within regions. The covariables are AGE (age of section), ADT (average daily traffic), WTV (weather variables), and CSV (construction variables).

# **Regression Analysis**

# Formulation of Prediction Model

In general, this analysis is intended to find the best general linear regression model of the type

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k + \varepsilon$$

to describe the relationship between the frictional performance of seal-coat surfaces (y) and all of the independent variables involved  $(x_1, x_2, \ldots, x_k)$ . Since an explanation of causal effects of each independent variable is the primary thrust of this investigation, the stepwise regression procedure, an option in the SAS program, is used for building the prediction model.

# Multicollinearity

Multicollinearity is a problem that arises when two or more of the independent variables are found to be highly correlated to one another. When such a problem is encountered, the respective individual contribution of the correlated variables to the reduction in the error sum of squares cannot be determined. If two variables contribute overlapping information, the first  $\beta$  parameter may be overestimated, whereas  $\beta_2$  tends to be underestimated (56). In fact, multicollinearity may even cause the algebraic sign of one or more regression parameter estimates to be contrary to logic. Thus, if a multicollinearity problem arises, two approaches may be employed (56,58). The first is to drop one of the two correlated variables from the equation and to reestimate it; this can cause bias in the reestimated model, but it may be justified if the bias can be argued to be small. The second approach is to combine the two variables into an index variable by standardizing their effects; the variables should be conceptually and theoretically related for this approach to be used.

# Measuring the Goodness-of-Fit of the Model

The term  $R^2$ , the multiple coefficient of correlation, provides a measure of the fit of the multivariable regression model.

That is,  $R^2$  gives the proportion of the total sum of squares that is explained by the predictor variables. The remainder is explained by the omission of important information-contributing variables from the model, an incorrect formulation of the model, and experimental error.  $R^2$  takes values in the interval  $0 \le R^2 \le 1$ .

A small value of  $R^2$  means that the predictor variables contribute very little information for the prediction of frictional performance; a value of  $R^2$  near 1 means that the predictor variables provide almost all the information necessary for the prediction of frictional performance.

A relatively poor fit of the model (a small  $R^2$ ) may result if the predictor variables are not entered properly into the model (perhaps interaction terms  $x_1x_2$ ,  $x_1x_3$ ,  $x_2x_3$ , etc., and quadratic terms  $x_1^2$ ,  $x_2^2$ ,  $x_3^2$ , etc., should be included), or perhaps frictional performance is a function of many other variables besides the ones already considered. Interaction terms are considered in the analysis for their ability to contribute information for the prediction of frictional performance, and residual analysis is performed to identify new variables that may improve the fit of the model.

# Residual Analysis

Residual analysis, a capability of the SAS program, examines the degree to which the model satisfies the random-error assumption of multivariable regression analysis and thereby suggests the inclusion of additional variables that may improve the fit of the model (56,58). The analysis involves plotting the residuals against each independent variable. In some cases a residual plot might suggest the inclusion of a second-order term, say  $x_2^2$ , into the analysis as an additional independent variable. Another plot might depict a case where the variance in the response (frictional performance) increases proportionally to the independent variable  $x_3$ . Usually the addition of the variable  $\log(x_3)$  will accommodate this problem (53).

# Investigation of Other Relationships

Regression analysis is used to investigate the relationships that may exist among the field responses, FN, BPN, and ATD, in order to determine how microtexture and macrotexture, reflected by BPN and ATD respectively, influence the traditional friction measure, FN. Also, a friction curve for each aggregate source is developed that shows how friction decreases with accumulation of traffic. The curves are generated by regressing friction against accumulative traffic and weather-related variables.

# USE AND SIGNIFICANCE OF FINDINGS

The prediction model will be of value in three ways:

- 1. It can be used to estimate the mean value of frictional performance for given values of the predictor variables.
- 2. It can be used to predict some future value of frictional performance for given values of the predictor variables.
  - 3. If the predictor model provides a good fit to the set of

data (if  $R^2$  is large) and the number of predictor variables is not too large, the model will help the engineer or researcher understand the relationship between the predicted value of frictional performance and the set of predictor variables. For example, if two predictor variables, say PV and four-cycle soundness, are found to be interacting, the model will show how the relationship between frictional performance and PV is dependent on the soundness loss.

The significance of the model lies in the fact that it will provide a method for nominating, at the planning or design phase of seal-coat projects, the physical properties and petrographic characteristics of the aggregate as well as the design values of the construction variables needed to provide or maintain a given acceptable frictional resistance under a certain projected traffic volume in a specific climatic region. In addition, the findings will likely have an immediate impact on specifications relating to aggregate laboratory preconditioning methods for the benefit of frictional resistance.

#### CONCLUSIONS AND RECOMMENDATIONS

The following conclusions can be drawn at this stage of the study:

- In Texas, the PV test is the most widely used method for evaluating the polish susceptibility of coarse aggregates used in pavements. The four-cycle soundness test is used along with the PV test in most of the surveyed districts.
- The soundness of the aggregates was found to be an important characteristic affecting the frictional resistance of highway surfaces. The findings of the survey of Texas districts and the analysis of obtained friction data indicated that aggregates with high PVs but inadequate soundness did not have good frictional performance.
- Petrographic tests may prove to be very useful in selection of aggregates if such tests or a combination of test results can be correlated with field performance.
- According to the literature, long-term and short-term seasonal changes are a major cause of variation in the frictional resistance of highway surfaces. In addition, in many studies the rejuvenating effects of wet periods appeared to offset the polishing effects of dry periods in that the curves of frictional performance showed no consistent upward or downward trend for the annual minimum levels after about only 2 yr of exposure to traffic. The magnitude of variation was found to be strongly associated with traffic volume and type of aggregate. Pennsylvania State University has developed models that treat these seasonal variations, but it has been suggested that those models be used only in the geographical area in which the investigation was conducted.
- There is not yet any relationship that can reliably predict field frictional resistance from aggregate properties. This is largely because no research has attempted to relate field friction to microtexture and macrotexture laboratory properties. The literature repeatedly stated that the inclusion of a field-measured macrotexture variable in predicting models would not serve any purpose in design (the current art of construction methods cannot assure a predesigned macrotexture). Another reason for the lack of reliable friction-predicting models

is that the effects of seasonal variations have never been corrected.

- Considering all of the laboratory and petrographic tests relevant to determining the aggregate properties that hypothetically influence the frictional performance of seal coats may well suggest which tests (or properties) ought to be used to evaluate an aggregate.
- Because macrotexture's effect on frictional resistance is undisputed and including it in the formulation of prediction models decreases their design value, the methodology of this study introduced, instead, the factors that contribute to the formation of macrotexture (aggregate gradation and shape, application rates of asphalt and aggregate) and those believed to govern the rate of wear in such texture under traffic exposure (resistance to abrasion, soundness, petrographic properties, and others).
- The randomized selection of the seal-coat projects and aggregate sources for the construction of test sections points to the areas in which the results of this study may usefully be implemented.
- The construction of test sections end-to-end as number of different aggregates used is very convenient for performing field testing. Sections 1,000-ft long are of sufficient length to make five friction and texture measurements.
- The climatological data being collected and the tentative segmentation of a year into dry and wet periods are expected to help account for the variability in frictional resistance caused by short- and long-term seasonal variations, respectively.

Only a few recommendations can be made at this stage of the study.

- Until a reliable relationship is established between field frictional resistance and aggregate characteristics, the selection or evaluation of aggregates on the basis of the PV and soundness requirements, along with the frictional performance history (if available), should be continued.
- When considering whether to resurface an existing pavement, decision makers should rely upon friction measurements taken in the dry period or periods of the climatic division where the roadway is located.

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# REFERENCES

 J. L. Beaton. Providing Skid Resistant Pavements. In Transportation Research Record 622, TRB, National Research Council, Washington, D.C., 1976, pp. 39-50.

F. Brillet. Skid Resistance of Highway Pavements: Assessments
of Fourteen Years of National Skid-Resistance Surveying. Bulletin de Liaison des Laboratoires des Ponts, Dec. 1984.

3. K. D. Hankins et al. The Degree of Influence of Certain Factors

Pertaining to the Vehicle and the Pavement on Traffic Accidents Under Wet Conditions, Research Report No. 133-3F, Texas High-

way Department, Austin, Sept. 1970.

4. J. R. Hosking. The Effect of Aggregate on the Skidding Resistance to Bituminous Surfacings: Factors Other Than Resistance to Polishing. TRRL LR 553, Transport and Road Research Laboratory, Crowthorne, England.

5. S. Q. Kidd. Skid Resistance and Wear Properties of Aggregates for Asphalt Paving Mixtures. MSHD-RD-85-55. Mississippi State

Highway Department, Jackson, Feb. 1985.

6. Test Method Tex-438-A-1983: Accelerated Polish Test for Coarse Aggregate. In Manual of Test Methods, Texas State Department of Highways and Public Transportation, Austin, Jan. 1983

 ANSI/ASTM E 274-79: Standard Test Method for Skid Resistance of Paved Surfaces Using a Full-Scale Tire. In Annual Book of ASTM Standards, ASTM, Philadelphia, Pa., 1979.

V. Adam and S. C. Shah. Evaluation of Open-Graded Plant-Mix Seal Surfaces for Correction of Slippery Pavements. In Transportation Research Record 523, TRB, National Research Council, Washington, D.C., 1974, pp. 88-96.

9. B. M. Gallaway and J. A. Epps. Conventional Chip Seals as Corrective Measures for Improved Skid Resistance. In Transportation Research Record 523, TRB, National Research Council,

Washington, D.C., 1974, pp. 97-109.

10. NCHRP Research Results Digest 89: Guidelines for Skid-Resistant Highway Pavement Surfaces. TRB, National Research Council, Washington, D.C., 1976.

11. K. D. Hankins. How to Provide Needed Skid Resistance: State of the Art, State of the Practice. Report No. SS 11.11. Texas Highway Department, Austin, Jan. 1975.

12. D. C. Mahone and R. K. Shaffer. Corrective Programs for Improving Skid Resistance. In Highway Research Record 376, HRB, National Research Council, Washington, D.C., 1971, pp. 54-58.

13. B. F. McCullough and K. D. Hankins. Skid Resistance Guidelines for Surface Improvements on Texas Highways. Research Report No. 45-2. Texas Highway Department, Austin, 1965.

- 14. C. Papaleontiou, A. H. Meyer, and D. W. Fowler. Evaluation of the 4-Cycle Magnesium Sulfate Soundness Test. Research Report 438-1F. Center for Transportation Research, University of Texas at Austin, Nov. 1987.
- 15. F. C. Benson et al. Analysis of Field Skid Resistance Performance of Different Aggregates from District 15. Research Report No. 286-3, Texas Transportation Institute, Texas A&M University, College Station, Feb. 1984.
- 16. S. H. Dahir. Review of Aggregate Selection Criteria for Improved Wear Resistance and Skid Resistance of Bituminous Surfaces. ASTM Journal of Testing and Evaluation, Vol. 7, No. 5, Sept. 1979, pp. 245-253.

17. M. M. Davis and J. A. Epps. Skid Resistant Surfaces. Research Report No. 214-7, Texas Transportation Institute, Texas A&M

University, College Station, Dec. 1975.

- W. Eppensteiner et al. Assessment of Mineral Aggregates in Bituminous Pavement Construction. The Relation Between Laboratory Tests and Performance in Practice. Part 2: Long Term Observations. Ministry of Building and Technology, Strassenforschung, Austria, 1979.
- 19. T. C. Ferrara, J. P. Mahoney, and J. A. Epps. Skid Resistance of Different Gravel Surface in District 25. Research Report No. 214-24. Texas A&M University, College Station, July 1981
- 20. R. W. Miller and W. P. Chamberlin. Skid Resistance of Bituminous Pavements Built with Carbonate Aggregates. Report FHWA-NY-RR-80-77. FHWA, U.S. Department of Transportation, Apr. 1980.

21. J. P. Underwood. Field Friction Performance of Several Experimental Test Sections. Research Report No. 126-1. Texas High-

way Department, Austin, March 1971.

22. H. E. Desolminiah, W. R. Hudson, and E. Ricci. Serviceability Ratings of Texas Highway System for Pavement Management. Research Report 400-1F. Center for Transportation Research, University of Texas at Austin, Aug. 1986.

ANSI/ASTM E 303-83: Standard Test Method for Aggregate Durability Index. In Annual Book for ASTM Standards, ASTM,

Philadelphia, Pa., 1983.

24. Test Method Tex-436-A-1983: Measurement of Texture Depth

- by the Sand-Patch Method. In Manual of Test Methods, Texas State Department of Highways and Public Transportation, Austin, Jan. 1983.
- 25. S. H. Dahir et al. Seasonal Skid Resistance Variations. Report FHWA/PA 80/75-10. FHWA, U.S. Department of Transportation, Aug. 1979.
- 26. J. P. Underwood, K. D. Hankins, and E. Garana. Aggregate Polishing Characteristics: The British Wheel Test and the Insoluble Residue Test. Research Report No. 126-2. Texas Highway Department, Austin, Jan. 1971.
- 27. J. L. Burchett and R. L. Rizenbergs. Seasonal Variations in the Skid Resistance of Pavements in Kentucky. In Transportation Research Record 788, TRB, National Research Council, Wash-

ington, D.C., 1980, pp. 6-14. NCHRP Synthesis 14: Skid Resistance. HRB, National Research

Council, Washington, D.C., 1972.

29. J. A. Epps, B. M. Gallaway, and C. H. Hughes. Field Manual on Design and Construction of Seal Coats. Research Report 214-25. Texas Transportation Institute, Texas A&M University, College Station, July 1981.

30. J. A. Epps, A. H. Meyer, I. E. Larrimore, and J. L. Jones. Roadway Maintenance Evaluation User's Manual. Research Report 151-2. Texas Transportation Institute, College Station, Sept. 1974.

31. Test Method Tex-401-A-1983: Sieve Analysis of Fine and Coarse Aggregate. In Manual of Test Methods, Texas State Department of Highways and Public Transportation, Austin, Jan. 1983.

32. Test Method Tex-403-A-1983: Saturated Surface-Dry Specific Gravity and Absorption of Aggregates. In Manual of Test Methods, Texas State Department of Highways and Public Transportation, Austin, Jan. 1983.

33. Test Method Tex-433-A-1983: Absorption and Dry Bulk Specific Gravity of Synthetic Coarse Aggregate. In Manual of Test Methods, Texas State Department of Highways and Public Transpor-

tation, Austin, Jan. 1983

34. Test Method Tex-406-A-1983: Materials Finer Than No. 200 Sieve in Mineral Aggregates (Decantation Test for Concrete Aggregates). In Manual of Test Methods, Texas State Department of Highways and Public Transportation, Austin, Jan. 1983.

ANSI/ASTM D 3042-79: Standard Test Method for Insoluble Residue in Carbonate Aggregates. In Annual Book for ASTM

Standards, ASTM, Philadelphia, Pa., 1979.

Test Method Tex-413-A-1983: Determination of Deleterious Material and/or Crushed Particles in Mineral Aggregate. In Manual of Test Methods, Texas State Department of Highways and Public Transportation, Austin, Jan. 1983.

37. Test Method Tex-411-A-1978: Soundness of Aggregate by Use of Sodium Sulphate or Magnesium Sulphate. In Manual of Test Methods, Texas State Department of Highways and Public Trans-

portation, Austin, Jan. 1978.

38. Test Method Tex-432-A-1983: Coarse Aggregate Freeze-Thaw Test. In Manual of Test Methods, Texas State Department of Highways and Public Transportation, Austin, Jan. 1983.

- Test Method Tex-410-A-1983: Abrasion of Coarse Aggregate by Use of the Los Angeles Machine. In Manual of Test Methods, Texas State Department of Highways and Public Transportation, Austin, Jan. 1983.
- ANSI/ASTM D 3744-79: Standard Test Method for Aggregate Durability Index. In Annual Book for ASTM Standards, ASTM, Philadelphia, Pa., 1979.
- S. H. Dahir and W. G. Mullen. Factors Influencing Aggregate Skid-Resistance Properties. Highway Research Record 376, HRB, National Research Council, Washington, D.C., 1971, pp. 136-
- 42. A. Ubach. Study of the Susceptibility to Polishing of Aggregate Mixtures of Different Petrological Composition. Presented at Transportation and Soil Mechanics Lab, Spain, July 1970.
- 43. R. R. Hegmon. Seasonal Variations in Pavement Skid Resistance-Are These Real? Public Roads, Sept. 1978, pp. 55-62.
- B. J. Hill and J. J. Henry. Short-Term, Weather-Related Skid Resistance Variations. Transportation Research Record 836, TRB, National Research Council, Washington, D.C., 1981, pp. 76-81.
- J. T. Carr, Jr. The Climate and Physiography of Texas. TWDB Report 53, Texas Water Development Board, Austin, Mar. 1969.
- 46. T. J. Larkin and G. W. Bomar. Climatic Atlas of Texas. TDWR

- LP-192, Texas Department of Water Resources, Austin, Dec. 1983
- 47. J. M. Rice. Seasonal Variations in Pavement Skid Resistance. *Public Roads*, Mar. 1977, pp. 160-166.
- J. M. Clifford. Skid-Resistance Measurements Made on Several Segmental Block Pavements. RP/13/83, National Institute for Transport and Road Research, Transvaal, South Africa, May 1983.
- D. C. Colony. Overview of Skid Resistance on Ohio Pavements. Journal of Transportation Engineering, Vol. 111, No. 5, Sept. 1985, pp. 546-560.
- S. H. Dahir, W. E. Meyer, and R. R. Hegmon. Laboratory and Field Investigation of Bituminous Pavement and Aggregate Polishing. *Transportation Research Record* 584, TRB, National Research Council, Washington, D.C., 1976, pp. 1-14.
- J. J. Emery, M. A. Lee, and N. Kamel. Skid Resistance Predictive Models for Asphaltic Concrete Surface Courses. In Pavement Surface Characteristics and Materials, ASTM STP 763 (C. M. Hayden, ed.), ASTM, Philadelphia, Pa., 1982, pp. 61-72.
- L. J. Wylde. Literature Review: Crushed Rock and Aggregate for Road Construction: Some Aspects of Performance, Test Methods and Research Needs. Australian Road Research Board, Vermont South, Jan. 1976.

- 53. V. L. Anderson and R. A. McLean. Design of Experiments: A Realistic Approach. Marcel Dekker, Inc., New York, 1974.
- S. K. Kachigan. Statistical Analysis: An Interdisciplinary Introduction to Univariate & Multivariate Methods. Radius Press, New York, 1986.
- S. Weisberg. Applied Linear Regression. John Wiley and Sons, Inc., New York, 1980.
- W. Mendenhall and J. E. Reinmuth. Statistics for Management and Economics. Duxbury Press, Boston, 1978.
- 57. Y. Dodge. Analysis of Experiments with Missing Data. John Wiley and Sons, Inc., New York, 1985.
- R. S. Pindyck and D. L. Rubinfeld. Econometric Models and Economic Forecasts. McGraw-Hill Book Company, St. Louis, 1981

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