Dynamic Assignment in Three-Dimensional Time Space

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In traditional assignment models, cars are assigned to a route and are therefore present on all links on that route simultaneously. Calculations from this type of model give few positive results. If the assignment is done in space with time as a third dimension, this problem can be overcome. The first part of the paper gives a simple example of the equilibrium assignment model showing that, in some parts of the network, congestion is unrealistically calculated as a consequence of bottlenecks upstream. The second part of the paper gives a description of the three-dimensional assignment models. The proposed algorithm conforms with existing two-dimensional assignment models, although details in the algorithm are different. The effect of improving the capacity of bottlenecks on congestion downstream is shown. A computer model of the assignment model works under MS-DOS on a microcomputer.

In traditional assignment models two-dimensional (2-D) origin–destination (O-D) matrices are assigned to two-dimensional networks. Cars between each O-D pair are assigned to the links belonging to a certain route. Because these links do not have a time dimension, the implicit assumption is made that cars are present on all links at the same time. So cars that in reality are caught in a particular bottleneck can also be considered in the calculation the cause of congestion downstream. To improve the assignment process, a time dimension is added to the traditional 2-D assigned space. A three-dimensional (3-D) O-D matrix is assigned to a three-dimensional network.

The following are discussed in this paper:

1. The problems with the 2-D assignment,
2. The principles of dynamic assignment in 3-D time space,
3. The principles of the algorithm used,
4. The increase in capacity downstream from the bottleneck, and
5. A few remarks about computing.

TRADITIONAL ASSIGNMENT MODELS

In traditional 2-D assignment models [e.g., that of van Vliet (*1*)], networks are defined by links. These links connect two models (e.g., j and k). Each node j (1, 2, 3, . . .) and k (1, 2, 3, . . .) has coordinates $x_j;y_j$ and $x_k;y_k$. Each link has a certain length (z_{jk}) with a distance, time, or generalized time dimension. In this paper, time will be used as a dimension. The shortest routes are calculated between each O-D pair. In the all-or-nothing assignment program, all cars between each O-D pair are assigned to the shortest route.

The equilibrium method (*2,3*) can be used if there are overloaded links in a network. The time on every link jk (z_{jk}) is calculated by using a delay function:

$$z_{jk} = F(q_{jk}, C_{jk}, z_{jk0}) \qquad (1)$$

where

q_{jk} = the traffic flow on link jk,
C_{jk} = the capacity of link jk,
z_{jk0} = the time of a link jk in an unloaded network, and
z_{jk} = the time of link jk in a loaded network.

See Brandston's overview (*4*).

The value of q_{jk} is calculated by an iterative process. Equilibrium will be reached when the flow on all routes in use is equal and when there are no more unused links (Wardrop's principle). To reach equilibrium, the linear approximation method can be used (*3*). The flow in iteration i (q^i_{jk}) is calculated as a linear combination of q^{i-1}_{jk} and q^+_{jk}. The value q^+_{jk} is the assigned traffic to the shortest routes in the network with $z^{i-1}_{jk} = F[q^{(i-1)}_{jk}, C_{jk}]$.

The next example was inspired by the traffic system southwest of Rotterdam where a bridge limits the traffic crossing the river. The O-D matrix in Table 1 was assigned to the network in Figure 1. The traffic flows run from right to left. Figure 1 shows an all-or-nothing assignment and an equilibrium assignment. The equilibrium model shows that part of the cars are assigned to routes 4-10-9 and 2-1-8. This assignment is made because of congestion on links 3-2 and 5-8. In reality this congestion does not appear because the cars are held in bottleneck 7-6. The equilibrium assignment model gives fewer satisfactory results in this example.

ASSIGNMENT IN TIME SPACE

The main problem of the assignment models is that traffic is assigned to a network without a time dimension. To improve these methods, a time dimension has been added. Links are defined by nodes jk and period p.

Instead of time on a link jk, the time on link jk is introduced during period p. A period capacity is used instead of an hour capacity. The traffic flows are also defined by nodes jk and period p. The routes are calculated on the surface and in space, so a 3-D time space is used.

If a link is overloaded, then the path (a) will switch to a route along other nodes as in the 2-D space, (b) will switch to a route in a later period, or (c) both.

The delay on the links is also determined in time space and

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TABLE 1 ORIGIN-DESTINATION MATRIX

From / To	4	7	sum
8	1250	3750	5000
9	1250	3750	5000
sum	2500	7500	

Flows in overloaded links

link	Flow	capacity
7 - 6	7500	4000
6 - 5	7500	4000
3 - 2	5000	4000
2 - 9	5000	4000
8 - 5	5000	4000

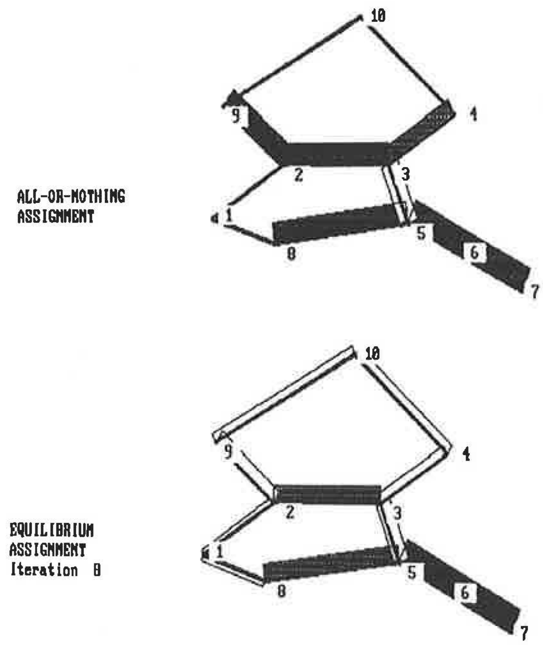

FIGURE 1 Assignment of O-D matrix in Table 1 to a network with the all-or-nothing method (*top*) and the equilibrium method (*bottom*). (In the following figures, links loaded between 85 and 95 percent of capacity are lightly shaded, and links that are loaded more than 95 percent of capacity are shaded more darkly.)

may be different from period to period. At the end a 3-D O-D matrix is assigned to a 3-D network. This method, "Dynamic Assignment in the Three-Dimensional Time-space," was first published in 1987 (*11*).

An Example

The example in Figure 1 is now represented in 3-D time space. Figure 2 gives the flows during the successive eight periods. Traffic is held in the upstream bottleneck, links 7-6 and 6-5. There is no congestion in links 3-2 and 5-8 downstream, as in 2-D space. The less logical routes of the equilibrium assignment in 2-D space do not appear in 3-D time space; consequently, the difficulties with the 2-D equilibrium assignment model are solved in 3-D space.

Other Methods

The essential differences between the present method and the well-known CONTRAM and SATURN methods are the following:

- In the SATURN method (*5,6*), trip-dependent O-D matrices are assigned to independent networks for various periods.
- In the CONTRAM assignment method (*7,8*), a limited number of cars are sequentially assigned to independent networks. Overloaded links lead to an overflow to the links in another period.

Kroes et al. (*9*) mentioned a method called "equilibrium assignment in the timespace," which was also developed in the Netherlands. In addition to the road network, a network with shadow links was made that represented the alternative of driving at other than peak-period times. With a 2-D equilibrium model, part of the traffic is assigned to this network. Although the name is similar, this method is different from the method presented in this paper.

In the method proposed by Ben-Akiva et al. (*10*), an equilibrium method is used to change the departure times and link times. The method can be used only for a very small hypothetical network. The 3-D assignment method in this paper uses 3-D O-D matrices and networks with departure times that are not affected by congestion. A study has been started to integrate our method with those of Kroes et al. (*9*) and Ben-Akiva et al. (*10*).

THE ALGORITHM

Three-dimensional assignment can be formulated as a 3-D equilibrium model. The algorithm consists of the following steps:

1. Read a 2-D network.
2. Determine the 3-D O-D matrix.
3. Determine the period capacity of the links.
4. Calculate the delay in the links.
5. Calculate the shortest routes in 3-D space.
6. Assign the 3-D O-D matrix to the shortest routes.

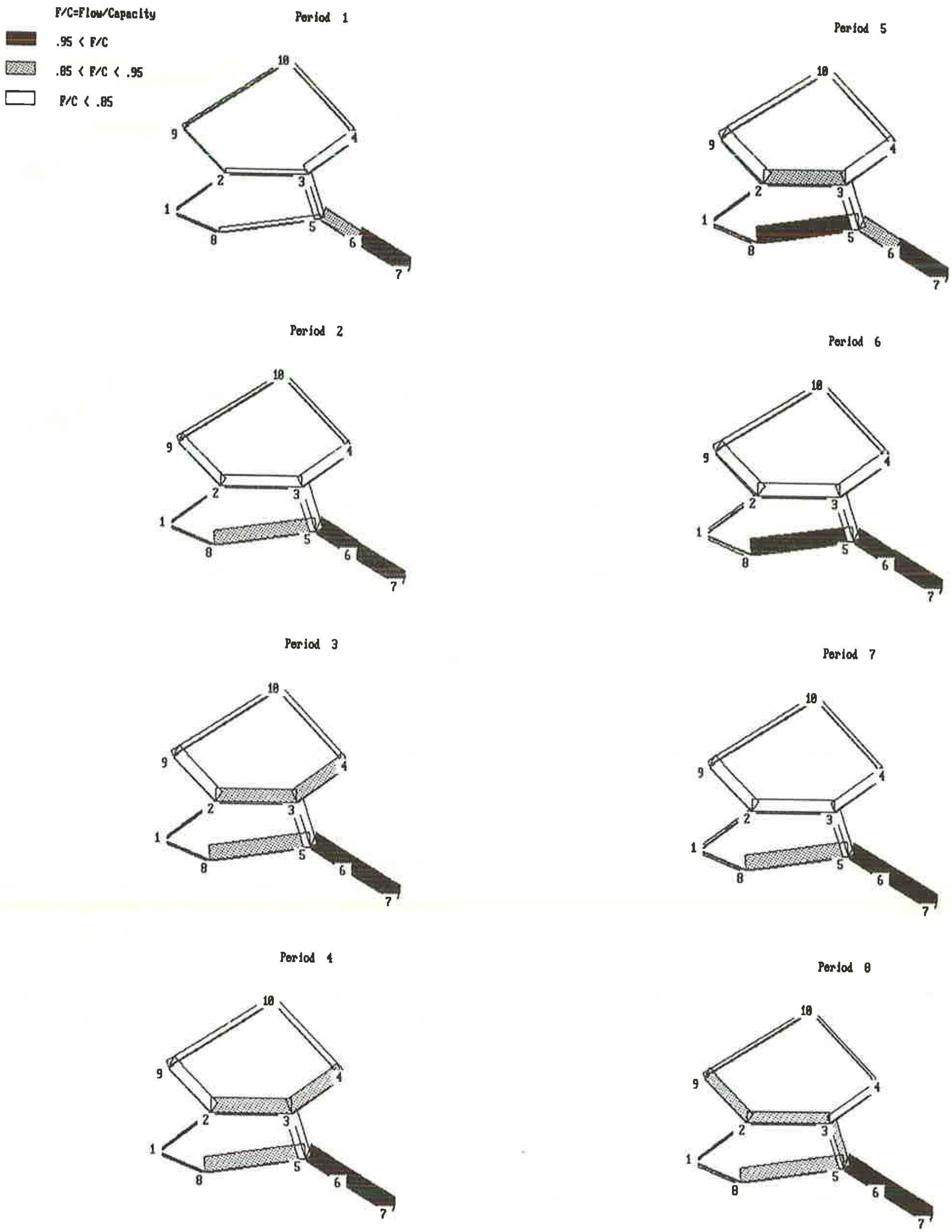

FIGURE 2 Example of an assignment in time space of the O-D matrix of Table 1.

7. Load the network.
8. If the stop criterion has not been reached, return to Step 4.

Although the 3-D algorithm, generally speaking, is similar to the algorithm in 2-D space, there are some important differences on a more detailed level:

1. Read the 2-D O-D matrix and the 2-D network. Existing 2-D networks can be used in the 3-D calculations; consequently there is no need for conversion or extra input of data. This factor is a practical advantage of the method.
2. Determine the 3-D O-D matrix. The 3-D matrix is determined by splitting up the 2-D O-D matrix into periods defined by the departure time fractions. This system is a good way to approximate the peak periods. For longer periods (e.g., holiday traffic), more complicated methods should be used to determine the 3-D matrix.
3. Determine the period capacity. The period capacity of the link can be determined as a fraction of the hourly capacity. The capacity is multiplied by the ratio of period length and 60 min. It is also possible to reduce some of the period capacities to account for delays caused by highway construction.
4. Determine the link delay. A 3-D delay function is used to determine the delay in 3-D links. This function is similar to that in 2-D space.

$$z_{jkp} = F(q_{jkp}, Q_{jkp}, C_{jkp}, z'_{jkp}) \quad (2)$$

where

q_{jkp} = the number of cars on link kj during period p,
Q_{jkp} = the number of waiting cars from previous periods,
C_{jkp} = the capacity (cars per unit) during period p,
z'_{jkp} = the time on the unloaded link jk during period p, and
z_{jkp} = the time on the loaded link jk during period p.

In general, z_{jkp} will have different values for the various periods. The overloaded links in *previous* periods influence the delay in the *later* periods.

5. Determine the shortest routes in 3-D space. Figure 3 shows a 3-D network with a string of links. Link 2-3 has a lower capacity than the other links. The Y-axis is the time scale. The links of the successive time periods are shown. The dashed lines are the 3-D paths of the first and last cars in each period. The last car in the first period is the same as the first car in the second period, and so on.

The departure time of the first car in the first period equals zero. This car uses links in period 1. The departure time of the last car in the first period equals 10. This car uses the links in the second period. Some of the cars that depart between the first and last cars are using links during the first and the second periods. The departure time of the first car of period 3 is 20 min. Because of a delay in node 2-3 caused by congestion, the car arrives in node 3 more than 10 min later. This car also uses links in different periods.

So the departure times are established in increments of 10 (0, 10, 20, 30, etc.), instead of zero as in 2-D assignment. The point at which the nodes are passed depends on delays, which may be different from period to period. In addition, the cars use links in different periods.

In 3-D space, the determination of routes from the origin is similar to that in 2-D space. It is possible to use the Moore

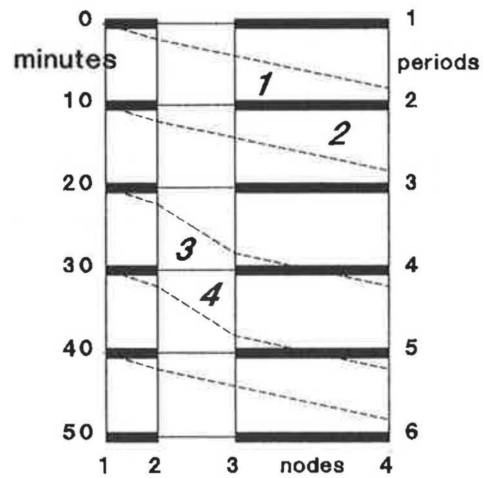

FIGURE 3 Example of the 3-D assignment in time space of a network.

or the Dijkstra algorithms. The difference is that this route determination is done for all the periods, rather than just one period, as in 2-D space. Another important difference is that the routes in 3-D space are found by comparing space paths and time paths simultaneously, which enables a comparison of routes between origin and destination.

6. Assign the 3-D O-D matrix to the shortest 3-D routes. The important difference is that in the 2-D space all car trips of an O-D pair are assigned to all links along the shortest route. Because in the 3-D space links are used during different periods, the cars must be assigned to different periods. The cars that depart in the first period use link 3-4 in the first and second periods. The ratio of the car trips that are assigned to link 3-4 in the first and second periods is proportionate to the areas marked *1* and *2* (Figure 3). The car trips of the third period are partly assigned to link 2-3 in the third and fourth periods. The ratio is proportionate to the areas marked *3* and *4*.

7. Load the network. Loading the network is done by part of the all-or-nothing assignment flows just calculated and with flows from the previous iteration. As in the 2-D space, it can be done in various ways.

Two methods have been tested. The first method is similar to the linear approximation method of the equilibrium method. The first experience with this method was not very successful, as was reported at the UTSG conference in London (*12*) in 1988. However, the method is being improved, so linear approximation may be useful after all. Some research is still required to make this suitable for publication.

The second algorithm uses the equation

$$q^i_{jkp} = q^+_{jkp} \cdot g^i + q^{i-1}_{jkp} \quad (3)$$

The value of g^i depends on the number of iterations (i) and will also be chosen in such a way that there are no overloaded links.

$$g^i = \min[1/(i + 1), (q^{i-1}_{jkp} - C_{jkp})/q^+_{jkp}] \quad (4)$$

EXAMPLE OF UPSTREAM CONGESTION

It is possible to gain insight into the problem of new congestion that appears after improvements upstream. The example in

Figure 2 is used to demonstrate the effect of increased capacity of links 7-6 and 6-5. Although the congestion on these links disappears, new congestion arises on links 2-3 and 5-8 downstream. Some traffic uses routes 3-4-10-9-2 and 2-1-8 (in period 3, 4, 6, 7) to avoid the congestion.

The calculation shows the influence of upstream bottlenecks on downstream congestion. It seems possible that downstream congestion can be prevented by delaying the traffic feeding onto links upstream. The 3-D assignment technique can give better insight into the ability of this method to improve the working of the traffic system.

COMPUTING

The system runs as part of the TFTP workbench on OLIVETTI M21 and PC-AT and PC-386 with EGA cards for small networks (*13*). The assignment of large networks is also possible. However, 3-D calculation needs more computer time than 2-D assignment. The calculation time is the product of

- The number of iterations,
- The number of time periods, and
- The time necessary for the calculation of an all-or-nothing assignment.

The calculation time necessary will be about 100 times a 2-D all-or-nothing assignment or 10 times an equilibrium assignment.

To improve the calculation speed, a special processor is being developed so the system can be used for very large networks. The first prototype of this processor is about 200 times faster than a Microvax. An even faster execution is possible (*14*). Because of these improvements it is expected that a longer calculation time will not be required for very large networks.

CONCLUDING REMARKS

Since the traditional 2-D assignment methods have some shortcomings, a time dimension has been introduced to improve this method. The algorithm, generally speaking, is similar to the 2-D variant. However, on a detailed level there are some differences that cannot be neglected: the method can be used for large networks; the existing 2-D networks can be used as input for the calculations; the calculation time is longer; and the development of computer hardware makes the method suitable for very large networks.

In conclusion, the dynamic assignment in 3-D time space can be used for the following purposes:

- A more realistic assignment of traffic on congested networks;
- Acquisition of new insights into new downstream congestion after improving capacity upstream;
- Ability to calculate, based on downstream congestion, the influence of decreases in capacity caused by such factors as road construction, road maintenance, and accidents;
- Ability to calculate the areawide effect of feeding cars into a network system on certain strategic chosen links; and
- Ability to use the program as part of a delay warning system during road congestion.

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