Bridge Service Life Prediction Model Using the Markov Chain

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This paper describes the application of Markov chain technique in estimating bridge service life. The change of bridge conditions is a stochastic process and, therefore, the service life of bridges is related to the probabilities of condition transitions. A bridge service life prediction model, using the Markov chain, was developed to reflect the stochastic nature of bridge condition and service life. The paper includes a discussion on the concept of Markov chain, the development and application of the service life prediction model using the Markov chain, and the comparison of service life predictions by statistical and Markov chain approaches.

The service life of a bridge is one of the most important factors for bridge managers to estimate or predict. This paper presents a bridge service life prediction model using Markov chain. The model was developed as a continuation of the earlier work on bridge performance analysis (*1*). The earlier paper involved the use of Markov chain in developing bridge performance curves. The present paper deals with prediction of bridge service life on the basis of performance analysis.

Bridge service life can be predicted by a regression analysis of bridge age versus bridge condition. However, the prediction is restricted to average or mean service life of a number of bridges. The Markov chain model provides a tool for predicting not only the average service life, but also the service life of any individual bridge.

This paper describes the bridge service life prediction model using the Markov chain. The Markov chain concept is introduced and the development and application of the model are discussed. The comparison is made between the approaches of statistical regression and Markov chain.

BRIDGE SERVICE LIFE

All federally supported bridges in Indiana have been inspected every 2 years beginning in 1978. The inspection includes rating of individual components such as deck, superstructure and substructure, as well as of the overall bridge condition. According to the FHWA bridge rating system, bridge inspectors use a range from 0 to 9, with 9 being the maximum rating number for a new condition (*2*). When the rating reaches a value of 3, a bridge has to be repaired or replaced, otherwise it should be closed. Therefore, the time span between a bridge being built and its condition reaching the rating of 3 was defined as the service life of the bridge. Two approaches of estimating bridge service life—statistical and Markov chain—are discussed.

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STATISTICAL APPROACH

The performance curves of bridge components, such as deck, superstructure, and substructure, were developed separately for concrete and steel bridges on both Interstate and non-Interstate highways. Two of these curves, the performance curves of substructures of concrete and steel bridges on non-Interstate highways, are discussed to demonstrate the prediction methods.

The objective of developing performance curves was to find the relationship between condition rating and bridge age. A third-order polynomial model was used to obtain the regression function of the relationship. The polynomial model is expressed by the following formula (*3*):

$$Y_i(t) = \beta_0 + \beta_1 t_i + \beta_2 t_i^2 + \beta_3 t_i^3 + \varepsilon_i \tag{1}$$

where $Y_i(t)$ is the condition rating of a bridge at age t, t_i is the bridge age, and ε_i is the error term. This equation indicates that the condition rating of a bridge, $Y_i(t)$, depends on the bridge age of the bridge, t_i.

For a new bridge (age 0), the recorded condition rating was found always to be 9; therefore β_0 was specified as 9 to make the intercept of the regression line an integer and meaningful in practice.

When a regression model is selected for an application, it is usually not possible to make certain in advance that the model is appropriate for that application. Therefore, two regression assumptions, the constancy and normality of residual distribution, should be tested (*3*).

The Statistical Analysis System (SAS) statistical package was used for the test of aptness of polynomial model and for regression analysis (*4*). Residual plots were obtained to check the constancy of variance and the Kolomogorov-Smirnov test was used to test the normality of residual distribution. It was found that the polynomial regression function on raw data of concrete bridges met the two aptness requirements, the constancy and normality of residual distribution. However, the regression function on raw data of steel bridges did not meet the requirements. A transformation of $y' = \sqrt{y}$ on the raw data of steel bridges satisfied the necessary normality and constancy requirements.

The complete data base included about 5,700 state-owned bridges in Indiana. To evaluate the effects of the climate, traffic volume, highway system, and bridge type on bridge performance, bridges were divided into groups such as steel and concrete bridges; bridges with high, medium, or low average daily traffic; bridges in northern and southern regions; and bridges on Interstate and non-Interstate highways. Because the factors of traffic volume and climatic region were found not statistically significant, the final factors considered in the regression analysis were highway class (Interstate and non-Interstate) and bridge type (steel and concrete). For example, to develop the performance curves of bridge substructures on non-Interstate highways, 90 concrete bridges and 90 steel bridges on non-Interstate highways were randomly selected from the complete data base. The recorded data of condition ratings and corresponding ages of these bridges from 1978 to 1986 were used to perform the regression analysis. The resulting regression function for substructure of concrete bridges was

$$Y_i(t) = 9.0 - 0.28877329t_i + 0.0093685t_i^2 - 0.00008877t_i^3 \quad (2)$$

The regression function for substructure of steel bridges was

$$Y_i(t) = \left(\sqrt{Y_i(t)}\right)^2 = (3.0 - 0.051696t_i + 0.001715t_i^2 - 0.000021t_i^3)^2 \quad (3)$$

Equation 3 was obtained using the transformed data and so it had different form from Equation 1.

The regression function can be used to predict service life; that is, the value of t_i corresponding to $Y_i(t) = 3$ is nothing but the estimated bridge substructure service life. Figures 1 and 2 show the curves of the regression functions and the estimated service lives obtained by solving the functions. The predicted service lives of concrete and steel bridge substructures are both 54 years, as shown in Figures 1 and 2, where *SL* represents service life and the subscripts *c* and *s* denote concrete and steel bridges.

In reality, service life varies from bridge to bridge. However, by the definition of regression analysis (*3*), $Y(t)$ is the average condition rating at bridge age *t*, that is, $\bar{r} = Y(t)$, so the service life obtained from the performance function is actually the mean or average service life of bridge substructures on non-Interstate highways, that is, $\bar{T} = SL$. As shown in Figure 3, A and B are two steel bridges with the same

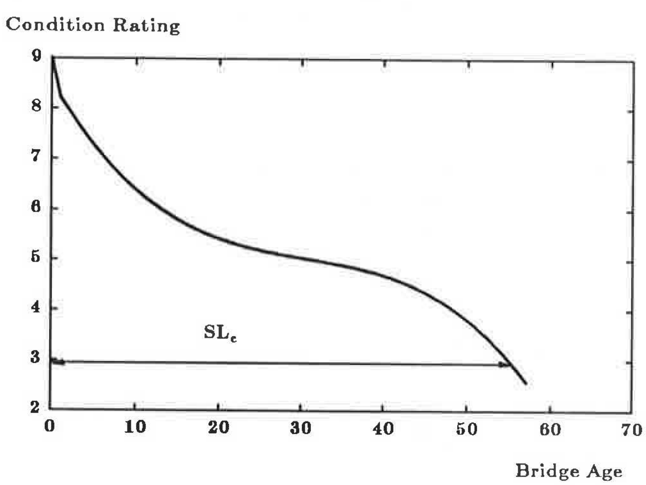

FIGURE 1 Performance curve of substructure condition of concrete bridges.

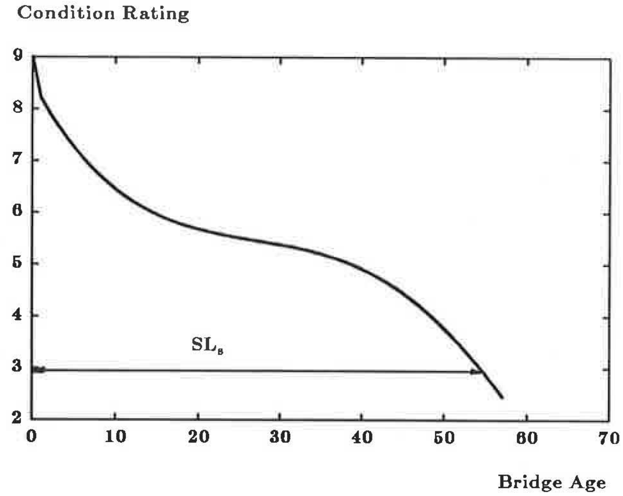

FIGURE 2 Performance curve of substructure condition of steel bridges.

bridge ages but different substructure condition ratings. The condition ratings are r_a and r_b for bridges A and B, respectively. Substructure service life of Bridge A may be predicted by the performance function because Point A is on the performance curve. Thus, it may be expected that substructure service life of bridge A equals to $\bar{T}$, the average service life of the substructures, that is, $t_1 + t_{2a} = \bar{T}$; where t_1 is the bridge age and t_{2a} is the remaining substructure service life of Bridge A. The remaining substructure service life of Bridge B, t_{2b}, cannot be estimated from the performance function because Point B is not on the performance curve. It can be guessed, however, that t_{2b} would be less than t_{2a}. It is necessary for a bridge manager to estimate t_{2b} as well as t_{2a}. Therefore, the Markov chain technique was applied to fulfill the task.

MARKOV CHAIN APPROACH

Introduction to Markov Chain

The Markov chain as applied to bridge service life prediction is based on the concept of defining states in terms of bridge condition ratings and obtaining the probabilities of bridge condition transiting from one state to another. These probabilities are represented in a matrix form that is called the transition probability matrix or simply, transition matrix, of the Markov chain. If the present state of bridge conditions or the initial state is known, the future condition and the time needed to change condition from one rating to another can be predicted through multiplications of initial state vector and the transition matrix.

Seven bridge condition ratings are defined as seven states with each condition rating corresponding to one of the states. For example, Condition 9 is defined as State 1, Rating 8 as State 2, and so on. Without repair or rehabilitation, the bridge condition rating decreases as the bridge age increases. Therefore, there is a probability of condition transiting from one state, say *i*, to another state, *j*, during a 1-year period, which is denoted by $p_{i,j}$. Table 1 shows the correspondence of condition ratings, states, and transition probabilities. Because the

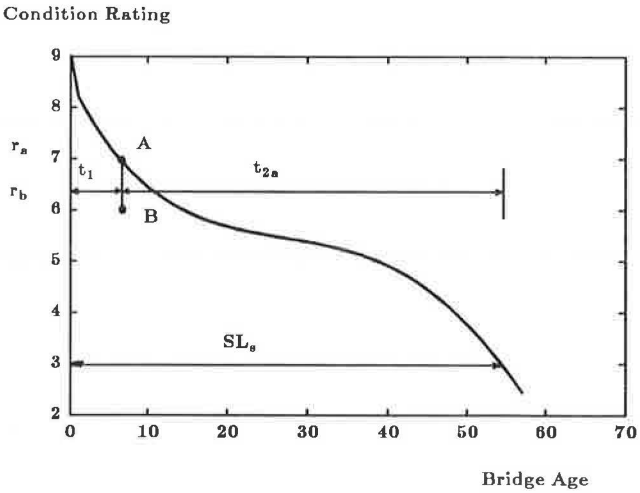

FIGURE 3 Example of service life predictions.

TABLE 1 CORRESPONDENCE OF CONDITION RATINGS, STATES, AND TRANSITION PROBABILITIES

		R=9	R=8	R=7	R=6	R=5	R=4	R=3
		S=1	S=2	S=3	S=4	S=5	S=6	S=7
R=9	S=1	$p_{1,1}$	$p_{1,2}$	$p_{1,3}$	$p_{1,4}$	$p_{1,5}$	$p_{1,6}$	$p_{1,7}$
R=8	S=2	$p_{2,1}$	$p_{2,2}$	$p_{2,3}$	$p_{2,4}$	$p_{2,5}$	$p_{2,6}$	$p_{2,7}$
R=7	S=3	$p_{3,1}$	$p_{3,2}$	$p_{3,3}$	$p_{3,4}$	$p_{3,5}$	$p_{3,6}$	$p_{3,7}$
R=6	S=4	$p_{4,1}$	$p_{4,2}$	$p_{4,3}$	$p_{4,4}$	$p_{4,5}$	$p_{4,6}$	$p_{4,7}$
R=5	S=5	$p_{5,1}$	$p_{5,2}$	$p_{5,3}$	$p_{5,4}$	$p_{5,5}$	$p_{5,6}$	$p_{5,7}$
R=4	S=6	$p_{6,1}$	$p_{6,2}$	$p_{6,3}$	$p_{6,4}$	$p_{6,5}$	$p_{6,6}$	$p_{6,7}$
R=3	S=7	$p_{7,1}$	$p_{7,2}$	$p_{7,3}$	$p_{7,4}$	$p_{7,5}$	$p_{7,6}$	$p_{7,7}$

Note: R = Condition Rating

S = State

$p_{i,j}$ = Transition Probability from State i to State j

lowest recorded rating number in the data base was 3, indicating that the bridges are usually repaired or replaced at rating not less than 3, condition ratings less than 3 were not included in the transition matrix.

As a bridge age increases, the deteriorating rate of bridge conditions changes. That is, the process of condition transition is not homogeneous with respect to bridge age. To meet the homogeneity requirement of Markov chain, bridge age was divided into groups, and within each group the Markov chain was assumed to be homogeneous. Age groups consisting of 6 years were used and each group had its own transition matrix that was different from those of remaining groups.

An assumption was made that the bridge condition rating would not drop more than 1 in a single year. Thus, the bridge condition would either stay in its current rating or transit to the next lower rating in 1 year. The transition matrix has, therefore, the form

$$P = \left| \begin{matrix} p(1) & q(1) & 0 & 0 & 0 & 0 & 0 \\ 0 & p(2) & q(2) & 0 & 0 & 0 & 0 \\ 0 & 0 & p(3) & q(3) & 0 & 0 & 0 \\ 0 & 0 & 0 & p(4) & q(4) & 0 & 0 \\ 0 & 0 & 0 & 0 & p(5) & q(5) & 0 \\ 0 & 0 & 0 & 0 & 0 & p(6) & q(6) \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{matrix} \right| \quad (4)$$

where $q(i) = 1 - p(i) \cdot p(i)$ corresponds to $p_{i,i}$ and $q(i)$ to $p_{i,i+1}$ in Table 1. Therefore, $p(1)$ is the transition probability from Rating 9 (State 1) to Rating 9, and $q(1)$, from Rating 9 to Rating 8, and so on.

It should be noted that the lowest rating number before a bridge is repaired or replaced is 3. Consequently, the corresponding transition probability $p(7)$ equals 1.

Development of Transition Matrix

To estimate the transition probabilities for each age group, the following nonlinear programming objective function was formulated:

$$\min \sum_{t=1}^{N} | Y(t) - E(t, P) | \quad (5)$$

subject to

$$0 \leq p(i) \leq 1 \qquad i = 1, 2, \ldots, I$$

where

N = 6, the number of years in one age group,
I = 6, the number of unknown probabilities,
P = a vector of length I equal to $[p(1), p(2), \ldots, p(I)]$,
$Y(t)$ = the average of condition ratings at time t, estimated by regression function, and
$E(t,P)$ = estimated value of condition rating by Markov chain at time t.

By Markov chain, the state vector for any time t, $Q_{(t)}$, can be obtained by the multiplication of initial state vector $Q_{(0)}$ and the transition probability matrix P raising the power to t (5):

$$Q_{(t)} = Q_{(0)} * P^t \quad (6)$$

Let R be a vector of condition ratings, $R = [9\ 8\ 7\ 6\ 5\ 4\ 3]$, and R' be the transform of R, then the estimated condition rating by Markov chain is,

$$E(t, P) = Q_{(t)} * R' \quad (7)$$

The objective function was to minimize the absolute distance between the average bridge condition rating at a certain age and the predicted bridge condition rating for the corresponding age generated by the Markov chain with the probabilities obtained by the nonlinear programming. The values of the regression function were taken as the average conditions, $Y(t)$, to solve the nonlinear programming or Equation 5.

A new bridge is almost always given a condition rating of

9 for all of its components, deck, superstructure, and substructure. In other words, a bridge at age 0 has a condition rating 9 for its components with unit probability. Thus, the initial state vector $Q_{(0)}$ for deck, superstructure, and substructure of a new bridge is always [1 0 0 . . . 0], where the numbers are the probabilities of having a condition rating of 9, 8, 7, . . ., and 3 at age 0, respectively. That is, the initial state vector of the first age group is known. Therefore, Equation 5 can be solved for Age Group 1. Age Group 2 takes the last state vector of Age Group 1 as its initial state vector, Equation 5 can also be solved for Age Group 2. In the same manner, the transition probabilities for all the age groups were obtained, as shown in Tables 2 and 3. For example, $p(1) = 0.717$ in Group 1 (Age 6 years or less) of Table 3 represents the transition probability from State 1 to State 1 in a 1-year period for substructure of steel bridges with age less or equal to 6 years, and the transition probability from State 1 to State 2 in a 1-year period is, therefore, $q(1) = 1 - 0.717 = 0.283$. The solution to this function was obtained by the Quasi-Newton method (*6*).

Application of Markov Chain Model

Once the transition matrices are obtained, the prediction of bridge service life can be conducted using Equations 6 and 7. By this approach, the service life prediction is not restricted to the average service life of bridges. Instead, the prediction can be made for any individual bridge or bridge component at a given bridge age with any condition rating. For the two bridges in Figure 3, the Markov chain model can be used to predict service life for both Bridge A and Bridge B.

For demonstration purpose, we assume Bridges A and B are both 9-year-old steel bridges. The substructure condition rating is 7 for Bridge A and 6 for Bridge B. From Table 3, the corresponding transition matrix for Age 9 (in Age Group 2) is:

$$P = \begin{vmatrix} 0.366 & 0.634 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.715 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.285 & 0.030 & 0 & 0 & 0 \\ 0 & 0 & 0.970 & 0.814 & 0.186 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.574 & 0.426 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.359 & 0.641 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.000 \end{vmatrix} \quad (8)$$

The initial state vector of Bridge A is $Q_{(0)} = [0\ 0\ 1\ 0\ .\ .\ .\ 0]$, with the unit probability corresponding to Rating 7 ($r_a = 7$). Therefore, the state vector of Bridge A in the future can be predicted by Equation 6. The state vectors and corresponding condition ratings are as follows:

$$R = [9\ 8\ 7\ 6\ 5\ 4\ 3]$$

$$Q_{(0)} = [0\ 0\ 1\ 0\ 0\ 0\ 0]$$

$$E(0, P) = Q_{(0)} * R' = 7$$

$$Q_{(1)} = Q_{(0)} * P = [0.00\ 0.00\ 0.97\ 0.03\ 0.00\ 0.00\ 0.00]$$

$$E(1, P) = Q_{(1)} * R' = 6.97 \approx 7$$

$$Q_{(2)} = Q_{(0)} * P^2 = [0.00\ 0.00\ 0.94\ 0.05\ 0.01\ 0.00\ 0.00]$$

$$E(2, P) = Q_{(2)} * R' = 6.94 \approx 7$$

$$Q_{(3)} = Q_{(0)} * P_3 = [0.00\ 0.00\ 0.89\ 0.08\ 0.03\ 0.00\ 0.00]$$

$$E(3, P) = Q_{(3)} * R' = 6.87 \approx 7$$

Then, $Q_{(3)}$ obtained above is taken as the initial state vector of Age Group 3 (from Age 13 to Age 18) and the transition matrix for Age Group 3 is used to continue the procedure.

TABLE 3 TRANSITION PROBABILITIES FOR SUBSTRUCTURE CONDITIONS—STEEL BRIDGES, NON-INTERSTATE

Bridge Age	p(1)	p(2)	p(3)	p(4)	p(5)	p(6)
0 - 6	0.717	0.727	0.950	0.664	0.692	0.700
7 - 12	0.366	0.715	0.970	0.814	0.574	0.359
13 - 18	0.700	0.507	0.950	0.653	0.950	0.766
19 - 24	0.700	0.707	0.950	0.950	0.950	0.950
25 - 30	0.700	0.700	0.950	0.950	0.950	0.950
31 - 36	0.200	0.200	0.200	0.950	0.696	0.447
37 - 42	0.200	0.200	0.200	0.950	0.950	0.950
43 - 48	0.200	0.200	0.200	0.950	0.790	0.722
49 - 54	0.200	0.200	0.200	0.916	0.050	0.050
55 - 60	0.200	0.200	0.200	0.050	0.050	0.050

TABLE 2 TRANSITION PROBABILITIES FOR SUBSTRUCTURE CONDITION—CONCRETE BRIDGES, NON-INTERSTATE

Bridge Age	p(1)	p(2)	p(3)	p(4)	p(5)	p(6)
0 - 6	0.705	0.818	0.810	0.802	0.801	0.800
7 - 12	0.980	0.709	0.771	0.980	0.980	0.856
13 - 18	0.638	0.639	0.748	0.980	0.980	0.980
19 - 24	0.798	0.791	0.788	0.980	0.870	0.824
25 - 30	0.794	0.810	0.773	0.980	0.980	0.980
31 - 36	0.815	0.794	0.787	0.980	0.980	0.737
37 - 42	0.800	0.798	0.815	0.980	0.850	0.980
43 - 48	0.800	0.800	0.309	0.938	0.980	0.050
49 - 54	0.800	0.800	0.800	0.711	0.707	0.768
55 - 60	0.800	0.800	0.800	0.050	0.050	0.050

In general, Group n takes the last state vector of Group $n - 1$ as its initial state vector.

By this procedure, $E(45, P) = 3$ is obtained, therefore t_{2a} in Figure 1 is estimated to be 45 years. The service life of Bridge A, T_A, is predicted:

$$T_A = t_1 + t_{2a} = 9 + 45 = 54 \text{ years}$$

As expected, the value of T_A is the same as the one obtained by regression function, $SL_s = 54$, because Point A is on the performance curve.

Applying the Markov chain technique to Bridge B, $E(23, P) = 3$ is obtained. That is, t_{2b} is 23 and the service life of Bridge B is

$$T_B = t_1 + t_{2b} = 9 + 23 = 32 \text{ years}$$

As we expected, T_B is less than T_A. Similarly, for any possible value of Condition Rating r at any time, the service life can be predicted by the Markov chain. However, these predictions cannot be made by the regression function for condition ratings other than $r = 7$.

Test of Accuracy

The accuracy of the service life prediction depends on the closeness of the values of condition ratings predicted by the Markov chain and by the regression function. The chi-square goodness of fit test (7) is used to measure the closeness of the predicted values of condition ratings. The computed chi-square is given by:

$$\chi^2 = \sum_{i=1}^{k} \frac{(E_i - Y_i)^2}{E_i} \tag{8}$$

where

χ^2 = has a chi-square distribution with $k - 1$ degrees of freedom,
E_i = value of condition rating in Year i predicted by the Markov chain,
Y_i = value of condition rating in Year i predicted by the regression function, and
k = the number of years predicted.

As an example, the results of the chi-square test for Bridge A are presented. The chi-square test was performed by using the values of $E(t, P)$ and $Y(t)$ from $t = 10$ to $t = 54$ ($k = t_{2a} = 45$) and the results indicated that the difference between the values of the Markov chain and regression function was not significant at $\alpha = 0.05$, as shown here:

$$\chi^2 = \sum_{i=1}^{45} \frac{(E_i - Y_i)^2}{E_i} = 3.111$$

$$CHI^2_{44} (\chi^2 \geq 3.111) > 0.995 > \alpha = 0.05$$

Therefore, the values of condition ratings predicted by the two approaches, Markov chain and regression function, were very close.

TABLE 4 PREDICTED SUBSTRUCTURE SERVICE LIVES OF CONCRETE BRIDGES FOR DIFFERENT CONDITION RATINGS

Age	$\bar{r}_i$	r_i	Δr_i	Predicted SL	Overestimation of SL
0	9	9	0	54	0
4	8	9	1	54	0
		8	0	54	0
		7	-1	54	0
		6	-2	54	0
		5	-3	35	19
9	7	8	1	54	0
		7	0	54	0
		6	-1	54	0
		5	-2	40	14
		4	-3	19	35
18	6	8	2	54	0
		7	1	54	0
		6	0	54	0
		5	-1	42	12
		4	-2	22	32
42	5	7	2	55	-1
		6	1	54	0
		5	0	54	0
		4	-1	43	11
52	4	6	2	56	-2
		5	1	55	-1
		4	0	54	0

Note: $\bar{r}_i$ = Average Condition Rating
r_i = Actual Condition Rating
$\Delta r = r_i - \bar{r}_i$
SL = Service Life
(Average Service Life = 54 Years)

TABLE 5 PREDICTED SUBSTRUCTURE SERVICE LIVES OF STEEL BRIDGES FOR DIFFERENT CONDITION RATINGS

Age	$\bar{r}_i$	r_i	Δr_i	Predicted SL	Overestimation of SL
0	9	9	0	54	0
4	8	9	1	54	0
		8	0	54	0
		7	-1	53	1
		6	-2	25	29
		5	-3	8	46
9	7	8	1	54	0
		7	0	54	0
		6	-1	32	22
		5	-2	24	30
		4	-3	10	44
25	6	8	2	55	-1
		7	1	55	-1
		6	0	54	0
		5	-1	34	20
		4	-2	30	24
44	5	7	2	56	-2
		6	1	56	-2
		5	0	54	0
		4	-1	47	7
52	4	6	2	56	-2
		5	1	54	0
		4	0	54	0

Note: $\bar{r}_i$ = Average Condition Rating
r_i = Actual Condition Rating
$\Delta r = r_i - \bar{r}_i$
SL = Service Life
(Average Service Life = 54 Years)

COMPARISON OF SERVICE LIFE PREDICTIONS USING PERFORMANCE CURVE AND MARKOV CHAIN

The difference between the average service life of bridges, $\overline{T}$, and the predicted service life of a bridge, T_i, are defined as the overestimation of the bridge's service life by the performance curve developed through statistical regression, ΔT_i, that is, $\Delta T_i = \overline{T} - T_i$. Thus, the overestimation of service life of Bridge A and that of Bridge B in Figure 3 are:

$$\Delta T_A = \overline{T} - T_A = 54 - 54 = 0 \text{ years}$$

$$\Delta T_B = \overline{T} - T_B = 54 - 32 = 22 \text{ years}$$

where $\overline{T} = SL_s$.

When the results are examined, it can be found that the overestimation of a bridge's service life is related to the difference of the actual condition rating and the average condition rating. The r_a is equal to 7, which is the same as the value of the regression function, or average condition rating, corresponding to the bridge age of 9. Thus, the difference of r_a and the average condition rating, $\bar{r} = Y(9)$, is

$$\Delta r_a = r_a - \bar{r} = 7 - 7 = 0$$

which causes no overestimation of the bridge's service life, that is, $\Delta T_A = 0$.

However, r_b is equal to 6 and the condition rating difference is,

$$\Delta r_b = r_b - \bar{r} = 6 - 7 = -1$$

Therefore, $\Delta T_B = 22$ can be considered as the overestimation of service life by performance curve resulting from overestimating condition rating by one unit. The effect of Δr_i on ΔT_i is explained by the fact that $Q_{(0)}$ is determined by the actual condition rating of a bridge and the Markov chain equation (Equation 6). Once the transition matrix, P, is known, the state vector, $Q_{(t)}$, is determined by the initial state vector, $Q_{(0)}$.

In order to analyze this effect, the service lives of bridge substructures were obtained by the Markov chain model for various possible condition ratings. Tables 4 and 5 show the results and the corresponding values of Δr and ΔT. Comparing values of the two tables, it can be found that the values of service life overestimations for substructures of concrete and steel bridges are different. For example, for $\bar{r}_i = 7$ and $\Delta r_i = 1$, the overestimation of service life is 0 for concrete bridge substructures and 22 for steel bridge substructures.

The values of bridge substructure service life overestimations for all possible combinations of $\bar{r}_i$ and Δr_i are presented in Table 6. The paired data in Table 6 are the overestimations of service lives for steel and concrete bridge substructures. For example, for $\bar{r}_i = 6$ and $\Delta r_i = -1$, the corresponding data (20, 12) represent that the overestimation of substructure service life is 20 for a steel bridge and 12 for a concrete bridge, or $\Delta T_{si} = 20$ and $\Delta T_{ci} = 12$. The differences of the values in the table were used to perform the paired-t statistical test (*3*):

$$x_i = \Delta T_{si} - \Delta T_{ci}$$

where ΔT_{si} and ΔT_{ci} are the overestimations of service lives of steel and concrete bridge substructures by the performance function. Because $t^* = 2.03 > t(0.95,25) = 1.708$, the mean difference in overestimation of steel and concrete substructure service lives is significant at $\alpha = 0.05$. That is, the overestimation of service life of steel bridge substructure is more sensitive to overestimation of condition rating with respect to actual condition rating.

As can be seen, the service life of a bridge is affected by bridge type and the difference of actual condition rating and average condition rating. The average condition rating is

TABLE 6 PREDICTIONS OF OVERESTIMATION OF SERVICE LIVES FOR SUBSTRUCTURES OF STEEL AND CONCRETE BRIDGES

		Actual Rating - Average Rating					
		2	1	0	-1	-2	-3
Average Rating	9	-	-	(0,0)*	-	-	-
	8	-	(0,0)	(0,0)	(1,0)	(29,0)	(46,19)
	7	-	(0,0)	(0,0)	(22,0)	(30,14)	(44,35)
	6	(-1,0)	(-1,0)	(0,0)	(20,12)	(24,32)	(36,36)
	5	(-2,-1)	(-2,0)	(0,0)	(7,11)	(10,12)	-
	4	(-2,-2)	(0,-1)	(0,0)	(2,2)	-	-

*: (a, b) -- a is the overestimation of service life of steel bridge

b is the overestimation of service life of concrete bridge

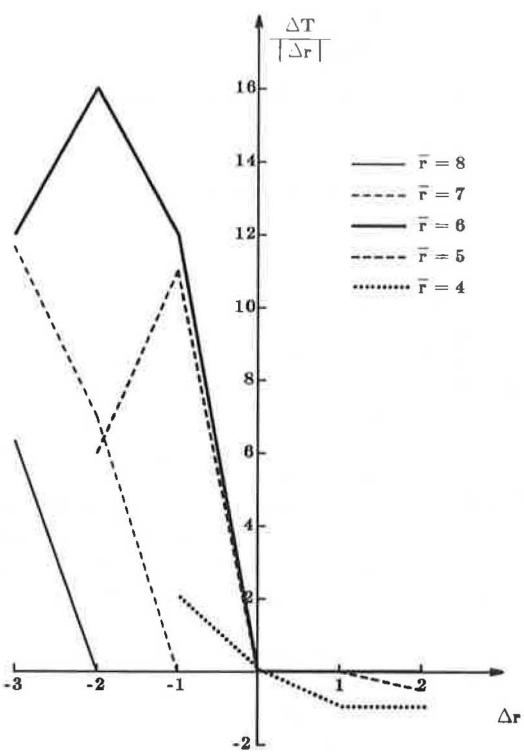

FIGURE 4 Service life sensitivity analysis for substructure of concrete bridges.

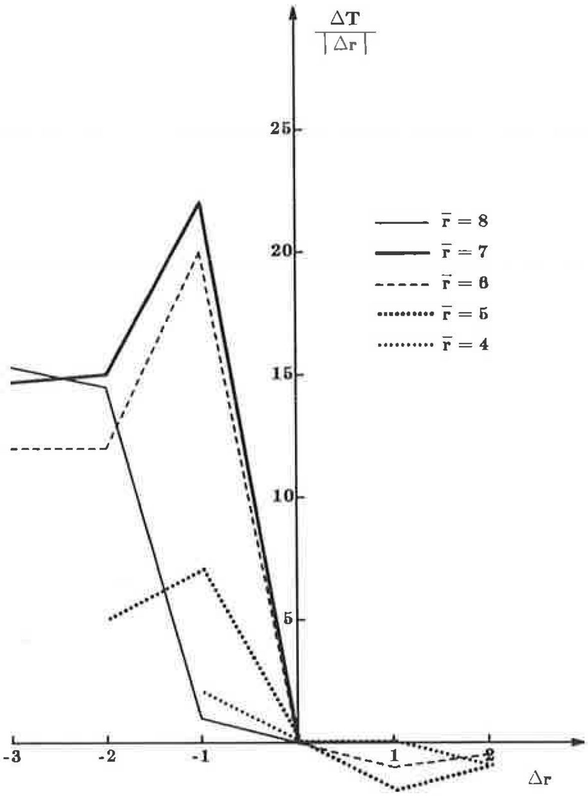

FIGURE 5 Service life sensitivity analysis for substructure of steel bridges.

obtained by regression function, which is the relationship between bridge age and condition rating. Therefore, the sensitivity of overestimation of bridge service life to bridge age can be studied by analyzing the sensitivity of overestimation of bridge service life to average condition rating.

Figures 4 and 5 present the curves of Δr versus $\Delta T/|\Delta r|$ for substructures of concrete and steel bridges, where $\Delta T/|\Delta r|$ represents the overestimation of service life per unit change of condition rating from average condition rating. Figure 4 shows that the curve for the average condition rating $\bar{r}_i = 6$, for substructure of concrete bridges, has the highest value of $\Delta T/\Delta r$. The average age of concrete bridge substructures corresponding to the rating of 6 is 25. Therefore, the rating given at this age can be considered to be most sensitive with respect to service life estimation. From Figure 4, it can be seen that when the average performance curve for concrete bridge substructures reaches a condition rating of 6 (Age 25), if a lower-than-average condition rating is given, the service life of the bridge would be overestimated by as much as 16 years for every unit of lower-than-average condition rating. It means that, for concrete bridge substructures, the expected overestimation of service life per unit of lower-than-average condition rating is the highest when the performance curve reaches the condition rating of 6. From Figure 5, the corresponding most critical level for steel bridge substructures is when the performance curve reaches the condition rating of 7 (Age 9). The expected overestimation in this case is 22 years. The implications of these results are that average performance functions can only be used for making macroscopic decisions at network level. However, for making decisions at a project level, Markov chain is a better tool.

CONCLUSIONS

The application of the Markov chain provides bridge managers a powerful and convenient tool for estimating bridge service life. Service life prediction by Markov chain has the advantage over the statistical regression approach in that it can be used not only to estimate the average service life of a number of bridges, but also the service life of any individual bridge. Furthermore, the Markov chain prediction is based on the current condition and age of bridges, therefore, it is simple and can be updated by new information of condition rating and bridge age. However, it should be noted that this study was based on statistical analysis of condition ratings. Condition ratings are subjective judgments, which may in themselves be biased, and therefore may affect the results of service life predictions. To reduce the bias of human judgments, a bridge condition evaluation model has been developed using the theory of fuzzy sets (*8*); the reliability and accuracy of service life predictions could be greatly enhanced by applying the condition evaluation model in the process of bridge condition inspection.

The theory of the Markov chain is well developed and based on simple multiplications of matrices. As compared with the regression method, the Markov chain model, a probability-based method, reflects better the stochastic nature of bridge service life. The model provides a mathematical tool for predicting bridge service life. The comparison of service life predictions of the two models enables bridge managers to study the effects of bridge types, condition ratings, and bridge ages on service lives of bridges.

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