Comparison of Macroscopic Models for Signalized Intersection Analysis

Lawrence T. Hagen and Kenneth G. Courage

The signalized intersection methodology presented in the 1985 Highway Capacity Manual (HCM) introduced a new delay model, which naturally invites comparison with the delay models contained in the existing traffic signal timing and analysis techniques. This paper compares the HCM delay computations with those performed by the Signal Operations Analysis Package (SOAP) and by TRANSYT-7F Release 5. The paper focuses on the effect of the degree of saturation, the peak-hour factor, and the period length on delay computations and on the treatment of left turns opposed by oncoming traffic. All of the models agreed closely at volume levels below the saturation point. When conditions became oversaturated, the models diverged; however, they could be made to agree by the proper choice of parameters. The computed saturation flow rates for left turns opposed by oncoming traffic also agreed closely. However, the treatment of protected plus permitted left turns produced substantial differences. It was concluded that neither SOAP nor the HCM treats this case adequately. Therefore, an alternative model based on a deterministic queuing process was proposed and evaluated.

The 1985 Highway Capacity Manual (HCM) (1) introduced a new analytical technique for modeling the operation of traffic at signalized intersections. This represents a significant change from the previous HCM methodology. The change was required to support the newly adopted use of vehicular delay as a measure of the level of service experienced by drivers at traffic signals. Previous techniques were based on the relationship between traffic volume and intersection capacity.

Traffic signal operations have received extensive analytical treatment in the past. It should not be surprising that several computerized models already exist; it is also natural that the introduction of a new model would invite comparisons with the established methodology. This paper attempts to make such a comparison using two traffic signal timing design and analysis programs that have been employed extensively in the United States. The comparisons are limited to isolated traffic signal operations. In other words, no consideration is given to the effect of coordination with adjacent signalized intersections, which has been discussed in a previous paper (2).

The two existing programs considered are the Signal Operations Analysis Package (SOAP) (3) and the Traffic Network Study Tool (TRANSYT) (4). SOAP is concerned only with single, isolated intersections, while TRANSYT treats such intersections as a special case in a traffic signal network, which is its primary area of application. The version of TRANSYT used is TRANSYT-7F, Release 5.

DELAY MODEL COMPARISON

Delay is an important measure in traffic engineering since it represents a direct cost of fuel consumption and an indirect cost of the time lost to motorists. Delay results when traffic is impeded by factors beyond the motorist's control. It may be due to interference from other motorists or attributable to the traffic control devices themselves.

Webster's Delay Model

Before comparing the SOAP and TRANSYT models with the HCM model, it is important to understand what they represent and where they originated. Both models are variations of one first proposed by Webster in 1958 (5). Webster's delay model for intersections controlled by pretimed signals is given in the following equation:

\[
D = \frac{C(1 - \lambda)^2}{2(1 - \lambda \lambda_1)} + \frac{X^2}{2q(1 - X)} - 0.65 \left(\frac{C}{q^2}\right)^{1/3} [X^{1/2 + 1/3}] \tag{1}
\]

where

\[D = \text{average delay per vehicle on the subject approach or movement},\]
\[\lambda = \text{proportion of the cycle that is effectively green for the approach or movement (g/C)},\]
\[X = \text{degree of saturation [volume/capacity (V/C)}],\]
\[C = \text{cycle length},\]
\[q = \text{flow rate (the average number of vehicles passing a given point on the road in the same direction per unit of time), and}\]
\[g = \text{effective green time.}\]

The first term of Webster's equation represents the average delay per vehicle, assuming the vehicles arrive at a uniform rate (q) throughout the cycle. The second term attempts to account for the fact that the vehicles arrive randomly, not uniformly. This term increases rapidly with the degree of saturation. The third term is an adjustment factor developed semiempirically to provide a better mathematical fit to the theoretical curve. The upper limit for Webster's model was generally considered to be slightly less than full saturation.
Enhancements to Webster’s Model

All techniques considered in this paper use a two-component version of Webster’s original three-component model. The first term accounts for the delay experienced by traffic with uniform arrivals from cycle to cycle, and the second term provides for the additional delay caused by randomness in the arrival patterns.

The first terms differ as follows:

- SOAP uses Webster’s first term directly.
- The HCM uses Webster’s first term divided by 1.3 to reflect only the stopped delay portion of the total delay.
- TRANSYT divides the cycle into 60 equal steps and accumulates arrivals over each step. This produces results identical to Webster’s first term for isolated operation.

So, in effect, all of the models use the same formulation for the first term.

The second terms have the following differences:

- SOAP and TRANSYT both use Robertson’s model (4), which was originally incorporated into TRANSYT to account for oversaturation. The length of the oversaturated period is an important factor in the result produced by this model. Neither SOAP nor TRANSYT uses a peak-hour factor (PHF) since peaking characteristics are treated implicitly in the model.

- The HCM uses a different formulation to account for oversaturation. In this case, the period of oversaturation is assumed to be 15 min. The PHF has a strong influence on the results.

So, each model uses one parameter that is not recognized by the other. By choosing the proper combination of values for these parameters, it should be possible to produce more or less equivalent results.

Example Problem

A simple example is presented to determine how the HCM and SOAP differ when they are analyzing the same conditions. The example deals with the intersection of two two-lane, two-way streets with only through movements on all approaches.

The delay models were implemented over a wide range of volumes for the same capacity. The results for undersaturated conditions are summarized in Figure 1, which shows that the two models are very similar for PHF = 1.0. For PHF = 0.9, the delay values from SOAP and the HCM are significantly different because the PHF increases the flow rate only in the HCM.

Figure 2 reveals a noticeable difference in delay values for oversaturated conditions because SOAP uses a 60-min period and the HCM is based on a 15-min period. Therefore, SOAP

![Graph comparing SOAP and HCM delay values for undersaturated conditions.](attachment:figure1.png)
permits left turns. This is accomplished through the use of the PHF.

In Figure 4, the HCM's delay values for PHF = 1 are compared with SOAP's delay for various period lengths. The figure shows that SOAP's delay values for a period length of 40 min agree well with the HCM for large V/C values. It is also apparent that the period length is a significant factor in SOAP's delay equation.

Figures 3 and 4 demonstrate that the HCM's delay values for PHF = 1.0 compare well with SOAP's delay values for 60-min period lengths provided the intersection is undersaturated. For oversaturated conditions, however, a small difference in PHF or period length leads to significantly different results.

PERMITTED LEFT TURNS

One of the most difficult traffic movements to analyze at a signalized intersection is the permitted left turn. Through and unobstructed right-turning movements can essentially proceed at their saturation flow rate throughout the green phase. On the other hand, permitted left turns must filter through gaps in the oncoming traffic at a much lower saturation flow rate. The saturation flow rate for opposed left turns is primarily a function of the opposing traffic volume.

To analyze this complex situation, both SOAP and the HCM compute the unsaturated green time for the opposed left turns based on the opposing volumes. In the HCM, a supplemental worksheet for permitted left turns is used. This worksheet contains a series of complex equations to calculate the effects of the opposing traffic on the left-turning traffic. It also computes the impact of the left-turning traffic on through vehicles in shared lanes.

Example Problem

An example is used to demonstrate the comparison of the permitted left-turn models. It is similar to the example for the previous section except that these approaches include exclusive left-turn lanes.

The intersection configuration is shown in Figure 5. To ensure that any differences that arise are due to differences in the models themselves, it is assumed that there are no trucks, buses, pedestrians, or grades at the subject intersection.

All of the streets shown in the figure are three-lane, two-way streets with a center left-turn lane and through lanes on the outside. The intersection has no grades on any approach. Since this paper has already established that the through
FIGURE 3 Comparison of SOAP's delay model for a 60-minute period and the HCM delay model for various peak-hour factor values.

FIGURE 4 Comparison of the SOAP and HCM delay models for various period lengths.
movements behave similarly in the two models, the example only considers the effects of opposing traffic on the left-turning capacity.

Comparison of Results

Figure 6 compares the permitted left-turn capacities for the two programs. While the models have different shapes, they are still quite similar. Both of them converge to two sneakers per cycle when the opposing traffic reaches 100 percent saturation. The values are alike because both models use the same equation for unsaturated green time and because the models for the permitted saturation flow rate are very similar.

The HCM defines the left-turn factor for a shared left-through lane or an exclusive left-turn lane by the following equation:

\[
f_m = \frac{g_t}{g} + \frac{g_u}{g} \left[ \frac{1}{1 + P_L(E_L - 1)} \right] + \frac{2}{g} (1 + P_L)
\]  

where

- \( f_m \) = left-turn factor,
- \( g_t \) = portion of the green phase during which through vehicles may move in a shared lane until the first left-turn vehicle arrives,
- \( g_u \) = portion of green not blocked by clearing of the opposing queue,
- \( P_L \) = proportion of left turns in a shared lane, and
- \( E_L \) = through vehicle equivalent for opposed left turns.

FIGURE 5 Intersection layout and phasing pattern for the permitted left-turn example.

FIGURE 6 Comparison of permitted left-turn capacities for SOAP and the HCM.
However, for exclusive left-turn lanes, \( P_L = 1.00 \) and \( g_f = 0 \). Therefore, the equation simplifies to the following:

\[
    f_m = \frac{g_o}{g} \left( \frac{1,400 - V_o}{1,800} + \frac{4}{g} \right)
\]

where \( V_o \) is the opposing volume.

In SOAP, the saturation flow rate for unprotected left turns is computed based on the Netsim model, which was developed by Nemeth and Mekenson (6). The Netsim model considers the opposing traffic and the number of opposing lanes as follows:

For a single opposing lane:

\[
    S_L = 1,404 - 1.632 V_o + 0.0008347 V_o^2
    - 0.000002138 V_o^3
\]

and

For multiple opposing lanes:

\[
    S_L = 1,393 - 1.734 V_o + 0.0009173 V_o^2
    - 0.000001955 V_o^3
\]

where \( S_L \) is the permitted left-turn saturation flow rate.

The unsaturated flow rate is defined by the following equation in SOAP:

\[
    g_u = g_o - g_{so}
\]

where

\[
    g_o = \text{unsaturated green time}
\]

\[
    g_{so} = \text{green time for the opposing traffic, and}
\]

\[
    g_{so} = \text{saturated green time for the opposing queue.}
\]

In Equation 6, \( g_{so} \) is defined by

\[
    V_o(C - g_u)/(S_o - V_o)
\]

where \( S_o \) is the saturation flow rate of the opposing traffic. Therefore,

\[
    g = g_o - V_o(C - g_u)/(S_o - V_o)
\]

which simplifies to

\[
    Y_o = V_o/S_o
\]

where \( Y_o \) is the flow ratio of the opposing traffic.

This equation is exactly the same as the unsaturated green model for the HCM provided that \( g_o = g \). In cases where \( g_o \neq g \), the HCM does not specify which green time to use; however, it seems to suggest using the green time of the permitted movement.

It was concluded from this example that the left-turn saturation flow rates for SOAP and the HCM are more or less equivalent. Because TRANSYT-7F bases its treatment of permitted left turns on a time scan deterministic simulation, no comparisons with that model were performed.

PROTECTED PLUS PERMITTED LEFT TURNS

When a left turn is made on both a protected phase and a permitted phase, the modeling process becomes complex. The three techniques differ substantially:

- The HCM requires a user-specified split of the left-turning volume between the permitted and protected phases.
- SOAP makes an internal adjustment to increase the effective green time in proportion to the saturation flow rates for the two phases. This eliminates the need for a user-specified volume split, by equalizing the degree of saturation on both phases.
- TRANSYT accumulates arrivals on a step-by-step basis and releases the left-turning vehicles based on the instantaneous volume of the conflicting movement. This is a detailed deterministic simulation process.

Of the three models, the TRANSYT technique appears to offer the most rational formulation, but it can only be implemented by a relatively complex iterative algorithm that is not suited to the HCM format.

Problems With Existing Models

The treatment of this situation is one of the significant weaknesses of the HCM. An iterative technique is suggested, beginning with the assumption that all flow occurs during the protected phase. If this results in a V/C ratio that is too high, some volume may be assigned to the permitted interval (up to the capacity of the permitted phase).

However, vehicles assigned to the permitted phase are subsequently ignored in the delay calculations. Furthermore, the g/C ratio used in the delay calculations is the g/C total of both the protected and permitted phases. This underestimates the delay of these left-turning vehicles and results in an overly optimistic level of service for the intersection as a whole.

FHWA’s Highway Capacity Software User’s Manual (HCS) (7) allows three options for the protected plus permitted phasing:

1. Assign no vehicles to the permitted phase,
2. Assign the maximum number of vehicles to the permitted phase, or
3. Assign the vehicles to the permitted phase such that the V/C ratios for the permitted and the protected phases are equal.

Of these three options, the third seems the most defensible. The first option ignores the additional capacity of the permitted movement and therefore uses a V/C value that is too high. The second option ignores a volume equal to the capacity of the permitted movement calculated as if the left-turn movement was unopposed (i.e., the upper limit of the permitted capacity). The equation for the capacity of the permitted phase multiplies the saturation flow rate for permitted left turns by the g/C ratio for the entire permitted phase rather than using the ratio of \( g_o \) (the unsaturated green) to the cycle length. As noted by Bonneson and McCoy (8), this yields a permitted capacity that is unreasonably high unless the opposing volume is negligible.
Unlike the HCM, SOAP does not ignore the permitted turning vehicles. It modifies the saturation flow rate in the protected phase to account for the flow during the permitted phase when computing the capacity of the protected plus permitted left turn. The saturation flow rate for the permitted left-turning movement is computed in the same manner as that for permitted-only left turns. This rate is then multiplied by the g/C ratio for the permitted movement to obtain the permitted capacity. The protected capacity is then adjusted for the permitted capacity, and the flow is assumed to occur in the protected interval. The saturation flow rate of the protected interval is modified, and this higher saturation flow rate is then used in conjunction with the g/C ratio of the protected interval to determine the left-turn delay. This method seems more reasonable than ignoring the permitted flow; however, by using the g/C ratio of the protected interval only, the delay values are much higher.

A Deterministic Queuing Model

Both SOAP and the HCM could improve their analysis of protected and permitted operations by using a deterministic queuing approach of the type shown in Figure 7. This queuing model computes only the uniform delay component. It assumes that the second term of the HCM delay equation is appropriate and proposes a new first term for the case of protected plus permitted left turns. The model further assumes that the methodology of the supplemental worksheet for permitted left turns applies to the permitted portion of the protected plus permitted left movement. The queuing model’s principal advantage over the other models is that it accounts realistically for the complexity of this type of movement.

Figure 8 shows the components of the protected plus permitted left-turn movement:

- The red portion of the cycle, when the arriving vehicles are queued;
- The protected left-turn interval when the queued vehicles discharge at their saturation flow rate;

\[
D_1 = \frac{1}{2} (R + L + V)
\]

\[
D_2 = \frac{1}{2} (S_p \cdot V) g_p
\]

\[
D_3 = (g_p + g_o) \left( (R + L) V + (S_p \cdot V) s_p \right)
\]

\[
D_4 = \frac{1}{2} V g_o^2
\]

\[
D_5 = \frac{1}{2} (S_u \cdot V) g_u^2
\]

**FIGURE 8** Components of the protected plus permitted delay model.

- The beginning of the permitted interval, when the vehicles must yield to the oncoming traffic that is discharging from the opposing queue at the saturation flow rate; and
- The portion of the permitted interval during which the vehicles must accept gaps in the oncoming traffic that is entering the intersection at the arrival rate.

To help implement this complex model, a worksheet similar to the one for permitted left turns was developed (see Figure 9). The model involves the following steps:

1. **Input Variables.** The first nine rows of the worksheet summarize the data needed to compute the delay for protected plus permitted left turns:
   a. Cycle length, C (sec);
   b. Red time, R (sec);
   c. Start-up lost time, L (sec);
   d. Effective green time for the protected interval, g_p (sec);
   e. Effective green time for the permitted interval, g (sec);
   f. Left-turn flow rate, V (vph);
   g. Saturation flow rate for the protected left turns, S_p (vphg);
   h. Opposing volume, V_o (vph); and
   i. Opposing saturation flow rate, S_o (vphg).

2. **Computations.** The lower portion of the worksheet contains a series of equations to compute the delay for the various portions of the cycle:
   a. Flow ratio of the opposing traffic (Y_o):
   \[
   Y_o = \frac{V_o}{S_o}
   \]
   b. Unsaturated green time (g_u):
   \[
   g_u = \frac{g - CY_o}{1 - Y_o}
   \]
## SUPPLEMENTAL WORKSHEET FOR PROTECTED PLUS PERMITTED LEFT TURNS

<table>
<thead>
<tr>
<th>Input Variables</th>
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### COMPUTATIONS

\[
Y_o = \frac{V_o}{S_o}
\]

\[
g_u = \frac{g - CY_o}{1 - Y_o}
\]

\[
g_Q = g - g_u
\]

\[
f_{LT} = \frac{g_u}{\frac{1400 - V_o}{1800} + \frac{4}{g}}
\]

\[
S_u = f_{LT} \cdot S_p
\]

\[
g_p = \frac{V(R+L)}{S_p \cdot V}
\]

\[
g_u = \frac{(R+L) V \cdot (S_o - V) g_p^2 + g_o V}{S_u \cdot V}
\]

\[
D_1 = \frac{1}{2} (R + L)^2 V
\]

\[
D_2 = \frac{1}{2} (S_p - V) g_p^2
\]

\[
D_3 = (g_p + g_Q) \left[ (R+L) V - (S_p - V) g_p \right]
\]

\[
D_4 = \frac{1}{2} V g_Q^2
\]

\[
D_5 = \frac{1}{2} (S_u - V) g_o^2
\]

\[
d_1 = 0.76 \left( \sum_{i=1}^{5} D_i \right) / VC
\]

### NOTES:
- IF \( g_p > g_p \) THEN \( g_p' = g_p \)
- IF \( D_3 < 0 \) THEN \( D_3 = 0 \)
- IF \( g_u' > g_u \) THEN \( g_u' = g_u \)

### FIGURE 9
Worksheet for protected plus permitted left turns.
The green time used in this equation is the green time for the opposing traffic ($g_o \geq 0$).

c. Green time that is blocked by the clearance of the opposing queue ($g_o$):

$$g_o = g - g_u$$

d. Permitted left-turn factor ($f_{LT}$):

$$f_{LT} = \frac{g_u (1.400 - V_p)}{g} + \frac{4}{g}$$

e. Saturation flow rate for the permitted left portion ($S_p$):

$$S_u = f_{LT} * S_p$$

f. The portion of the protected green time that is used, $g'_p$, is computed as follows:

$$g'_p = \frac{V(R + L)}{S_p - V}$$

(Note: $g'_p$ must be in the range $0 \leq g'_p \leq g_p$)

g. The portion of the unsaturated green time that is used, $g'_u$:

$$g'_u = \frac{(R + L)V - (S_p - V)g'_p + g_o V}{S_u - V}$$

(Note: $g'_u$ must be in the range $0 \leq g'_u \leq g_u$)

h. First delay term ($D_1$):

$$D_1 = \frac{1}{2} (R + L) V$$

i. Second delay term ($D_2$):

$$D_2 = \frac{1}{2} (S_p - V) g'_p^2$$

j. Third delay term ($D_3$):

$$D_3 = (g_p + g_o) [(R + L) V - (S_p - V) g_p]$$

(Note: $D_3 = 0$ if $D_3 < 0$)

k. Fourth delay term ($D_4$):

$$D_4 = \frac{1}{2} V g'_p^2$$

l. Fifth delay term ($D_5$):

$$D_5 = \frac{1}{2} (S_u - V) g'_u^2$$

m. The total delay must be converted to the average stopped delay per vehicle to be consistent with the HCM methodology. The average stopped delay ($d_s$) is computed from the following equation:

$$d_s = .76 \left( \frac{\sum_{i=1}^{5} D_i}{VC} \right)$$

Since the queuing model attempts to account for all components of the protected plus permitted left movement, it should yield much more realistic delay values than SOAP or the HCM.

This model assumes protected plus permitted left turns (leading left-turn protection). A similar model can be used for permitted plus protected turns (lagging left-turn protection). The worksheet for lagging protection is shown in Figure 10.

A comparison of leading protection and lagging protection is shown in Figure 11. The figure shows that the delay values for lagging protection would be much higher than those for leading protection. While the first, second, fourth, and fifth terms are identical, the third term is much greater for lagging protection. This is true because one cycle of the leading protection case can essentially be viewed as two short cycles (red-green-effective and red-green), and shorter cycles yield reduced delays in the first term of the delay equation. However, neither SOAP nor the HCS computes any difference in delay between leading and lagging protection. This issue is critical since the delay values can differ significantly and many models used to design and evaluate signal timing do not consider this difference.

**Example Problem**

To compare the left-turn models used in SOAP and the HCS with the queuing model for protected plus permitted left turns, a simple example is considered (see Figure 12). This is the same intersection as in the permitted left-turn example, but the phasing has been changed to add left-turn protection. For this example, all through movements have a volume of 500 vph.

**Comparison of Results**

The results from SOAP, the HCS, TRANSYT, and the queuing model are shown in Figure 13 for leading protection and in Figure 14 for lagging protection.

Neither SOAP nor the HCM compares favorably with the proposed model. The first protected plus permitted treatment from the HCS becomes oversaturated quickly since the permitted capacity is totally ignored. For the second HCS option, the delay stays constant over the full range of left-turning volume. This is because the permitted capacity is overestimated by incorrectly using the entire permitted green time (as mentioned above). The third case from the HCS is more reasonable than the other HCS solutions but is significantly lower than either TRANSYT or the queuing model. Again, this is due to the overestimation of the permitted capacity.
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<td>( D_2 = \frac{1}{2} \cdot (S_p - V) \cdot g_p^2 )</td>
<td></td>
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<tr>
<td>( D_3 = (R + L) \cdot g_o \cdot V + (S_p - V) \cdot g_u \cdot g_p^2 )</td>
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<tr>
<td>( D_4 = \frac{1}{2} \cdot V \cdot g^2 )</td>
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<tr>
<td>( D_5 = \frac{1}{2} \cdot (S_u - V) \cdot g_u^2 )</td>
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<tr>
<td>( d_1 = 0.76 \sum_{i=1}^{5} D_i / V )</td>
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</tbody>
</table>

**NOTES:**
- IF \( g_p^* > g_p \) THEN \( g_p^* = g_p \)
- IF \( g_u^* > g_u \) THEN \( g_u^* = g_u \)
- IF \( g_u^* < g_u \) THEN \( g_p^* = 0 \)

**FIGURE 10**  Worksheet for permitted plus protected left turns.
The queuing model does seem to compare well with TRANSYIT. The general shape of the curves is similar and, more important, both models reflect the difference between leading and lagging protection. In contrast, both SOAP and the HCS results are identical (incorrectly) for both left-turn protection schemes.

The general concept of the queuing model is not new. A similar formulation was suggested by Bonneson and McCoy (8) and the PASSER II-87 signal optimization program (9). It is also the basis for delay calculation in SICAP (10), a computer program that implements the HCM calculations on a microcomputer.
CONCLUSIONS AND RECOMMENDATIONS

This research resulted in some interesting observations. It is apparent that the delay model used in the 1985 Highway Capacity Manual agrees very well with the SOAP and TRANSYT models for undersaturated operation. There were significant differences when conditions were oversaturated; however, it was possible to make the delay values agree by the proper choice of period length and peak-hour factor. It is also clear that neither of these models should be used to estimate the delay for long periods of oversaturation or high degrees of oversaturation.

The TRANSYT model, because of its detailed breakdown of the cycle, appeared to treat the case of permitted and protected left turns in a more reasonable manner than either SOAP or the HCM. While it is not practical to incorporate the TRANSYT model in either of the other techniques, it is relatively easy to incorporate the queuing model described in this paper. The queuing model offers two important advantages over the HCM signalized intersection methodology:

1. It eliminates the need for user-specified volume split.
2. It differentiates between leading and lagging protection.

With this in mind, it is recommended that the models described in this paper for protected plus permitted left turns and permitted plus protected left turns be incorporated into the HCM methodology.

REFERENCES


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