Testing Delay Models with Field Data for Four-Way, Stop Sign-Controlled Intersections

MARK ZION, GEORGE F. LIST, AND CHARLES MANNING

Four-way, stop sign-controlled intersections are a relatively common phenomenon, especially in urban networks, yet little analysis has been devoted to determining their capacity and delay characteristics. This paper presents the results of new field studies and compares data collected from two recent delay models. Generally, findings are that delay increases as the intersecting volumes increase; intersections with balanced volumes have lower delays than those without; and the percentage of left turns has a noticeable effect on delay. Statistical analyses suggest that one of the two models considered in this research may provide satisfactory delay estimates.

Four-way, stop sign controlled intersections (FWSC) are a relatively common phenomenon, especially in urban networks, yet little analysis has been done to determine their capacity and delay characteristics (*1–6*). This paper reports the results of recent field studies on delay and compares data collected with predicted delays for two proposed models.

One of the first analyses of capacity and delay for FWSCs was conducted by Hebert (*1*), who collected data from three intersections in the Chicago area. He determined that, among other things, delay increased as volume increased and that the capacity of FWSC intersections increased as the volume split between the two streets became even (i.e., approached a 50/50 split). As the data in Table 1 show, capacity ranged from 1,900 veh/hr for an intersection with a 50/50 split to 1,500 veh/hr for one with a 70/30 split.

More recently, Richardson (*5*) developed a model, based on Hebert's work, that predicts the stopped delay for FWSC intersections. Richardson's model makes certain simplifying assumptions, but it does produce capacity estimates nearly identical to those found by Hebert (see Table 2). The model assumes that no turning movements occur; that a certain prescribed pattern (arguably plausible) operates at the intersection; and that the volumes on the opposing approaches are equal.

Assuming the two streets are labeled *a* and *b* (these labels can also be applied to the approaches on those streets), Richardson's model asserts that the average length of queue (vehicles), L_a, can be obtained by applying the Pollaczek-Kyintchine formula, as follows:

$$L_a = \frac{2u_a - (u_a)^2 + (x_a)^2V(s_a)}{2(1 - u_a)} \quad (1)$$

M. Zion and G. F. List, Department of Civil Engineering, Rensselaer Polytechnic Institute, Troy, N.Y. 12180–3590. C. Manning, Roger Creighton Associates, Inc., Delmar, N.Y. 12054.

where

x_a = average arrival rate for approach *a* (veh/sec),
s_a = average service time for approach *a* (sec/veh),
$V(s_a)$ = variance of service time for approach *a*, and
u_a = utilization ratio (unitless) for approach *a* = (arrival rate)*(service time).

Substituting subscript *b* for *a* throughout yields the average length of queue for approach *b*.

The average "time in system" (in seconds) for vehicles on approach *a*, w_a, (in this instance, the stopped delay) is given by

$$w_a = L_a/x_a \quad (2)$$

Again, substituting *b* for *a* yields the stopped delay for approach *b*.

TABLE 1 CAPACITY OF A TWO-LANE BY TWO-LANE, FOUR-WAY STOP SIGN INTERSECTION (FROM *HIGHWAY CAPACITY MANUAL*)

Demand Split	Capacity (vph)
50/50	1900
55/45	1800
60/40	1700
65/35	1600
70/30	1500

TABLE 2 CAPACITY OF A TWO-LANE BY TWO-LANE, FOUR-WAY STOP SIGN INTERSECTION AS PREDICTED BY RICHARDSON

Demand Split	Capacity (vph)
50/50	1900
55/45	1760
60/40	1650
65/35	1600
70/30	1560
80/20	1520
90/10	1570
100/0	1800

Moreover, the service time (in seconds, and in a queuing sense), s_a, for approach a is given by the expected value of the length of time the vehicle will have to wait before crossing the intersection:

$$s_a = t_m P_b(0) + T_c P_b(1) \quad (3)$$

where

t_m = minimum possible headway—the service time on approach a when there is no vehicle waiting to cross the intersection at the stopline on approach b,
T_c = impeded service time—the service time for a vehicle on approach a when a vehicle is already waiting to cross the intersection at the stopline on approach b,
$P_b(0)$ = probability of no vehicle being at the stopline on approach b, and
$P_b(1)$ = probability that there is a vehicle at the stopline on approach b.

For Equation 3, Richardson assumes that t_m is 4.0 sec for all approaches and that T_c is given by 2 * t_c where t_c, the clearance time on a single conflicting approach, is given by 3.6 sec plus 0.1 sec times the number of crossflow lanes that must be crossed (two in the case of a two-lane by two-lane intersection, resulting in a value of 3.8 for t_c and 7.6 for T_c).

Probabilities $P_b(0)$ and $P_b(1)$ depend on the utilization ratio, u_b, for approach b:

$$P_b(0) = 1 - u_b$$

$$P_b(1) = u_b \quad (4)$$

Inserting Equations 4 into Equation 3 gives the average service time (sec/veh) for approach a:

$$s_a = t_m(1 - u_b) + T_c u_b \quad (5)$$

Similarly, the average service time (sec/veh) for approach b is given by

$$s_b = t_m(1 - u_a) + T_c u_a \quad (6)$$

These equations (with four unknowns, s_a, s_b, u_a, and u_b) can be solved by substituting $x_a s_a$ for u_a in Equation 6 (in general $u = xs$) and then substituting $x_b s_b$ for u_b in Equation 5—where the expression used for s_b is obtained from Equation 6. This new Equation 5, when solved for s_a, yields the following:

$$s_a = \frac{x_a t_m T_c + t_m - x_b (t_m)^2}{1 - x_a x_b[(T_c)^2 - 2t_m T_c + (t_m)^2]} \quad (7)$$

Replacing b for a, and vice versa, throughout, yields s_b instead.

Once s_a has been found, $V(s_a)$, the variance of s_a, can be determined as follows:

$$V(s_a) = (t_m)^2(T_c - s_a)/(T_c - t_m) + (T_c)^2(s_a - t_m)/(T_c - t_m) - (s_a)^2 \quad (8)$$

The value for $V(s_a)$ and the value for s_a from Equation 7 can then be used in Equations 1 and 2 to determine L_a (the average length of queue on approach a) and w_a (the average time in system for approach a—here, the average stopped delay. Similar logic applies for finding L_b and w_b. Figure 1 demonstrates a sample calculation.

The major phenomena predicted by this model are the following:

- Delay increases at a greater-than-linear rate as volume increases.
- For a given total volume, the level of delay decreases as the flows become balanced. That is, an intersection with a 50/50 split in volume has lower delays than one with an uneven split.
- The foregoing phenomenon, conversely put, implies that intersection capacity (the point at which stopped delay reaches infinity) increases as the volume split approaches 50/50, as Hebert found. (A comparison of Tables 1 and 2 show, however, that the capacities are slightly different.)

A second model, developed by Chan et al. (6), also predicts delays for FWSC intersections. In this instance, a multivariable log-linear regression equation was fitted to field data and augmented by computer simulation to determine the predictive equation. The field data were collected at five intersections located in Idaho and Washington State.

The average delay/veh (sec/veh), D, is given by

$$D = a * \exp[(bS + cH + dT + e)V] \quad (9)$$

where

V = total arrival volume (veh/hr),
S = volume split factor (percent),
H = street-width factor (percent),
T = turning movement factor (percent), and
a, b, c, d, and e = calibration constants.

The volume split factor, S, is found by

$$S = s_1 - s_2 \quad (10)$$

where s_1 is the percentage of traffic on the major flow (percent) and s_2, the minor flow (percent).

The street width factor H (nondimensional) is given by

$$H = \max\,[0, (h - 30)/h] \quad (11)$$

where h is the road width (ft) and, in effect, 30 ft is the width (for a two-lane street) and no width adjustment is necessary.

The turning factor T (nondimensional) is given by

$$T = LTV/TV \quad (12)$$

where LTV is the left-turn volume and TV is the total volume for all four approaches. Sample calculations using these formulas are shown in Figure 2.

Three predictions given by this model are as follows:

- The average delay/veh rises rapidly as volume approaches capacity.

Given: Approach Vol. = 876 veh/hr
Vol. Split = 52.1/47.9
Balanced opposing volumes on both streets

Find: The time in system for approach a using the Richardson model:

$t_m = 4.0$ sec.
$t_c = 3.6$ sec. $+ 0.1(2) = 3.8$ sec.
$T_c = 2t = 2(3.8 \text{ sec.}) = 7.6$ sec.

$x_a = .521\ (876 \text{ veh/hr})/2 = 228.1$ veh/hr
$= 0.0634$ veh/sec
$x_b = .479\ (876 \text{ veh/hr})/2 = 209.8$ veh/hr
$= 0.0583$ veh/sec

$$s_a = \frac{x_b t_m T_c + x_b t_m^2}{1 - x_a x_b\,(T_c^2 - 2t_m T_c + t_m^2)} \quad (7)$$

$$= \frac{(0.0583)\ (4.0)\ (7.6) + (4.0) \quad - \quad (0.0583)\ (4.0)^2}{1 - (0.0583)\ (0.0634)\ \ ((7.6)^2 - 2(4.0)(7.6) + (4.0)^2)}$$

$$= 5.083 \text{ sec.}$$

$$V(s_a) = t_m^2(T_c - s_a)/(T_c - t_m) + T_c^2(s_a - t_m)/(T_c - t_m) - s_a^2$$

$$= (4.0)^2(7.6 - 5.083)/(7.6 - 4.0) + (7.6)^2(5.083 - 4.0)/(7.6 - 4.0) - (5.083)^2$$

$$= 2.725 \text{ sec} \quad (8)$$

$$u_a = s_a x_a = (5.083)(0.0634) = 0.322$$

$$L_a = \frac{2u_a - u_a^2 + x_a^2 V(s_a)}{2(1 - u_a)} \quad (1)$$

$$= \frac{2(0.322) - (0.322)^2 + (0.0634)^2(2.725)}{2(1 - 0.322)}$$

$$= 0.407$$

$$w_a = L_a/x_a = 0.407/0.0634 = 6.418 \text{ sec.} \quad (2)$$

time in system for approach a = 6.418 sec.

FIGURE 1 Sample calculation using the Richardson model (note: equation numbers refer to text).

- The capacity of a FWSC intersection decreases as its volume split approaches 50/50.
- Intersection capacities for the sites Chan studied ranged between 600 and 1,400 veh/hr (see Table 3).

DATA COLLECTION AND ANALYSIS

The data collected for this study were obtained from three intersections near Albany, N.Y. Three criteria were applied in selecting these sites: First, there had to be two-lane streets on all four approaches to ensure compatibility with the intersections used in the studies and models cited above; second, the sites needed a variety of volume splits so that variations in that parameter could be considered; and, last, the sites' volumes had to be high enough that delays of significance would be observed. The intersections selected were—

- Site 1: Moe Road and Clifton Park Center Road, Clifton Park, New York (this intersection has since been signalized);
- Site 2: an intersection internal to the Colonie Shopping Center, Colonie, New York; and
- Site 3: Pine and Lodge Streets, Albany, New York.

Following a methodology suggested by Box and Oppenlander (*7*), data were collected at Site 1 for 115 min during

Given: Approach Vol. = 876 veh/hr
Vol. Split = 52.1/47.9
% Left Turns = 27.4
Street Width = 24 feet

Find: The delay time using the Chan model

$$S = (s_1 - s_2)/100 \quad (11)$$
$$= (52.1 - 47.9)/100 = 0.042$$
$$H = \max[0, (h-30)/h] = \max[0, (24-30)/24] = 0$$
$$T = (\%\text{ left turns})/100 = .274 \quad (12)$$
$$V = 876 \text{ veh/hr}$$
$$d = .186 \exp((-.007455\ S + .01333\ T + .004037)\ V) \quad (10)$$
$$= .186 \exp((-.007455\ (.042) + .01333(.274) + .004037)(876))$$
$$= 119.1 \text{ sec}$$

delay time = 119.1 sec.

FIGURE 2 Sample calculation using the Chan model (note: equation numbers refer to text).

TABLE 3 CAPACITY OF A TWO-LANE BY TWO-LANE, FOUR-WAY STOP SIGN INTERSECTION AS PREDICTED BY CHAN

Demand Split	Capacity (vph)
50/50	1076
60/40	1489
70/30	2419

an afternoon peak; at Site 2 for 50 min during the heaviest shopping hour of an afternoon; and at Site 3 for 70 min during an afternoon rush hour.

At each site, and for each 5-min time period observed, one person counted traffic (turning movement counts by approach and total) and a second collected delay data. The observation time, 5 min, was chosen as it was long enough to provide sample sizes adequate for statistically reliable estimates of delay and it allowed an opportunity to see the effects of volume split changes as the traffic volumes fluctuated. The person monitoring delays used a stopwatch to determine how long it took vehicles to pass through the intersection. (Vehicles were selected so that approximately equal numbers of observations would be made on each approach.) The stopwatch was started when the vehicle began braking and it was stopped when the vehicle cleared the intersection. (These observations thus included more delay than predicted by either the Richardson or Chan models; these delay estimates therefore should be bounded from below by the model predictions of delay.)

The data collected were processed to determine, for each 5-min time period, the volume split, the percentage of left turns, and the average delay (based on the way the data were collected and on the vehicles observed). The resulting statistics are presented in Table 4.

ANALYSIS AND OBSERVATIONS

The first and simplest observation made is that measured delays clearly increase as volume increases, as shown in Figure 3. Moreover, Table 5, which summarizes the average delays by site, shows Site 2 with the highest volumes and greatest delays, and Site 3 with the lowest volumes and lowest delays.

Table 6 also shows delays increasing as the volume split becomes unbalanced, which again agrees with Hebert and Richardson.

A stepwise linear regression analysis of the data collected generates the following results:

$$D = 12.361V \text{ (no constant allowed)} \quad (13)$$
$$(\text{adjusted } R^2 = 0.9168)$$

$$D = 21.24V - 8464 \text{ (adjusted } R^2 = 0.5813) \quad (14)$$

$$D = 8.517V + 16.74L \text{ (no constant allowed)} \quad (15)$$
$$(\text{adjusted } R^2 = 0.9281)$$

$$D = 16.91V + 13.16L - 7212 \quad (16)$$
$$(\text{adjusted } R^2 = 0.6210)$$

where, for a given 5-min time period, D is an estimate of total vehicular delay (the average observed delay times total volume), V is the total volume passing through the intersection (vehicles), and L is the total number of vehicles making left turns.

The implications of these regression results are as follows:

- Including a constant in the predictive equation lowers the adjusted R^2 value considerably. Therefore, the constants should be omitted from the regression equations.
- Including the left-turn volume in the predictive relationship always increases the adjusted R^2, whether the constant

TABLE 4 FIELD DATA

Moe Road and Clifton Park Center Road:

Data entry*	Vol. (veh/hr)	Vol. Split	% Left Turns	Ave. Delay** (sec)
1	984	58.5/41.4	20.9	13.27
2	444	59.5/40.5	32.4	8.19
3	672	55.4/44.6	23.2	10.25
4	960	50.0/50.0	30.0	13.97
5	828	60.9/39.1	14.5	8.31
6	684	57.9/42.1	31.6	12.92
7	876	52.1/47.9	27.4	10.32
8	960	60.0/40.0	25.0	10.91
9	900	56.0/44.0	30.7	13.93
10	900	57.3/42.7	30.7	8.89
11	912	59.2/40.8	27.6	9.52
12	888	50.0/50.0	27.0	12.14
13	948	53.2/46.8	22.8	14.17
14	1140	60.0/40.0	31.6	15.50
15	1188	51.5/48.5	27.3	13.30
16	864	59.7/40.3	27.8	14.47
17	1128	60.6/39.4	24.5	12.17
18	1032	52.3/47.7	24.4	17.00
19	1188	50.5/49.5	25.3	15.37
20	984	51.2/48.8	28.0	8.81
21	996	56.6/43.4	28.9	11.32
22	804	49.3/50.7	31.3	9.55
23	708	52.5/47.5	25.4	14.22

Colonie Center:

Data entry*	Vol. (veh/hr)	Vol. Split	% Left Turns	Ave. Delay** (sec)
24	1104	62.0/38.0	23.9	10.86
25	828	70.0/30.0	24.6	8.38
26	1104	70.0/30.0	31.5	26.30
27	1068	67.4/32.6	25.8	12.59
28	792	62.1/37.9	31.8	14.76
29	1044	57.9/42.5	33.3	12.31
30	1008	71.4/28.6	27.4	9.62
31	1044	64.4/35.6	29.9	17.22
32	1152	58.3/41.7	35.4	13.64
33	972	67.9/32.1	30.9	10.43

Pine Street and Lodge Street:

Data entry*	Vol. (veh/hr)	Vol. Split	% Left Turns	Ave. Delay** (sec)
34	888	71.6/28.4	13.5	9.12
35	816	57.4/42.6	14.7	8.32
36	924	76.6/23.4	7.8	7.39
37	1020	62.4/37.6	10.6	8.29
38	1152	56.2/43.8	15.6	16.37
39	1116	53.8/46.2	8.6	14.51
40	660	50.9/49.1	9.1	7.04
41	696	75.9/24.1	5.2	5.83
42	732	82.0/18.0	4.9	6.72
43	864	56.9/43.1	8.3	11.84
44	1056	67.0/33.0	11.4	13.27
45	888	63.5/36.5	9.5	9.49
46	888	60.8/39.2	4.1	8.73
47	408	52.9/47.1	5.9	6.61

* Each entry contains many readings taken over a 5 min. period.

** Ave. of all delay readings during 5 min. period.

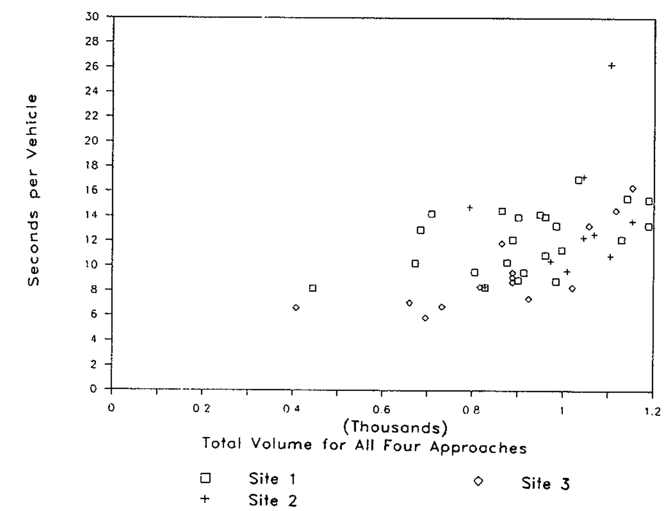

FIGURE 3 Delay versus volume for the three intersections studied.

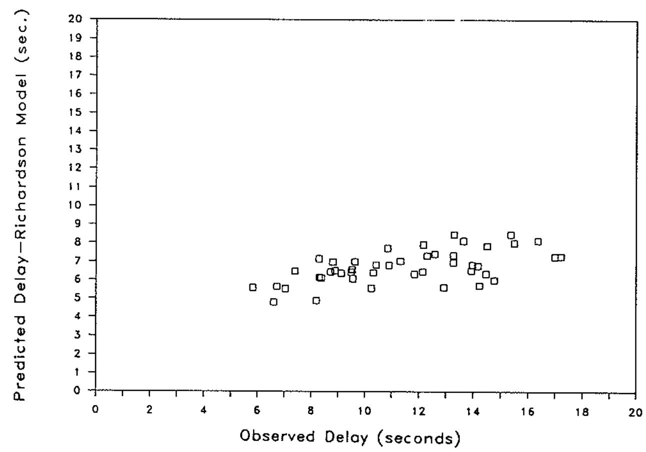

FIGURE 4 Correlation between the Richardson model and the delays observed.

TABLE 5 AVERAGE VOLUMES AND AVERAGE DELAYS

Site	Intersection	Average Volume	Percent Lefts	Average Delay
1	Moe and Clifton Center Roads	913	26.9%	12.11 sec
2	Colonie Center	1012	29.5%	13.61 sec
3	Pine and Lodge Streets	865	9.2%	9.54 sec

TABLE 6 DELAY TIMES FOR VARIOUS DEMAND SPLITS AS MEASURED IN FIELD (APPROACH VOL. = 900)

Demand Split	Delay Time
50/50	11.10
55/45	11.27
60/40	11.70
65/45	12.49
70/30	12.96

is included or not. This suggests that left turns do indeed have an effect on intersection delays.

- The best relationship among those tested involves the total volume and the left-turn volume.

The coefficients of Equation 15, in particular, imply that each vehicle passing through the intersection adds, on average, 8.517 sec to the total intersection delay (including deceleration time and the time required to cross the intersection) and each left turn makes the total intersection delay increase by an additional 16.74 sec.

The data can also be compared with the delay predictions given by the Richardson and Chan models, as shown in Figures 4 through 7. In Figure 4, the correlation between the observed delays and the Richardson delay estimates seems clear even though the correlation coefficient is low (adjusted $R^2 = 0.3232$), while in Figure 5 no such correlation is apparent (the adjusted R^2 is 0.0693). In Figures 6 and 7, the delay estimates and the observed delays from the models are plotted against volume. It is clear that the Richardson model does indeed fit the data observed in that it provides a lower bound on all of the observations. (This is because the field data include stopped delay plus deceleration time and the time required to cross the intersection.) In fact, it appears that the additional delay buried in the field observations is somewhere between 0 and 8 sec. The Chan model, as Figure 5 shows, does not provide such a bound, at least for the three intersections examined in this study.

Further analysis of the data, breaking the observations down into groups based on their volume splits, reveals additional insights. To conduct this analysis, each data point shown in Table 4 is placed in one of seven classes, based on the split category to which it most closely corresponds: 50/50, 55/45, 60/40, 65/35, 70/30, 75/25, and 80/20. For three of these classes, sufficient data are available to meaningfully plot the observed delays against the delay estimates from the Richardson and Chan models. As shown in Figures 8, 9, and 10, the Richardson model provides a consistent lower bound for the delays observed while the Chan model does not. In addition, the delays predicted by the Chan model increase more quickly than do those observed.

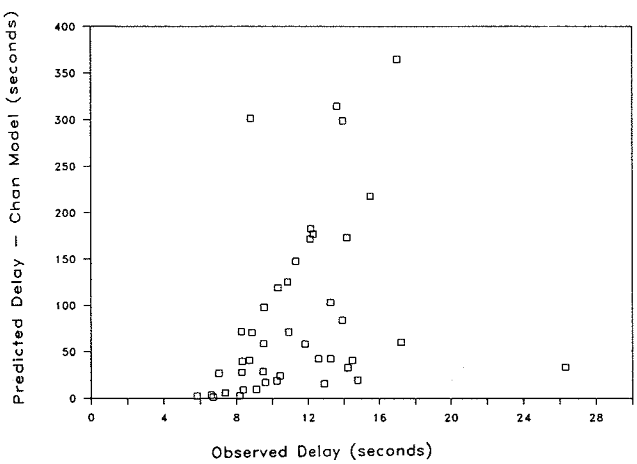

FIGURE 5 Correlation between the Chan model and the delays observed.

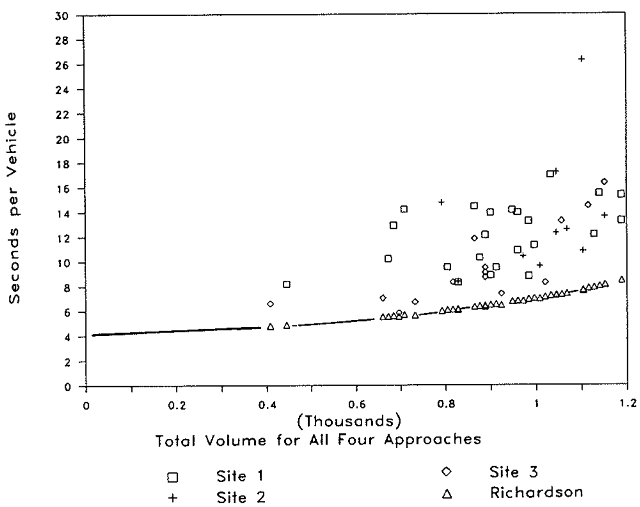

FIGURE 6 The Richardson model and the delays observed plotted against total intersection volume.

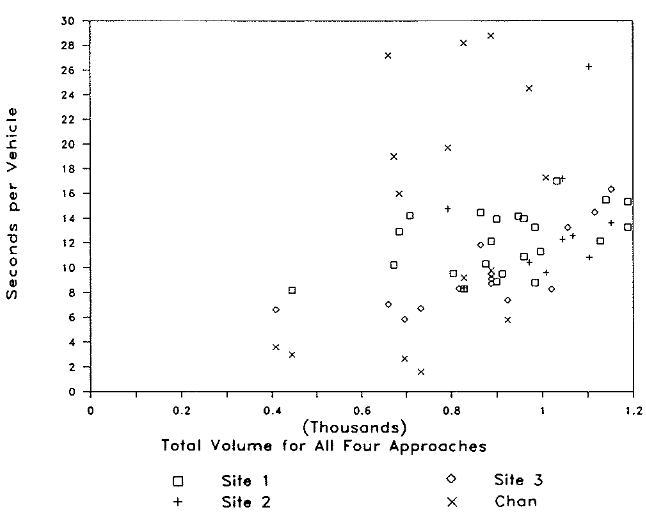

FIGURE 7 The Chan model and the delays observed plotted against total intersection volume.

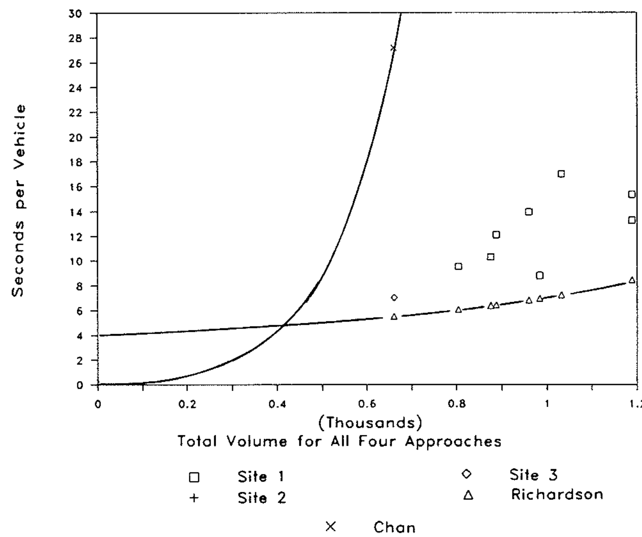

FIGURE 8 Delay versus volume for the observations with a 50/50 split in approach volumes.

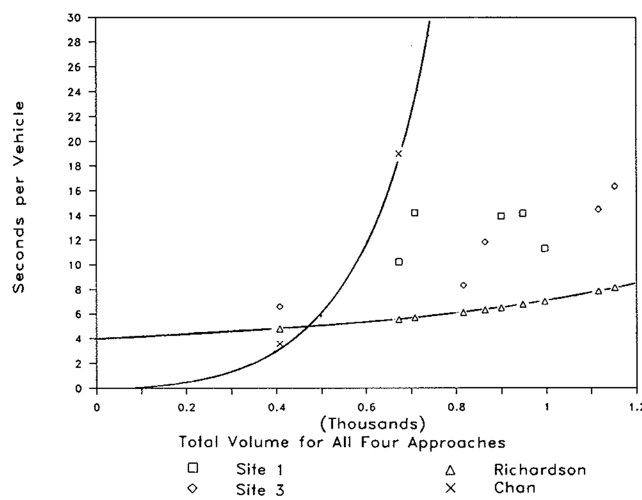

FIGURE 9 Delay versus volume for the observations with a 55/45 split in approach volumes.

CONCLUSION

Clearly, it should not be assumed that the data and analyses presented here can be used to defend definitive statements about the nature of delays at FWSC intersections. What the data do suggest, however, is that the following trends should hold in further analyses:

- Average delays should decrease as volume splits near a balanced 50/50 split.
- The percent of left-turning traffic should affect the delays observed.
- The Richardson model may provide a credible estimate of stopped delay.

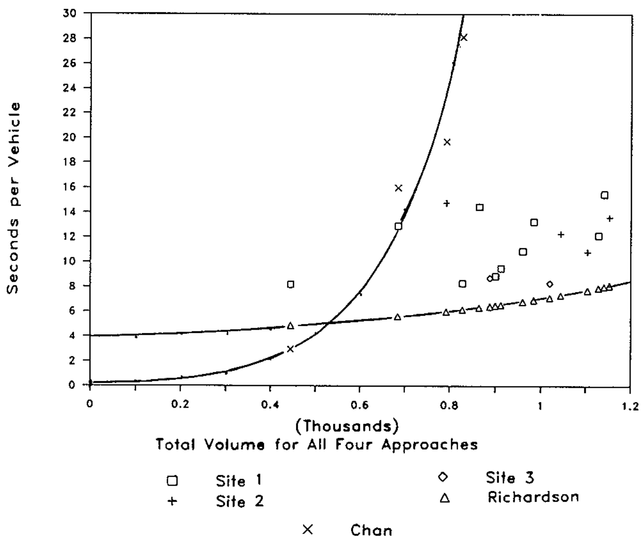

FIGURE 10 Delay versus volume for the observations with a 60/40 split in approach volumes.

The more extensive studies currently under way in support of revisions to Chapter 11 of the *Highway Capacity Manual* (*8*) will expand upon the work presented here and provide information sufficient to determine whether the hypotheses presented above are indeed true.

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