Use of Three-Dimensional Conjugate Directions Search Method To Improve **TRANSYT-7F Computational Efficiency**

HUEL-SHENG TSAY AND KWO-TSAUR WANG

A modification of the computer program TRANSYT-7F has been developed to reduce computational time and improve the performance index by using the conjugate directions search method in three dimensions. The original TRANSYT-7F uses the hillclimbing method to perform a two-step optimization, This type of optimizing procedure has been used in the TRANSYT program for many years, and even TRANSYT-7F's new version, 6.0, still applies the same algorithm. In this paper, a new search method is developed to obtain simultaneously the final cycle length, split, and offset. It is a one-step optimization algorithm. From tests of 21 cases on a PC/AT, this modifïed TRANSYT-7F reduces computational time significantly and improves the performance index slightly compared with the new TRANSYT-7F. It also allows the user to consider the spillover effect, perform arterial priority or link maximum-allowed delay, and fix the offsets for designated intersections. Currently, this program can be used not only as a detailed off-line signal-timing analysis tool but also as a part of computing software for four newly developed traffic control systems in Taiwan to generate on-line signal-timing plans.

TRANSYT is a popular computer program used around the worid to optimize the signal timing of networks with coordinated intersections. It identifies optimal offsets and phase splits through the minimization of a performance index (PI) a linear combination of stops and delays. The weight of each stop equivalent to delay is supplied by the user network-wide or on an individual link basis. To find the minimum PI, TRANSYT uses the hill-climbing method to perform a twostep optimization. It usually uses the "quick" step size to obtain the best cycle length and then applies the "normal" step size for the split and offset optimization. The magnitude of the searching step size is given as part of the TRANSYT program but can easily be altered by the user from input Card 'Iype 4. This is an iterative, gradient search technique that requires extensive numerical computation by the computer. It has been used in TRANSYT as the optimization procedure for many years, and the new TRANSYT-7F version, 6.0, still uses the same algorithm. Although the current TRANSYT-7F provides users with a sensitivity parameter to improve its computing efficiency, the hill-climbing method requires considerable computation to obtain the final signal-timing plan.

In 1986, Foulds (1) developed another search approach (a modification of the Fibonacci search) in place of the hillclimbing procedure. The Fibonacci search significantly improves TRANSYT, both in terms of the PI and computational time. Later, Chen (2) coded a computer program and performed several tests based on this method. He concluded that it is difficult to improve the computational time and PI at the same time through the Fibonacci search. This is probably because the Fibonacci search is a one-dimensional search. The offset and split of coordinated signalized intersections cannot be optimized simultaneously in the second phase of TRANSYT-7F after obtaining the final cycle time through the first step.

Furthermore, the PI function does not always obey the condition of strictly-increase-monotonically or strictly-decreasemonotonically, or the combinations of both. It has some variations in the form of local maxima and minima. This means that the one-dimensional search procedure may not be the best strategy for finding the global optimum of TRANSYT-7F. After numerous tests of six combined strategies proposed by Chen (2), it is suggested that the Fibonacci method only be considered in the first step for cycle optimization and that the hill-climbing method still be used to perform the offset and split optimization in the second step. This selected strategy can reduce the computational time by 30 percent but with a worse PI value (3 percent on average) than the new TRANSYT-7F.

In this paper, a three-dimensional, nonlinear search technique—the conjugate directions search method—is developed to replace the traditional hill-climbing method. This new method obtains the final cycle length, offset, and split simultaneously without the two-step process. It is a one-step optimization procedure and can be extended to solve special types ofintersections, such as signalized circles, and signalized junctions of expressway offramps and surface arterials. The following sections first focus on the theory of TRANSYT-7F related to signal-timing optimization. Then the framework and theory of the conjugate directions search method in three dimensions is presented and discussed. Finally, this new method is compared with the hill-climbing and modified Fibonacci search methods.

CHARACTERISTICS OF TRANSYT.TF OBJECTIVE FUNCTION

To find a suitable and effective method of optimizing the objective function of TRANSYT-7F, the characteristics of PI must first be discussed. When optimizing, TRANSYT mini-

H-S. Tsay, Department of Transportation, Taipei City Government, Taiwan. K-T. Wang, Institute of Transportation and Traffic, National Chiao Tung University, Taipei, Taiwan.

mizes the PI. The optimization formulation of TRANSYT-7F follows. All variables and symbols are based on the original TRANSYT-7F (3), unless otherwise specified.

$$
PI = \sum_{i=1}^{n} (d_i + KS_i)
$$
 (1)

where

- d_i = delay on link i (of n links) (veh-hr/hr),
- S_i = stops on link *i* (stops/sec), and
- $K = a$ user input coefficient to express the importance of stops relative to delay.

Delay in Equation 1 is composed of uniform delay (d_u) , random delay (d_r) , and oversaturation delay (d_r) . The uniform delay is calculated by averaging the queue length (m) over the cycle for any step t times the cycle length, as shown in Equation 2 :

$$
d_u = \frac{C}{3,600N^2} \sum_{i=1}^{n} m_i
$$
 (2)

where

- d_u = uniform delay (veh-hr/hr),
- $c =$ cycle length (sec),
- m_t = queue length during step t, and

 $N =$ number of steps in the cycle.

Then random delay accrues due to the random arrivals of vehicles. TRANSYT computes the combined effect of random delay and saturation delay through the following equation:

$$
d_s = \left[\left(\frac{B_n}{B_d} \right)^2 + \frac{X^2}{B_d} \right]^{1/2} - \frac{B_n}{B_d} \tag{3}
$$

where

- d_s = random and saturation delay, $B_n = 2(1 - X) + XZ,$ $B_{d}^{n} = 4Z - Z^{2},$ $Z = (2X/V) * 60/T,$ $X = \text{degree of saturation},$
- $V =$ volume on the link, and
- $T =$ period length.

Therefore, the total delay in veh-hr/hr (D) can be computed AS

$$
D = d_u + d_s \tag{4}
$$

The number of stopped vehicles estimated in TRANSYT-7F is equal to the number of vehicles arriving when a queue is present. This is based on empirical studies by the Transport and Road Research Laboratory (TRRL), and even partial stops are counted through a reduction curve if the delay to such vehicles is small (3) . Based on Equations 1 to 4, the variables considered in the PI include the queue length, degree of saturation, volume, and cycle length. Since volume is an external input provided by the user, the queue length and degree of saturation become two major factors that need to be determined through the cycle length, split, and offset. In other words, the cycle length, split, and offset will be the

major elements affecting the vaiue of the objective function PI. These variables, however, have implicit functional forms that should be calculated internally from simulation to obtain the minimum PI value.

Generally speaking, there are four types of methods for solving unconstrained nonlinear optimization problems. These nonlinear problems can be categorized as single variabìe requiring derivatives (Type A), single variable without derivatives (Type B), several variables requiring derivatives (Type C), and several variables without derivatives (Type D). The solution methods related to each type of nonlinear problem are summarized in Table 1. According to the above discussion, the variables related to the objective function PI of TRAN-SYT-7F are nonlinear, unconstrained, and have no derivative form with the existence of implicit functions. Therefore, the objective function PI cannot be solved through nonlinear methods that require derivatives.

For nonlinear optimization problems without derivatives, the effective method for dealing with a single variable and several variables may be the Fibonacci search and the conjugate directions search, respectively $(4,5)$. Use of the Fibonacci search instead of the hill-climbing procedure used in TRANSYT-7F was proposed by Foulds (I) and extensively analyzed by Chen (2). Therefore the following section focuses on the theory of the conjugate directions search method in three dimensions and its applications.

CONJUGATE DIRECTIONS SEARCH METHODS IN THREE DIMENSIONS

The conjugate directions search method in three dimensions was first presented by Powell (4) and then by Zangwill (6) and Brent (7) . It is a direction set method that has the characteristics of quadratic termination when used to solve quadratic function problems. The procedure takes at most n steps to obtain the final solution when a quadratic function with n variables is considered. This nonlinear search method has an advantage over the other methods with its faster convergence. Since the objective function of TRANSYT-7F considers three implicit variables, the three-dimensional conjugate directions method can further be applied to solve for the cycle length, offset, and split.

Definition of Conjugate Directions

Given an $N \times N$ symmetric matrix C, the directions $S^{(1)}$, $S^{(2)}$, $S^{(3)}$, ..., $S^{(r)}$, $r \le N$, are said to be C conjugate if (a) the directions are linearly independent, and (b) $S^{(i)}CS^{(j)} = 0$ for all $i \neq j$.

Theorem: Parallel Subspace Property (8)

Given a quadratic function $q(x)$, two arbitrary but distinct points $x^{(1)}$ and $x^{(2)}$, and a direction d, if $y^{(1)}$ is the solution to min $q(x^{(1)} + d)$ and $y^{(2)}$ is the solution to min $q(x^{(2)} + \lambda d)$, the direction $(y^{(2)} - y^{(1)})$ is conjugate to d.

This property is illustrated in Figure 1 in two dimensions. It can be seen that a single-variable search from $y^{(1)}$ or $y^{(2)}$ along the direction ($y^{(2)} - y^{(1)}$) will produce the minimum (8). This method can be further extended to three dimensions to find the optimization value of TRANSYT-7F, which is known as the application of the extended parallel subspace property. Using this extension, the construction shown in Fig-

	Types	Methods			
Α.	Single Variable Requiring Derivatives	1. Newton-Raphson Method			
		2. Bisection Method			
		3. Secant Method			
		4. Cubic Search Method			
Β.	Single Variable Without Derivatives	1. Interval Halving Method			
		2. Golden Section Search Method			
		3. Fibonacci Search Method			
С.	Several Variables Requiring Derivatives	1. Steepest Descent Method			
		2. Newton's Method			
		3. Modified Newton's Method			
		4. Quasi-Newton Method			
		5. Conjugate Gradient Method			
D.	Several Variables Without Derivatives	1. Univariate Method			
		2. Simplex Method			
		3. Pattern Search Method			
		4. Conjugate Directions Method			

TABLE 1 SUMMARY OF FOUR TYPES OF NONLINEAR OPTIMIZATION PROBLEMS AND SOLUTION METHODS

FIGURE 1 Conjugacy in two dimensions.

ure 1 can immediately be generalized to higher dimensions. The search procedure is illustrated through the example given in Figure 2 (8) .

From the upper part of this figure, the search begins with coordinate directions $\mathcal{D}, \mathcal{D},$ and $\mathcal{D},$ which represent the split, offset, and cycle length, respectively. The initial signal timing plan $x^{(0)}$ is obtained from the subroutine STAR1 of TRAN-SYT-7F by considering the minimization of degree of saturation. From $x^{(0)}$, a series of line searches is made along $\circled{0}$, 10, 2, and again 3. At the conclusion of this cycle, the directions \circledcirc and $x^{(4)}-x^{(1)}$ will be conjugate. The new search

direction designated \circledast in Figure 2 then replaces \circledast . A new cycle of line searches is executed using directions $\mathcal{D}, \mathcal{D}, \mathcal{D}$, and again \circledast . Through the extended parallel subspace property, the new direction $(x^{(8)}-x^{(5)})$, designated \circledcirc in the figure, will be conjugate not only to \circledast but also to \circledast . Hence, the set of directions \circledcirc , $(x^{(4)}-x^{(1)})$, and $(x^{(8)}-x^{(5)})$ are mutually conjugate. Therefore, if one additional line search is executed from $x^{(8)}$ along $(x^{(8)}-x^{(5)})$, the point $x^{(9)}$ is found. This point must be the optimum if $f(x)$ is a three-dimensional quadratic, since three mutually conjugate directions have been reached in turn. In other words, nine line searches using only function values are required to determine the exact optimum of a quadratic function in three dimensions. This construction is easily generalized and will require N^2 line searches in N dimensions to optimize a quadratic function.

The proposed method, in fact, comprises three subproblems of a one-dimensional search. It is important to determine the functional characteristics of each variable with the PI value. Several networks of Keelung and Tainan cities were tested comprehensively by the simulation procedure to derive the functional forms of cycle length, offset, and split versus PI. The relationships between the PI and each variable can be observed from Figures 3 through 5.

From the six examples shown in Figure 3, the functional form of PI related to the offsets is a type of discrete function. Most functions obey the characteristics of the sine or cosine pattern, except with some variations. It is not a unimodal function. As far as the computational efficiency is concerned, the Fibonacci search procedure approximately follows the sine or cosine curve. This procedure depends on a numerical

FIGURE 2 Construction of conjugate directions search in three dimensions.

sequence called Fibonacci numbers. The procedure successively reduces the interval in which the minimum of a nonlinear function must lie. The computational procedure requires the function value of only one point instead of two points for each reduced interval after obtaining the first'interval from computing the value of two points. Such a searching procedure can reduce the computational time significantly. Since the unimodality of the sine or cosine function does not hold, the Fibonacci search needs to be modified. Full details of the Fibonacci search procedure and its modification were given by Foulds. In this paper, these searching steps are further considered and partially modified to find the final value of offsets in approaching the optimal solution through the conjugate directions search method.

The relationships between PI and different green splits can be seen from Figure 4. In these six cases, the green split seems to have the form of a unimodal function. Since this is a rather important element, more tests are needed before a final conclusion can be drawn. The variables considered in the objective function PI have implicit functions, which are probably the only way to determine the relationships between the green split and PI by using the simulation result. Although a theoretical proof of the unimodal function is difficult, the optimal value of splits can be obtained directly from the Fibonacci search method with the assumption of satisfying unimodality. This nonlinear function may become a constrained condition if the user sets the minimum green split for the designated approach through input Card Type 2X in TRANSYT-7F. Since the Fibonacci search method solves only the unconstrained case, a split-adjusting procedure needs to be considered and has been included in this proposed method to satisfy the constrained situation.

Figure 5 represents the relationships between PI and various cycle lengths. The PI increases gradually (Figures 54, 5C, and 5D) or decreases first and then increases (Figure 58) as the cycle length increases. There are some exceptions (Figures 5E and 5F) where PI decreases rapidly to a minimum value when the cycle length increases to a certain extent. To seek an effective search method for obtaining the cycle length, three kinds of searching rules may be considered. First, the Fibonacci search method is used directly to calculate its optimum value. The elimination interval needs to be recomputed if it is greater than 10 sec. Second, the entire cycle length is divided into several 5-sec intervals. In each interval the PI value is calculated; these values are compared to find the smallest PI and its corresponding interval. In the final interval the Fibonacci search procedure is applied to obtain the cycle length. Third, the PI value is evaluated at each cycle length increment, which is part of the input provided by the user.

After several tests, it was found that the first and second searching rules need considerable computational time and cannot be guaranteed to give the optimal cycle length. The third rule, however, comes close to the optimum value if the cycle increment is gradually decreased. Hence the proposed method uses the third rule to calculate the final cycle length. More research is needed to obtain the exact functional form of the PI value with various cycle lengths.

The proposed conjugate directions search method is composed of three incremental vectors. To approach the global optimum, each linear search method obviously needs to be modified. Otherwise, the solution may be restricted in the local minimum because it lacks the unimodal property. Using the above searching process, the suitable moving distance must be determined after a good search direction is found. In this paper, the ratios of the offset and split variations versus the cycle length increment are computed first. Then, the offsets and splits equal to each ratio times the actual cycle increment are calculated. The split value is discarded if it violates the preset minimum green split. Finally, the final cycle length, split, and offset can be determined until line searches of all conjugate directions have been completely executed.

COMPARISONS OF COMPUTATIONAL TIME AND PI VALUE

The hill-climbing search, modification of the Fibonacci search, and three-dimensional conjugate directions search are used to compare 21 cases with different traffic volumes from Keelung, Taichung, and Tainan cities in Taiwan. To make a consistent comparison, the same network information and traffic flows were used to prepare the inputs for the three search methods. TRANSYT-7F has been separately programmed

 (B)

 (C)

ÿ.

 \mathbb{R}^2

 $\frac{f}{\lambda}$

Â, \ldots

 $\hat{\boldsymbol{\beta}}$

 $\hat{\sigma}_{\rm eff}$

 $\ddot{\cdot}$ $\frac{1}{2}$

Ķ

 \sim $\frac{1}{2}$

FIGURE 4 Relationships between PI and different green splits.

FIGURE 5 Relationships between PI and various cycle lengths.

FIGURE 5 (continued)

with different search methods and can be run on an IBM PC, CDC, or VAX. In Table 2, the results of the computational time and PI are displayed for 21 coordinated intersections. Times are given for the same PC/AT with math coprocessor 80287-10. In all cases, the conjugate directions method reduces the computational time significantly more than the hill-climbing method but a little less than the Fibonacci method. The conjugate directions search method improves the PI in 20 out of 21 cases when compared with the hill-climbing method. It is also better than the Fibonacci method in terms of the PI in most cases. It should be noted that the modified Fibonacci method and program used here are based on Chen's study (2). In other words, the Fibonacci search is considered only at the first step for cycle optimization and the hill-climbing method is still applied to find the final offset and split at the second step.

Since the modified TRANSYT-7F gives a rather fast computing capability and better performance values with the con-

jugate directions search method, it has been used as a part of computing software to generate on-line signal-timing plans in a newly developed traffic control system known as TRUSTS (Traffic Responsive and Uniform Surveillance Timing System) (9). Two TRUSTS systems have been installed in the cities of Keelung and Taichung, and another two are expected to be completed in Taoyuan and Chiayi by the end of June i989. TRUSTS calculates signal-timing plans through ^a 32-bit PC with math coprocessor 80387-16, which can handle up to 40 intersections for on-line timing plan generation and table selection. Part of the on-line output for six intersections from this modified TRANSYT-7F program is given in Figure 6.

A NEW FORMULA FOR SPILLOVER PENALTY

The modified TRANSYT-7F also considers the effect of spillover. A new formula for the spillover penalty has been derived

Cases	No. of	Cycle	Cycle			Time(sec, PC/AT)	ΡI
	Intersections	Ranges (sec)	Increments (sec)		CDS ^a FS ^b	HC ^c	CDS FS HC
$\mathbf{1}$	4	$60 - 150$	\overline{c}	700	694	992	769.08 770.81 792.75
$\mathbf{2}$	4	$60 - 150$	5	332	306	452	767.73 771.98 767.73
3	4	60-150	10	211	187	277	767.73 771.98 767.91
4	4	$60 - 150$	\overline{c}	202	182	305	191.12 199.96 190.83
5	4	$60 - 150$	5	135	125	170	189.59 190.19 190.19
6	4	$60 - 150$	10	114	88	125	190.19 190.19 190.19
7	4	60-90	\overline{c}	206	174	291	126.78 144.33 128.57
8	4	$60 - 90$	5	139	115	162	126.78 131.60 128.57
9	4	60-90	10 [°]	119	90	120	126.78 131.60 128.57
10	$\overline{7}$	$60 - 90$	$\mathbf{2}$	389	494	681	105.02 105.02 105.02
11	$\overline{7}$	$60 - 90$	5	238	274	367	106.49 103.73 107.43
12	$\overline{7}$	60-90	10	191	200	260	106.49 107.73 107.43
13	$\overline{7}$	$60 - 90$	\overline{c}	475	482	684	357.92 359.61 358.20
14	$\overline{7}$	60-90	5	314	279	378	357.92 362.32 357.92
15	7	$60 - 90$	10	260	215	275	357.92 362.32 357.92
16	8	$90 - 120$	\overline{c}	450	485	550	264.92 277.20 302.50
17	8	$90 - 120$	5	301	294	338	264.92 261.16 264.92
18	8	$90 - 120$	10	256	227	292	264.92 264.92 264.92
19	15	$90 - 120$	\overline{c}		1457 1957 2274		211.64 222.40 215.85
20	15	$90 - 120$	5		875 1052 1226		211.64 211.92 216.25
21	15	$90 - 120$	10	699	766	835	211.64 211.92 211.92

TABLE 2 COMPARISONS OF CONJUGATE DIRECTIONS SEARCH WITH FIBONACCI SEARCH AND HILL-CLIMBING SEARCH FOR 21 CASES

CDS (Conjugate DirectÍons Search in Three Dimensions)a b

FS (Fibonacci Search)

 \mathcal{C}

HC (Hill-CIimbing Search)

 $0.1 - 17.50$

DIAGRAM>> $<< T$ IME - SPACE

FIGURE 6 On-line output of the modified TRANSYT-7F for six intersections.

and combined with the number of stops and delay to form ^a new PI:

$$
PI = \sum_{i=1}^{n} \left[d_i + kS_i + \delta D_Q \left(\frac{Q_{\text{max}} - Q_{\text{cap}}}{Q_c} \right)_i \right] \tag{5}
$$

where

- $(Q_{\text{max}})_i$ = maximum queue length within a cycle considering 5 percent variation of flow on link i (veh/lane),
- $(Q_{\text{cap}})_i$ = maximum allowable queue capacity on link i (veh/lane),
- $(Q_c)_i$ = allowable queue vehicles for the width of the upstream intersection on link i (vehicles),
	- D_Q = penalty value for the spillover, and

 $\delta = 0$ if $Q_{\text{max}} \leq Q_{\text{cap}}$, 1 if $Q_{\text{max}} > Q_{\text{cap}}$.

The definitions of variables related to spillover can be seen in Figure 7. These values are represented by the equivalent number of queue vehicles instead of the queue length. After comparisons of system performance from several saturated networks, it is concluded that the proposed spillover equation has a better performance value than the one used in SIGOP-III (10) . Therefore, this new formula is included in the modified TRANSYT-7F to prevent the overflow problem through Field 11 of input Card Type 10.

OTHER SPECIAL DEVELOPMENT

The modified TRANSYT-7F allows the user to determine either arterial priority or maximum allowed link delay. The arterial priority option is similar to recent research (11) but uses different concepts of input and performance require_ ment. The term "arterial priority" is used because the user can specify which arterial links will receive higher priority to

proceed under the constraint of setting the maximum allowable delay for the minor street. The unit of the delay value used considers the seconds per vehicle instead of the degree of saturation. To determine the maximum allowed link delay, any link may be assigned with a reasonable and different maximum allowed delay value. The final signal-timing plan satisfies all setting delay restrictions on the designated links. Through this option, some saturated links on arterial and minor streets can be avoided. This is a good way to design arterial signal-timing plans for urban streets.

The modified TRANSYT-7F can fix the offsets for certain intersections through the three-dimensional conjugate direc_ tions technique during the optimization phase. It can perform the optimizing search of the cycle length and splits simultaneously under the preset offsets, given network geometry, and traffic flows. This is a rather useful tool when designing special types of intersections, such as signalized circles, signalized junctions of expressway offramps and surface arteri_ als, and progression of coordinated signals. A summary of these special functions related to the newly added input Card Type is shown in Table 3. In addition to the on-line capability, the modified TRANSYT-7F can perform a detailed off-line signal-timing analysis like that of the original TRANSYT-7F but with more functions and less computational time.

CONCLUSIONS

This paper has discussed how a new conjugate directions search method in three dimensions can be used instead of the traditional hill-climbing search method to optimize the signaltiming plan in TRANSYT-7F. After comparing three different search methods, it is concluded that, in most cases, the proposed method significantly reduces computational time and slightly improves PI when compared with the TRANSyT-7F. The modified TRANSYT-7F based on the proposed search

FIGURE 7 Graphic representation of variables in the spillover formula.

Card Type	Field	Column	Description
10	11	$46 - 50$	0-10000 (Spillover penalty range)
31		$1 - 5$	31
	\mathfrak{p}	$6 - 10$	0 or 1 ("0" for arterial priority control, "1" for link maximum-allowed delay control)
	3	$11 - 15$	Link number
	4	$16 - 20$	Link maximum-allowed delay in sec/veh
	$5 - 16$	$21 - 80$	Same as Fields 3 and 4, Repeat
32		$1 - 5$	32 (When this Card Type 32 is used, the modified TRANSYT-7F will perform the optimization of the cycle length and split)
	2	$6 - 10$	99

TABLE 3 SPECIAL FUNCTIONS OF MODIFIED TRANSYT-7F RELATED TO THE NEW ADDED INPUT CARD TYPE

technique has been developed and programmed in four different types of computer systems. It has a new PI formula that considers the spillover penalty and allows the user to perform arterial priority or link maximum-allowed delay and to fix the offsets for certain intersections. This program can be used as a detailed off-line analysis like the new TRANSYT-7F. Furthermore, it has been applied on a newly developed traffic control system-TRUSTS-as a part of computing software for generating on-line signal-timing plans. Thus, it is recommended that this modified TRANSYT-7F be used to obtain on-line or off-line signal-timing plans of a given network if the TRANSYT-7F type of PI function is desired. Finally, the search method for each conjugate direction remains a fertile area for future research to further reduce computational time and improve the PI.

ACKNOWLEDGMENT

The authors would like to acknowledge the assistance of Ru-Miaw Hwang and Jen-Yao Wu in preparing this manuscript and in performing several tests of different computer programs.

REFERENCES

1. L. R. Foulds. TRANSYT Traffic Engineering Program Efficiency Improvement Via FIBONACCI Search. Transportation Research, Vol.20A, No;4,1986, pp. 331-335.

- 2. C. H. Chen. Development of a Modified TRANSYT-7F Program Through the Modification of Fibonacci Search. Master's thesis. Graduãte School of Transportation and Communication Management Science, National Cheng Kung University, Taiwan, 1987.
- 3. *TRANSYT-7F User's Manual, Release 5.0*. FHWA, U.S. Department of Transportation, 1987.
- 4. M. J. D. Powell. An Efficiency Method for Finding the Minimum of a Function of Several Variables Without Calculating Derivatives. Computer Journal, 1964, pp. 155-162.
- 5. M. Avriel. Nonlinear Programming Analysis and Methods. Prcntice-Hall, Lnc.,1976.
- W. I. Zangwill. Minimization a Function Without Calculating Derivatives. Computer Journal, 10, 1967, pp. 293-296.
- R. P. Brent. Algorithms for Minimization Without Derivatives. Prentice-Hall, Inc., 1973.
- 8. G. V. Reklaitis, A. Ravindran, and K. M. Ragsdell. Engineering Optimization Methods and Applications. John Wiley & Sons, Inc., 1983.
- 9. H. S. Tsay. A New Type of Urban On-line Computerized Traffic Control System. Compendium of Technical Papers. Institute of Transportation Engineers, 1988, pp. 101-106.
- 10. SIGOP-III User's Manual. FHWA, U.S. Department of Transportation, 1983.
- 11. M. J. Moskaluk and P. S. Parsonson. Arterial Priority Option for the TRANSYT-7F Traffic-Signal-Timing Program. ln Trans' portation Research Record 1181, TRB, National Research Council, Washington, D.C., i988.

Publication of this paper sporsored by Committee on Traffic Flow Theory and Characteristics.