Analysis of Full-Depth Asphalt Concrete Pavements Using Shakedown Theory

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Full-depth asphalt concrete pavements are generally designed to resist fatigue and rutting when subjected to repeated traffic loads. Of particular importance in this case is whether such pavements will exhibit increased accumulation of plastic strains under long-term, repeated loading conditions that will eventually lead to incremental collapse or whether the accumulation of plastic strains will cease and a shakedown condition is reached. In this paper, the shakedown theory is used in the analysis of full-depth asphalt concrete pavements overlying clay subgrade. The proposed numerical algorithm incorporates the stress-dependent resilient behavior of the subgrade. The influences of stiffness and strength of the subgrade, and thickness and temperature of the asphalt concrete on shakedown behavior are illustrated. Results are used to develop shakedown-limiting criteria in terms of vertical stresses and strains acting on top of the subgrade layer. Moreover, comparisons among shakedown, fatigue, and rutting predictions are presented.

Full-depth asphalt concrete pavements are generally designed to account for fatigue and rutting. Design criteria, in terms of maximum allowable values for both the tensile strain on the underside of the asphalt concrete layer and vertical strain on top of the subgrade, have been established and are used as the basis for selecting the design thickness (*1, 2*). Of particular importance is whether such pavements will exhibit increased accumulation of plastic strains under long-term, repeated loading conditions that may lead to eventual collapse or whether the accumulation of plastic strains will cease and a shakedown condition will be reached.

The shakedown theory was first presented by Melan (*3*). According to this theory, a system will shake down under repeated cyclic loads if a self-equilibrated, time-independent, residual-stress field could be found such that equilibrium conditions, boundary conditions, and yield conditions are satisfied within the system. In this case, the material is assumed to be elastic-ideally plastic with convex yield surface, applicable normality condition, and negligible viscous and inertia effects. The theory has been applied to discrete structures (*4, 5*) and more recently to general continua, including pavements (*6–8*).

In this paper, an attempt is made to use the shakedown theory to analyze full-depth asphalt concrete pavements overlying clay subgrade. The proposed algorithm incorporates the stress-dependent resilient behavior of the subgrade. The influences of stiffness and strength of the subgrade, and thickness and temperature of the asphalt concrete layer on shakedown behavior are illustrated, and results are used to develop shakedown-limiting criteria in terms of vertical stresses and strains acting on top of the subgrade layer. Moreover, comparisons of shakedown, fatigue, and rutting predictions are presented.

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PROPOSED ANALYTICAL MODEL

In the proposed method of analysis, the two-layer system is discretized into a series of rectangular finite elements (Figure 1). A quasi-static analysis is implemented, whereby inertia and viscous effects are assumed negligible. If stress states σ^o, σ^s, and σ^a correspond respectively to body forces, P^o, statically applied loads, f^s, and repeated loads, f^a, then the system will not collapse under repeated loads—provided a stress increment, $\Delta\sigma$, can be found such that equilibrium conditions, boundary conditions, and yield conditions are satisfied. If the system under consideration is assumed to be elastic-ideally plastic with convex yield surface (i.e., the Mohr-Coulomb yield criterion is adopted in this case) and applicable normality condition, then the determination of the shakedown load for a plane strain or a plane stress reduces to an optimization

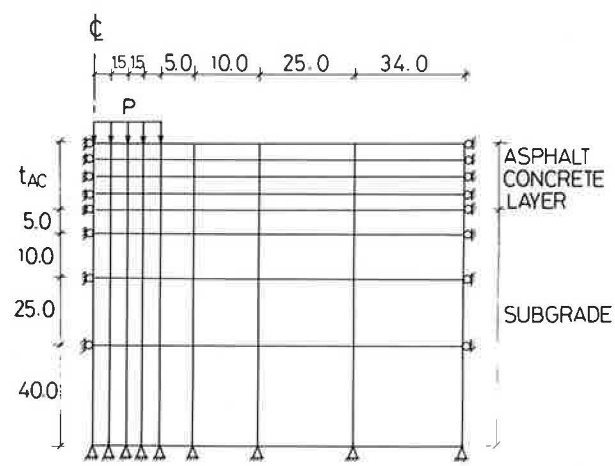

FIGURE 1 Finite element representation of the pavement structure (P = applied surface load (psi); t_{ac} = thickness of asphalt concrete layer; all indicated dimensions are in inches).

problem stated mathematically as follows:

Minimize

$$Q = -\alpha + \sum_{i=1}^{NP} (S_{xi})^2 + \sum_{i=1}^{NP} (S_{yi})^2 \qquad (1)$$

Subject to the following constraints:

$$\alpha > 0 \qquad (2)$$

$$f(\sigma) \leq 0 \qquad (3)$$

$$\sigma_3 \geq -2\,C \tan(45 - \phi/2) \qquad (4)$$

where

NP = number of nodal points,
α = load multiplier associated with repeated loads, f^a, and

$$\sigma = (\sigma ij)o + (\sigma ij)s + \alpha\,(\sigma ij)a + \Delta\sigma ij \qquad (5)$$

where

$(\sigma_{ij})_o$, $(\sigma_{ij})_s$, and $(\sigma_{ij})_a$ = stresses due to body forces, P^o, statically applied forces, f^s, and repeated loads, f^a, respectively at the center of a given element;
$\Delta\sigma_{ij}$ = arbitrary stress increment applied at the center of each element;
S_{xi}, S_{yi} = resultant forces in the x and y directions at a nodal point with respect to a global set of coordinates x-y; and
f = yield function with yield occurring when $f \geq 0$.

In this case, f represents the Mohr-Coulomb failure criterion as given by

$$f = \sigma_1 - \sigma_3 \tan^2(45 + \phi/2) - 2\,C \tan(45 + \phi/2) \qquad (6)$$

where

σ_1 and σ_3 = major and minor principal stresses,
C = cohesion, and
ϕ = angle of friction.

The optimization procedure for minimizing Q and obtaining the shakedown solution is described elsewhere (*8, 9*). More recently, the procedure has been extended to include the nonlinear, stress-dependent resilient properties of granular and subgrade layers in pavements (*10, 11*). A typical representation of the subgrade resilient modulus with repeated deviator stress is shown in Figure 2. A series of iterative steps using finite element analysis is conducted so that the stresses at the center of each element satisfy the stress-dependent modulus relationships. A new shakedown load is then calculated using the newly determined moduli at the center of elements. The procedure is repeated until convergence is attained whereby the shakedown load in two consecutive steps reaches essentially the same value; hence, shakedown conditions are satisfied simultaneously with the stress-dependent moduli relations.

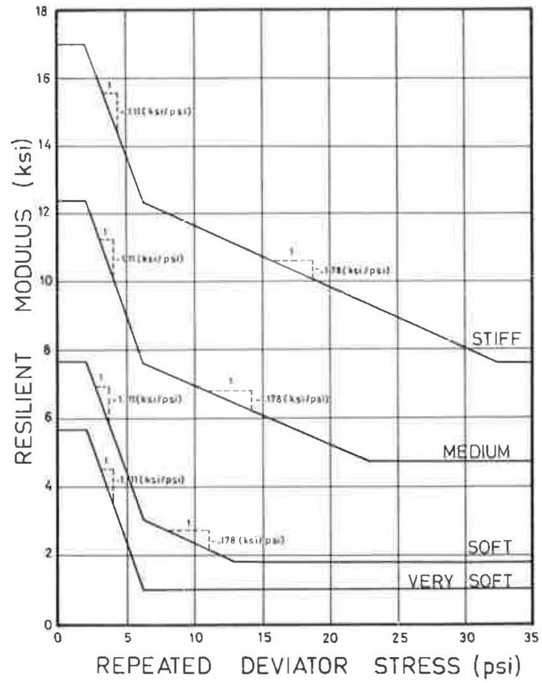

FIGURE 2 Resilient properties of subgrade (*15*).

APPLICATIONS

Materials Characterization

The asphalt concrete mix properties considered in the analysis correspond to mix M27 as designated by Salam (*12*). The mix aggregates consisted of 63.5 percent fine aggregate and 36.5 percent coarse aggregate of crushed Watsonville granite. A 7-percent asphalt cement with a 60 to 70 penetration grade was used. The average air void content was 0.82 percent and the average specific gravity was 2.53. The volume concentrations of aggregate and asphalt were 0.84 and 0.16, respectively. Specimens prepared using the designated mix were tested for the purpose of determining strength, stiffness, and fatigue properties. The variations of stiffness and strength parameters (cohesion, C_{ac}, and angle of friction ϕ_{ac}) with mix temperature are shown in Figures 3 and 4, respectively.

The subgrade was assumed to exhibit stress-dependent resilient properties. Typical relations between the deviator stress and resilient modulus (defined as the ratio of repeated stress to recoverable or resilient strain) are shown in Figure 2.

A summary of asphalt concrete and subgrade properties for all the cases used in the study is presented in Tables 1 and 2, respectively.

Shakedown Behavior

Results of analyses to investigate the influence of the asphalt concrete layer thickness and temperature, and subgrade stiffness and strength on shakedown behavior could be summarized as follows:

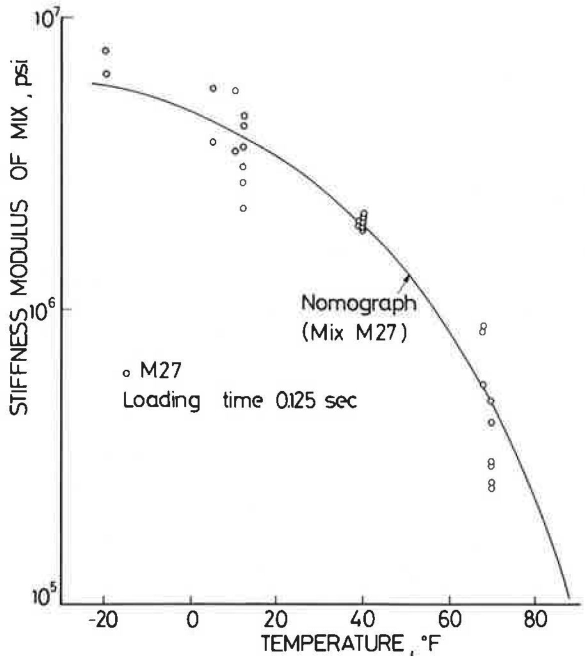

FIGURE 3 Variation of mix stiffness with temperature (*12*).

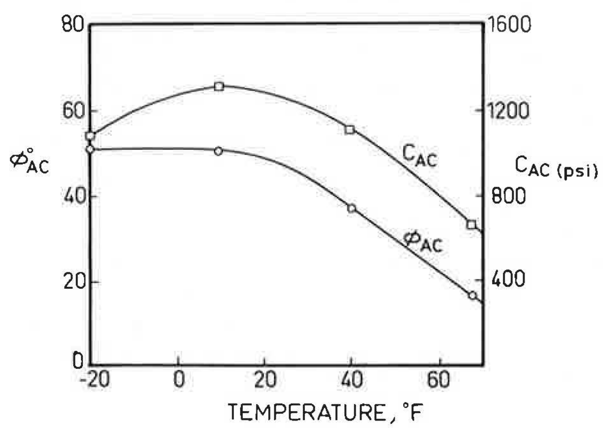

FIGURE 4 Variation of strength parameters with mix temperature (*12*).

TABLE 1 ASPHALT CONCRETE MIX PROPERTIES USED IN SHAKEDOWN ANALYSIS

Layer Temperature (°F)	Strength Parameter C_{ac} (psi)	Strength Parameter ϕ_{ac} (°)	Modulus of Elasticity (psi)
10	1300	50	3.0×10^6
40	1100	35	1.0×10^6
68	650	17	5.0×10^5
90	400	10	1.0×10^5

NOTE: Poisson's ratio was assumed equal to 0.35. Density used was 140 lb/ft^3.

TABLE 2 SUBGRADE PROPERTIES USED IN SHAKEDOWN ANALYSIS

Subgrade	Strength Parameters C_s (psi)	Strength Parameters ϕ_s(°)
Soft	3	0
Medium	6	0
Stiff	12	0

NOTE: Resilient properties for soft, medium, and stiff subgrade are presented in Figure 2. Poisson's ratio was assumed equal to 0.45. Density was equal to 115 lb/ft^3. The at-rest coefficient of earth pressure was assumed equal to 0.5.

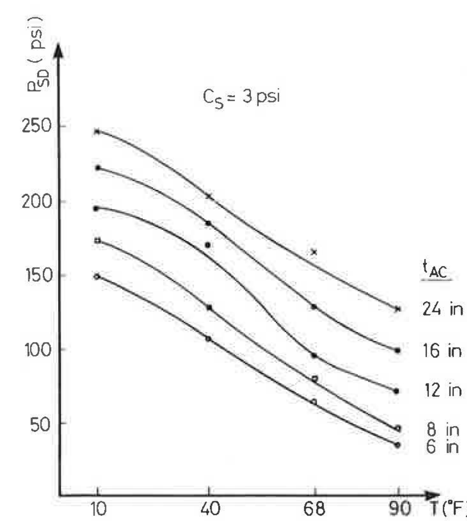

FIGURE 5 Variation of shakedown load for soft subgrade conditions.

1. The shakedown load seems to decrease with increase in temperature of the asphalt concrete layer above 40°F. Such behavior reflects probably the predominant effect of the subgrade on shakedown. An increase in asphalt concrete temperature would reduce the stiffness of the asphalt concrete layer and would, therefore, result in a larger transfer of applied load to the subgrade (Figures 5–7). On the other hand, for values of asphalt concrete layer temperature under 40°F, shakedown behavior becomes influenced essentially by the asphalt concrete layer. A decrease in layer temperature in this case would be reflected in an increase in its stiffness and strength. The asphalt concrete layer would, therefore, carry a larger proportion of the applied load, which could result in a lower shakedown capacity—particularly for thinner surfaces and stiffer subgrades as illustrated in Figures 6 and 7.

2. The shakedown load increases with increasing subgrade strength as illustrated in Figures 8 and 9. However, the beneficial effect of subgrade strength on shakedown behavior is less for thinner asphalt concrete sections and lower asphalt concrete temperature. The shakedown capacity in this case is limited by the strength of the surface layer.

3. The influence of thickness of asphalt concrete layer on shakedown is shown in Figures 10 and 11. An increase in thickness of the asphalt concrete layer results in an increase in the shakedown load. This increase would be greater for a stiff subgrade support than for soft subgrade conditions. It should be noted that, in case of the stiff subgrade (Figure 11) and for thicknesses of the asphalt concrete layer less than about 20 in., the shakedown load increases with increasing layer temperature above 10°F but decreases as the tempera-

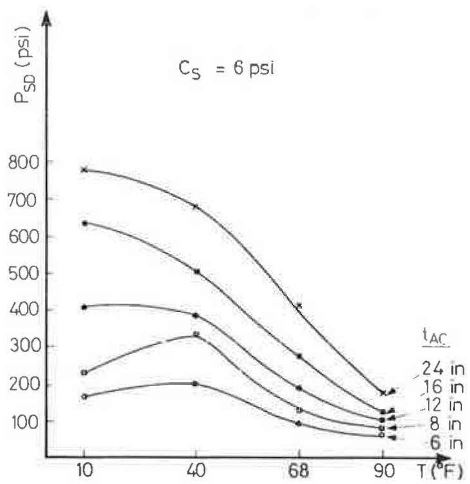

FIGURE 6 Variation of shakedown load for medium subgrade conditions.

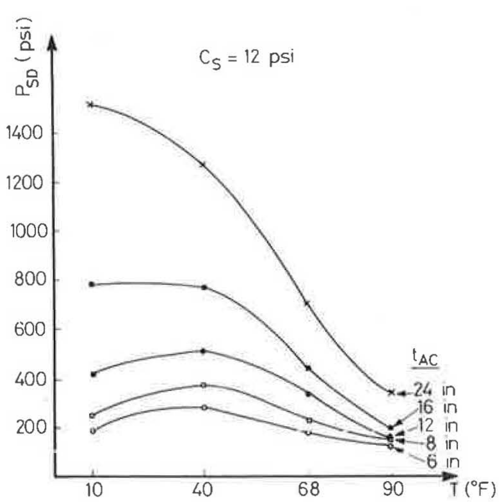

FIGURE 7 Variation of shakedown load for stiff subgrade conditions.

ture exceeds 40°F. On the other hand, for asphalt concrete layer thicknesses greater than about 20 in., an increase in layer temperature would result in a decrease in shakedown capacity.

Limiting Criteria

In many situations, limiting values in the critical response parameters have been proposed as means of predicting pavement performance. Specifically, in full-depth asphalt concrete pavements, the accumulation of plastic strains (i.e., rutting) has been linked to vertical strains and/or vertical stresses on the top part of the subgrade. An attempt is made in this paper to determine limiting criteria associated with shakedown behavior. Analyses were conducted to determine the shakedown loads for asphalt concrete layer thicknesses of 6, 8, 12, 16, and 24 in. and for the subgrade and temperature conditions summarized in Tables 1 and 2. Plane strain finite element analyses were then performed using the nonlinear stress-dependent resilient properties of the subgrade in order to determine the response of pavement sections under the applied shakedown load. Limiting criteria in terms of subgrade vertical stress, σ_v, and vertical subgrade strain, ε_v, were determined. Attempts to develop other limiting criteria using pavement response parameters, such as surface and subgrade deflections, and tensile stresses and strains on the underside of the asphalt concrete layer were not conclusive.

Results presented in Figure 12 indicate that the ratio of subgrade normal stress, σ_v, to subgrade shear strength, C_s, lies essentially between 3 and 4 for all the asphalt concrete layer thicknesses and subgrade stiffnesses considered—provided the asphalt concrete layer temperature is greater than 40°F. However, for cases where the temperature drops to 10°F and for asphalt concrete layer thicknesses less than 12 in., the ratio σ_v/C_s drops to a value between 2 and 3. This reflects the greater mobilization of tensile stresses in the surface layer that would result in lower values of vertical stresses acting on the top of the subgrade at shakedown. The average values of critical normal stress in this case are in the range of 10 to 40 psi.

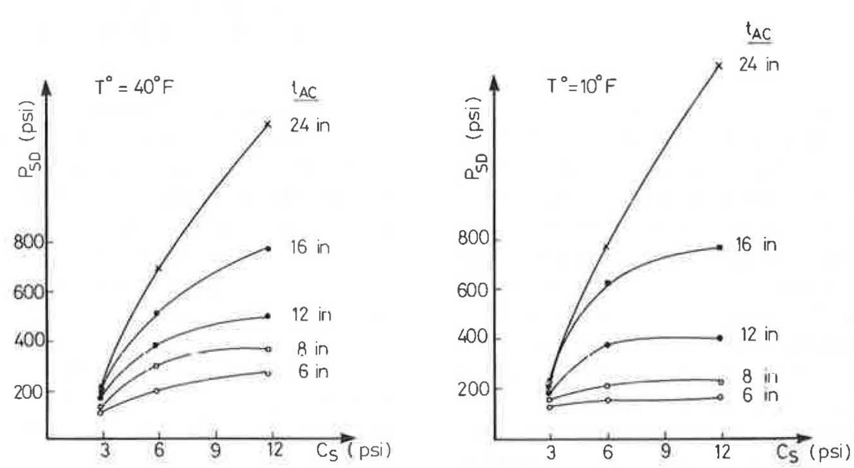

FIGURE 8 Influence of subgrade strength on shakedown for pavement temperatures of 40°F and 10°F.

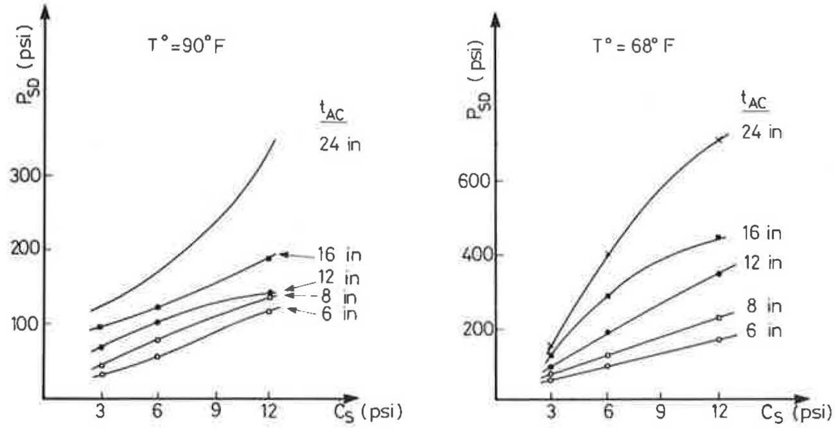

FIGURE 9 Influence of subgrade strength on shakedown for pavement temperatures of 90°F and 68°F.

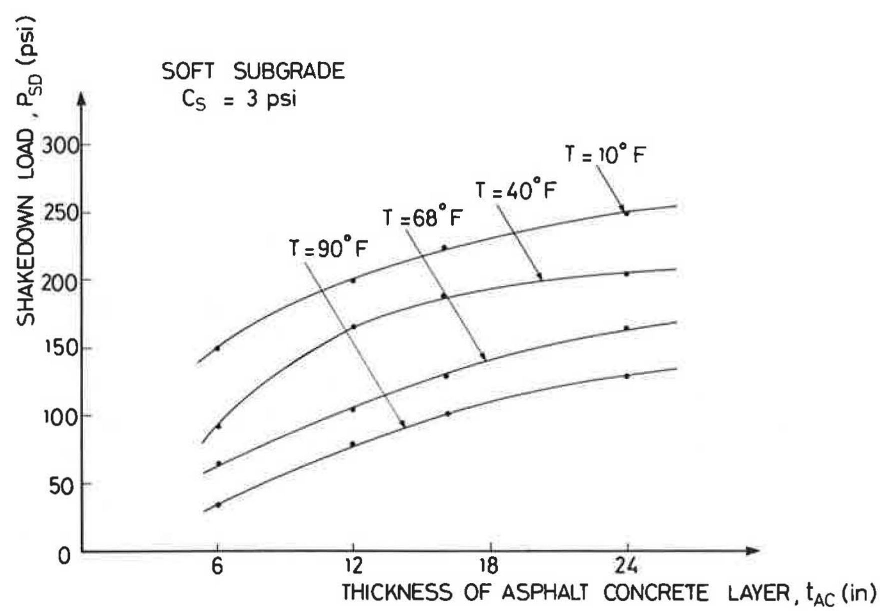

FIGURE 10 Influence of asphalt concrete layer thickness on shakedown for soft subgrade.

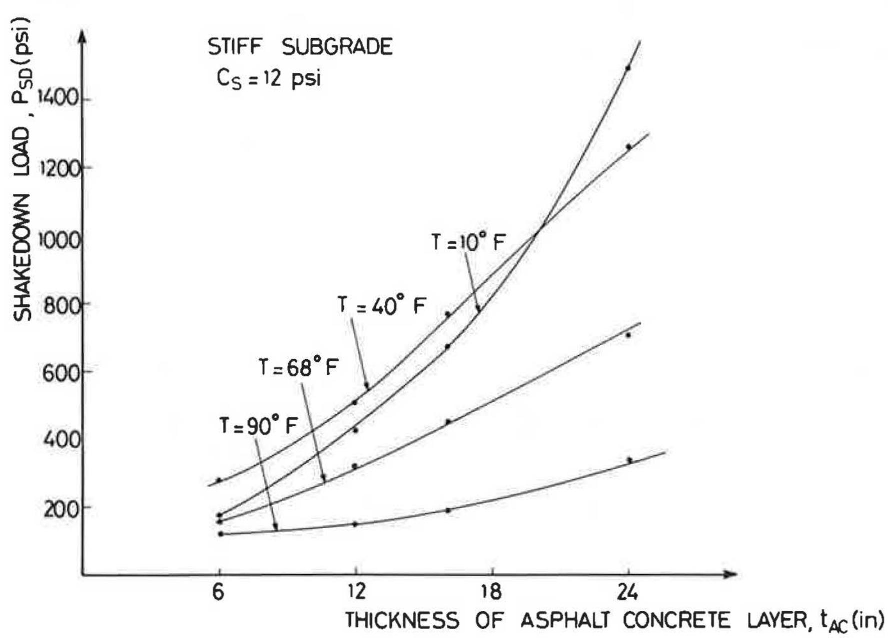

FIGURE 11 Influence of asphalt concrete layer thickness on shakedown for stiff subgrade.

Available subgrade normal stress criteria include work by the British Railway (*13*) and Peattie (*14*). The British Railway determined through repetitive triaxial testing that, in general, most subgrade soils exhibit a threshold stress of 20 psi or less. Peattie proposed allowable values for subgrade normal stress for 1 million stress applications in the range of 2 to 22 psi for subgrade California Bearing Ratio (CBR) values between 3 and 20, respectively. These values are generally lower than those obtained from shakedown loading.

Results of shakedown normal strains on top of the subgrade are presented in Figure 13. Values indicate that the limiting vertical strain, ε_v, on top of the subgrade is dependent on the stiffness of the subgrade. The limiting vertical strain, ε_v, is given in this case as

$$1 \times 10^{-3} < \varepsilon_v < 2.5 \times 10^{-3} \text{ for soft subgrade} \quad (7)$$

$$1.5 \times 10^{-3} < \varepsilon_v < 3.0 \times 10^{-3} \text{ for medium subgrade} \quad (8)$$

$$2.5 \times 10^{-3} < \varepsilon_v < 4.0 \times 10^{-3} \text{ for stiff subgrade} \quad (9)$$

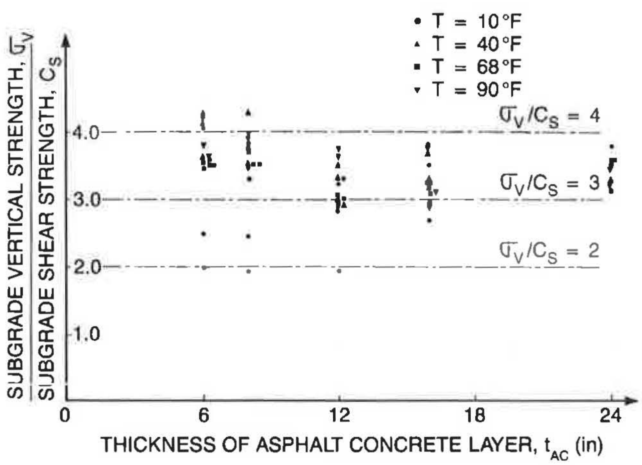

FIGURE 12 Limiting subgrade normal stress criteria.

Data summarizing available limiting subgrade normal strain criteria were presented by Figueroa (*15*). The data presented indicate that normal strain values associated with 10^6 repetitions lie in the range of 0.10×10^{-3} to 1.5×10^{-3}. These values are generally lower than the shakedown normal subgrade strains. According to the criteria presented by Figueroa, the shakedown normal strain values in constraints 7, 8, and 9 would probably induce failure after 10^3 to 10^4 strain repetitions.

The larger values of subgrade normal stress and strain obtained from shakedown analysis could indicate a more severe loading condition or could be a result of the conservative interpretation of pavement serviceability data when determining the existing criteria. Although shakedown loading could result in low pavement serviceability, it may nevertheless be used as an upper bound which, if exceeded, would lead to incremental collapse. Moreover, if the applied loads are kept below the shakedown limit, pavement maintenance will be more effective, because the rate of accumulation of plastic strains will eventually cease and the pavement system will exhibit a stable response.

Shakedown Versus Fatigue

It is of practical significance to determine whether a given pavement under existing or projected fatigue loading will shakedown. In this respect, if the shakedown limit is not exceeded, the rate of accumulation of permanent strains and the associated pavement distress will be less and pavement maintenance will be more effective in comparison with the case where shakedown conditions are exceeded. To illustrate this, analyses were performed to compare shakedown and fatigue behavior. The asphalt concrete was analyzed assuming layer temperature values of 10°F and 68°F for stiff and soft subgrade conditions. The fatigue criterion used was proposed by Monismith et al. (*16*) and is illustrated in Figure 14.

Results of analyses presented in Figures 15 through 18 indicate a significant influence of subgrade conditions on pavement performance. For the case of a stiff subgrade, fatigue of the asphalt concrete layer seems to be the predominant

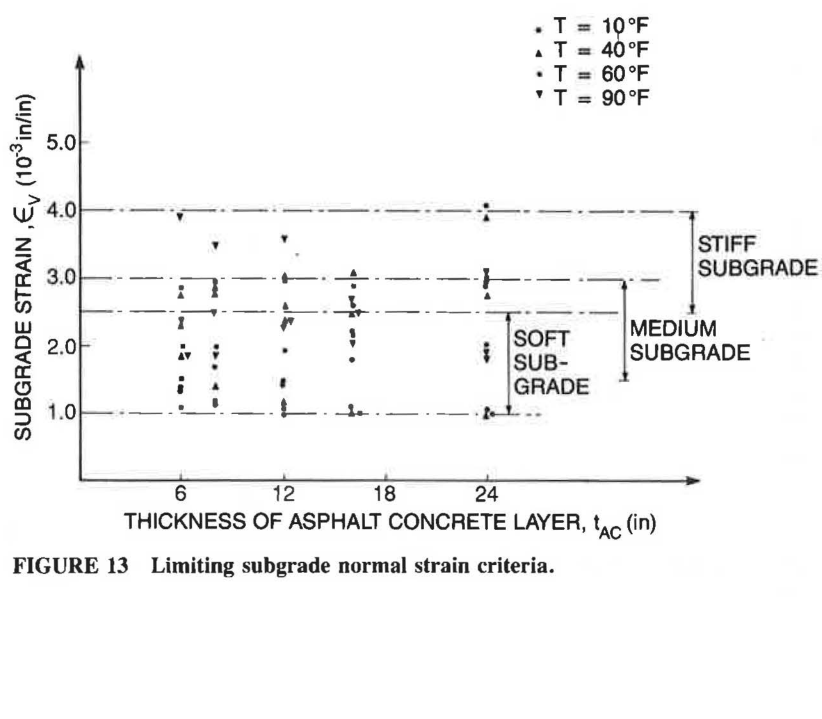

FIGURE 13 Limiting subgrade normal strain criteria.

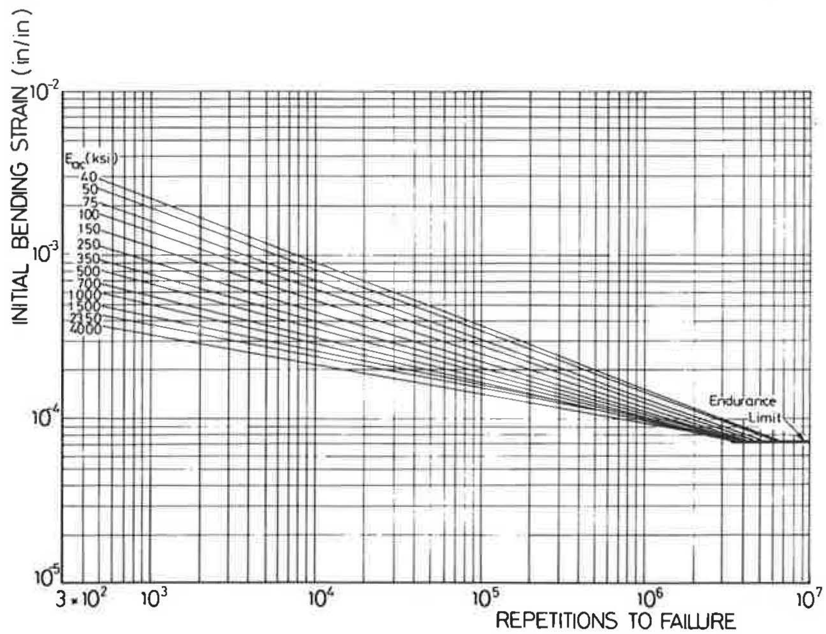

FIGURE 14 Fatigue failure criteria for asphalt concrete mixes (*16*).

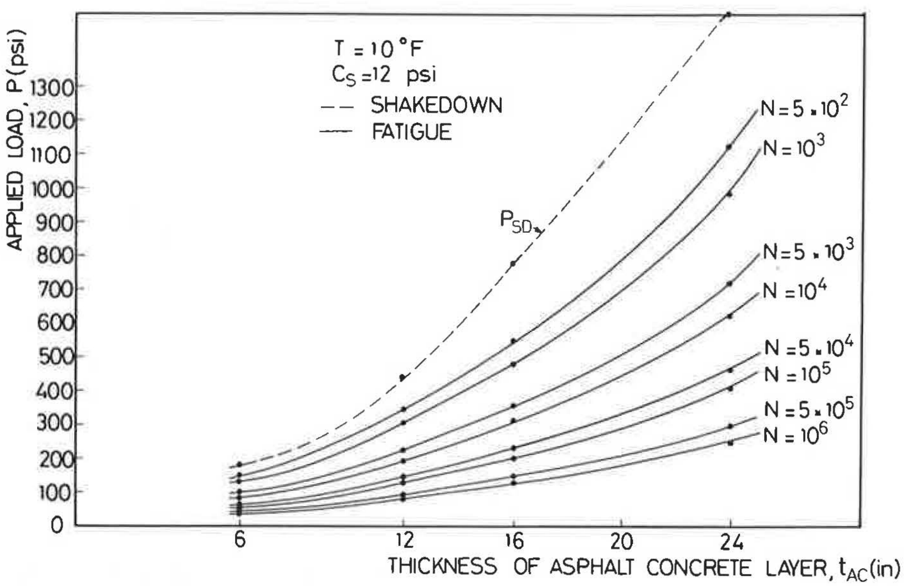

FIGURE 15 Fatigue and shakedown loadings for stiff subgrade conditions and T = 10°F.

distress mode, and the loads required to induce fatigue failure are smaller than the shakedown loads for all the pavement temperatures and thicknesses considered (Figures 15 and 16). On the other hand, for the case of a soft subgrade, fatigue and/or rutting could be the governing modes of distress depending on pavement thickness, pavement temperature, and magnitude and repetitions of the applied load (Figures 17 and 18). For a given applied load in this case, an increase in pavement thickness will increase the resistance to fatigue and rutting. Moreover, for a given temperature, pavement sections designed to carry a certain number of load repetitions in fatigue may or may not shakedown depending on pavement thickness. For example, assuming a pavement temperature of 68°F and a design number of load repetitions equal to 10^4, a 6-in.-thick asphalt concrete (design load equal to 35 psi) will stabilize and attain shakedown, whereas a 24-in.-thick asphalt concrete layer (design load equal to 195 psi) will not shakedown and will exhibit incremental collapse (Figure 18). Moreover, an increase in pavement temperature reduces both its fatigue and shakedown capacity.

It is interesting to note that if pavement overloading occurs and reaches a value equal to or greater than the shakedown limit, then fatigue and rutting for relatively thin pavement sections (thickness less than 8 inches) and temperature greater than 68°F could take place after only a few load repetitions (fewer than 1,000).

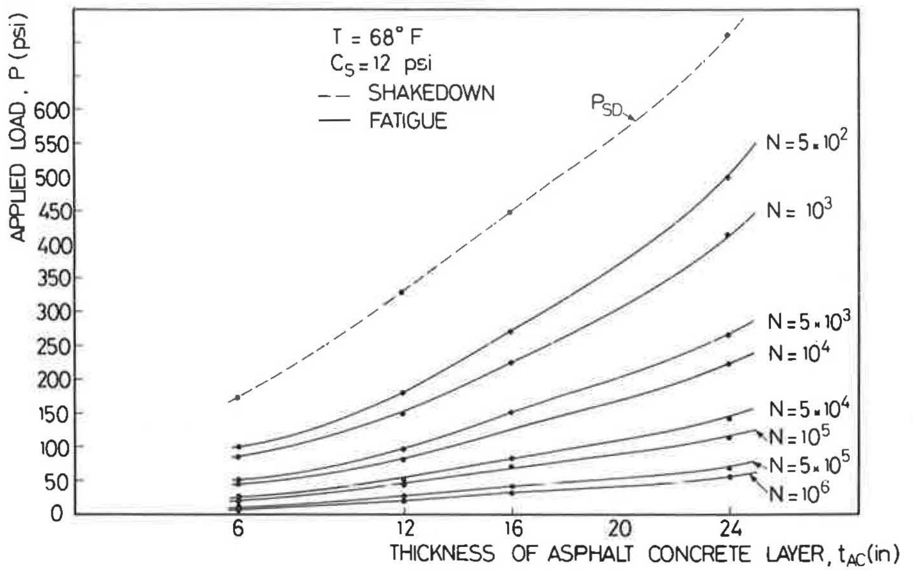

FIGURE 16 Fatigue and shakedown loadings for stiff subgrade conditions and T = 68°F.

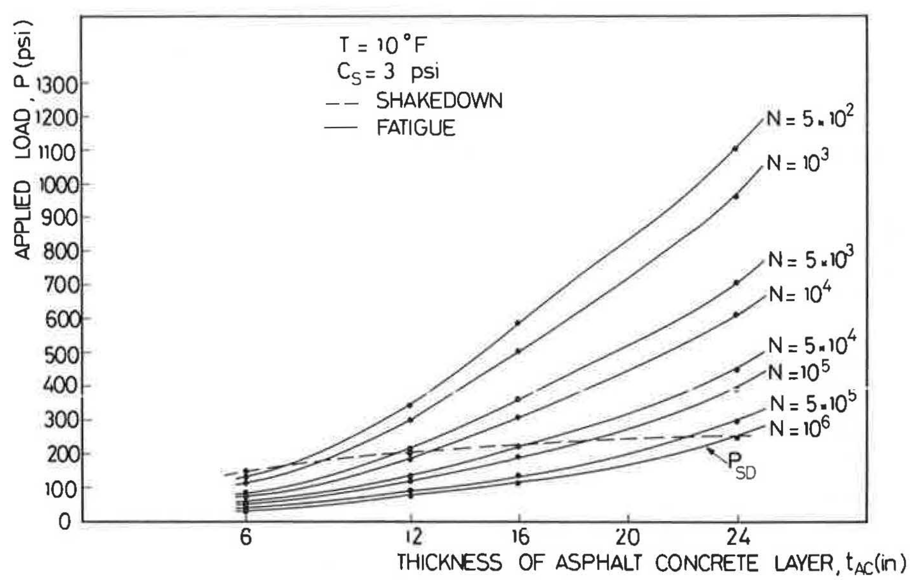

FIGURE 17 Fatigue and shakedown loadings for soft subgrade conditions and T = 10°F.

Shakedown Versus Rutting

To assess the magnitude and rate of rutting associated with repeated shakedown load applications, a typical example—consisting of an 8-in. asphalt concrete layer over a stiff subgrade—was considered. The section was subjected to a variable number of load applications of magnitude equal to 0.75 P_{sd}, P_{sd}, and 1.25 P_{sd}, where P_{sd} is equal to the shakedown load. The asphalt concrete layer was subdivided into two layers of equal thickness. Weather data used by Monismith et al. (*17*) were used in this case to compute pavement monthly temperature distribution. The corresponding traffic-weighted mean stiffness induced by load repetitions was determined based on repeated flexural beam test data presented by Monismith et al. (*16*). A summary of temperature, stiffness, and strength characteristics of the asphalt concrete layer is presented in Table 3. The influence of temperature variation over a typical year's time was considered in determining an effective shakedown load using Miner's cumulative damage hypothesis. The effective shakedown load, P_{sd}, could be obtained as follows:

$$\frac{1}{n}\sum_{i=1}^{n} P_{sd}/P_{sdi} = 1 \qquad (10)$$

where P_{sdi} equals shakedown load determined for a representative period (i) and n equals total number of representative periods.

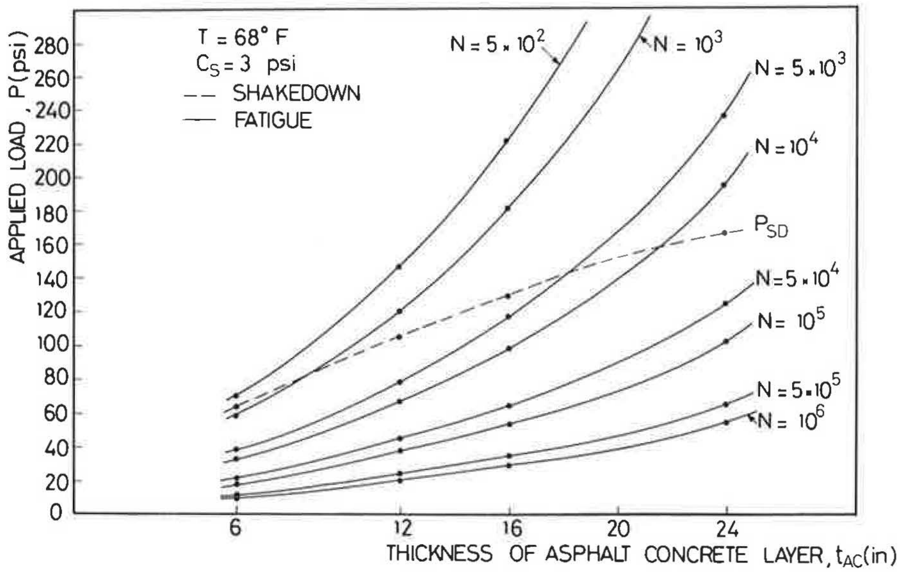

FIGURE 18 Fatigue and shakedown loadings for soft subgrade conditions and T = 68°F.

In this example, each month of the year was considered a representative period, and the total number, n, was equal to 12. The variation of shakedown load, P_{sdi}, over a 1-year period is shown in Figure 19. The stress state was then estimated for loading conditions corresponding to $0.75P_{sd}$, P_{sd}, and $1.25\ P_{sd}$ for every month of the year. Plane strain finite element analysis that incorporates the stress-dependent resilient properties of the subgrade was used for this purpose.

The magnitude of vertical permanent strains, ε_z, under the center of the applied wheel load was determined using the method proposed by Monismith et al. (*17*). In this case

$$\varepsilon_z = R\,[\sigma_z - \nu(\sigma_x + \sigma_y)] \tag{11}$$

where

$\sigma_z, \sigma_y, \sigma_x$ = stresses in vertical, radial, and tangential directions, respectively,
ν = Poisson's ratio, and
R = permanent strain parameter.

The corresponding rut depth, D, is then expressed as

$$D = \sum_{i=1}^{s} \varepsilon_{zi}\, h_i \tag{12}$$

where

h_i = thickness of sublayer (i),
ε_{zi} = permanent strain at center of sublayer (i), and
s = number of sublayers considered.

For the asphalt concrete layer,

$$R_b = \Delta(T)\, N^{\alpha}\, \sigma^{\beta-1}\, t \tag{13}$$

where

$$\Delta(T) = B\, T\, e^{-A/T} \tag{14}$$

α, β, B, A = material coefficients determined experimentally,
t = loading time (sec),
N = number of load repetitions,
T = absolute temperature, and
$\sigma = \sigma_z - \nu\,(\sigma_x + \sigma_y)$.

For the subgrade,

$$R_s = \frac{1}{l - m\sigma}\left(\frac{N}{N_o}\right)^{b} \tag{15}$$

where l, m, b equal material coefficients and N_o equals number of repetitions at which coefficients are determined. Material coefficients [determined by Monismith et al. (*17*)] for the asphalt concrete layer used in the analysis are summarized as follows (maximum $m\sigma$ in Equation 15 used for $0.75P_{sd}$ and P_{sd} was 0.90; maximum $m\sigma$ in Equation 15 used for $1.25P_{sd}$ was 0.95):

Coefficient	*Amount*
l	0.27×10^{-4}
m	0.027
b	0.24
N_o	10,000

Material coefficients (*17*) for the subgrade used in the analysis are as follows (time of loading assumed was 0.10 sec):

Coefficient	*Amount*
A	1.02×10^4
B	9.24×10^6
α	0.44
β	0.82

Results of analyses illustrating the variation of rut depth, D, and rate of rutting with applied load repetitions, dD/dN, are presented in Figures 20 and 21, respectively.

Results indicate that most rutting occurs in the subgrade (Figure 20). Significant increase in rutting occurs when the

TABLE 3 SUMMARY OF DATA USED IN RUTTING EXAMPLE

Month	Lay. N_o	Pavt. Temp. °F	S_{mix} (10^2) x 10^6 psi	S_{mix} (10^3) x 10^6 psi	S_{mix} (10^4) x 10^6 psi	S_{mix} (10^5) x 10^6 psi	C_{mix} psi	$\phi°_{mix}$	Δ(T) x10^{-7}
Jan	1	44.5	1.75	1.642	1.40	0.805	1046	34	3.88
	2	43.6	1.80	1.688	1.44	0.828	1054	35	3.63
Feb	1	50.5	1.36	1.276	1.088	0.626	954	30	6.03
	2	49.3	1.43	1.341	1.144	0.658	966	31	5.52
Mar	1	53.1	1.24	1.163	0.992	0.570	908	28	7.27
	2	51.6	1.31	1.229	1.048	0.603	931	29	6.53
Apr	1	58.1	0.98	0.919	0.784	0.451	815	24	10.38
	2	56.0	1.08	1.013	0.864	0.497	846	25	8.95
May	1	65.2	0.65	0.61	0.52	0.299	708	18	17.0
	2	62.7	0.76	0.713	0.608	0.350	754	20	14.3
Jun	1	73.0	0.40	0.375	0.32	0.184	546	12	28.8
	2	70.0	0.485	0.455	0.388	0.223	600	14	23.6
July	1	85.5	0.147	0.138	0.118	0.068	339	3	65.1
	2	80.8	0.225	0.211	0.180	0.103	415	6	48.1
Aug	1	82.3	0.200	0.188	0.16	0.092	395	5	53.0
	2	78.3	0.265	0.248	0.212	0.122	460	9	40.9
Sept	1	73.9	0.37	0.347	0.296	0.170	544	11	30.6
	2	70.9	0.45	0.422	0.360	0.207	590	14	25.1
Oct	1	63.3	0.70	0.657	0.560	0.322	730	18	15.2
	2	61.6	0.77	0.722	0.616	0.354	770	20	13.3
Nov	1	52.8	1.26	1.182	1.008	0.580	924	27	7.12
	2	51.7	1.30	1.219	1.040	0.600	937	28	6.58
Dec	1	44.9	1.71	1.604	1.368	0.787	1040	32	4.0
	2	44.2	1.76	1.651	1.408	0.810	1052	35	3.79

Notes

- C_{mix}, ϕ_{mix} are cohesion and friction of the asphalt concrete mix respectively
- S_{mix} is the stiffness of the asphalt concrete mix. Reduction of S_{mix} is determined for average monthly load repetitions of 10^2, 10^3, 10^4, and 10^5.

shakedown load, P_{sd}, is increased by 25 percent. The pavement experiences a rut depth of about 2 inches after 1 million repetitions when the applied load is equal to the shakedown load, whereas the predicted rut depth reaches 6 in. for the same number of repetitions if the shakedown load is increased by 25 percent. Moreover, the rate of accumulation of permanent deflections, dD/dN, seems to decrease with the number of load applications as shown in Figure 21. However, when the applied load is equal to 1.25 P_{sd}, then the rate of accumulation of rutting, dD/dN, will cease to decrease at about 1 million repetitions—indicating an increased accumulation of plastic strains that will eventually lead to incremental collapse.

SUMMARY AND CONCLUSIONS

An attempt has been made to apply the shakedown theory in the analysis of full-depth asphalt concrete pavements overlying clay subgrade using a numerical algorithm that incor-

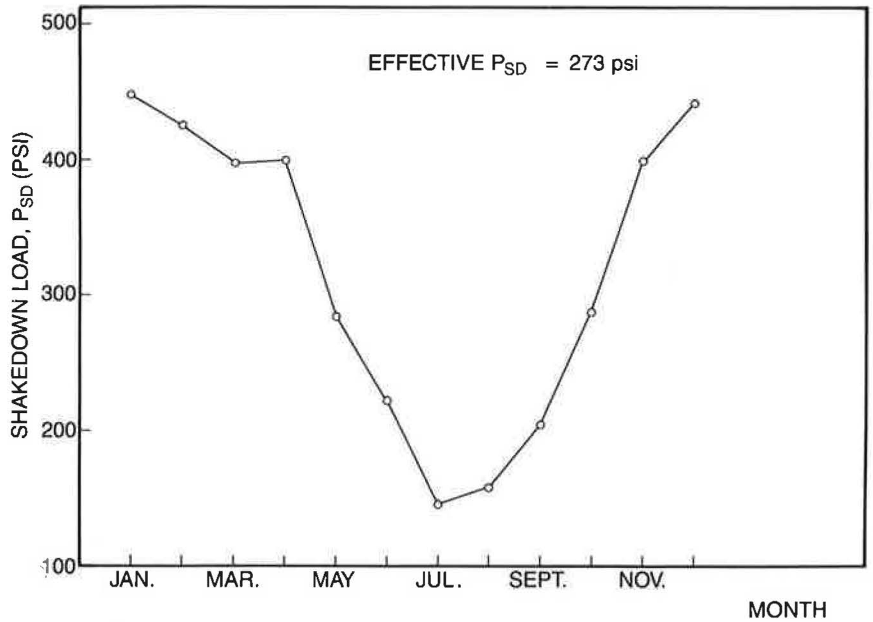

FIGURE 19 Variation of shakedown load over a 1-year period.

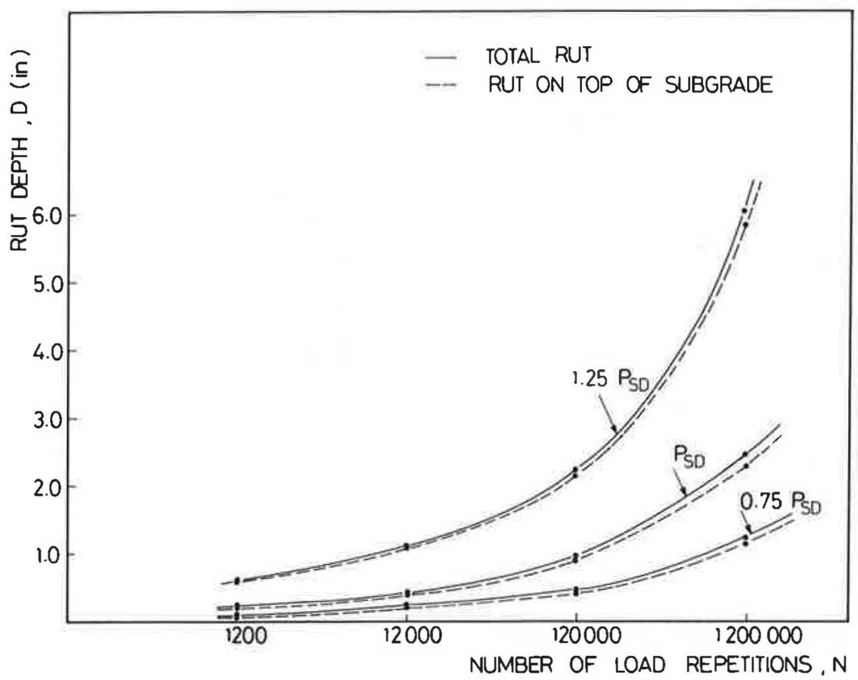

FIGURE 20 Rut depth variation with magnitude and number of load applications.

porated the stress-dependent resilient properties of the subgrade. The influences of stiffness and strength of the subgrade, and thickness and temperature of the asphalt concrete layer on shakedown behavior were investigated and several conclusions were reached:

1. The effect of subgrade conditions in terms of stiffness and strength has a predominant effect on the shakedown behavior of full-depth asphalt concrete pavements. The shakedown load increases with increase in subgrade stiffness, and the increase is more pronounced for thick pavement sections at higher temperatures. The shakedown load also increases with increase in thickness of the asphalt concrete layer. The increase is greater for stiffer subgrades. Moreover, for pavements overlying stiff subgrades, the loads required to induce fatigue failure are smaller than shakedown loads, whereas for soft subgrades, fatigue and/or rutting could be the governing modes of failure.

2. An increase in pavement temperature reduces the shakedown capacity of the pavement. However, for stiff subgrades

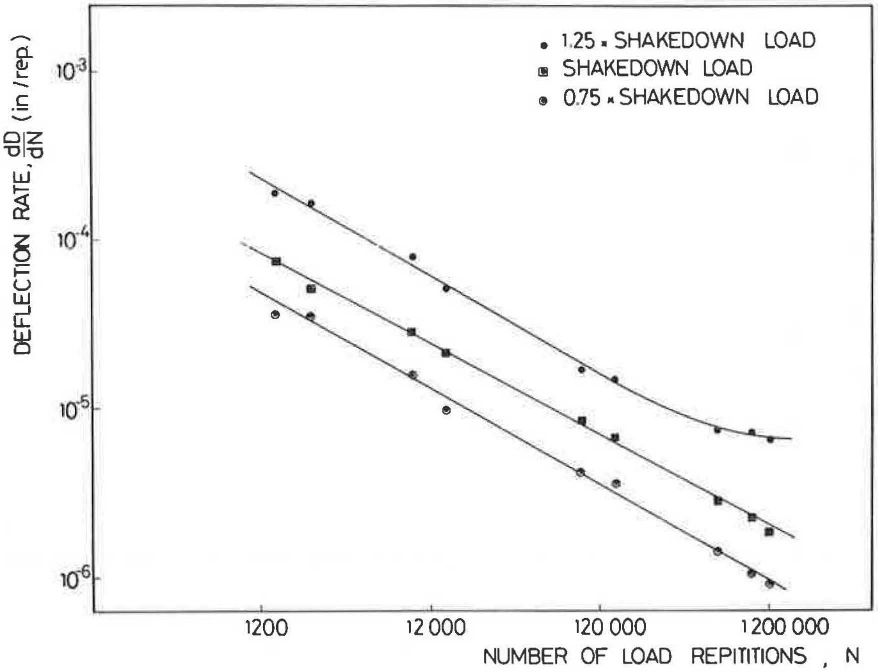

FIGURE 21 Influence of repeated load magnitude on rate of permanent deflections.

and pavement thicknesses less than 20 in. the shakedown load increases as the temperature increases in the range of 10°F to 40°F.

3. Limiting criteria in terms of subgrade normal stress and normal strain have been developed using plane strain finite element analysis of full-depth asphalt concrete pavements subjected to shakedown loads. The limiting ratio, σ_v/C_s, ranges between 2 and 4, whereas the limiting normal strains, ε_v, depend on subgrade stiffness and vary between 10^{-3} and 4.0×10^{-3}.

4. Exceeding the shakedown limit would increase significantly the magnitude of rut depth and the rate of accumulation of permanent deflections, thereby leading to incremental collapse of the pavement structure.

The results presented in this study reflect the significance of shakedown capacity on pavement performance. However, additional research is needed to verify analytical predictions and to assess pavement serviceability under shakedown loading conditions for the purpose of developing improved pavement design and maintenance procedures.

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